

Table 1 Previous models and correlations to predict the interfacial friction factor.

Author(s)	Correlation	Remarks
Wallis-type correlations		
Wallis(1969)	$f_i = 0.005 \left(1 + 300 \frac{h}{D} \right)$	
Moeck(1970)	$f_i = 0.005 \left[1 + 1458 \left(\frac{h}{D} \right)^{1.42} \right]$	ID:19,25,53mm; Steam/water
Whalley and Hewitt(1978)	$f_i = 0.079 \text{Re}_c^{-0.25} \left[1 + 24 \left(\frac{\rho_l}{\rho_c} \right)^{\frac{1}{3}} \frac{h}{D} \right]$	
Asali et al. (1985)	$\frac{f_i}{f_g} = 1 + 0.45 (h_g^* - 4) \text{Re}_{sg}^{-0.2} \quad h_g^* = 0.19 \text{Re}_{if}^{0.7} \frac{v_l}{v_g} \left(\frac{\rho_l \tau_c}{\rho_g} \right)^{0.5}$	ID:22.9,42mm; air/water, air/glycerol; p:1~2bar; μ_l : 1.1~5cP
Klausner and Chao(1991)	$\frac{f_i}{f_g} = 1 + 0.039 h^* \quad h^* = \frac{h}{v_g} \sqrt{\frac{\tau_c}{\rho_g}}$	air/water, air/glycerol; ID:16~42mm; vapor density 1.2~9.7kg/m ³ ; vapor viscosity 0.011~0.019cP, Fluid viscosity 0.44~5.6cP
Ambrosini et al. (1991)	$\frac{f_i}{f_g} = 1 + 13.8 W e^{0.2} \text{Re}_{sg}^{-0.6} \left(h_g^* - 200 \sqrt{\frac{\rho_g}{\rho_l}} \right) \quad h_g^* = \frac{h}{v_g} \sqrt{\frac{\tau_l}{\rho_g}}$	ID:10.26~42.2 mm; air/water, nitrogen /water, air/various hydrocarbons; p: 0.2~1.9bar
Fukano & Furukawa(1998)	$f_i = 0.425 \left(12 + \frac{v_l}{v_w} \right)^{-1.33} \left[1 + 12 \left(\frac{h}{D} \right) \right]^8$	ID: 26,19.2mm; fluid:air/water, air/glycerol; u_{sg} :10~50m/s; u_{sl} :0.04~0.3m/s; p:1.03~1.17bar; T:27~29°C; μ_l : 0.85×10^{-6} ~ 8.6×10^{-6} m ² /s

Holt(1999)	$\frac{f_i}{f_g} = 1 + 13.8 We^{0.2} Re_{sg}^{-0.6} \left(h_i^* - 200 \sqrt{\frac{\rho_g}{\rho_l}} \right) h_i^* = \frac{h}{v_i} \sqrt{\frac{\tau_i}{\rho_l}} \quad m < 100 \text{kgm}^{-2} \text{s}^{-1}$ $\frac{f_i}{f_g} = 1 + 13.8 We^{0.175} Re_{sg}^{-0.7} \quad m > 100 \text{kgm}^{-2} \text{s}^{-1}$	ID:5, 10mm, rectangular tube 7.7×2.6mm, laddertron 2×7×4.4mm; air/water, helium/water, nitrogen /water, air/glycerol; p:0.2~1.5bar
Fore et al. (2000)	$f_i = 0.005 \left\{ 1 + 300 \left[\left(1 + \frac{17500}{Re_g} \right) \frac{h}{D} - 0.0015 \right] \right\}$	5.08×101.6mm rectangular tube;p: 3.4~17atm; T:38~93°C;ρ:4~20kg/m³; μ:0.3×10 ⁻³ ~0.7×10 ⁻³ kg/ms; u _{sg} :4~30m/s; u _{sl} :0.06~1.0m/s.air/water, air/glycerol; ID: 50.8,42,22.9mm; T:18~21°C; p: 1~2atm
Belt et al. (2009)	$f_i = 2 \times (3.413 \times 10^{-4} + 1.158 \frac{h}{D})$	ID:50,19mm; air/water; p: 1bar; u _{sg} :22~42 m/s; u _{sl} :0.01~0.08 m/s
Aliyu et al. (2017)	$\frac{f_i}{f_g} = \left[1 + 0.3 \left(\frac{h}{D} \right)^{0.12} Re_{sg}^{0.54} Fr_g^{-1.2} \right]^{1.5}$	ID:16~127mm; p:1~6bar; u _{sl} :0.01~1.5m/s; u _{sg} : 7~150m/s
Non-Wallis-type correlations		
Henstock and Hanratty (1976)	$\frac{f_i}{f_g} = 1 + 1400 F \quad F = \frac{\gamma(Re_{if})}{Re_{sg}^{0.9}} \frac{v_l}{v_g} \sqrt{\frac{\rho_l}{\rho_g}}$ $\gamma(Re_{if}) = \left[(0.707 Re_{if}^{0.5})^{2.5} + (0.0379 Re_{if}^{0.9})^{2.5} \right]^{0.4}$	ID:50.8,63.5, 25.4mm; Re _{sg} :5000~255000, Re _{if} :20~15100

