Unsteady Couette Flow of Dusty Fluid Past Between Two Riga Plates

M. R. Islam[1] and S. Nasrin[2]

[1]*Department of Mathematics, Bangabandhu Sheikh Mujibur Rahman Science and Technology University, Gopalganj-8100, Bangladesh*
[2]*Department of Mathematics, Jagannath University, Dhaka-1100, Bangladesh*
Corresponding author email: mribsmrstu@yahoo.com

KEYWORDS

Dusty Fluid,
Riga Plate,
Couette Flow,
Finite Difference

Abstract

The present study is carried out on the unsteady laminar heat transferable dusty fluid flow past between two parallel Riga plates. The lower plate is kept fixed while the upper plate keeps moving with a constant velocity. A uniform Lorentz forces induced by Riga plates and a constant pressure gradient is applied on the fluid. The governing equations are derived from Navier-Stokes equation, Energy equation. Boundary layer approximations have been employed. The motion of the dust particles is governed by Newton's second law. The non-dimensional equations are solved by using the explicit finite difference method. The effects of necessary parameters on the velocity and temperature distributions as well as the shear stress and Nusselt number of clean fluid particle and dust particles have been discussed in detail.

1. Introduction

Riga plate is the combination of electrodes and permanent magnets that create a plane surface instead of polarity and magnetization. This order produces the electromagnetic hydrodynamic fluid behavior. It can be used to minimize the friction and pressure drag of submarines. The Riga plate is first induced by Grailities and Lielas [1] to generate a wall Lorentz force to control the fluid flow. Further, the Grinberg-term was introduced with the most important feature that the boundary layer of the momentum equation is exponentially decreased normal to the plate. After that the researchers regained the interest in the Gailitis-Lielaus actuator, when the Lorentz force was detected over a Riga plate. Over the past few decades many authors have been interested in contributing their views on the Riga Plate.

Pantokratoras and Magyari [2] proposed an electromagnetic actuator or Riga plate of an electro-magneto hydrodynamic free convection flow of a conducting fluid. Thereafter Pantokratoras [3] investigated on the Riga-plate moves with constant velocity (Sakiadis flow) or the Riga-plate is situated in a constant free stream (Blasius flow). Wahidunnisa et al. [4] studied on the heat source of nanofluid flow through a Riga plate with viscous dissipation. Anjum et al. [5] explained the thermally stratified viscous fluid with stagnation point flow dominated by a Riga plate. Ahmad [6] studied the effect of the Powell - Eyring and Reiner-Phillipoff fluid flow on the Riga plate. Hayat et al. [7] studied on the characteristics of nano fluid boundary layer flow occupied with a Riga plate. Iqbal et al. [8] and [9] investigated an electrically conducting Riga-plate on viscous nanofluid with the viscous dissipation, thermal radiation and melting heat. They proposed the erratic thickness of the stagnation point flow over the Riga plate. Ayub et al. [10] examined the effect of EMHD nanofuid flow along a Riga plate. Ahmed et al. [11] discussed on the mixed convection boundary layer flow along a vertical Riga plate in the presence of strong suction of a nanofluid.

The dusty fluid flow along a Riga plate has significant applications in engineering and geophysical procedures, refrigeration, polluted soil, air and water, dust or fumes in the gas cooling system, in agriculture, crude oil purifying, polymer technology, dye systems, combustion and blood flows in animals body. Dusty fluid flows have a special two-phase nature that occurs when raindrops have been fallen then the combination of small dust particles in air with water, extraction of oil and gas from the ground are the perfect examples. Last few years, some of the authors [12–16] have been analyzed the phenomenon of dust particles of Newtonian and non-Newtonian fluids with or without heat transfer between parallel plates. Eguia et al. [17] achieved an accurate prediction of the flow and heat transfer of dusty fluid flow between parallel plates with the presence of magnetic field. Hazarika and Hazarika [18] investigated the dusty fluid flow along a moving plate with thermal conductivity and variable viscosity. Dusty fluid flow along a channel with a magnetic field also has significant applications in engineering, chemical industry, purification air and oil, wear systems, pumps and generators [19–21]. The authors [22–24] studied on dusty fluid with heat transfer on stretching surface for a wide range of uses in engineering, industrial and chemical processing, in air conditioning, refrigeration and nuclear reactors. Yabo et al. [25] considered the unsteady free convection Couette flow with the effect of the transverse magnetic field and the thermal radiation.

From the above discussion it follows that no author has previously given any idea about the dusty fluid flow over the Riga plate. Our main investigation is the unsteady laminar heat transferable Couette flow of dusty fluid through two parallel Riga plates. The behavior of the flow properties is discussed and presented graphically.

