A new Non-wallis-type empirical correlation to predict the interfacial friction factor in vertical annular pipes flow

Zhibin Wang[1,2*] Xin Yao[1]

[1]*State Key Laboratory of Oil & Gas Reservoir Geology and Exploitation, Southwest Petroleum University, Chendu, China*

[2]*State Key Laboratory of Multiphase Flow in Power Engineering, Xi'an Jiaotong University, Xi'an, China*

Abstract: Accurate prediction of the interfacial friction factor is the basis to calculate the pressure drop in annular pipe flow. Many previous empirical correlations based on the superficial gas Reynolds number perform not so well in condition with a high pressure or a large liquid velocity. Analysis on the collected experimental data show that the modified Weber number is better than the superficial gas Reynolds number, at considering the effect of the liquid velocity and pressure on the interfacial friction factors simultaneously. So a new correlation was proposed based on the modified Weber number, the form of which is different from that of wallis-type correlation. The new correlation consider the effect of the gas velocity, gas density, liquid velocity and liquid viscosity and pipe diameter. Evaluation against 414 experiment data show that the new correlation works better than any other evaluated correlation with a mean absolute error of 17.77%.

Keywords*: annular pipes flow; interfacial friction factor; empirical correlations; modified Weber number*

1 Introduction

Annular flow is generally agreed to be one of the most frequently flow patterns encountered in nuclear, chemical and petroleum industries. The annular flow is characterized by liquid film flowing along the inner wall of the tube, while the gas core with droplets flows through the center of the tube at a higher velocity. Accurate prediction of the interfacial friction factor is the basis to calculate the pressure drop and heat transfer in annular pipe flow.

Numerous correlations have been proposed to predict the interfacial friction factor, as shown in **Table 1**. These correlations fall into two categories. One is Wallis-type. **Wallis' correlation (1969)** was proposed earliest and used most widely, in which the liquid film was considered to be a rough wall and the interfacial friction factor is linearly increased with the liquid film thickness. However, studies show that the Wallis correlation is applicable to a small range of liquid film thickness, so many studies have been conducted to modified the Wallis correlation.**Moeck (1970)** raised an exponential correlation to satisfied the experimental data of steam/water. **Fukano and Furukawa (1998)** introduced the viscosity ratio (liquid viscosity to water viscosity) to consider the effect of fluid viscosity. **Whalley and Hewitt (1978)** introduced the density ratio (liquid density to gas core density) to account for the effect of the entrained droplets. **Fore et al. (2000)**

[*]Corresponding author: Professor Zhibin Wang, E-mail address:swpuwzb@163.com

found that the interfacial friction factor decreased with the gas Reynolds number in a certain range, so the gas Reynolds number was introduced to constitute a new correlation.

The above mentioned correlations were fited from experimental data of pipe diameter smaller than 100mm. In order to make the correlation applicable for a larger range of pipe diameter, **Aliyu et al. (2017)** collected a broad range of data and fit a new correlation by using relative film thickness *h/D*, gas Reynolds number and gas Froude number.

Asali et al. (1985) proposed a new Wallis-type correlation, in which dimensionless liquid film thickness h_g^+ (its defination shown in **Table 1**) instead of *h/D* was used. Meanwhile, **Asali et al. (1985)** introduced the gas Reynolds number. **Ambrosini et al. (1991)** introduced the weber number into **Asali's correlation (1985)** to consider the effect of the droplet entrainment on the interfacial friction factor, they also introduce $200\sqrt{\rho_g / \rho_l}$ to consider the effect of gravity for lower gas velocities. Later, **Holt et al. (1999)** revised **Ambrosini's** correlation, by replacing h_g^+ by h_l^+ for low mass flux, and eliminated the gravity item and h_g^+ for high mass flux. **Klausner & Chao (1991)** modified the **Asali et al. (1985)** correlation by replacing h_g^+ with h^+.

Another group of correlations is non-Wallis type (**Henstock & Hanratty,1976; Hori et al. ,1978; Wongwises and Kongkiatwanitch,2001; Aliyu et al.,2015; Fukano et al.,1991**) , its form is different from that of **Wallis' correlation (1969)**. These correlations were comprised of several dimensionless variables.

Model evaluation conducted in this study find that previous correlations perform not so well in condition with a higher pressure or a larger liquid velocity. The objective of this study is to develop a new non-wallis-type empirical correlation which would work better in a wide range of flow conditions.

2 Experimental data collection

A total of 414 data points were collected, which cover a wide range of gas velocity, liquid velocity, pressure, liquid viscosity and pipe diameter. Key information about the data sources were given in **Table 1**.

