Received ; | Revised ; | Accepted

DOI: xxx/xxxx

ARTICLE TYPE

Analysis of a fractional mathematical model for Zika virus under the framework of singular and nonsingular kernels

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In this paper, we investigate the dynamics of a fractional Zika virus model (ZIKV) with Caputo, Caputo-Fabrizio-Caputo (CFC) and Atangana-Baleanu-Caputo (ABC) derivatives. Firstly the basic properties of the classical integer order model are furnished followed by the equilibrium points and basic reproduction number. Furthermore, with respect to the Caputo, CFC and ABC derivatives, we establish via a fixed point technique that under certain conditions the fractional ZIKV model admits a unique system of solutions. The Adams-Bashforth numerical scheme incorporating the fractional order parameter is then used to obtain numerical schemes for the approximate solutions of the fractional ZIKV model with respect to each of the considered fractional differential operators. Finally, with a view to visualize the behaviour of the approximate solutions to fractional ZIKV model with respect to each of the fractional differential operators, we do some numerical simulations for distinct values of the fractional order parameter.

KEYWORDS:
Fractional derivatives and integrals. Equilibria, Existence and uniqueness, Zika virus, Singular and nonsingular kernels, Adams-Bashforth

1 | INTRODUCTION

Zika virus (ZIKV) is an infectious vector-borne disease belonging to the family of *Flaviviridea*. Its name comes from the Zika forest in Uganda, where the virus was first discovered in 1947 and isolated from a Rhesus Macaque population during a research study on Yellow fever[1]. Among the human population, ZIKV was first identified in Nigeria in 1954.[1,2,3] Its mode of transmission to humans is via the bites of infected female mosquitoes from the *Aedes* genus (these include *Aedes aegypti, Aedes africanus, Aedes apicoargenteus, Aedes furcifer, Aedes hensilli, Aedes luteocephalus and Aedes vitattus*)[2,4] which are known to be predominant in the tropical and subtropical regions and are also responsible for the transmission of closely related and well-known notorious pathogen such as Dengue, Japanese encephalitis, Chikungunya, Yellow fever virus and West Nile virus. It has also been established that unprotected sexual relations, blood transfusions, transplacental transmission during child delivery are other potential ZIKV transmission routes from already infected individuals.[2,5,6,7] In humans, ZIKV infection usually causes less severe symptoms like mild fever, maculopapular rash, loss of appetite, conjunctivitis, muscle and joint pain and headache which have very short duration of about 2-7 days. However, some infected individuals do not develop symptoms. Among infected pregnant women, ZIKV may lead to newly born babies having small heads with abnormal brain development and muscle weakness which affects the nervous system. Research findings have shown that ZIKV increases the chances of congenital brain anomalies including microcephaly which occur in fetuses of infected pregnant woman.[5,8,9,10] as well as the

[0]**Abbreviations:** ANA, anti-nuclear antibodies; APC, antigen-presenting cells; IRF, interferon regulatory factor

Guillain-Barré syndrome (GBS).[11,12] Unfortunately, up until now, there is no vaccine, specific treatment, or fast diagnostic test for the treatment, prevention, or diagnosis of ZIKV infection.

In 1952, sporadic transmission of ZIKV among humans were recorded in many countries in Southeast Asia and Africa. In April 2007 and October 2013, major outbreaks were also recorded on Yap Island, Federated States of Micronesia, in the North Pacific[7] and in French Polynesia, South Pacific,[12,13] respectively. In April 2015, the ZIKV outbreak in Brazil rapidly spread to many American and Caribbean countries with over 140,000 suspected and confirmed cases by the end of February 2016.[2] Between October 2015 and February 2016, nearly 6,000 suspected cases of microcephaly (including 139 deaths) among newborns with possible links to the ZIKV infections in Brazil was reported. Furthermore, in Colombia (and El Salvador), over 200 ZIKV related GBS cases (and 118 GBS cases) were reported December 2015 (and February 2016). In 2016, the World Health Organization[14] declared ZIKV as a Public Health Emergency of International Concern.

In an attempt to have deeper understanding of the transmission dynamics of a wide range of infectious diseases, a variety of deterministic mathematical models, based on systems of ordinary differential equations with classical integer-order derivatives have been formulated by different authors over the past decades. These models have not only facilitated the mathematical studies of infectious diseases from both qualitative and quantitative perspectives, they have also helped in the determination of adequate control mechanisms to curtail the spread of these diseases as well as to eradicate them from the community. On the mathematical perspectives of the transmission dynamics of ZIKV we refer to the works[2,4,15,16,17,18,19,20] where the various authors studied different mathematical models describing ZIKV in the framework of classical systems of integer-order ordinary differential equations. However, models with integer-order derivatives do not adequately account for hereditary and memory effects associated with many biological processes. Advances in the field in recent times have yielded mathematical models with fractional (or arbitrary or non-integer) order differential operators which have became a central area of study as they effectively and adequately incorporate the evolution-related realities and evidences of the systems they model.

As a generalization of the classical integer calculus, fractional calculus encompass the notions, properties and applications of fractional order differential and integral operators.[21,22,23,24,25] It provides a mathematical framework in the form of fractional differential or integral equations for modeling and exploring the complex dynamics associated with real phenomena. Among the several advantages over the integer order operators, the fractional order operators incorporate hereditary properties and provides good description of the memory effects associated with many physical systems. In literature, a variety of fractional order models, with different types of fractional operators (Caputo[21,22,23], Caputo-Fabrizio-Caputo (CFC)[24] and Atangana-Baleanu-Caputo (ABC)[25]) have been used to explore infectious disease dynamics. Unlike the Caputo fractional derivative, the kernels of the CFC and ABC derivatives do not have singularities. The CFC derivative uses the exponential law as nonsingular kernel while the ABC derivative use the Mittag-Leffler law as nonlocal and non-singilular kernel. We refer the reader to the works[26,27,28,29,30,31,32,33,34,35] and the references therein, where a variety of fractional order models arising in mathematical biology have been investigated by different authors.

In this paper, we study a fractional mathematical model for ZIKV. The fractional derivatives for the proposed model are taken in the Caputo, CFC, and ABC sense. The ZIKV model considered divides the human population into four sub-classes, namely, $S_h(t)$, $E_h(t)$, $I_h(t)$, $R_h(t)$ while the vector (mosquitoes) population is divided into three sub-classes, namely, $S_v(t)$, $E_v(t)$, $I_v(t)$. For each type of the mentioned fractional derivatives, we investigate questions on existence and uniqueness of solutions via fixed point technique. Furthermore, motivated by the two-step fractional Adams-Bashforth (FAB) numerical scheme,[36] we investigate the behaviour of solutions to the proposed fractional ZIKV model with respect to the Caputo, CFC and ABC fractional derivatives, respectively.

The organization of this is paper is as follows: In Section 2, we collect some important information about the fractional differential and integral operators related to those of Caputo, Caputo-Fabrizio and Atangana-Baleanu types. In Section 3, we formulate an integer order mathematical model for ZIKV. The basic solution properties of the constructed model are also investigated. Basic system properties such as non-negativity of of solutions, invariant region and system equilibrium points are also discussed in this section. In Section 4, we introduce the corresponding fractional ZIKV model in Caputo, Caputo-Fabrizio and Atangana-Baleanu derivatives. In Section 5, the existence and uniqueness of solutions to the fractional ZIKV model in the Caputo, CFC and ABC derivatives are investigated. In Section 6, we employ the two-step fractional Adams-Bashforth method to investigate the behaviour of solutions to the model with respect to each type of the considered fractional differential operator while the conclusion is presented in Section 7.

2 | PRELIMINARIES

In the present section, we present some fundamental definitions and properties on fractional differential and integral operators related to those of Caputo, Caputo-Fabrizio and Atangana-Baleanu types. In the sequel, we denote by $\sigma > 0$ the fractional order parameter, $\Gamma(\cdot)$ the gamma function and $E_\sigma(\cdot)$ the Mittag-Leffler function [37].

Definition 1. A real valued function $g(t),\ t > 0$ is said to be in the space $C_\mu,\ \mu \in \mathbb{R}$ if there exist a real number $\rho(> \mu)$ such that $g(t) = t^\rho g_1(t)$, where $g_1 \in C[0, \infty)$, and is said to be in the space C_μ^m if $g^m \in C_\mu,\ m \in \mathbb{N} \cup \{0\}$.

Definition 2. [23,21,22] The Riemann-Liouville fractional integral of order σ of a function $g \in C_\mu,\ \mu \geq -1$ is defined as

$$I_t^\sigma[g(t)] = \begin{cases} g(t), & \sigma = 0,\ t > 0, \\ \dfrac{1}{\Gamma(\sigma)} \displaystyle\int_0^t (t-\tau)^{\sigma-1} g(\tau) d\tau, & \sigma > 0,\ t > 0, \end{cases} \tag{1}$$

where $0 < \sigma < 1$.

Definition 3. [23,21,22] The Caputo fractional derivative of order of order σ of a function $g \in C_{-1}^m,\ m \in \mathbb{N} \cup \{0\}$ is defined as

$${}_0^C D_t^\sigma[g(t)] = I_t^{m-\sigma} D_t^m[g(t)] = \begin{cases} g^{(m)}(t) := \dfrac{d^m g(t)}{dt^m}, & \sigma = m, \\ \dfrac{1}{\Gamma(m-\sigma)} \displaystyle\int_0^t g^{(m)}(t)(t-\tau)^{m-\sigma-1} d\tau, & m-1 < \sigma < m. \end{cases} \tag{2}$$

Definition 4. [24] For a given function $g \in H^1(a, b),\ b > a$, the Caputo-Fabrizio-Caputo (CFC) derivative of order σ $(0 < \sigma \leq 1)$ is defined as

$${}_a^{CFC} D_t^\sigma g(t) = \frac{\mathbb{M}[\sigma]}{1-\sigma} \int_a^t g'(\tau) \exp\left[-\frac{\sigma}{1-\sigma}(t-\tau)\right] d\tau. \tag{3}$$

where $\mathbb{M}[\sigma]$ is a normalization functions satisfying $M(0) = M(1) = 1$. If $g \notin H^1(a, b)$ then the CFC derivative (3) can be expressed as

$${}_a^{CFC} D_t^\sigma g(t) = \frac{\sigma \mathbb{M}[\sigma]}{1-\sigma} \int_a^t \left[g(t) - g(\tau)\right] \exp\left[-\frac{1-\sigma}{\sigma}(t-\tau)\right] d\tau. \tag{4}$$

Remark 1. Suppose $\nu = \frac{1-\sigma}{\sigma} \in [0, \infty),\ \sigma = \frac{1}{1+\nu} \in [0, 1]$, then the CFC derivative (4) has the following representation

$${}_a^{CFC} D_t^\sigma \omega(x, t) = \frac{\mathbb{M}(\nu)}{\nu} \int_a^t \omega'(x, t) \exp\left[-\frac{t-\tau}{\nu}\right] d\tau, \ \text{ with } \ \mathcal{M}(0) = \mathcal{M}(\infty) = 1. \tag{5}$$

In addition,

$$\lim_{\nu \to 0} \frac{1}{\nu} \exp\left[-\frac{t-\tau}{\nu}\right] = \delta(t-\tau) \tag{6}$$

where $\delta(t-\tau)$ is the Dirac delta function.

Definition 5. [38] The fractional integral related to the CFC derivative is defined by

$${}_a^{CFC} I_t^\sigma g(t) = \frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]} g(t) + \frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]} \int_0^t g(\tau) d\tau, \ \ 0 < \sigma < 1, \ \ t \geq 0. \tag{7}$$

Remark 2. According to Losada and Nieto [38], the equality

$$\frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]} + \frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]} = 1, \tag{8}$$

must hold for the fractional (7). This implies $\mathbb{M}[\sigma] = \frac{2}{2-\sigma}, \ 0 < \sigma < 1$.