2. Problem Formulation

Consider an incompressible laminar flow of viscous dusty fluid between two horizontal parallel Riga plates, one of which moving and other is at rest. Let the lower plate is kept fixed at $\tilde{y}=0$ and the upper plate keeps moving at a distance $\tilde{y}=d$ with a velocity $\frac{\pi\upsilon}{l}$. Let the direction of the flow be taken along the $\tilde{x}$-axis, $\tilde{y}$-axis is perpendicular to the flow and plates are parallel to the $\tilde{x}\tilde{z}$-plane. A constant pressure gradient $\frac{\partial\tilde{p}}{\partial\tilde{x}}\ (=\mathrm{P})$ is applied to the fluid. The velocity components $\tilde{v}$ and $\tilde{w}$ are zero everywhere at the plate. For dusty particle, $\tilde{v}_p$ and $\tilde{w}_p$ are also zero everywhere. The two plates are fixed at two constant temperatures T_1 for the lower plate and T_2 for the upper plate, where $T_2 > T_1$. The initial temperatures of the fluid and dusty particles are assumed to be equal to the temperature of the lower plate T_1. The physical model is shown in Fig.1.

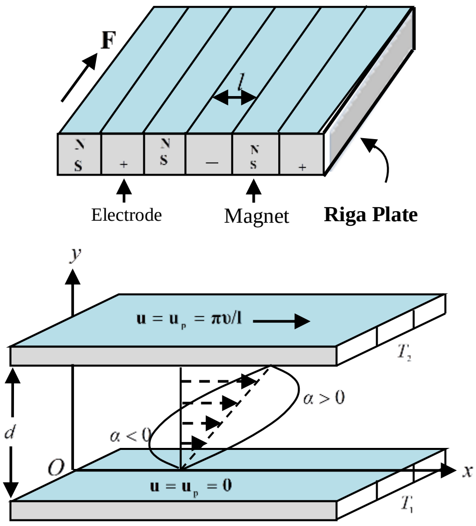

Fig.1: Physical model of the system

A uniform magnetic force is generated by the Riga plate. According to the Grinberg hypothesis the Lorentz force $\bar{f} = \mathbf{J}\wedge\mathbf{B} \approx \sigma(\mathbf{E}\wedge\mathbf{B})$ is defined as a magnetic force as follows:

$$\bar{f} = \mathbf{J}\wedge\mathbf{B} = \left(\frac{\pi}{8}J_0M_0e^{-\frac{\pi}{l}\tilde{y}}\ ,\ 0\ ,\ \frac{\pi}{8}J_0M_0e^{-\frac{\pi}{l}\tilde{y}}\right)$$

Under the above assumptions of Couette flow and boundary layer approximation, the continuity equation reduces for fluid phase to $\tilde{u}=\tilde{u}(\tilde{y},\tilde{t})$ and for dusty phase to $\tilde{u}_p=\tilde{u}_p(\tilde{y},\tilde{t})$ and the dimensional forms of the momentum and energy equations are reduced for the clean fluid and the dust particle as follows:

$$\frac{\partial\tilde{u}}{\partial\tilde{t}} = -\frac{\mathrm{P}}{\rho} + \upsilon\frac{\partial^2\tilde{u}}{\partial\tilde{y}^2} + \frac{\pi}{8\rho}J_0M_0e^{-\frac{\pi}{l}\tilde{y}} - \frac{1}{\rho}KN(\tilde{u}-\tilde{u}_p) \qquad (1)$$

$$m_p\frac{\partial\tilde{u}_p}{\partial\tilde{t}} = KN(\tilde{u}-\tilde{u}_p) \qquad (2)$$

$$\frac{\partial\tilde{T}}{\partial\tilde{t}} = \frac{k}{\rho c_p}\frac{\partial^2\tilde{T}}{\partial\tilde{y}^2} - \frac{\rho_p c_s}{\rho c_p\gamma_T}\left(\tilde{T}-\tilde{T}_p\right) \qquad (3)$$

$$\frac{\partial\tilde{T}_p}{\partial\tilde{t}} = \frac{1}{\gamma_T}(\tilde{T}-\tilde{T}_p) \qquad (4)$$

The corresponding boundary conditions are

$$\tilde{t}\le 0:\quad \tilde{u}=\tilde{u}_p=0,\ \ \tilde{T}=\tilde{T}_p=\tilde{T}_1 \quad \text{for all } \tilde{y}\ge 0$$

$$\tilde{t}>0:\begin{cases}\tilde{u}=0,\ \tilde{u}_p=0,\ \tilde{T}=\tilde{T}_1,\ \tilde{T}_p=\tilde{T}_1 \ \text{ at } \ \tilde{y}=0\\ \tilde{u}=\frac{\pi\upsilon}{l},\ \tilde{u}_p=\frac{\pi\upsilon}{l},\ \tilde{T}=\tilde{T}_2,\ \tilde{T}_p=\tilde{T}_2 \ \text{at } \ \tilde{y}=d\end{cases} \qquad (5)$$