A majority of the collected data come from air-water two-phase flow, except for those of **Fore & Dukler (1995), Asali (1983) and Fukano & Furukawa (1998)**, in which mixtures of air/glycerine with a dynamic viscosity of 6 cP, 2~6 cp and 3.4~10 cP, respectively.

The fluid used in study conducted by **Zabaras et al. (1986)** is a 1 Mole solution of sodium hydroxide together with 0.005 Mole potassium ferricyanide and 0.005 Mole potassium ferrocyanide. Values of the liquid dynamic viscosity is 1.04cP.

Fig.1 shows the distribution of the collected data against the **Hewitt and Roberts (1969)** flow patterns map. It is clear from **Fig. 1** that all the collected data fall in the annular flow patterns.

3 Model establishment

3.1. Determination of interfacial friction factor from experimental measurements

From the foregoing discussion, the data obtained for this study were measured in annular flow. These data were used to calculate the interfacial friction factor from definition correlation.

$$f_i = \frac{2\tau_i}{\rho_g u_{sg}^2} \tag{1}$$

When droplet is entrained in the gas core, gas core density and gas core velocity should be used.

$$f_i = \frac{2\tau_i}{\rho_c \left(u_g - u_f\right)^2} = \frac{2\tau_i}{\rho_c u_{sg}^2}\left(1 - 2\frac{h}{D}\right)^4 \tag{2}$$

The interfacial shear stress τ_i is obtained from the momentum balance of the gas core.

$$\tau_i = \frac{D - 2h}{4}\left(-\frac{dp}{dz} - \rho_c g\right) \tag{3}$$

The gas core velocity u_c is estimated from following correlation, which can be get from mass balance.

$$u_c = \frac{\left(u_{sg} + u_{sl} f_E\right) D^2}{\left(D - 2h\right)^2} \tag{4}$$

Where f_E is the entrained liquid fraction. When f_E was not available from experiment data, it was calculated from the correlation of **Cioncolini & Thome (2010)** . Since there is numerous droplet entrained in the gas core, the gas density would be replaced by the gas core density:

$$\rho_c = \left(1 - \alpha_c\right)\rho_l + \alpha_c \rho_g \tag{5}$$

In annular flow, the slip between the gas and the entrained droplet can be ignored, the gas core void fraction α_c can be calculated as follows:

$$\alpha_c = \frac{u_{sg}}{u_{sg} + u_{sl} f_E} \tag{6}$$

3.2 Analysis on the effect of the interfacial friction factor

3.2.1 Gas velocity

The superficial gas Reynolds number is used to consider the effect of the gas initial and viscosity by some investigators (**Fore et al.,2000; Aliyu et al. ,2017; Holt,1999, et al.**), as shown in **Table 1**.

Fig. 2 presents the variation of the superficial gas Reynolds number $\boldsymbol{Re_{sg}}$ with the interfacial friction factor, it is clear from **Fig.2** that the interfacial friction factor increases with the $\boldsymbol{Re_{sg}}$ decreases under the same liquid velocity; and it increases with the pressure at the same $\boldsymbol{Re_{sg}}$. The reason for this variation is that the gas inertial force increase with the increase of the gas velocity and gas density, more and more liquid film is atomized into liquid droplet, and so the liquid film become more and more thin, interfacial friction factor decreases correspondingly.

Meanwhile, it is clear from **Fig.2** that only $\boldsymbol{Re_{sg}}$ can't consider the effect of the liquid velocity and pressure on the interfacial friction factors simultaneously; however, the modified Weber number $\boldsymbol{We_g}$ could, as shown in **Fig. 3**. In another words, $\boldsymbol{We_g}$ is better than $\boldsymbol{Re_{sg}}$ at presenting the variation of the liquid velocity and pressure on the interfacial friction factors. Therefore, the $\boldsymbol{We_g}$ is used to constitute a new correlation in this study. The modified Weber number for the gas phase is the ratio of the gas inertia to its surface tension, could be written as follows.

$$We_g = \frac{\rho_g u_{sg}^2 D}{\sigma}\left(\frac{\Delta\rho}{\rho_g}\right)^{1/4} \tag{7}$$

The modified Weber number characterize the ability of the liquid atomization caused by the gas phase. The bigger is the gas inertia force, the higher is the quality of the droplet entrained in the gas core, and therefore, the modified Weber is closely related to the quality of the droplet entrainment.