Definition 6. [25] Let $u \in H^1(a, b), \ b > a$ be a given function. The Atagana-Baleanu fractional derivative in Caputo sense (ABC) and in the Riemann-Liouville sense (ABR) ate defined as

$$ {}^{ABC}_{a}D^{\sigma}_{t} g(t) = \frac{\mathbb{B}[\sigma]}{1-\sigma} \int_{a}^{t} g'(\tau) E_{\sigma}\left[-\frac{\sigma}{1-\sigma}(t-\tau)^{\sigma}\right] d\tau, \ \ t > 0, \tag{9} $$

and

$$ {}^{ABR}_{a}D^{\sigma}_{t} g(t) = \frac{\mathbb{B}[\sigma]}{1-\sigma} \frac{d}{dt} \int_{a}^{t} g(\tau) E_{\sigma}\left[-\frac{\sigma}{1-\sigma}(t-\tau)^{\sigma}\right] d\tau, \ \ t > 0, \tag{10} $$

respectively, where $\mathbb{B}[\sigma] := 1 - \sigma + \frac{\sigma}{\Gamma(\sigma)}$ denotes the normalization functions satisfying $B(0) = B(1) = 1$.

Definition 7. [25] The fractional integral related to the Atagana-Baleanu derivative is defined as

$$ {}^{AB}_{a}I^{\sigma}_{t}[g(t)] = \frac{1-\sigma}{\mathbb{B}[\sigma]} g(t) + \frac{\sigma}{\mathbb{B}[\sigma]\Gamma(\sigma)} \int_{a}^{t} (t-\tau)^{\sigma-1} g(\tau) d\tau. \tag{11} $$

Remark 3. The CFC derivative uses the exponential law as nonsingular kernel while the ABC derivative use the Mittag-Leffler law as nonlocal and non-singilular kernel. Unlike the Caputo fractional derivative, the kernels of the CFC and ABC derivatives do not have singularities at $t = \tau$.

3 | MATHEMATICAL FORMULATION AND BASIC MODEL ANALYSIS

In formulating the Zika virus dynamics, we take into account the human to human infection as well as the vector (mosquito) to human transmission. The total human population $N_h(t)$ is subdivided into four compartments, namely, susceptible humans $S_h(t)$, exposed humans $E_h(t)$, infected humans $I_h(t)$, and recovered humans $R_h(t)$, while the entire vector (mosquito) population $N_v(t)$ is subdivided into three compartments, namely, susceptible vectors $S_v(t)$, exposed vectors $E_v(t)$ and infected mosquito $I_v(t)$ so that

$$ N_h(t) = S_h(t) + E_h(t) + I_h(t) + R_h(t) \quad \text{and} \quad N_v(t) = S_v(t) + E_v(t) + I_v(t), \tag{12} $$

respectively. Susceptible humans and mosquitoes are recruited into the susceptible compartments S_h and S_v at rates rates Π_h and Π_v, respectively. We represent by $\lambda S_h = (\lambda_1 + \lambda_2) S_h$ the incidence rate of infection in the human population where $\lambda_1 = \beta_h I_v$ is the rate at which susceptible individuals acquire infection due to effective contact with an infected vector and $\lambda_2 = \rho\beta_h I_h$ is the rate at which susceptible individuals acquire infection due to sexual interaction with infected individuals. Here, β_h is the effective contact rate between susceptible humans and infected mosquitoes while ρ is a modification parameter that accounts for the relative infectiousness of individuals in the I_h relative to those in the I_v compartment. Similarly, we represent by $\beta_v I_h S_v$ the incidence rate of the susceptible vector population where β_v denotes the transmission rate from infected humans to susceptible mosquito. The disease induced mortality rate is denoted by δ. Natural mortality rates due for the human and vector subpopulations are denoted by μ_h and μ_v respectively. Lastly, γ and τ are the natural and treatment rates.

Following the above description for the interrelationship between compartments, we arrive at the following coupled system of nonlinear ordinary differential equations describing the Zika virus dynamics:

$$ \begin{cases} D_t S_h(t) = \Pi_h - \beta_h S_h (I_v + \rho I_h) - \mu_h S_h \\ D_t E_h(t) = \beta_h S_h (I_v + \rho I_h) - (\mu_h + \chi) E_h \\ D_t I_h(t) = \chi E_h - (\mu_h + \gamma + \tau) I_h \\ D_t R_h(t) = \gamma I_h - \mu_h R_h \\ D_t S_v(t) = \Pi_v - \beta_v S_v I_h - \mu_v S_v \\ D_t E_v(t) = \beta_v S_v I_h - (\mu_v + \delta) E_v \\ D_t I_v(t) = \delta E_v - \mu_v I_v. \end{cases} \tag{13} $$

3.1 | Basic model analysis

Since the model (13) describes the dynamics of living species (human and vectors), it will be considered biologically meaningful if all system parameters as well as system variables are non-negative for all time $t \geq 0$. In other words, solution with positive initial data will remain positive for all time. Indeed, let us denote by $\lambda(t) = \beta_h(I_v + \rho I_h)$ the force of infection. By using $\exp\left\{\mu_h t + \int_0^t \lambda(\tau)d\tau\right\}$ as an integrating factor in the S_h-equation of (13), we get

$$S_h(t) = \exp\left\{-\mu_h t - \int_0^t \lambda(\tau)d\tau\right\} S_h(0) + \Pi_h \exp\left\{-\mu_h t - \int_0^t \lambda(\tau)d\tau\right\} \int_0^t \exp\left\{\mu_h \vartheta + \int_0^\vartheta \lambda(\tau)d\tau\right\} d\vartheta > 0,$$

after some manipulations where $S_h(0)$ is the initial condition for $S_h(t)$ at $t = 0$. Since $S_h(0) > 0$ and the exponential function is always nonnegative for any exponent, the last equation above therefore guarantees the positivity of $S_h(t)$. In a similar manner, it can be shown that $E_h > 0$,$I_h > 0$,$R_h > 0$,$S_v > 0$,$E_v > 0$ and $I_v > 0$ for all $t > 0$.

Lemma 1. Let $\Omega_h = \left\{(S_h(t), E_h(t), I_h(t), R_h(t)) \in \mathbb{R}^4_+ : N_h(t) \leq \frac{\Pi_h}{\mu_h}\right\}$ and $\Omega_v = \left\{(S_v(t), E_v(t), I_v(t)) \in \mathbb{R}^3_+ : N_v(t) \leq \frac{\Pi_v}{\mu_v}\right\}$. Then the closed set $\Omega = \Omega_h \cup \Omega_v \subset \mathbb{R}^4_+ \times \mathbb{R}^3_+$ positively invariant with respect to the model (13).

Proof. In view of both equations in (12) in relation to the system of equations (13), the total human and vector populations satisfy the following differential equalities

$$\frac{dN_h}{dt} = \Pi_h - \mu_h N_h - \tau I_h \Rightarrow \frac{dN_h}{dt} \leq \Pi_h - \mu_h N_h, \quad \text{and} \quad \frac{dN_v}{dt} = \Pi_v - \mu_h N_v, \tag{14}$$

respectively, and it follows that $\frac{dN_h}{dt} \leq 0$ and $\frac{dN_v}{dt} \leq 0$ if $N_h \geq \frac{\Pi_h}{\mu_h}$ and $N_v \geq \frac{\Pi_v}{\mu_v}$, respectively. Hence, by standard comparison theorem, it can be shown $N_h(t) \leq N_h(0)e^{-\mu_h t} + \frac{\Pi_h}{\mu_h}\left[1 - e^{-\mu_h t}\right]$ and $N_v(t) \leq N_v(0)e^{-\mu_v t} + \frac{\Pi_v}{\mu_v}\left[1 - e^{-\mu_v t}\right]$, respectively, where $N_h(0) = S_h(0) + E_h(0) + I_h(0) + R_h(0)$ and $N_v(0) = S_v(0) + E_v(0) + I_v(0)$. In particular, $N_h(t) \leq \frac{\Pi_h}{\mu_h}$ and $N_v(t) \leq \frac{\Pi_v}{\mu_v}$ if $N_h(0) \leq \frac{\Pi_h}{\mu_h}$ and $N_v(0) \leq \frac{\Pi_v}{\mu_v}$, respectively. Thus, the closed set $\Omega = \Omega_h \cup \Omega_v \subset \mathbb{R}^4_+ \times \mathbb{R}^3_+$ is positively invariant. Furthermore, if $N_h(0) > \frac{\Pi_h}{\mu_h}$ and $N_v(0) > \frac{\Pi_v}{\mu_v}$ then either the solutions of (13) enter the region Ω in finite time or $N_h(t)$ approaches $\frac{\Pi_h}{\mu_h}$ and $N_v(t)$ approaches $\frac{\Pi_v}{\mu_v}$ as $t \to \infty$. The region Ω is therefore an attraction set for all solutions of (13) in $\mathbb{R}^7_+$. It follows directly from Hethcote[39] that the model (13) is both epidemiologically well-posed as well as biologically feasible in the invariant region Ω.Thus a qualitative investigated on the model (13) can be sufficiently initiated in this region as demonstrated in[16]. □

3.2 | Equilibrium points and basic reproduction number

Using standard approach, we obtain the disease free equilibrium as $\mathbb{E}^0 = (S_h^0, E_h^0, I_h^0, R_h^0, S_v^0, E_v^0, I_v^0) = \left(\frac{\Pi_h}{\mu_h}, 0, 0, 0, \frac{\Pi_v}{\mu_v}, 0, 0\right)$. At the disease free equilibrium, the basic reproduction number

$$\mathcal{R}_0 = \frac{\beta_h \rho \Pi_h \chi}{2\mu_h k_1 k_2} + \sqrt{\frac{\rho^2 {\Pi_h}^2 {\beta_h}^2 \chi^2}{4{\mu_h}^2 {k_1}^2 {k_2}^2} + \frac{\Pi_h \Pi_v \beta_h \beta_v \chi \delta}{k_1 k_2 k_3 \mu_h {\mu_v}^2}}$$

is obtained via the next generation matrix (see. for instance,[40]). Moreover, by solving the associated steady state problem we obtain the disease endemic equilibrium $\mathbb{E}^* = (S_h^*, E_h^*, I_h^*, R_h^*, S_v^*, E_v^*, I_v^*)$ as where

$$S_h^* = \frac{\Pi_h \mu_v (\beta_v I_h^* + \mu_v)(\mu_v + \delta)}{\mu_v(\mu_v + \delta)(\rho\beta_h I_h^* + \mu_h)(\beta_v I_h^* + \mu_v) + \beta_h \beta_v \Pi_v \delta I_h^*}, \quad E_h^* = \frac{\beta_h \Pi_h I_h^* (\beta_v \delta \Pi_v + \rho \mu_v (\mu_v + \delta)(\beta_v I_h^* + \mu_v))}{\mu_v(\mu_v + \delta)(\mu_h + \chi^*)(\rho\beta_h I_h^* + \mu_h)(\beta_v I_h^* + \mu_v) + \beta_h \beta_v \delta \Pi_v I_h^*},$$

$$R_h^* = \frac{\gamma I_h^*}{\mu_h}, \quad S_v^* = \frac{\Pi_v}{\beta_v I_h^* + \mu_v}, \quad E_v^* = \frac{\beta_v \Pi_v I_h^*}{(\beta_v I_h + \mu_v)(\mu_v + \delta)}, \quad I_v^* = \frac{\beta_v \delta \Pi_v I_h^*}{\mu_v(\beta_v I_h^* + \mu_v)(\mu_v + \delta)},$$

with I_h^* satisfying $I_h^*(a_1 {I_h^*}^2 + a_2 I_h^* + a_3) = 0$ where $a_1 = \rho\mu_v\beta_h\beta_v k_1 k_2 k_3$, $a_2 = \beta_h\beta_v\left(\Lambda_v \delta_v k_1 k_2 - \rho\,\Lambda_h \chi_h \mu_v k_3\right) + \mu_v\left(\rho\,\beta_h\mu_v + \beta_v\mu_h\right)k_1 k_2 k_3$ and $a_3 = \mu_h \mu_v^2 k_1 k_2 k_3\left[\mathcal{R}_0^2 + \frac{\rho\beta_h\chi_h\Lambda_h}{\mu_h k_1 k_2}(1 - \mathcal{R}_0)\right]$.