Hori(1978)	$f_i = 1.13 \text{Re}_{sg}^{-0.889} \text{Re}_{sl}^{0.678} Fr_g^{0.252} Fr_l^{-0.452} \left(\frac{\mu_l}{\mu_w} \right)^{0.768}$	ID: 13,19.8,26mm; air/water, air/glycerol; u_{sg} :17~58m/s; u_{sl} :0.003~0.026 m/s
Fukano et al. (1991)	$\frac{f_i}{f_g} = 1 + 8.53 \times 10^{-4} X^{2.82} \text{Re}_{sg}^2 / \text{Re}_{sl}$	ID: 10,16,26 mm; p: 1.02~1.35 bar, u_{sg} :20~60m/s; u_{sl} :0.06~0.1m/s; Re_{sl} :67~2900; Re_{sg} :12720~99218
Wongwises and Kongkiatwanitch (2001)	$f_i = 17.172 \text{Re}_{sg}^{-0.768} \left(\frac{h}{D} \right)^{-0.253}$	ID:29mm; air/water; p:1bar; u_{sl} :0.05m/s~0.2m/s; u_{sg} :10~34m/s
Aliyu et al. (2015)	$f_i = 6059 \text{Re}_{sg}^{-0.05} \text{Re}_{sl}^{-0.38} Fr_g^{-1.6} \left(\frac{h}{D} \right)^{0.7}$	ID:101.6,127mm; u_{sg} :1.42~28.87m/s; u_{sl} :0.01~1.0m/s; p: 0.9~1.2bar

Table 2 Summary of the collected experimental data

Authors	D [mm]	Test pressure [bara]	L/D	u_{sl} range [m/s]	u_{sg} range [m/s]	Number of data points
Fore & Dukler (1995) ^[1]	50.8	1	69	0.006-0.06	16-36	35
Fore & Dukler (1995) ^[1]	50.8	1	69	0.006-0.06	16-36	24
Aliyu et al. (2017) ^[2]	101.6	1-1.4	46	0.1-1	13-29	18
Skopich et al. (2015) ^[3]	50.8,101.6	0.9-1.2	58-2	0.01-0.05	18-30	16
Shearer & Nedderman(1965) ^[4]	16,32	1.1	133-267	0.003-0.16	26-122	24
Wolf et al.(2001) ^[5]	31.8	2.38	343	0.01-0.12	23-50	28
Asali(1983) ^[6]	42,22.9	1-2	109,213	0.001-0.1	20-60	116
Asali(1983) ^[6]	42	1	109	0.001-0.1	20-60	66
Belt et al.(2009) ^[7]	50	1	120, 140	0.01-0.08	22-42	20
Fukano & Furukawa(1998) ^[8]	26	1.03-1.17	133	0.04-0.3	10-50	24
Kaji & Azzopardi(2010) ^[9]	19	1.2	300	0.03-0.1	12-35	28
Zabaras et al.(1986) ^[10]	50.8	1.4	31	0.006-0.06	15-38	15
total						414

Table 3 Statistical comparison between predictions of proposed and previous correlations.

Correlation	Statistic		
	MAE	MSE	Percentage of points within $\pm 50\%$ error band
New	17.77	0.000021	91.79
Wallis(1969)	29.32	0.000163	85.99
Fore et al(2000)	26.59	0.000071	87.68
Henstock & Hanratty(1976)	56.22	0.001691	67.63
Asali et al(1985)	38.57	0.000137	78.02
Ambrosini et al(1991)	80.24	0.000714	68.60
Belt et al(2009)	33.84	0.000124	80.68
Fukano et al(1991)	961.32	0.277006	50.80
Fukano&Furukawa(1998)	116.87	0.001374	40.58
Aliyu et al(2017)	647.97	0.214735	81.16
Aliyu et al(2015)	35.50	0.000452	58.70
Moeck(1970)	21.40	0.000131	90.72
Hori et al(1976)	31.97	0.000196	84.78
Holt et al(1999)	42.36	0.000561	60.14
Wongwises&Kongkiatwanitch(2001)	44.92	0.000399	62.56
Klausner&Chao(1991)	49.78	0.000392	81.88

MAE: the mean absolute percentage error $\frac{1}{n} \sum_i^n \left| \frac{f_{i,exp} - f_{i,pred}}{f_{i,exp}} \right| \times 100$

MSE: the mean square error $\frac{1}{n} \sum_i^n (f_{i,exp} - f_{i,pred})^2$