where, $\tilde{u},\tilde{v},\tilde{w}$ are the clean fluid velocity components, $\tilde{u}_p,\tilde{v}_p,\tilde{w}_p$ are the dust particles velocity components, υ is the kinematic viscosity of the clean fluid, $\mathbf{J}=(J_x,J_y,J_z)$ is the current density, $\mathbf{B}=(B_x,B_y,B_z)$ is the induced magnetic field vector, N is the number of dust particles per unit volume, K is the Stokes constant $= 6\pi\rho\upsilon a$; a is the average radius of the of dust particles, m_p is the average mass of the of dust particles, ρ_p is the material density (or mass per unit volume) of dust particles, c_s is the is the specific heat capacity of the particles, T is the temperature of the fluid, T_p is the temperature of the dust particles, k is thermal conductivity of the fluid, c_p is the specific heat capacity at constant pressure, γ_T is the temperature relaxation time which may defined as $\gamma_T = \dfrac{\rho_p c_s}{4k\pi a N}$ or $\dfrac{3\rho\upsilon\rho_p c_s}{2kKN}$, as the energy equation (3) leads to the equation

$$\frac{\partial \tilde{T}}{\partial \tilde{t}} = \frac{k}{\rho c_p}\frac{\partial^2 \tilde{T}}{\partial \tilde{y}^2} - \frac{2kKN}{3\rho^2 c_p \upsilon}\left(\tilde{T} - \tilde{T}_p\right)$$

Similarity Analysis: To make the non-dimensional form of the equations (1)-(5), it is introduced the non-dimensional variables are as follows:

$$x = \frac{\pi}{l}\tilde{x},\quad y = \frac{\pi}{l}\tilde{y},\quad u = \frac{l}{\pi\upsilon}\tilde{u},\quad u_p = \frac{l}{\pi\upsilon}\tilde{u}_p,$$

$$t = \frac{\pi^2\upsilon}{l^2}\tilde{t},\quad \theta = \frac{\tilde{T} - \tilde{T}_1}{\tilde{T}_2 - \tilde{T}_1},\quad \theta_p = \frac{\tilde{T}_p - \tilde{T}_1}{\tilde{T}_2 - \tilde{T}_1}$$

Applying these into the equations (1)-(5), yields

$$\frac{\partial u}{\partial t} = \alpha + \frac{\partial^2 u}{\partial y^2} + H_r\, e^{-y} - R\,(u - u_p) \tag{6}$$

$$\frac{\partial u_p}{\partial t} = \frac{1}{G}\left(u - u_p\right) \tag{7}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} - \frac{2R}{3P_r}\left(\theta - \theta_p\right) \tag{8}$$

$$\frac{\partial \theta_p}{\partial t} = L_0\left(\theta - \theta_p\right) \tag{9}$$

The corresponding boundary conditions are

$$t \le 0:\quad u = u_p = 0,\ \theta = \theta_p = 0 \text{ for all } y \ge 0$$

$$t > 0: \begin{cases} u = 0,\ u_p = 0,\ \theta = 0,\ \theta_p = 0 \text{ at } y = 0 \\ u = 1,\ u_p = 1,\ \theta = 1,\ \theta_p = 1 \text{ at } y = d \end{cases} \tag{10}$$

where,

$\alpha = -\dfrac{l^3 \mathrm{P}}{\rho\upsilon^2\pi^3}$ = Dimensionless pressure gradient;

$H_r = \dfrac{l^3 J_0 M_0}{8\rho\upsilon^2\pi^2}$ = Modified Hartmann Number;

$R = \dfrac{KNl^2}{\rho\upsilon\pi^2}$ = Fluid concentration parameter;

$G = \dfrac{m_p\upsilon\pi^2}{K l^2}$ = Particle mass parameter;

$P_r = \dfrac{\rho c_p \upsilon}{k}$ = Prandtl number;

$L_0 = \dfrac{l^2}{\upsilon\pi^2\gamma_T}$ = Temperature relaxation time parameter.