4 | FRACTIONAL ZIKA VIRIUS MODEL

In the present section, we redefine the ZIKV model (13) by replacing the classical time derivative by the time fractional derivative in the sense of Caputo, CFC and ABC to obtain

$$\begin{cases} {}^{\Xi(\sigma)}_{0}D^{\sigma}_{t} S_h(t) = \Pi_h - \beta_h S_h(I_v + \rho I_h) - \mu_h S_h, \\ {}^{\Xi(\sigma)}_{0}D^{\sigma}_{t} E_h(t) = \beta_h S_h(I_v + \rho I_h) - (\mu_h + \chi) E_h, \\ {}^{\Xi(\sigma)}_{0}D^{\sigma}_{t} I_h(t) = \chi E_h - (\mu_h + \gamma + \tau) I_h, \\ {}^{\Xi(\sigma)}_{0}D^{\sigma}_{t} R_h(t) = \gamma I_h - \mu_h R_h, \\ {}^{\Xi(\sigma)}_{0}D^{\sigma}_{t} S_v(t) = \Pi_v - \beta_v S_v I_h - \mu_v S_v, \\ {}^{\Xi(\sigma)}_{0}D^{\sigma}_{t} E_v(t) = \beta_v S_v I_h - (\mu_v + \delta) E_v, \\ {}^{\Xi(\sigma)}_{0}D^{\sigma}_{t} I_v(t) = \delta E_v - \mu_v I_v, \end{cases} \tag{15}$$

subject to the initial conditions

$$S_h(0) = S_{h0}, E_h(0) = E_{h0}, I_h(0) = I_{h0}, , R_h(0) = R_{h0}, S_v(0) = S_{v0}, E_v(0) = E_{v0}, I_v(0) = I_{v0}, \tag{16}$$

where ${}^{\Xi(\sigma)}_{0}D^{\sigma}_{t}$ denotes the fractional differential operator of order $0 < \sigma \leq 1$ either in the Caputo, CFC or ABC sense (we refer th reader to Table 1 for the definition of ${}^{\Xi(\sigma)}_{0}D^{\sigma}_{t}$). For the sake of convenience in subsequent sections, we make the following notations for the right hand terms appear in (15):

$$\begin{cases} \mathscr{F}_1(t, S_h(t)) = \Pi_h - \beta_h S_h(I_v + \rho I_h) - \mu_h S_h, \\ \mathscr{F}_2(t, E_h(t)) = \beta_h S_h(I_v + \rho I_h) - (\mu_h + \chi) E_h, \\ \mathscr{F}_3(t, I_h(t)) = \chi E_h - (\mu_h + \gamma + \tau) I_h, \\ \mathscr{F}_4(t, R_h(t)) = \gamma I_h - \mu_h R_h, \\ \mathscr{F}_5(t, S_v(t)) = \Pi_v - \beta_v S_v I_h - \mu_v S_v, \\ \mathscr{F}_6(t, E_v(t)) = \beta_v S_v I_h - (\mu_v + \delta) E_v, \\ \mathscr{F}_7(t, I_v(t)) = \delta E_v - \mu_v I_v. \end{cases} \tag{17}$$

Table 1: *Definitions of* ${}^{\Xi(\sigma)}_{0}D^{\sigma}_{t}$, $\Xi_1(\sigma)$, $\Xi_2(\sigma)$ *and* $\boldsymbol{H}(t, \tau)$

${}^{\Xi(\sigma)}_{0}D^{\sigma}_{t}$	$\Xi_1(\sigma)$	$\Xi_2(\sigma)$	$\mathbf{H}(t, \tau)$
${}^{C}_{0}D^{\sigma}_{t}$	0	$\frac{1}{\Gamma(\sigma)}$,	$(t-\tau)^{\sigma-1}$
${}^{CFC}_{0}D^{\sigma}_{t}$	$\frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}$	$\frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}$	1
${}^{ABC}_{0}D^{\sigma}_{t}$	$\frac{1-\sigma}{\mathbb{B}[\sigma]}$	$\frac{\sigma}{\mathbb{B}[\sigma]\Gamma(\sigma)}$	$(t-\tau)^{\sigma-1}$

5 | EXISTENCE AND UNIQUENESS OF SOLUTIONS

There is no existing straightforward procedure for obtaining exact solutions to the nonlinear nonlocal time-fractional system of equations (15). However, under certain conditions, the existence and uniqueness of solutions to the model with respect to each type of fractional derivative is assured via a fixed-point technique.

Applying any of the Riemann-Liouville integral operator (see Definition 1) or the Caputo-Fabrizio integral operator (see Definition 5) or the Atagana-Baleanu integral operator (see Definition 7) on both sides of each equation in (15) yields the

following equivalent system of Volterra-type fractional integral equations:

$$\begin{cases} S_h(t) - S_h(0) = \Xi_1(\sigma)\mathscr{F}_1(t, S_h(t)) + \Xi_2(\sigma)\displaystyle\int_0^t \mathbf{H}(t,\tau)\mathscr{F}_1(\tau, S_h(\tau))d\tau, \\ E_h(t) - E_h(0) = \Xi_1(\sigma)\mathscr{F}_2(t, E_h(t)) + \Xi_2(\sigma)\displaystyle\int_0^t \mathbf{H}(t,\tau)\mathscr{F}_2(\tau, E_h(\tau))d\tau, \\ I_h(t) - I_h(0) = \Xi_1(\sigma)\mathscr{F}_3(t, I_h(t)) + \Xi_2(\sigma)\displaystyle\int_0^t \mathbf{H}(t,\tau)\mathscr{F}_3(\tau, I_h(\tau))d\tau, \\ R_h(t) - R_h(0) = \Xi_1(\sigma)\mathscr{F}_4(t, R_h(t)) + \Xi_2(\sigma)\displaystyle\int_0^t \mathbf{H}(t,\tau)\mathscr{F}_4(\tau, R_h(\tau))d\tau, \\ S_v(t) - S_v(0) = \Xi_1(\sigma)\mathscr{F}_5(t, S_v(t)) + \Xi_2(\sigma)\displaystyle\int_0^t \mathbf{H}(t,\tau)\mathscr{F}_5(\tau, S_v(\tau))d\tau, \\ E_v(t) - E_v(0) = \Xi_1(\sigma)\mathscr{F}_6(t, E_v(t)) + \Xi_2(\sigma)\displaystyle\int_0^t \mathbf{H}(t,\tau)\mathscr{F}_6(\tau, E_v(\tau))d\tau, \\ I_v(t) - I_v(0) = \Xi_1(\sigma)\mathscr{F}_7(t, I_v(t)) + \Xi_2(\sigma)\displaystyle\int_0^t \mathbf{H}(t,\tau)\mathscr{F}_7(\tau, I_v(\tau))d\tau, \end{cases} \tag{18}$$

where $\Xi_1(\sigma)$, $\Xi_2(\sigma)$ and $\mathbf{H}(t,\tau)$ are as defined in Table 1.

Theorem 1. The kernels $\mathscr{F}_1,\ \mathscr{F}_2, \cdots,\ \mathscr{F}_7$ defined in (17) satisfy the Lipschitz condition and contractions provided the inequality $0 \leq \mathcal{K}_1,\ \mathcal{K}_2,\ \cdots,\ \mathcal{K}_7 < 1$ holds, where $\mathcal{K}_1,\ \mathcal{K}_2,\ \cdots,\ \mathcal{K}_7$ are the respective Lipschitz constants for $\mathscr{F}_1,\ \mathscr{F}_2, \cdots,\ \mathscr{F}_7$.

Proof. Firstly, we consider the kernel

$$\mathscr{F}_1(t, S_h(t)) = \Pi_h - \beta_h S_h(t)(I_v(t) + \rho I_h(t)) - \mu_h S_h(t).$$

Let $S_h^*(t)$ and $S_h^{**}(t)$ be two functions, then by Cauchy's inequality we have

$$\begin{aligned} \|\mathscr{F}_1(t, S_h^*(t)) - \mathscr{F}_1(t, S_h^{**}(t))\| =& \| - \beta_h(I_v(t) + \rho I_h(t))(S_h^*(t) - S_h^{**}(t)) - \mu_h(S_h^*(t) - S_h^{**}(t))\| \\ \leq& \left[\beta_h\Big(\|I_v(t)\| + \rho\|I_h(t)\|\Big) + \mu_h\right]\|S_h^*(t) - S_h^{**}(t)\| \\ \leq& \left[\beta_h\Big(\tau_2 + \rho\tau_1\Big) + \mu_h\right]\|S_h^*(t) - S_h^{**}(t)\|. \end{aligned}$$

Taking $\mathcal{K}_1 := \left[\beta_h\left(\tau_2 + \rho\tau_1\right) + \mu_h\right]$ where $\tau_1 = \max_{t\in I}\|I_h(t)\|$ and $\tau_2 = \max_{t\in I}\|I_v(t)\|$ are bounded functions, then we have

$$\|\mathscr{F}_1(t, S_h^*(t)) - \mathscr{F}_1(t, S_h^{**}(t))\| \leq \mathcal{K}_1\|S_h^*(t) - S_h^{**}(t)\|.$$

Similarly, it can be shown that the following inequalities

$$\begin{aligned} &\|\mathscr{F}_2(t, E_h^*(t)) - \mathscr{F}_2(t, E_h^{**}(t))\| \leq \mathcal{K}_2\|E_h^*(t) - E_h^{**}(t)\|, \\ &\|\mathscr{F}_3(t, I_h^*(t)) - \mathscr{F}_3(t, I_h^{**}(t))\| \leq \mathcal{K}_3\|I_h^*(t) - I_h^{**}(t)\|, \\ &\|\mathscr{F}_4(t, R_h^*(t)) - \mathscr{F}_4(t, R_h^{**}(t))\| \leq \mathcal{K}_4\|R_h^*(t) - R_h^{**}(t)\|, \\ &\|\mathscr{F}_5(t, S_v^*(t)) - \mathscr{F}_5(t, S_v^{**}(t))\| \leq \mathcal{K}_5\|S_v^*(t) - S_v^{**}(t)\|, \\ &\|\mathscr{F}_6(t, E_v^*(t)) - \mathscr{F}_6(t, E_v^{**}(t))\| \leq \mathcal{K}_6\|E_v^*(t) - E_v^{**}(t)\|, \\ &\|\mathscr{F}_7(t, I_v^*(t)) - \mathscr{F}_7(t, I_v^{**}(t))\| \leq \mathcal{K}_7\|I_v^*(t) - I_v^{**}(t)\|, \end{aligned}$$

also hold. Hence, the Lipschitz condition is satisfied by the each kernel with $\mathcal{K}_i$ $(i = 1, 2, \cdots, 7)$ as Lipschitz constant. Additionally, contraction is implied if $0 \leq \mathcal{K}_i < 1$ $(i = 1, 2, \cdots, 7)$. This concludes the proof. □

5.1 | Existence and uniqueness of solution for model in Caputo derivative

In view of the system of integral equations (18), the fractional Zika virus model (15) with Caputo derivative suggests the following recursive formulations:

$$\begin{cases} S_{h,n}(t) = \frac{1}{\Gamma(\sigma)}\int_0^t (t-\tau)^{\sigma-1}\mathscr{F}_1(\tau, S_{h,n-1}(\tau)), \\ E_{h,n}(t) = \frac{1}{\Gamma(\sigma)}\int_0^t (t-\tau)^{\sigma-1}\mathscr{F}_2(\tau, E_{h,n-1}(\tau)), \\ I_{h,n}(t) = \frac{1}{\Gamma(\sigma)}\int_0^t (t-\tau)^{\sigma-1}\mathscr{F}_3(\tau, I_{h,n-1}(\tau)), \\ R_{h,n}(t) = \frac{1}{\Gamma(\sigma)}\int_0^t (t-\tau)^{\sigma-1}\mathscr{F}_4(\tau, R_{h,n-1}(\tau)), \\ S_{v,n}(t) = \frac{1}{\Gamma(\sigma)}\int_0^t (t-\tau)^{\sigma-1}\mathscr{F}_5(\tau, S_{v,n-1}(\tau)), \\ E_{v,n}(t) = \frac{1}{\Gamma(\sigma)}\int_0^t (t-\tau)^{\sigma-1}\mathscr{F}_6(\tau, E_{v,n-1}(\tau)), \\ I_{v,n}(t) = \frac{1}{\Gamma(\sigma)}\int_0^t (t-\tau)^{\sigma-1}\mathscr{F}_7(\tau, I_{v,n-1}(\tau)), \end{cases} \tag{19}$$

with the initial conditions $S_{h,0}(t) = S_h(0), E_{h,0}(t) = E_h(0), I_{h,0}(t) = I_h(0), R_{h,0}(t) = R_h(0), S_{v,0}(t) = S_v(0), E_{v,0}(t) = E_v(0), I_{v,0}(t) = I_v(0)$.