3.2.2. Liquid velocity

Fig.4 present the variation of the interfacial friction factors with the modified Weber number for different superficial liquid velocity and different tubing diameter. According to **Fig. 4(a)-(c)**, the interfacial friction factor decreases with the increase of We_g under the same superficial liquid velocity, and it would decrease to the value of the single gas phase ,0.005. This is because the liquid film become thinner and thinner with the gas velocity and its thickness finally decreases to zero.The variation of the interfacial friction factors with the modified Weber number would be fitted by a nagative exponential correlation. **Fig.4** also show that the interfacial friction factor increases with the superficial liquid velocity, which would be contributed to the fact that the film becom thicker and thicker with the increase of the superficial liquid velocity.

Fig. 5 show the variation of the expotentional index fited in Fig. 4 with the liquid velocity for different pipe diameter. According to **Fig. 5**, the index, ***a***, has a logarithmic relationship with the superficial liquid velocity, and the index increase with the pipe diameter. Comprensively from Fig. 4 and **Fig. 5,** a new correlation, e.g. $We_g^{D(a1+a2\ln(usl))}$ could be used to present this variation of the interfacial friction factors with the modified Weber number,liquid velocity and pipe diameter.

3.2.3. Viscosity

The effect of liquid viscosity on the interfacial friction factor is shown in **Fig. 6**. As shown in **Fig. 6**, for the same flow condition, the interfacial friction factor increases as the liquid viscosity increases. This is because the liquid film atomization become not so easy as for low viscosity liquid, and, in return the liquid film become not so thin as for low viscosity liquid.

3.2.4 Pipe diameter

From the study conducted by Aliyu et al. (2017), the pipe diameter also have strong effect on the interfacial friction factor. In this study, the form of Aliyu et al. (2017) correlation is adapted to consider the effect of the pipe diameter.

3.3. New correlation

Comprehensively considering the effects of gas velocity, gas density, liquid velocity and liquid viscosity and pipe diameter, a new correlation was proposed as follow

$$f_i = 0.005\left[1 + a\left(h/D\right)^b We_g^{D(c+d\ln u_{sl})} Fr_g^e(\mu_l/\mu_w)^f\right]^j \tag{8}$$

Where *a*, *b*, *c*, *d*, *e*, *f*, *j* are undetemined parameter; μ_w is the dynamic viscosity of water at 20°C.

Levenberg-Marquardt & General global optimization was used to fit all the collected experimental data. The final correlation as follows

$$f_i = 0.005\left[1 + 65.26\left(h/D\right)^{0.5} We_g^{D(-2.4-0.03\ln u_{sl})} Fr_g^{-0.55}(\mu_l/\mu_w)^{0.03}\right]^{3.44} \tag{9}$$

$$Fr = u/\sqrt{gD}$$

4. Model evaluation

The comparison of the measured interfacial friction factor with the calculated values from previous models and newly proposed correlation is shown in **Fig. 7**. From **Fig. 7**, it can be concluded that previous models can work well in all the collected data.

From **Fig. 7(a)-(c)**, correlations prosed by **Henstock & Hanratty (1976)**, **Fukano & Furukawa (1998) and Ambrosini et al. (1991)** over-predicted most of the experimental data, the reason for this discrepancy is that they fail to consider the influence of droplet entrainment in the gas core.

As shown in the **Fig. 7(c)**, **Ambrosini et al. (1991)** correlation is restricted to pipe diameter 10 ~50mm and so its performance is not so well in large pipe diameter (>100mm) as that in small pipe diameter.

Holt et al. (1999) correlation modified from the **Ambrosini et al. (1991)** correlation seriously under-predict the interfacial friction factor, as shown in **Fig. 7(d)**.

The correlation proposed by **Wallis(1969), Aliyu et al. (2017), Moeck(1970), Hori(1976), Wongwises & Kongkiatwanitch (2001), Whalley & Hewitt (1978) and Klausner & Chao (1991)** also substantially under-predict the interfacial friction factor when the gas velocity is low and the liquid films is thick and rough. However, from Fig. 7(q), the new correlation work better than any other correlation.

Table 3 list the error statistics of these correlations. The MAE (Mean Absolute Percentage Error) and MSE (mean square error) of the newly proposed correlation are both the lowest in all evaluated correlation. The new correlation works better than any other correlation.

For correlations proposed by Henstock & Hanratty(1976), Ambrosini et al. (1991), Fukano et al. (1991), Fukano & Furukawa (1998) and Aliyu et al.(2015) , the MAE is bigger than 50% and the number of points within the 50% error band are also very low.

Among previous correlations, the Moeck correlation has the lowest values of MSE and over 90% of the predicted points lie within the 50% error band, as show in Fig. 7(k) . However, when this correlation is used to predict the condition with a thick film and a large interfacial friction factor, the calculated value is much smaller than the measured value. Comparing with the Moeck correlation, the new correlation also work better in case of large interfacial friction factor.