3. Method of solutions:

The explicit finite difference method has been applied to solve the non-dimensional coupled partial differential equations (6)-(9) together with associated boundary conditions (10). It is considered the maximum length of the plate is $x_{\max}(=10)$ and distance between the plates $d = 2$ *i.e.* $y_{\max} = 2$ as the lower plate is fixed at $y = 0$. This means x varies from 0 to 10 and y varies from 0 to 2. The finite difference schemes for the problems are as follows:

$$\frac{U_{i,j}^{k+1} - U_{i,j}^{k}}{\Delta t} = \alpha + \frac{U_{i,j+1}^{k} - 2U_{i,j}^{k} + U_{i,j-1}^{k}}{\Delta y^2} + H_r\, e^{-y_i} - R\,(U_{i,j}^{k} - Up_{i,j}^{k})$$

$$\frac{Up_{i,j}^{k+1} - Up_{i,j}^{k}}{\Delta t} = \frac{1}{G}\left(U_{i,j}^{k} - Up_{i,j}^{k}\right)$$

$$\frac{\Theta_{i,j}^{k+1} - \Theta_{i,j}^{k}}{\Delta t} = \frac{1}{P_r}\left(\frac{\Theta_{i,j+1}^{k} - 2\Theta_{i,j}^{k} + \Theta_{i,j-1}^{k}}{\Delta y^2}\right) - \frac{2R}{3P_r}\left(\Theta_{i,j}^{k} - \Theta p_{i,j_{i,j}}^{k\ k}\right)$$

$$\frac{\Theta p_{i,j}^{k+1} - \Theta p_{i,j}^{k}}{\Delta t} = L_0\left(\Theta_{i,j}^{k} - \Theta p_{i,j}^{k}\right)$$

with boundary conditions

$$\left.\begin{matrix} U_{i,L} = 0,\ Up_{i,L} = 0,\ \Theta_{i,L} = 0,\ \Theta p_{i,L} = 0 \text{ at } L = 0 \\ U_{i,L} = 1,\ Up_{i,L} = 1,\ \Theta_{i,L} = 1,\ \Theta p_{i,L} = 1 \text{ at } L = 2 \end{matrix}\right\} \text{for all } k$$

Here, the subscripts i and j refer to x and y and the superscript k refers to time t.

Shear stresses, Nusselt number and Sherwood number:

The effects of pertinent parameters on the local and average shear stress from the velocity of the fluid phase and dust particle phase have investigated. The non-dimensional forms of the local and average shear stress for the fluid phase are given by the relations $\tau_L = \mu \left.\dfrac{\partial u}{\partial y}\right|_{y=0}$ and $\tau_A = \dfrac{1}{L}\int_0^L \mu \left.\dfrac{\partial u}{\partial y}\right|_{y=0} dx$ for the dust particle are given by $\tau_{pL} = \mu \left.\dfrac{\partial u_p}{\partial y}\right|_{y=0}$ and $\tau_{pA} = \dfrac{1}{L}\int_0^L \mu \left.\dfrac{\partial u_p}{\partial y}\right|_{y=0} dx$ respectively. The rate of heat transfer at the plate is defined as the Nusselt number. The local and average Nusselt numbers for the fluid phase are given by $Nu_L = -\mu \left.\frac{\partial \theta}{\partial y}\right|_{y=0}$ and $Nu_A = -\frac{1}{L}\int_0^L \mu \left.\frac{\partial \theta}{\partial y}\right|_{y=0} dx$ for the dust particle are given by $Nu_{pL} = -\mu \left.\frac{\partial \theta_p}{\partial y}\right|_{y=0}$ and $Nu_{pA} = -\frac{1}{L}\int_0^L \mu \left.\frac{\partial \theta_p}{\partial y}\right|_{y=0} dx$ respectively.

4. Results and Discussions

As a result of extensive numerical calculation, the graphical presentations have been carried out for the influence of the relevant non-dimensional parameters namely pressure gradient parameter (α), modified Hartmann number (H_r), fluid concentration parameter (R), particle mass parameter (G), Prandtl number (P_r) and temperature relaxation time parameter (L_0) on the velocity and temperature distribution of the fluid and of the dust particle. The effects of those parameters on some necessary profiles are investigated with the fixed values of $\alpha = 2$, $H_r = 1.0$, $R = 0.5$, $G = 0.5$, $P_r = 7.0$, $L_0 = 0.8$.

Steady-state solution:

To get the steady-state solutions, it is necessary to show the distributions for different times. Fig.2(a) and Fig.2(b) illustrate the fluid velocity distribution u and dust particle temperature θ_p for different time τ. The computations have been carried out for the different time such as $\tau = 2, 3, 4, 5, 6, 7$ for u and time $\tau = 4, 6, 8, 10, 11, 12$ for θ_p with the time step size $\Delta t = 0.0005$. It observed from figures that there is negligible change between time $\tau = 3$ and $\tau = 4$ for u and between time $\tau = 11$ and $\tau = 12$ for θ_p . Fig.2 (c) depicts the validity of the grid pairs on the temperature θ. It has shown the velocity distribution for three grid pairs $(m,n) = (40,40)$, $(m,n) = (50,50)$ and $(m,n) = (60,60)$ with time $\tau = 12$ and time step $\Delta t = 0.0005$. There is also negligible change among these grid pairs so that any one grid pair is acceptable to find the steady-state solution. It has seen that the same situation for the effects of each parameter on the others distributions. The steady-state solution has performed at least value $\tau \geq 4$ for u (or u_p) and $\tau \geq 11$ for θ (or θ_p). In the present analysis, the following graphs have established for the choice of time $\tau = 3$ with the grid pair $(m,n) = (50,50)$ and time step $\Delta t = 0.0005$. But for average shear stress and Nusselt number have performed for the time $\tau = 12$.