Define

$$\begin{cases} \Upsilon_{1,n}(t) := S_{h,n}(t) - S_{h,n-1}(t) = \frac{1}{\Gamma(\sigma)}\int_0^t (t-\tau)^{\sigma-1}\left[\mathscr{F}_1(\tau, S_{h,n-1}(\tau)) - \mathscr{F}_1(\tau, S_{h,n-2}(\tau))\right] d\tau, \\ \Upsilon_{2,n}(t) := E_{h,n}(t) - E_{h,n-1}(t) = \frac{1}{\Gamma(\sigma)}\int_0^t (t-\tau)^{\sigma-1}\left[\mathscr{F}_2(\tau, E_{h,n-1}(\tau)) - \mathscr{F}_2(\tau, E_{h,n-2}(\tau))\right] d\tau, \\ \Upsilon_{3,n}(t) := I_{h,n}(t) - I_{h,n-1}(t) = \frac{1}{\Gamma(\sigma)}\int_0^t (t-\tau)^{\sigma-1}\left[\mathscr{F}_3(\tau, I_{h,n-1}(\tau)) - \mathscr{F}_3(\tau, I_{h,n-2}(\tau))\right] d\tau, \\ \Upsilon_{4,n}(t) := R_{h,n}(t) - R_{h,n-1}(t) = \frac{1}{\Gamma(\sigma)}\int_0^t (t-\tau)^{\sigma-1}\left[\mathscr{F}_4(\tau, R_{h,n-1}(\tau)) - \mathscr{F}_4(\tau, R_{h,n-2}(\tau))\right] d\tau, \\ \Upsilon_{5,n}(t) := S_{v,n}(t) - S_{v,n-1}(t) = \frac{1}{\Gamma(\sigma)}\int_0^t (t-\tau)^{\sigma-1}\left[\mathscr{F}_5(\tau, S_{v,n-1}(\tau)) - \mathscr{F}_5(\tau, S_{v,n-2}(\tau))\right] d\tau, \\ \Upsilon_{6,n}(t) := E_{v,n}(t) - E_{v,n-1}(t) = \frac{1}{\Gamma(\sigma)}\int_0^t (t-\tau)^{\sigma-1}\left[\mathscr{F}_6(\tau, E_{v,n-1}(\tau)) - \mathscr{F}_6(\tau, E_{v,n-2}(\tau))\right] d\tau, \\ \Upsilon_{7,n}(t) := I_{v,n}(t) - I_{v,n-1}(t) = \frac{1}{\Gamma(\sigma)}\int_0^t (t-\tau)^{\sigma-1}\left[\mathscr{F}_7(\tau, I_{v,n-1}(\tau)) - \mathscr{F}_7(\tau, I_{v,n-2}(\tau))\right] d\tau, \end{cases} \tag{20}$$

as the difference between successive terms of each equation in (19). Then, it is easy to see that

$$\begin{cases} S_{h,n}(t) = \sum_{k=1}^{n} \Upsilon_{1(k)}(t), \\ E_{h,n}(t) = \sum_{k=1}^{n} \Upsilon_{2(k)}(t), \\ I_{h,n}(t) = \sum_{k=1}^{n} \Upsilon_{3(k)}(t), \\ R_{h,n}(t) = \sum_{k=1}^{n} \Upsilon_{4(k)}(t), \\ S_{v,n}(t) = \sum_{k=1}^{n} \Upsilon_{5(k)}(t), \\ E_{v,n}(t) = \sum_{k=1}^{n} \Upsilon_{6(k)}(t), \\ I_{v,n}(t) = \sum_{k=1}^{n} \Upsilon_{7(k)}(t). \end{cases} \tag{21}$$

Taking norm on both sides of $(20)_1$ and using the triangle inequality and the fact that the Lipschitz condition holds for $\mathscr{F}_1$ with Lipschitz constant $\mathcal{K}_1 > 0$, we have

$$\|\Upsilon_{1,n}(t)\| \leq \frac{1}{\Gamma(\sigma)} \int_0^t (t-\tau)^{\sigma-1} \left\| \mathscr{F}_1(\tau, S_{h,n-1}(\tau)) - \mathscr{F}_1(\tau, S_{h,n-2}(\tau)) \right\| d\tau \leq \frac{\mathcal{K}_1}{\Gamma(\sigma)} \int_0^t (t-\tau)^{\sigma-1} \|\Upsilon_{1,n-1}(\tau)\| d\tau. \tag{22}$$

Similarly, for the rest equations in (20), one gets

$$\begin{cases} \|\Upsilon_{2,n}(t)\| \leq \frac{\mathcal{K}_2}{\Gamma(\sigma)} \int_0^t (t-\tau)^{\sigma-1} \Upsilon_{2(n-1)}(t)\| d\tau, \\ \|\Upsilon_{3,n}(t)\| \leq \frac{\mathcal{K}_3}{\Gamma(\sigma)} \int_0^t (t-\tau)^{\sigma-1} \|\Upsilon_{3(n-1)}(t)\| d\tau, \\ \|\Upsilon_{4,n}(t)\| \leq \frac{\mathcal{K}_4}{\Gamma(\sigma)} \int_0^t (t-\tau)^{\sigma-1} \|\Upsilon_{4(n-1)}(t)\| d\tau, \\ \|\Upsilon_{5,n}(t)\| \leq \frac{\mathcal{K}_5}{\Gamma(\sigma)} \int_0^t (t-\tau)^{\sigma-1} \|\Upsilon_{5(n-1)}(t)\| d\tau, \\ \|\Upsilon_{6,n}(t)\| \leq \frac{\mathcal{K}_6}{\Gamma(\sigma)} \int_0^t (t-\tau)^{\sigma-1} \|\Upsilon_{6(n-1)}(t)\| d\tau, \\ \|\Upsilon_{7,n}(t)\| \leq \frac{\mathcal{K}_7}{\Gamma(\sigma)} \int_0^t (t-\tau)^{\sigma-1} \|\Upsilon_{7(n-1)}(t)\| d\tau. \end{cases} \tag{23}$$

Consequently, we prove the following result.

Theorem 2. Let the assertions in Theorem 1 be satisfied. Then the fractional Zika virus model (15) in Caputo derivative admits a unique solution provided that the inequality

$$\left(1 - \frac{t^\sigma}{\Gamma(\sigma)} \mathcal{K}_i\right) > 0 \quad i = 1, 2, \cdots, 7, \tag{24}$$

holds.

Proof. We know that the functions $S_h(t)$, $E_h(t)$, $I_h(t)$, $R_h(t)$, $S_v(t)$, $E_v(t)$ and $I_v(t)$ are bounded and we have already established that their respective kernels satisfy the Lipschitz condition. By considering (22)-(23), the recursive method yield the following inequalities

$$\begin{cases} \|\Upsilon_{1,n}(t)\| \leq \|S_{h,0}(t)\| \left[\dfrac{t^\sigma}{\Gamma(\sigma)}\mathcal{K}_1\right]^n, \\ \|\Upsilon_{2,n}(t)\| \leq \|E_{h,0}(t)\| \left[\dfrac{t^\sigma}{\Gamma(\sigma)}\mathcal{K}_2\right]^n, \\ \|\Upsilon_{3,n}(t)\| \leq \|I_{h,0}(t)\| \left[\dfrac{t^\sigma}{\Gamma(\sigma)}\mathcal{K}_3\right]^n, \\ \|\Upsilon_{4,n}(t)\| \leq \|R_{h,0}(t)\} \left[\dfrac{t^\sigma}{\Gamma(\sigma)}\mathcal{K}_4\right]^n, \\ \|\Upsilon_{5,n}(t)\| \leq \|S_{v,0}(t)\| \left[\dfrac{t^\sigma}{\Gamma(\sigma)}\mathcal{K}_5\right]^n, \\ \|\Upsilon_{6,n}(t)\| \leq \|E_{v,0}(t)\| \left[\dfrac{t^\sigma}{\Gamma(\sigma)}\mathcal{K}_6\right]^n, \\ \|\Upsilon_{7,n}(t)\| \leq \|I_{v,0}(t)\| \left[\dfrac{t^\sigma}{\Gamma(\sigma)}\mathcal{K}_7\right]^n. \end{cases} \tag{25}$$

which proves the existence and smoothness of the functions in (21). To establish that these functions are indeed a system of solutions to (15), we assume that

$$\begin{cases} S_h(t) - S_{h,0} = S_{h,n}(t) - H_{1,n}(t), \\ E_h(t) - E_{h,0} = E_{h,n}(t) - \Theta_{2,n}(t), \\ I_h(t) - I_{h,0} = I_{h,n}(t) - \Theta_{3,n}(t), \\ R_h(t) - R_{h,0} = R_{h,n}(t) - \Theta_{4,n}(t), \\ S_v(t) - S_{v,0} = S_{v,n}(t) - \Theta_{5,n}(t), \\ E_v(t) - E_{v,0} = E_{v,n}(t) - \Theta_{6,n}(t), \\ I_v(t) - I_{v,0} = I_{v,n}(t) - \Theta_{7,n}(t), \end{cases}$$

where $\Theta_{k,n}(x,t)$ $(k = 1, \cdots, 7)$ are the remainder terms of the series solution. The aim is to show that $\|H_{k(n)}(t)\| \to 0$ as $n \to \infty$, $n = 1, 2, \cdots, 7$. We have

$$\|H_{1,n}(t)\| \leq \frac{1}{\Gamma(\sigma)} \int_0^t (t-\tau)^{\sigma-1} \|\mathscr{F}_1(\tau, S_h(\tau)) - \mathscr{F}_1(\tau, S_{h,n-1}(\tau))\| d\tau \leq \frac{t^\sigma}{\Gamma(\sigma)} \mathcal{K}_1 \|S_h(t) - S_{h,n-1}(t)\|,$$

and a recursive repetition of the same procedure gives

$$\|H_{1,n}(t)\| \leq \|S_{h,0}\| \left[\frac{t^\sigma}{\Gamma(\sigma)}\right]^{n+1} \mathcal{K}_1^{n+1} M.$$

Applying limit, we have $\|H_{1,n}(t)\| \to 0$ as $n \to \infty$. Similarly, we have $\|\Theta_{2,n}(t)\| \to 0$, $\|\Theta_{3,n}(t)\| \to 0$, $\|\Theta_{4,n}(t)\| \to 0$, $\|\Theta_{5,n}(t)\| \to 0$, $\|\Theta_{6,n}(t)\| \to 0$, and $\|\Theta_{7,n}(t)\| \to 0$ as $n \to \infty$. Hence, the existence of solution is proved.