From Fig. 7(a)-(c), correlations prosed by Henstock & Hanratty (1976), Fukano & Furukawa (1998) and Ambrosini et al. (1991) over-predicted most of the experimental data, the reason for this discrepancy is that they fail to consider the influence of droplet entrainment in the gas core.

As shown in the Fig. 7(c), Ambrosini et al. (1991) correlation is restricted to pipe diameter 10 ~50mm and so its performance in large pipe diameter (>100mm) is not as better as that in small pipe diameter.

Holt et al. (1999) correlation seriously under-predict the interfacial friction factor. The correlation proposed by Wallis(1969), Aliyu et al. (2017), Moeck(1970), Hori(1976), Wongwises & Kongkiatwanitch (2001), Whalley & Hewitt (1978) and Klausner & Chao (1991) also substantial under-predict the interfacial friction factor when the gas velocity is low and the liquid films is thick and rough.

5 Conclusions

(1)These correlations, which ignore the influence of droplet entrainment factors, over-predicted the most of the experimental data.

(2)The modified Weber number could characterize the ability of the liquid atomization of gas phase. The bigger the gas inertia force, the higher the quantity of the droplet entrained in the gas core. So its value present the quantity of the droplet entrainment.

(3)The modified Weber number is better than superficial gas Reynolds number at describing the effect of the liquid velocity and pressure on the interfacial friction factors simultaneously. So it would be reasonable to use the modified Weber number to constitute correlation.

(4)Experimental data show that the interfacial friction factor increases with the superficial liquid velocity, liquid viscosity and pipe diameter, however decrease with gas velocity.

(5)A new non-wallis-type correlation to predict of the interfacial friction factor in vertical annular pipes flow was proposed, which comprehensively consider the effects of gas velocity, gas density, liquid velocity and liquid Viscosity and pipe diameter.

(6)The new correlation works better than any other evaluated correlation with a mean absolute error of 17.77%.

Acknowledgments

This study is financially supported by the National Natural Science Foundation (No. 51974263), National Science and Technology Major Project of China (No.2016ZX05048001-06-LH),Applied Basic Research Program of Science and Technology Department of Sichuan Province (No.2018JY0444).

Nomenclature

Roman

D	pipe internal diameter, m
f_E	entrained liquid fraction
F	modified Martinelli flow parameter
Fr	Froude number
f	interfacial friction factor
g	acceleration due to gravity, m/s^2
m	phase mass flow rate, kg/s
$-dP/dz$	pressure gradient, Pa/m
Re	Reynolds number
h	film thickness, m
h_g^+	dimensionless film thickness defined by Eq.
h^+	dimensionless film thickness defined by Eq.
H_L	liquid holdup
u	phase superficial velocity, m/s
We	Weber number
X	Martinelli parameter

Greek

α	void fraction
v	kinematic viscosity, m^2/s
μ	dynamic viscosity, Pa.s
ρ	density, kg/m^3

σ	surface tension, N/m
τ	shear stress, Pa

Subscripts

c	core
g	gas phase
i	interfacial
l	liquid phase
lf	liquid film
sg	superficial gas
sl	superficial liquid
w	water

References

[1]Aliyu A M , Baba Y D , Lao L , et al. Interfacial friction in upward annular gas–liquid two-phase flow in pipes[J]. Experimental Thermal and Fluid Science, 2017, 84(Complete), 90-109.

[2]Aliyu M Aliyu, Lao L, Yeung H, et al.,2015.A Comparative Analysis of Interfacial Friction Factor Correlations for Adiabatic Co-Current Gas Liquid Annular Two-Phase Flow in Large Diameter Pipes, Proceedings of the World Congress on Mechanical, Chemical, and Material Engineering. Barcelona, Spain, July, 20-21,

[3] Ambrosini, W., Andreussi, P., Azzopardi, B.J. ,1991. A physically based correlation for drop size in annular flow[J]. Int. J. Multiph. Flow, 17 (4), 497-507.

[4]Asali, J. C., 1983.Entrainment in Vertical Gas-Liquid Annular Flows[D],Ph.D. Thesis, Univ. of Illinois, Urbana.

[5] Asali, J.C. Hanratty, T.J.,1985.Andreussi, Interfacial drag and film height for vertical annular flow[J]. Am. Inst. Chem. Eng., 31 (6), 895-902.

[6] Belt, R.J.,Van't Westende,J.M.C., Portela, L.M. 2009.Prediction of the interfacial shear-stress in vertical annular flow[J]. Int. J. Multiph. Flow, 35 (7), 689-697.