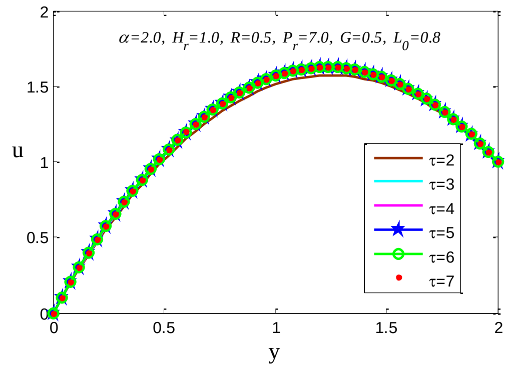

Fig.2(a): Time sensitivity on u

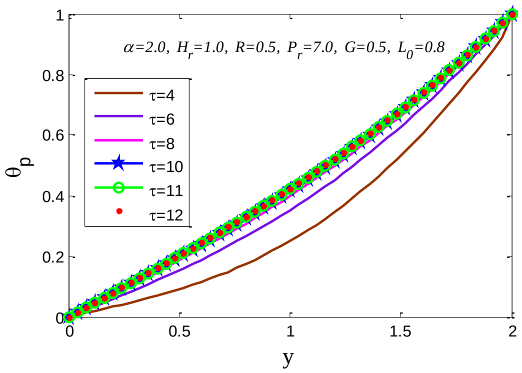

Fig.2(b): Time sensitivity on θ_p

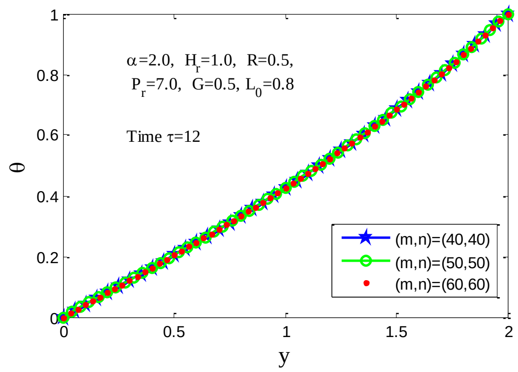

Fig.2(c): Mesh sensitivity on θ

Effects of various parameters:

To study the physical situation of the problem, it is mentioned that in each figure of Figs.3-8, the solid line plot has indicated the distribution for the fluid phase and the dotted line plot has indicated the distribution for the dust particle phase.

The effects of dimensionless pressure gradient α on the velocity for fluid phase and dust particle phase have shown in Figs.3(a)-(d). Since $\alpha = -l^3 \mathrm{P}/(\rho \upsilon^2 \pi^3)$, it means that $\alpha > 0$ whenever constant pressure gradient P decreasing in the direction of motion; in that case, the velocity u

$(\text{or } u_p)$ has increasing effect with the increase of α over the entire width between the plates. But $\alpha < 0$ influence that P increasing in the direction of motion and the velocity u $(\text{or } u_p)$ has occurred back-flow, which have shown in Fig.3(a) and 3(b). Specifically Fig.3(a) refers to the variations of u $(\text{or } u_p)$ for the time step $\tau = 3$. It has observed from figure that the velocity of the clean fluid particle faster than the dust particle. But Fig.3(b) refers to the steady-state solution for u $(\text{or } u_p)$. Here the dust particles coincide with the clean fluid particles, i.e. at steady-state position of the solution, the velocity of the clean fluid particle and dust particle have no difference. Figs.3(c) and 3(d) have shown the local shear stress and average shear stresses both are increased with the increase of α respectively. Moreover Figs.3 (d) has explained that average shear stresses of the dust particle coincides with fluid particle after the time $\tau = 4$ (approx.) and thereafter it has no changing values with time, i.e. it becomes at a steady-state position. The verified steady-state results of the effects of α on u $(\text{or } u_p)$ are shown in Fig.3(b).