Next, to establish uniqueness of solutions to the model (15), we assume the existence another set of solutions $S_h^*(t)$, $E_h^*(t)$, $I_h^*(t)$, $R_h^*(t)$, $S_v^*(t)$, $E_v^*(t)$, $I_v^*(t)$. Then By using the Lipschitz condition property satisfied by the kernel $\mathscr{F}_1$, we have

$$\|S_h(t) - S_h^*(t)\| \leq \frac{t^\sigma}{\Gamma(\sigma)} \mathcal{K}_1 \|S_h(\tau)) - S_h^*(\tau))\| \quad \Rightarrow \quad \|S_h(t) - S_h^*(t)\| \left(1 - \frac{t^\sigma}{\Gamma(\sigma)} \mathcal{K}_1\right) \leq 0. \tag{26}$$

With respect to (24), we have that (26) implies $\|S_h(t) - S_h^*(t)\| = 0$. Hence $S_h(t) = S_h^*(t)$. A similar argument also yield $E_h(t) = E_h^*(t)$, $I_h(t) = I_h^*(t)$, $R_h(t) = R_h^*(t)$, $S_v(t) = S_v^*(t)$, $E_v(t) = E_v^*(t)$, $I_v(t) = I_v^*(t)$. and the uniqueness of the system of solutions is thus established. This proves the theorem. □

5.2 | Existence and uniqueness of solutions for model in CFC derivative

Now, consider the fractional Zika virus model (15)with CFC derivative, i.e., ${}_0^{\Xi(\sigma)}D_t^{\sigma} = {}_0^{CFC}D_t^{\sigma}$. Then the system of integral equations (18) suggest the following system of recursive formula:

$$\begin{cases} S_{h,n}(t) = \frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathscr{F}_1(t, S_{h,n-1}(t)) + \frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\int_0^t \mathscr{F}_1(\tau, S_{h,n-1}(\tau)), \\ E_{h,n}(t) = \frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathscr{F}_2(t, E_{h,n-1}(t)) + \frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\int_0^t \mathscr{F}_2(\tau, E_{h,n-1}(\tau)), \\ I_{h,n}(t) = \frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathscr{F}_3(t, I_{h,n-1}(t)) + \frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\int_0^t \mathscr{F}_3(\tau, I_{h,n-1}(\tau)), \\ R_{h,n}(t) = \frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathscr{F}_4(t, R_{h,n-1}(t)) + \frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\int_0^t \mathscr{F}_4(\tau, R_{h,n-1}(\tau)), \\ S_{v,n}(t) = \frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathscr{F}_5(t, S_{v,n-1}(t)) + \frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\int_0^t \mathscr{F}_5(\tau, S_{v,n-1}(\tau)), \\ E_{v,n}(t) = \frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathscr{F}_6(t, E_{v,n-1}(t)) + \frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\int_0^t \mathscr{F}_6(\tau, E_{v,n-1}(\tau)), \\ I_{v,n}(t) = \frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathscr{F}_7(t, I_{v,n-1}(t)) + \frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\int_0^t \mathscr{F}_7(\tau, I_{v,n-1}(\tau)). \end{cases} \tag{27}$$

Using the same notations in (20), we obtain a corresponding system of equations for the difference between successive terms of each equation in (27). Furthermore, in view of the fact that the Lipschitz condition hold for the kernels $\mathscr{F}_i$ $(i = 1, 2, \cdots, 7)$, we apply triangular inequality to the obtained system of differences between successive terms as was done in (22) to get

$$\begin{cases} \|\Upsilon_{1,n}(t)\| \leq \frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_2\|\Upsilon_{2(n-1)}(t)\| + \frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_2\int_0^t \|\Upsilon_{2(n-1)}(t)\|d\tau, \\ \|\Upsilon_{3,n}(t)\| \leq \frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_3\|\Upsilon_{3(n-1)}(t)\| + \frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_3\int_0^t \|\Upsilon_{3(n-1)}(t)\|d\tau, \\ \|\Upsilon_{4,n}(t)\| \leq \frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_4\|\Upsilon_{4(n-1)}(t)\| + \frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_4\int_0^t \|\Upsilon_{4(n-1)}(t)\|d\tau, \\ \|\Upsilon_{5,n}(t)\| \leq \frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_5\|\Upsilon_{5(n-1)}(t)\| + \frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_5\int_0^t \|\Upsilon_{5(n-1)}(t)\|d\tau, \\ \|\Upsilon_{6,n}(t)\| \leq \frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_6\|\Upsilon_{6(n-1)}(t)\| + \frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_6\int_0^t \|\Upsilon_{6(n-1)}(t)\|d\tau, \\ \|\Upsilon_{7,n}(t)\| \leq \frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_7\|\Upsilon_{7(n-1)}(t)\| + \frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_7\int_0^t \|\Upsilon_{7(n-1)}(t)\|d\tau, \end{cases} \tag{28}$$

with the relations in (21) equally holding. Consequently, the following result is immediate:

Theorem 3. The Zika virus model (15) with fractional derivative in the CFC scene admits a system of solutions. Moreover this system of solutions is unique if

$$\left[1-\frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_i-\frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_i t\right]\geq 0 \quad i=1,2,\cdots,7. \tag{29}$$

Proof. Recall that the functions $S_h(t)$, $E_h(t)$, $I_h(t)$, $R_h(t)$, $S_v(t)$, $E_v(t)$ and $I_v(t)$ are bounded and we have earlier established that their kernels satisfy the Lipschitz condition. By considering (28), the recursive method yield the following inequalities

$$\begin{cases}
\|\Upsilon_{1,n}(t)\|\leq\|S_h(0)\|+\left[\frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_1+\frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_1\right]^n,\\
\|\Upsilon_{2,n}(t)\|\leq\|E_h(0)\|+\left[\frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_2+\frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_2\right]^n,\\
\|\Upsilon_{3,n}(t)\|\leq\|I_h(0)\|+\left[\frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_3+\frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_3\right]^n,\\
\|\Upsilon_{4,n}(t)\|\leq\|R_h(0)\}+\left[\frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_4+\frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_4\right]^n,\\
\|\Upsilon_{5,n}(t)\|\leq\|S_v(0)\|+\left[\frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_5+\frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_5\right]^n,\\
\|\Upsilon_{6,n}(t)\|\leq\|E_v(0)\|+\left[\frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_6+\frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_6\right]^n,\\
\|\Upsilon_{7,n}(t)\|\leq\|I_v(0)\|+\left[\frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_7+\frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_7\right]^n.
\end{cases} \tag{30}$$

This proves the existence and smoothness of the functions in (21) with respect to the fractional model with CFC derivative. To establish that these functions are indeed a system of solutions to the fractional model (15), we first assume that

$$\begin{cases}
S_h(t)-S_h(0)=S_{h,n}(t)-H_{1,n}(t),\\
E_h(t)-E_h(0)=E_{h,n}(t)-\Theta_{2,n}(t),\\
I_h(t)-I_h(0)=I_{h,n}(t)-\Theta_{3,n}(t),\\
R_h(t)-R_h(0)=R_{h,n}(t)-\Theta_{4,n}(t),\\
S_v(t)-S_v(0)=S_{v,n}(t)-\Theta_{5,n}(t),\\
E_v(t)-E_v(0)=E_{v,n}(t)-\Theta_{6,n}(t),\\
I_v(t)-I_v(0)=I_{v,n}(t)-\Theta_{7,n}(t),
\end{cases} \tag{31}$$

where $\Theta_{k,n}(x,t)$ $(k=1,\cdots,7)$ are the remainder terms of the series solution. Then for (31) we have

$$\begin{aligned}
\|\Theta_{1,n}(t)\|&=\left\|\frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\left[\mathscr{F}_1(t,S_h(t))-\mathscr{F}_1(t,S_{h,n-1}(t))\right]+\frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\int_0^t\left[\mathscr{F}_1(\tau,S_h(\tau))-\mathscr{F}_1(\tau,S_{h,n-1}(\tau))\right]d\tau\right\|\\
&\leq\frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\|\mathscr{F}_1(t,S_h(t))-\mathscr{F}_1(t,S_{h,n-1}(t))\|+\frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\int_0^t\|\mathscr{F}_1(\tau,S_h(\tau))-\mathscr{F}_1(\tau,S_{h,n-1}(\tau))\|d\tau\\
&\leq\frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_1\|S_h(t)-S_{h,n-1}(t)\|+\frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_1\|S_h(t)-S_{h,n-1}(t)\|t.
\end{aligned}$$

A recursive repetition of the same procedure gives

$$\|\Theta_{1,n}(t)\|\leq\left[\frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}+\frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}t\right]^{n+1}\mathcal{K}_1^{n+1}a,$$

and at $t=t_0$ we have

$$\|\Theta_{1,n}(t)\|\leq\left[\frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}+\frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}t_0\right]^{n+1}\mathcal{K}_1^{n+1}a.$$

Applying limit, we have $\|\Theta_{1,n}(t)\| \to 0$ as $n \to \infty$. Similarly, we have $\|\Theta_{2,n}(t)\| \to 0$, $\|\Theta_{3,n}(t)\| \to 0$, $\|\Theta_{4,n}(t)\| \to 0$, $\|\Theta_{5,n}(t)\| \to 0$, $\|\Theta_{6,n}(t)\| \to 0$, and $\|\Theta_{7,n}(t)\| \to 0$ as $n \to \infty$. Hence, the existence of solution is proved.

Next, to establish uniqueness of solution, we assume the existence of another system of solutions, namely, $(S_h^*(t),\ E_h^*(t),\ I_h^*(t),\ R_h^*(t),\ S_v^*(t),\ E_v^*(t),\ I_v^*(t))$. Then

$$S_h(t) - S_h^*(t) = \frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\left[\mathscr{F}_1(t, S_h(t)) - \mathscr{F}_1(t, S_h^*(t))\right] + \frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\int_0^t \left[\mathscr{F}_1(t, S_h(\tau)) - \mathscr{F}_1(\tau, S_h^*(\tau))\right] d\tau. \tag{32}$$

Applying norm in (32) together with the fact that $\mathscr{F}_1$ satisfies the Lipschitz condition gives

$$\|S_h(t) - S_h^*(t)\| \leq \frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_1\|S_h(t)) - S_h^*(t))\| + \frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_1 t\|S_h(\tau)) - S_h^*(\tau))\|,$$

that is,

$$\|S_h(t) - S_h^*(t)\|\left(1 - \frac{2(1-\sigma)}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_1 - \frac{2\sigma}{(2-\sigma)\mathbb{M}[\sigma]}\mathcal{K}_1 t\right) \leq 0. \tag{33}$$

Hence, under the condition (29), we have from (33) that $\|S_h(t) - S_h^*(t)\| = 0 \implies S_h(t) = S_h^*(t)$. A similar argument also yield $E_h(t) = E_h^*(t)$, $I_h(t) = I_h^*(t)$, $R_h(t) = R_h^*(t)$, $S_v(t) = S_v^*(t)$, $E_v(t) = E_v^*(t)$, $I_v(t) = I_v^*(t)$. and the uniqueness of the system of solutions is thus established. This proves the theorem. □

5.3 | Existence and uniqueness of solutions for model in ABC derivative

Our task here is to investigate the existence of a unique solution to the fractional Zika virus model (15) with ABC derivative via a fixed point theory. To this end, we rewrite the fractional model equations (15)in the form

$$\begin{cases} {}_0^{ABC}D_t^{\sigma}\mathbf{X}(t) = \mathcal{G}(t, \mathbf{X}(t)), \ \ 0 < t < T < \infty, \\ \mathbf{X}(0) = \mathbf{X}_0, \end{cases} \tag{34}$$

where $\mathbf{X}(t) = (S_h, E_h, I_h, R_h, S_v, E_v, I_v)^{\top}$ denotes the vector consisting of the state variables, $\mathcal{G}$ is a real-valued continuous vector function defined as

$$\mathcal{G}(t, \mathbf{X}(t)) = \begin{pmatrix} \mathscr{F}_1(t, S_h(t)) \\ \mathscr{F}_2(t, E_h(t)) \\ \mathscr{F}_3(t, I_h(t)) \\ \mathscr{F}_4(t, R_h(t)) \\ \mathscr{F}_5(t, S_v(t)) \\ \mathscr{F}_6(t, E_v(t)) \\ \mathscr{F}_7(t, I_v(t)) \end{pmatrix} = \begin{pmatrix} \Pi_h - \beta_h S_h(I_v + \rho I_h) - \mu_h S_h \\ \beta_h S_h(I_v + \rho I_h) - (\mu_h + \chi)E_h \\ \chi E_h - (\mu_h + \gamma + \tau)I_h \\ \gamma I_h - \mu_h R_h \\ \Pi_v - \beta_v S_v I_h - \mu_v S_v \\ \beta_v S_v I_h - (\mu_v + \delta)E_v \\ \delta E_v - \mu_v I_v \end{pmatrix} \tag{35}$$

and $\mathbf{X}(0) = (S_h(0), E_h(0), I_h(0), R_h(0), S_v(0), E_v(0), I_v(0))^{\top}$ is a vector denoting the initial condition for the state variables. The existence of positive constants $\mathcal{K}_i$, $i = 1, 2, \cdots, 7$, such that each of the functions $\mathscr{F}_i$ in (35) satisfies the Lipschitz condition and contraction has already been shown in Theorem 1. Thus the it follows immediately that there exists a constant $\mathcal{K} = \max\{\mathcal{K}_1, \mathcal{K}_2, \cdots, \mathcal{K}_7\} > 0$ such that the function $\mathcal{G}(t, \mathbf{X}(t))$ in (35) satisfies

$$\|\mathcal{G}(t, \mathbf{X}_1(t)) - \mathcal{G}(t, \mathbf{X}_2(2))\| \leq \mathcal{K}\|\mathbf{X}_1(t) - \mathbf{X}_2(2)\|. \tag{36}$$