[7] Cioncolini A, Thome J R., 2010. Prediction of the entrained liquid fraction in vertical annular gas-liquid two-phase flow[J]. Int. J. Multiph. Flow, 36(4), 293-302.

[8]Fore L.B., Dukler, A.E., 1995.Droplet deposition and momentum transfer in annular flow[J]. AIChE Journal, 41.

[9] Fore, L.B., Dukler, A.E.,1995.Droplet deposition and momentum transfer in annular flow[J]. AIChE J. 41 (9), 2040-2046.

[10] Fore, L., Beus, S. Bauer, R., 2000.Interfacial friction in gas–liquid annular flow: analogies to full and transition roughness[J]. Int. J. Multiph. Flow, 26 (11), 1755-1769.

[11] Fukano, T.,et al., 1991.Interfacial shear stress and holdup in an air–water Annular two-phase

flow, in: Proceedings of the ASME-JSME Thermal Engineering Joint Conference, Reno, Nevada, March,17-22, Amer Society of Mechanical, pp. 217.
[12] Fukano, T., Furukawa, T.,1998.Prediction of the effects of liquid viscosity on interfacial shear stress and frictional pressure drop in vertical upward gas–liquid annular flow[J]. Int. J. Multiph. Flow 24 (4), 587-603.
[13] Henstock, W.H., Hanratty, T.J.,1976. The interfacial drag and the height of the wall layer in annular flows[J]. AIChE J. 22 (6),990-1000.
[14] Hewitt, G.F., Roberts, D.N.,1969. Studies of Two Phase Patterns by Simultaneous X-Ray and Flash Photography [J]. Harwell, Berkshire.
[15] Holt, A.J., Azzopardi, B.J., Biddulph, M.W., 1999.Calculation of two-phase pressure drop for vertical upflow in narrow passages by means of a flow pattern specific model[J]. Chem. Eng. Res. Des. 77 (1), 7-15.
[16] Hori, K. , Nakasamomi, M. Nishikawa, K., Sekoguchi, K.,1978.Study of ripple region in annular two-phase flow (third report, effect of liquid viscosity on gas-liquid interfacial character and friction factor), Trans. Jpn. Soc. Mech. Eng. 44 (387),3847-3856.
[17] Kaji, R. Azzopardi, B.J. 2010.The effect of pipe diameter on the structure of gas/liquid flow in vertical pipes[J]. Int. J. Multiph. Flow 36 (4), 303-313.
[18] Klausner, J.F., Chao, B.T., Soo, S.L., 1991.An improved correlation for two-phase frictional pressure drop in boiling and adiabatic downflow in the annular flow regime[C]. Proc. Inst. Mech. Eng. Part C: J. Mech. Eng. Sci. 205 (5), 317-328.
[19]Lopes, J.C.B., Dukler, A.E., 1986. Droplet entrainment in vertical annular flow and its contribution to momentum transfer[J]. AIChE J. 32, 1500-1515.
[20] Moeck, E.O.,1970. Annular-dispersed Two-phase Flow and Critical Heat Flux, pp. 206.
[21] Sawant, P.,2008.Dynamics of annular two-phase flow, Purdue University[D].West Lafayette, USA,Ph.D. thesis.
[22]Skopich, A., Pereyra, E., Sarica, C., & Kelkar, M.,2015. Pipe-Diameter Effect on Liquid Loading in Vertical Gas Wells[J]. SPE Production and Operations, 30(2). 164-76.
[23] Shearer, C.J., Nedderman, R.M,1965. Pressure gradient and liquid film thickness in co-current upwards flow of gas/liquid mixtures: application to film-cooler design[J]. Chem. Eng. Sci. 20 (7), 671-683.
[24] Wallis, G.B.,1969. One Dimensional Two-Phase Flow, null, McGraw-Hill, New York.
[25]Whalley, P., Hewitt, G.F., 1978. The correlation of liquid entrainment fraction and entrainment rate in annular two-phase flow[C]. UKAEA Report AERE-R9187.
[26] Wongwises, S.,Kongkiatwanitch, W.,2001.Interfacial friction factor in vertical upward gas-liquid annular two-phase flow[J]. Int. Commun. Heat Mass Transf. 28 (3), 323 336.
[27]Wolf, A., Jayanti, S., Hewitt, G.F., 2001. Flow development in vertical annular flow[J].

Chemical Engineering Science 56, 3221-3235.
[28]Zabaras, G., Dukler, A.E., Moalem-Maron, D.,1986.Vertical upward cocurrent gas–liquid annular flow[J]. AIChE J. 32, 829-843.