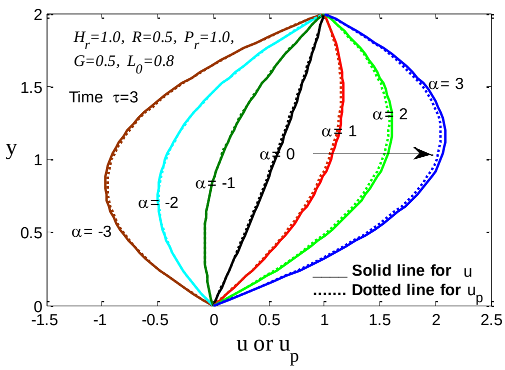

Fig.3(a): Effect of α on u $(\text{or } u_p)$ [Unsteady]

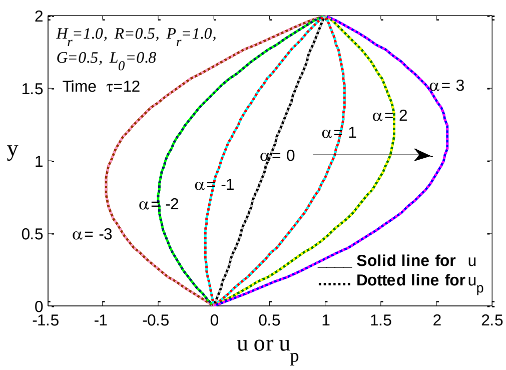

Fig.3(b): Effect of α on u $(\text{or } u_p)$ [Steady-state]

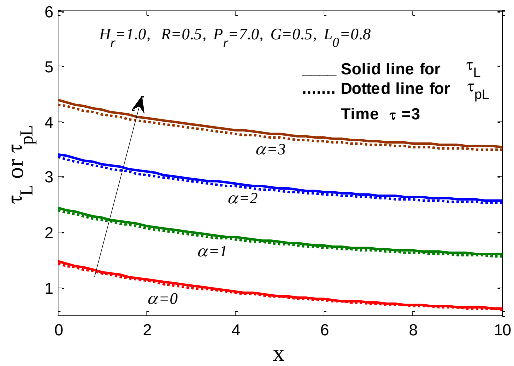

Fig.3(c): Effect of α on τ_L $(\text{or } \tau_{pL})$

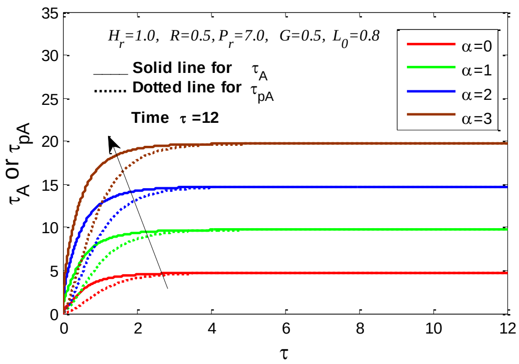

Fig.3(d): Effect of α on τ_A $(\text{or } \tau_{pA})$

Figs.4(a)-(c) depict the effect of modified Hartmann number H_r on the velocity u $(\text{or } u_p)$, local shear stress τ_L $(\text{or } \tau_{pL})$ and on the average shear stress τ_A $(\text{or } \tau_{pA})$ respectively. It is observed that u, τ_L and τ_A are all increased with the increase of H_r. For the dust particle u_p, τ_{pL} and τ_{pA} are also increased with the increase of H_r.

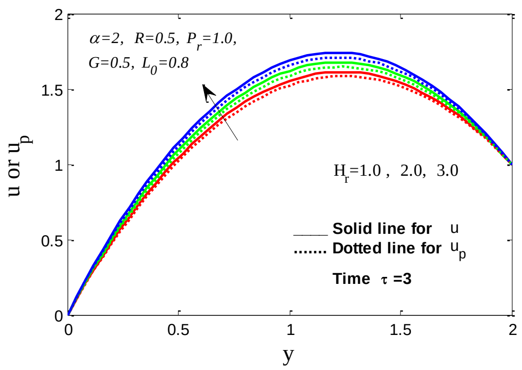

Fig.4(a): Effect of H_r on u $(\text{or } u_p)$

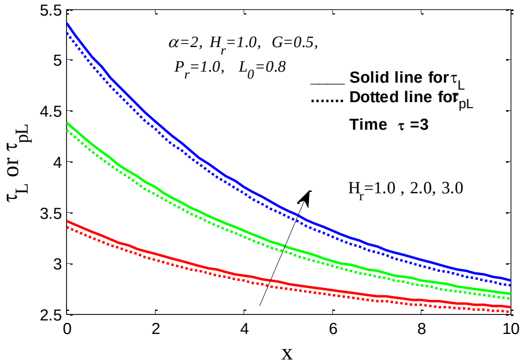

Fig.4(b): Effect of H_r on τ_L (or τ_{pL})

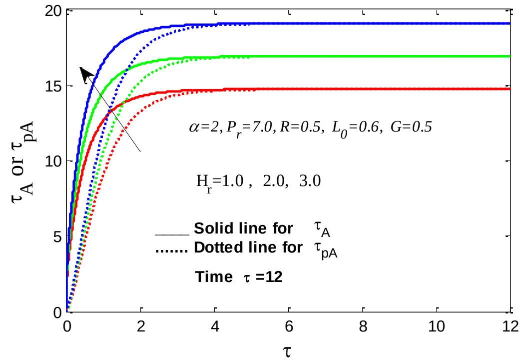

Fig.4(c): Effect of H_r on τ_A (or τ_{pA})

Figs.5(a)-(c) influence the effects of fluid concentration parameter R on the velocity u (or u_p), local shear stress τ_L (or τ_{pL}) and on the average shear stress τ_A (or τ_{pA}) respectively. It has shown that increasing values of R influence the decreasing effects on u, τ_L and τ_A. The dust particle shows the same results.