Next, we establish the existence and uniqueness of solution to the Zika virus model (15) with fractional derivative i the sense of ABC. To this end, we prove the following result:

Theorem 4. The fractional Zika virus model (15) with ABC derivative considered in the form (34) admits a unique solution under the condition we can find $T_{max} > 0$ such that

$$\frac{1-\sigma}{\mathbb{B}[\sigma]}\mathcal{K} + \frac{\sigma}{\mathbb{B}[\sigma]\Gamma(\sigma)}T_{\max}^{\sigma}\mathcal{K} < 1. \tag{37}$$

Proof. An application of the Atangana-Beleanu fractional integral (11) to both sides of (34) yields the following non-linear Volterra-type integral equation

$$\mathbf{X}(t) = \mathbf{X}(0) + \frac{1-\sigma}{\mathbb{B}[\sigma]}\mathcal{G}(t, \mathbf{X}(t)) + \frac{\sigma}{\mathbb{B}[\sigma]\Gamma(\sigma)}\int_0^t (t-\tau)^{\sigma-1}\mathcal{G}(\tau, \mathbf{X}(\tau))d\tau. \tag{38}$$

Let $\mathbb{J} = (0, T)$ and consider the operator $\Upsilon : C(\mathbb{J}, \mathbb{R}^7) \to C(\mathbb{J}, \mathbb{R}^7)$ defined by

$$\Upsilon[\mathbf{X}(t)] = \mathbf{X}(0) + \frac{1-\sigma}{\mathbb{B}[\sigma]}\mathcal{G}(t, \mathbf{X}(t)) + \frac{\sigma}{\mathbb{B}[\sigma]\Gamma(\sigma)}\int_0^t (t-\tau)^{\sigma-1}\mathcal{G}(\tau, \mathbf{X}(\tau))d\tau. \tag{39}$$

Then (38) can be read as

$$\mathbf{X}(t) = \Upsilon[\mathbf{X}(t)]. \tag{40}$$

With respect to the supremum norm $\|\mathbf{X}(t)\|_{\mathbb{J}} := \sup_{t\in\mathbb{J}} \|\mathbf{X}(t)\|$ on $\mathbb{J}$, $C(\mathbb{J}, \mathbb{R}^7)$ forms a Banach space. Moreover the following inequality holds

$$\left\| \int_0^t \mathcal{H}(t, \tau)\mathbf{X}(\tau)d\tau \right\|_{\mathbb{J}} \leq T\|\mathcal{H}(t, \tau)\|_{\mathbb{J}}\|\mathbf{X}(\tau)\|_{\mathbb{J}} \tag{41}$$

where $\mathcal{H}(t, \tau) \in C(\mathbb{J}^2, \mathbb{R})$ such that $\|\mathcal{H}(t, \tau)\|_{\mathbb{J}} = \sup_{t,\tau\in\mathbb{J}} |\mathcal{H}(t, \tau)|$ and $\mathbf{X}(t) \in C(\mathbb{J}, \mathbb{R}^7)$. In view of (36), (40), (41) and the triangle inequality, we have

$$\|\Upsilon[\mathbf{X}_1(t)] - \Upsilon[\mathbf{X}_2(t)]\|_{\mathbb{J}} \leq \left(\frac{1-\sigma}{\mathbb{B}[\sigma]}\mathcal{K} + \frac{\sigma}{\mathbb{B}[\sigma]\Gamma(\sigma)}\mathcal{K}T^{\sigma} \right) \|\mathbf{X}_1(t) - \mathbf{X}_2(t)\|_{\mathbb{J}}.$$

Equivalently, the above last inequality reads

$$\|\Upsilon[\mathbf{X}_1(t)] - \Upsilon[\mathbf{X}_2(t)]\|_{\mathbb{J}} \leq \kappa\|\mathbf{X}_1(t) - \mathbf{X}_2(t)\|_{\mathbb{J}},$$

where

$$\kappa = \left(\frac{1-\sigma}{\mathbb{B}[\sigma]}\mathcal{K} + \frac{\sigma}{\mathbb{B}[\sigma]\Gamma(\sigma)}\mathcal{K}T^{\sigma} \right).$$

With respect to the condition given by (37), the operator Υ will be a contraction on $C(\mathbb{J}, \mathbb{R}^7)$. Thus, the Banach fixed point theorem asserts that the fractional Zika virus model (15) in ABC derivative admits a unique solution. □

6 | NUMERICAL SCHEMES AND SIMULATIONS

Motivated by the two-step fractional Adams-Bashforth (FAB) numerical scheme developed by Atangana and Owolabi,[36] we present corresponding FAB schemes for the fractional ZIKV model (15) with fractional derivative in the Caputo, CFC and ABC sense, respectively. We refer the reader to the work[36] where the convergence and stability analysis for aforementioned the scheme for each of the above mentioned types of fractional derivatives is furnished in detail . Furthermore, we furnish graphical visualizations for the behaviour of the numerical solutions to the proposed fractional ZIKV model (15) for distinct values of the fractional order parameter σ. The simulation parameter values are taken as $\Pi_h = 0.8, \beta_h = 0.007, \rho = 0.05, \mu_h = 0.0028, \chi = 0.7, \gamma = 0.05, \tau = 0.08, \beta_v = 0.009, \Pi_v = 0.08, \mu_v = 0.071, \delta = 0.5$ while the initial values used are $S_{h0} = 100, E_{h0} = 10, I_{h0} = 30, R_{h0} = 20, S_{v0} = 10, E_{v0} = 50, I_{v0} = 10$.

6.1 | CASE I: Model in Caputo derivative

Using the fundamental theorem of integral calculus, we obtain the following corresponding nonlinear fractional Volterrra-type integral equation

$$S_h(t) - S_h(0) = \frac{1}{\Gamma(\sigma)}\int_0^t (t-\tau)^{\sigma-1}\mathcal{F}_1(\tau, S_h(\tau))d\tau. \tag{42}$$

for the S_h-equation (15) in Caputo derivative. At $t = t_{k+1}$ and $t = t_k$, $k = 0, 1, 2, \cdots$, (42) can be read as

$$S_h(t_{k+1}) - S_h(0) = \frac{\sigma}{\Gamma(\sigma)} \int_0^{t_{k+1}} (t_{k+1} - t)^{\sigma-1} \mathscr{F}_1(t, S_h(t))dt \qquad \text{and} \qquad S_h(t_k) - S_h(0) = \frac{\sigma}{\Gamma(\sigma)} \int_0^{t_k} (t_k - t)^{\sigma-1} \mathscr{F}_1(t, S_h(t))dt,$$

respectively. We easily see that,

$$S_h(t_{k+1}) - S_h(t_k) = \mathbf{X}_{\sigma,1} - \mathbf{X}_{\sigma,2} \tag{43}$$

where

$$\mathbf{X}_{\sigma,1} = \frac{1}{\Gamma(\sigma)} \int_0^{t_{k+1}} (t_{k+1} - t)^{\sigma-1} \mathscr{F}_1(t, S_h(t))dt \qquad \text{and} \qquad \mathbf{X}_{\sigma,2} = \frac{1}{\Gamma(\sigma)} \int_0^{t_k} (t_k - t)^{\sigma-1} \mathscr{F}_1(t, S_h(t))dt. \tag{44}$$

Over the interval $[t_k, t_{k+1}]$, the function $\mathscr{F}_1(t, S_h(t))$ can be approximated by the two-point Lagrange interpolation polynomial of the form

$$\begin{aligned} \mathscr{F}_1(t, S_h(t)) &\simeq \frac{t - t_{k-1}}{t_k - t_{k-1}} \mathscr{F}_1(t_k, S_h(t_k)) + \frac{t - t_k}{t_{k-1} - t_k} \mathscr{F}_1(t_{k-1}, S_h(t_{k-1})) \\ &= \frac{t - t_{k-1}}{h} \mathscr{F}_1(t_k, S_h(t_k)) - \frac{t - t_k}{h} \mathscr{F}_1(t_{k-1}, S_h(t_{k-1})), \end{aligned} \tag{45}$$

where $h = t_k - t_{k-1}$ is the step-size. Substituting (45) into the first and second integrals in (44) yield

$$\mathbf{X}_{\sigma,1} = \frac{\mathscr{F}_1(t_k, S_h(t_k))}{h\Gamma(\sigma)} \left[\frac{2ht_{k+1}^{\sigma}}{\sigma} - \frac{t_{k+1}^{\sigma+1}}{\sigma+1} \right] - \frac{\mathscr{F}_1(t_{k-1}, S_h(t_{k-1}))}{h\Gamma(\sigma)} \left[\frac{ht_{k+1}^{\sigma}}{\sigma} - \frac{t_{k+1}^{\sigma+1}}{\sigma+1} \right] \tag{46}$$

and

$$\mathbf{X}_{\sigma,2} = \frac{\mathscr{F}_1(t_k, S_h(t_k))}{h\Gamma(\sigma)} \left[\frac{ht_k^{\sigma}}{\sigma} - \frac{t_k^{\sigma+1}}{\sigma+1} \right] + \frac{\mathscr{F}_1(t_k, S_h(t_{k-1}))}{h\Gamma(\sigma)} \frac{t_k^{\sigma+1}}{\sigma+1}, \tag{47}$$

respectively, after some manipulations. By inserting (46) and (47) into (43), we obtain

$$\begin{aligned} S_h(t_{k+1}) = S_h(t_k) &+ \frac{\mathscr{F}_1(t_k, S_h(t_k))}{h\Gamma(\sigma)} \left[\frac{2ht_{k+1}^{\sigma}}{\sigma} - \frac{t_{k+1}^{\sigma+1}}{\sigma+1} - \frac{ht_k^{\sigma}}{\sigma} + \frac{t_k^{\sigma+1}}{\sigma+1} \right] \\ &+ \frac{\mathscr{F}_1(t_{k-1}, S_h(t_{k-1}))}{h\Gamma(\sigma)} \left[-\frac{ht_{k+1}^{\sigma}}{\sigma} + \frac{t_{k+1}^{\sigma+1}}{\sigma+1} - \frac{t_k^{\sigma+1}}{\sigma+1} \right] \end{aligned} \tag{48}$$

as the final two-step FAB scheme for the S_h-equation (15) with Caputo derivative. In the same way, we can obtain a similar scheme for each of the remaining equations in (15). In general, by setting $\mathbf{X}(t) = (S_h, E_h, I_h, R_h, S_v, E_v, I_v)^{\top}$ and $\mathcal{G}(t, \mathbf{X}(t))$ as defined in (35), the two-step FAB scheme for the fractional ZIKV model (15) with Caputo derivative is given as

$$\mathbf{X}(t_{k+1}) = \mathbf{X}(t_k) + \frac{\mathcal{G}(t_k, \mathbf{X}(t_k))}{h\Gamma(\sigma)} \left[\frac{2ht_{k+1}^{\sigma}}{\sigma} - \frac{t_{k+1}^{\sigma+1}}{\sigma+1} - \frac{ht_k^{\sigma}}{\sigma} + \frac{t_k^{\sigma+1}}{\sigma+1} \right] + \frac{\mathcal{G}(t_{k-1}, \mathbf{X}(t_{k-1}))}{h\Gamma(\sigma)} \left[-\frac{ht_{k+1}^{\sigma}}{\sigma} + \frac{t_{k+1}^{\sigma+1}}{\sigma+1} - \frac{t_k^{\sigma+1}}{\sigma+1} \right]. \tag{49}$$

6.2 | CASE II: Model in CFC derivative

Applying the fundamental theorem of integration on the S_h-equation (15) in CFC derivative we obtain the following corresponding fractional Volterrra-type integral equation

$$S_h(t) - S_h(0) = \frac{1 - \sigma}{\mathbb{M}[\sigma]} \mathscr{F}_1(t, S_h(t)) + \frac{\sigma}{\mathbb{M}[\sigma]} \int_0^t \mathscr{F}_1(\tau, S_h(\tau))d\tau. \tag{50}$$