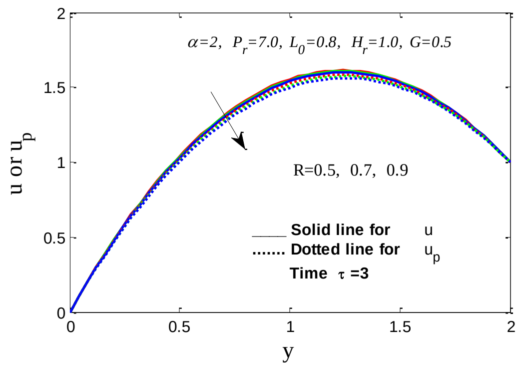

Fig.5(a): Effect of R on u (or u_p)

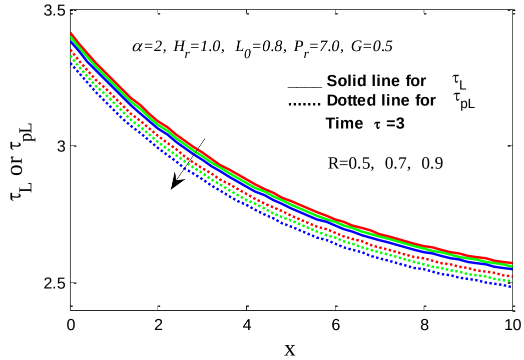

Fig.5(b): Effect of R on τ_L (or τ_{pL})

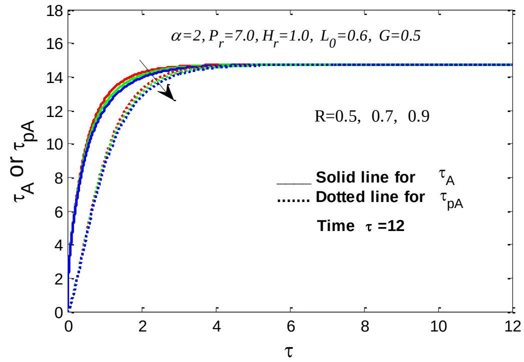

Fig.5(c): Effect of R on τ_A (or τ_{pA})

Figs.6(a)-(c) present the effects of particle mass parameter G on the velocity u (or u_p), local shear stress τ_L (or τ_{pL}) and on the average shear stress τ_A (or τ_{pA}) respectively. It has seen that u, τ_L and τ_A are all decreased with the increase of G. For the dust particle has also found the same results.

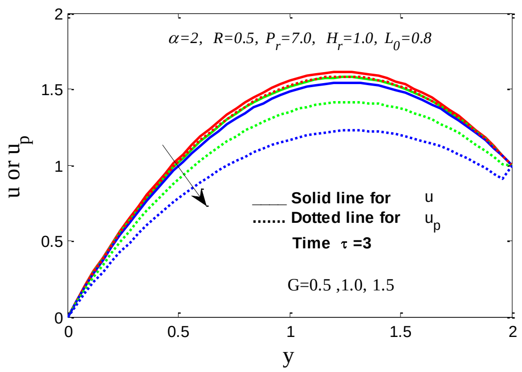

Fig.6(a): Effect of G on u (or u_p)

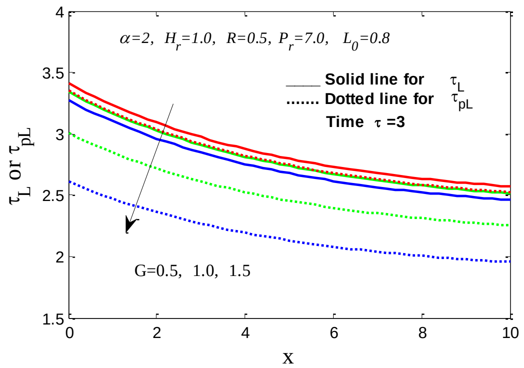

Fig.6(b): Effect of G on τ_L (or τ_{pL})