At $t = t_k$ and $t = t_{k+1}$, $n = 0, 1, 2, \cdots$, we have

$$S_h(t_k) - S_h(0) = \frac{1 - \sigma}{\mathbb{M}[\sigma]} \mathscr{F}_1(t_{k-1}, S_h(t_{k-1})) + \frac{\sigma}{\mathbb{M}[\sigma]} \int_0^{t_k} \mathscr{F}_1(t, S_h(t))dt,$$

and

$$S_h(t_{k+1}) - S_h(0) = \frac{1-\sigma}{\mathbb{M}[\sigma]}\mathcal{F}_1(t_k, S_h(t_k)) + \frac{\sigma}{\mathbb{M}[\sigma]}\int_0^{t_{k+1}} \mathcal{F}_1(t, S_h(t))dt,$$

respectively. Moreover,

$$S_h(t_{k+1}) - S_h(t_k) = \frac{1-\sigma}{\mathbb{M}[\sigma]}\Big[\mathcal{F}_1(t_k, S_h(t_k)) - \mathcal{F}_1(t_{k-1}, S_h(t_{k-1}))\Big] + \frac{\sigma}{\mathbb{M}[\sigma]}\int_{t_k}^{t_{k+1}} \mathcal{F}_1(t, S_h(t))dt \tag{51}$$

Over the interval $[t_k, t_{k+1}]$, the function $\mathcal{F}_1(t, S_h(t))$ can be approximated by the Lagrange polynomial (45) where $h = t_k - t_{k-1}$. Substituting (45) in the integral on the right hand side of (51) yields

$$\begin{aligned}\int_{t_k}^{t_{k+1}} \mathcal{F}_1(t, S_h(t))dt &= \int_{t_k}^{t_{k+1}} \left[\frac{t - t_{k-1}}{h}\mathcal{F}_1(t_k, S_h(t_k)) - \frac{t - t_k}{h}\mathcal{F}_1(t_{k-1}, S_h(t_{k-1}))\right] dt \\ &= \frac{3h}{2}\mathcal{F}_1(t_k, S_h(t_k)) - \frac{h}{2}\mathcal{F}_1(t_{k-1}, S_h(t_{k-1})).\end{aligned} \tag{52}$$

By inserting (52) into (51), we obtain

$$S_h(t_{k+1}) = S_h(t_k) + \left(\frac{1-\sigma}{\mathbb{M}[\sigma]} + \frac{3h}{2\mathbb{M}[\sigma]}\right)\mathcal{F}_1(t_k, S_h(t_k)) - \left(\frac{1-\sigma}{\mathbb{M}[\sigma]} + \frac{\sigma h}{2\mathbb{M}[\sigma]}\right)\mathcal{F}_1(t_{k-1}, S_h(t_{k-1})) \tag{53}$$

as the final two-step FAB scheme for the S_h-equation (15) with CFC derivative. In the same way, we can obtain similar schemes for the remaining equations in (15). In general, by setting $\mathbf{X}(t) = (S_h, E_h, I_h, R_h, S_v, E_v, I_v)^\top$ and $\mathcal{G}$ as defined in (35), the two-step FAB scheme for the fractional ZIKV model (15)with Caputo derivative is given as

$$\mathbf{X}(t_{k+1}) = \mathbf{X}(t_k) + \left(\frac{1-\sigma}{\mathbb{M}[\sigma]} + \frac{3h}{2\mathbb{M}[\sigma]}\right)\mathcal{G}(t_k, \mathbf{X}(t_k)) - \left(\frac{1-\sigma}{\mathbb{M}[\sigma]} + \frac{\sigma h}{2\mathbb{M}[\sigma]}\right)\mathcal{G}(t_{k-1}, \mathbf{X}(t_{k-1})). \tag{54}$$

6.3 | CASE III: Model in ABC derivative

An application of the fundamental theorem of integration, we obtain the following fractional Volterrra-type integral equation corresponding to the S_h-equation of (15) in ABC derivative:

$$S_h(t) - S_h(0) = \frac{1-\sigma}{\mathbb{B}[\sigma]}\mathcal{F}_1(t, S_h(t)) + \frac{\sigma}{\mathbb{B}[\sigma]\Gamma(\sigma)}\int_0^t (t-\tau)^{\sigma-1}\mathcal{F}_1(\tau, S_h(\tau))d\tau. \tag{55}$$

At $t = t_{k+1}$ and $t = t_k$, $k = 0, 1, 2, \cdots$, we have

$$S_h(t_{k+1}) - S_h(0) = \frac{1-\sigma}{\mathbb{B}[\sigma]}\mathcal{F}_1(t_k, S_h(t_k)) + \frac{\sigma}{\mathbb{B}[\sigma]\Gamma(\sigma)}\int_0^{t_{k+1}} (t_{k+1}-t)^{\sigma-1}\mathcal{F}_1(t, S_h(t))dt,$$

and

$$S_h(t_k) - S_h(0) = \frac{1-\sigma}{\mathbb{B}[\sigma]}\mathcal{F}_1(t_{k-1}, S_h(t_{k-1})) + \frac{\sigma}{\mathbb{B}[\sigma]\Gamma(\sigma)}\int_0^{t_k} (t_k-t)^{\sigma-1}\mathcal{F}_1(t, S_h(t))dt,$$

respectively. Moreover,

$$S_h(t_{k+1}) - S_h(t_k) = \frac{1-\sigma}{\mathbb{B}[\sigma]}\Big[\mathcal{F}_1(t_k, S_h(t_k)) - \mathcal{F}_1(t_{k-1}, S_h(t_{k-1}))\Big] + \frac{\sigma}{\mathbb{B}[\sigma]\Gamma(\sigma)}(\mathbf{X}_{\sigma,1} - \mathbf{X}_{\sigma,2}) \tag{56}$$

where

$$\mathbf{X}_{\sigma,1} := \int_0^{t_{k+1}} (t_{k+1}-t)^{\sigma-1}\mathcal{F}_1(t, S_h(t))dt, \quad \text{and} \quad \mathbf{X}_{\sigma,2} := \int_0^{t_k} (t_k-t)^{\sigma-1}\mathcal{F}_1(t, S_h(t))dt. \tag{57}$$

Over the interval $[t_k, t_{k+1}]$, the function $\mathscr{F}_1(t, S_h(t))$ can be approximated by the two-point Lagrange interpolation polynomial of the form (45) where $h = t_k - t_{k-1}$. Substituting (45) into the first and second integrals in (57) yield

$$\mathbf{X}_{\sigma,1} = \frac{\mathscr{F}_1(t_k, S_h(t_k))}{h}\left[\frac{2ht_{k+1}^{\sigma}}{\sigma} - \frac{t_{k+1}^{\sigma+1}}{\sigma+1}\right] - \frac{\mathscr{F}_1(t_{k-1}, S_h(t_{k-1}))}{h}\left[\frac{ht_{k+1}^{\sigma}}{\sigma} - \frac{t_{k+1}^{\sigma+1}}{\sigma+1}\right]. \tag{58}$$

and

$$\mathbf{X}_{\sigma,2} = \frac{\mathscr{F}_1(t_k, S_{h,k})}{h}\left[\frac{ht_k^{\sigma}}{\sigma} - \frac{t_k^{\sigma+1}}{\sigma+1}\right] + \frac{\mathscr{F}_1(t_{k-1}, S_{h,k-1})}{h}\frac{t_k^{\sigma+1}}{\sigma+1}, \tag{59}$$

respectively, after some manipulations. Then by inserting (58) and (59) into (56), we obtain

$$\begin{aligned} S_h(t_{k+1}) =& S_h(t_k) + \mathscr{F}_1(t_k, S_h(t_k))\left[\frac{1-\sigma}{\mathbb{B}[\sigma]} + \frac{\sigma}{h\mathbb{B}[\sigma]\Gamma(\sigma)}\left(\frac{2ht_{k+1}^{\sigma}}{\sigma} - \frac{t_{k+1}^{\sigma+1}}{\sigma+1} - \frac{ht_k^{\sigma}}{\sigma} + \frac{t_k^{\sigma+1}}{\sigma+1}\right)\right] \\ &+ \mathscr{F}_1(t_{k-1}, S_h(t_{k-1}))\left[\frac{\sigma-1}{\mathbb{B}[\sigma]} - \frac{\sigma}{h\mathbb{B}[\sigma]\Gamma(\sigma)}\left(\frac{ht_{k+1}^{\sigma}}{\sigma} - \frac{t_{k+1}^{\sigma+1}}{\sigma+1} + \frac{t_k^{\sigma+1}}{\sigma+1}\right)\right] \end{aligned} \tag{60}$$

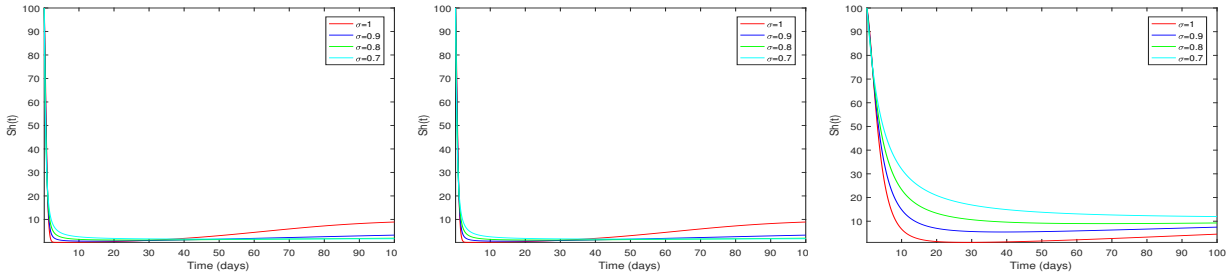
as the final two-step FAB scheme for the S_h-equation (15) with ABC derivative. In the same way, we can obtain a similar scheme for each of the remaining equations in (15). In general, by setting $\mathbf{X}(t) = (S_h, E_h, I_h, R_h, S_v, E_v, I_v)^{\top}$ and $\mathcal{G}$ as defined in (35), the two-step FAB scheme for the fractional ZIKV model (15)with Caputo derivative is given as

$$\begin{aligned} \mathbf{X}(t_{k+1}) =& \mathbf{X}(t_k) + \mathcal{G}(t_k, \mathbf{X}(t_k))\left[\frac{1-\sigma}{\mathbb{B}[\sigma]} + \frac{\sigma}{h\mathbb{B}[\sigma]\Gamma(\sigma)}\left(\frac{2ht_{k+1}^{\sigma}}{\sigma} - \frac{t_{k+1}^{\sigma+1}}{\sigma+1} - \frac{ht_k^{\sigma}}{\sigma} + \frac{t_k^{\sigma+1}}{\sigma+1}\right)\right] \\ &+ \mathcal{G}(t_{k-1}, \mathbf{X}(t_{k-1}))\left[\frac{\sigma-1}{\mathbb{B}[\sigma]} - \frac{\sigma}{h\mathbb{B}[\sigma]\Gamma(\sigma)}\left(\frac{ht_{k+1}^{\sigma}}{\sigma} - \frac{t_{k+1}^{\sigma+1}}{\sigma+1} + \frac{t_k^{\sigma+1}}{\sigma+1}\right)\right]. \end{aligned} \tag{61}$$

6.4 | Discussion

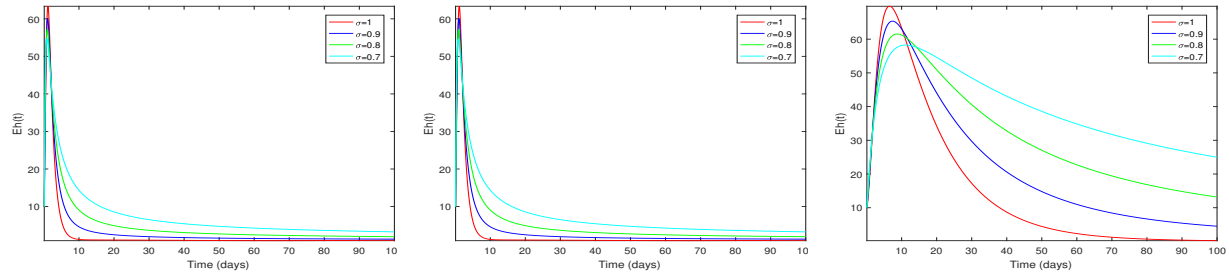
Using the above two-step FAB schemes (49), (54) and (61) we present graphical visualizations to demonstrate the behaviour of the approximate solutions to the fractional ZIKV model (15) with Caputo, CFC and ABC derivatives, respectively, for each system variable. The plots in each of the graphs are with respect to distinct values of the fractional order parameter σ with $\sigma = 1.0; 0.9; 0.8; 0.7$. The time level up to 100 days and the step size used in evaluating the approximate solutions is $h = 0.002$. The graphs for susceptible individuals $S_h(t)$, exposed individuals $E_h(t)$, infected individuals $I_h(t)$, recovered individuals $R_h(t)$, susceptible vectors $S_v(t)$, exposed vectors $E_v(t)$ and infected vectors $I_v(t)$ are presented in Figure 1-7, respectively. In each of the plots, it is observed that the magnitude of σ continuously affect the trend of each state variable for both the human and vector populations.

In Figure 1(a)-(c) the plots compares the dynamics of the susceptible individuals using the corresponding $S_h(t)$-schemes in (49), (54) and (61) for the Caputo, CFC and ABC derivatives, respectively. In each case, as the value of σ increases from 0.7 to 1, there is a considerable decrease in the number of susceptible individuals and then a gradual increase after some time until it steadies at equilibrium. In Figure 2(a)-(c) the plots demonstrates the dynamics of the exposed individuals for the case of the $E_h(t)$-equation of (15) in Caputo, CFC and ABC derivatives, respectively. The plots in each graph are presented for different values of σ. In the graphical representations in Figure 3(a)-(c), we compare the dynamics of Zika infected individuals by presenting plots for the approximate $I_h(t)$-solution using the corresponding $I_h(t)$-schemes of (49), (54) and (61) for the Caputo, CFC and ABC derivatives, respectively, for different values of σ in each case. In each of the plots, it is observed that as the fractional order parameter increases from 0.7 to 1 the number of infected individual decreases after some time. Figure 4(a)-(c) shows the behaviour of the recovered individual using the corresponding $R_h(t)$-schemes of (49), (54) and (61) for the Caputo, CFC and ABC derivatives, respectively, for distinct values of the fractional parameter. Using the corresponding $S_v(t)$-schemes in (49), (54) and (61) for the Caputo, CFC and ABC operators, respectively, Figure 5(a)-(c) demonstrate the dynamics of susceptible vectors. In Figure 6(a)-(c) the plots demonstrates the dynamics of the exposed vectors for the case of the $E_v(t)$-equation of (15) using the corresponding $E_v(t)$-schemes in (49), (54) and (61) for the Caputo, CFC and ABC operators, respectively. In the graphical representations in Figure 7(a)-(c), we compare the dynamics of Zika infected vectors by presenting plots for the approximate $I_v(t)$-solution using the corresponding $I_v(t)$-schemes of (49), (54) and (61) for the Caputo, CFC and



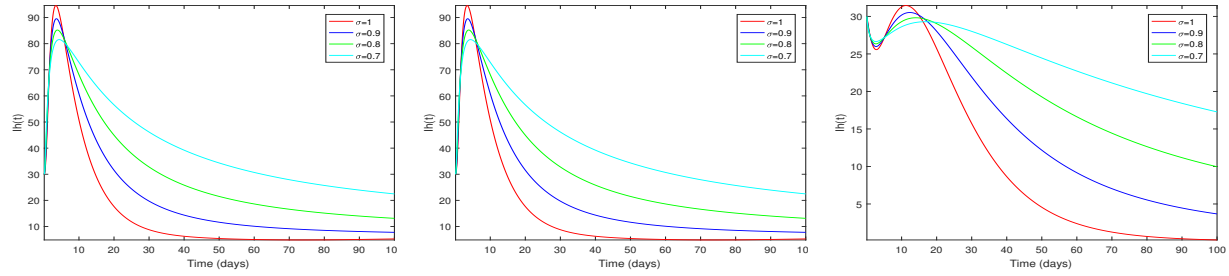
(a) $S_h(t)$-Caputo **(b)** $S_h(t)$-CFC **(c)** $S_h(t)$-ABC

FIGURE 1 Dynamics of susceptible individuals $S_h(t)$ for different values of σ ($\sigma = 1, \sigma = 0.9, \sigma = 0.8, \sigma = 0.7$): (a) $S_h(t)$ in Caputo derivative (b) $S_h(t)$ in CFC derivative (c) $S_h(t)$ in ABC derivative.



(a) $E_h(t)$-Caputo **(b)** $E_h(t)$-CFC **(c)** $E_h(t)$-ABC

FIGURE 2 Dynamics of exposed individuals $E_h(t)$ for different values of σ ($\sigma = 1, \sigma = 0.9, \sigma = 0.8, \sigma = 0.7$): (a) $E_h(t)$ in Caputo derivative (b) $E_h(t)$ in CFC derivative (c) $E_h(t)$ in ABC derivative.



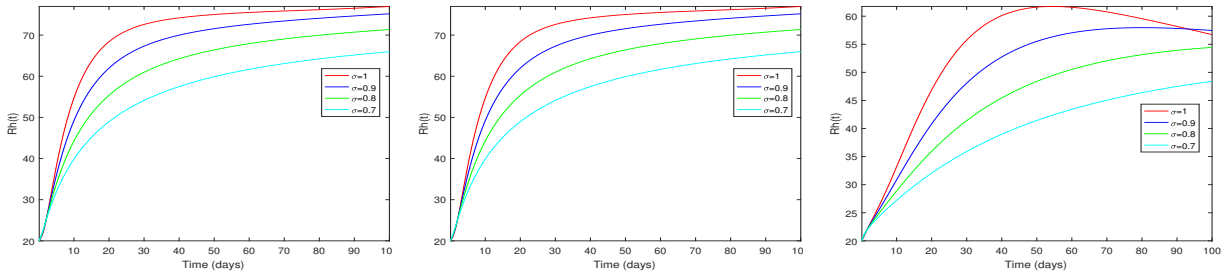
(a) $I_h(t)$-Caputo **(b)** $I_h(t)$-CFC **(c)** $I_h(t)$-ABC

FIGURE 3 Dynamics of infected individuals $I_h(t)$ for different values of σ ($\sigma = 1, \sigma = 0.9, \sigma = 0.8, \sigma = 0.7$): (a) $I_h(t)$ in Caputo derivative (b) $I_h(t)$ in CFC derivative (c) $I_h(t)$ in ABC derivative.

ABC derivatives, respectively, for different values of σ. In each of the plots, it is observed that as the fractional order parameter increases from 0.7 to 1 the population of infected vectors decreases after some time.

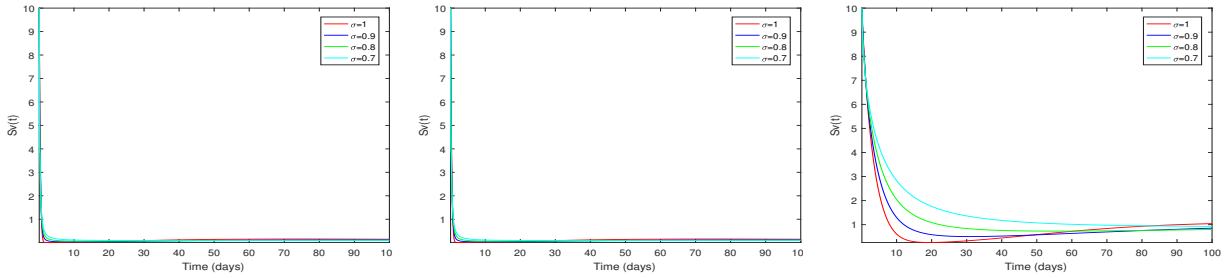
7 | CONCLUSION

In this work, we analyzed a fractional mathematical model for the transmission dynamics of ZIKV under the framework of singular and nonsingular kernels. Firstly, the solution set of classical model is shown to be non-negative and positively invariant. Next we determine the equilibrium points of the model and the basic reproduction number is determined via the next generation matrix technique. Existence and uniqueness of solutions to the fractional model with respect to Caputo, CFC and ABC fractional



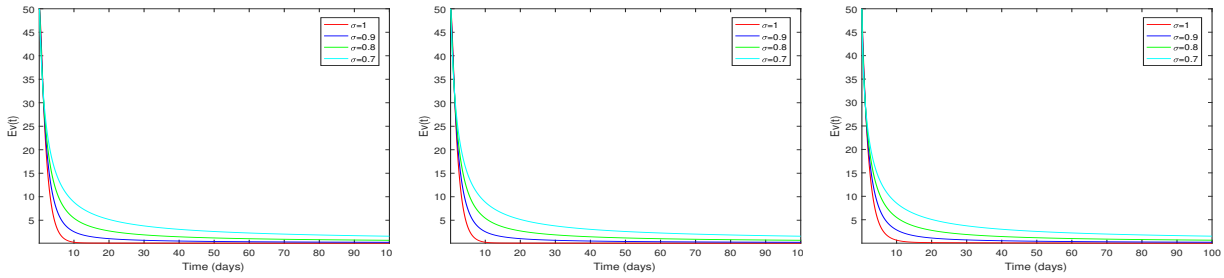
(a) $R_h(t)$-Caputo **(b)** $R_h(t)$-CFC **(c)** $R_h(t)$-ABC

FIGURE 4 Dynamics of recovered individuals $R_h(t)$ for different values of σ ($\sigma = 1, \sigma = 0.9, \sigma = 0.8, \sigma = 0.7$): (a) $R_h(t)$ in Caputo derivative (b) $R_h(t)$ in CFC derivative (c) $R_h(t)$ in ABC derivative.



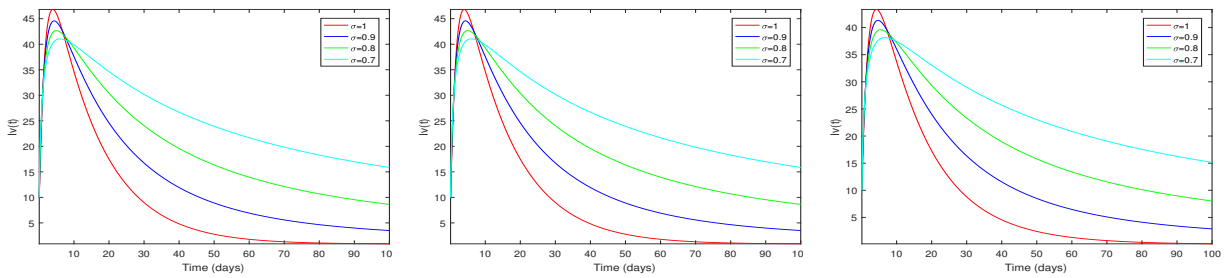
(a) $S_v(t)$-Caputo **(b)** $S_v(t)$-CFC **(c)** $S_v(t)$-ABC

FIGURE 5 Dynamics of susceptible vectors $S_v(t)$ for different values of σ ($\sigma = 1, \sigma = 0.9, \sigma = 0.8, \sigma = 0.7$): (a) $S_v(t)$ in Caputo derivative (b) $S_v(t)$ in CFC derivative (c) $S_v(t)$ in ABC derivative.



(a) $E_v(t)$-Caputo **(b)** $E_v(t)$-CFC **(c)** $E_v(t)$-ABC

FIGURE 6 Dynamics of susceptible vectors $E_v(t)$ for different values of σ ($\sigma = 1, \sigma = 0.9, \sigma = 0.8, \sigma = 0.7$): (a) $E_v(t)$ in Caputo derivative (b) $E_v(t)$ in CFC derivative (c) $E_v(t)$ in ABC derivative.



(a) $I_v(t)$-Caputo **(b)** $I_v(t)$-CFC **(c)** $I_v(t)$-ABC

FIGURE 7 Dynamics of susceptible vectors $I_v(t)$ for different values of σ ($\sigma = 1, \sigma = 0.9, \sigma = 0.8, \sigma = 0.7$): (a) $I_v(t)$ in Caputo derivative (b) $I_v(t)$ in CFC derivative (c) $I_v(t)$ in ABC derivative.

derivatives are established via a fixed point technique. Numerical investigations using the two-step Adams-Bashforth method for the fractional ZIKV model with respect to the three considered fractional differential operators are then carried out with a view to demonstrate the dynamics of each of the system variables for different values of the fractional order parameter. In light of the above last statement, we made comparisons on the obtained results for each system variables with respect to the Caputo, CFC and ABC fractional derivatives

ACKNOWLEDGMENTS

The authors are thankful to the referees for...

Conflict of interest

The authors declare no potential conflict of interests regarding the publication of this paper.

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How to cite this article: ., and . (–), –, –, –.