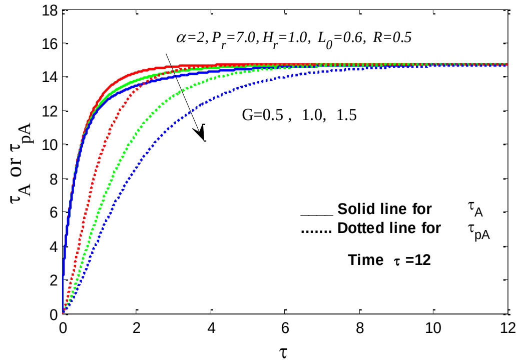

Fig.6(c): Effect of G on τ_A (or τ_{pA})

Figs.7-8 present the variation of the temperature distribution, shear stress and Nusselt number for the different values of temperature relaxation time parameter and Prandtl number. Fig.7(a) shows that temperature decreases with the increasing values of L_0 for the fluid phase, while it has an increasing effect for the dust particle. Local and average Nusselt number have described in Fig.7(b) and Fig.7(c) respectively. From these figures, it is found that the local and average Nusselt number both have reverse effects with temperature for clear fluid particle and dust particle. The effects of P_r on θ (or θ_p) have shown in Fig.8(a). The temperature θ (or θ_p) has been decreasing behavior with increasing values of P_r. But the local and average Nusselt number increases with an increase of P_r for both clear fluid particle and dust particle.

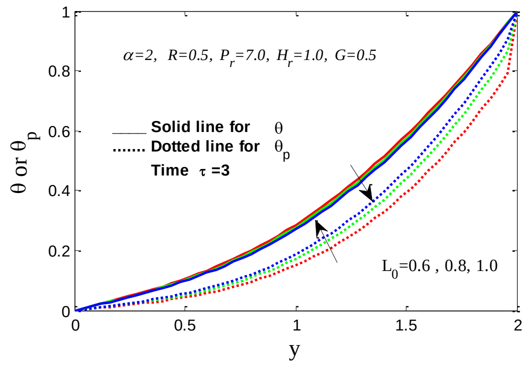

Fig.7(a): Effect of L_0 on θ (or θ_p)

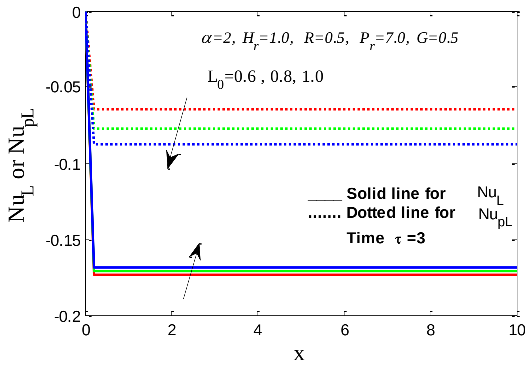

Fig.7(b): Effect of L_0 on Nu_L (or Nu_{pL})

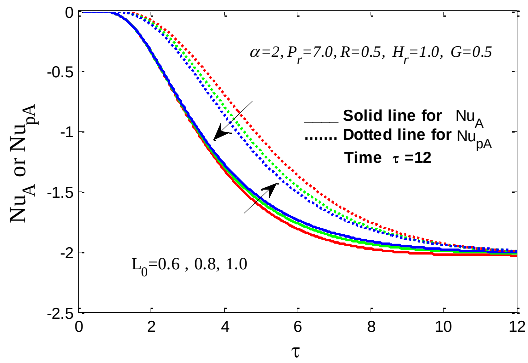

Fig.7(c): Effect of L_0 on Nu_A (or Nu_{pA})

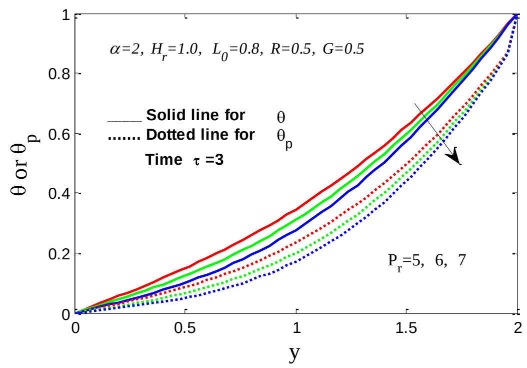

Fig.8(a): Effect of P_r on θ (or θ_p)

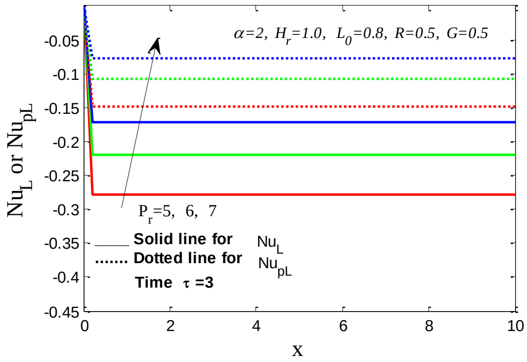

Fig.8(b): Effect of P_r on Nu_L (or Nu_{pL})

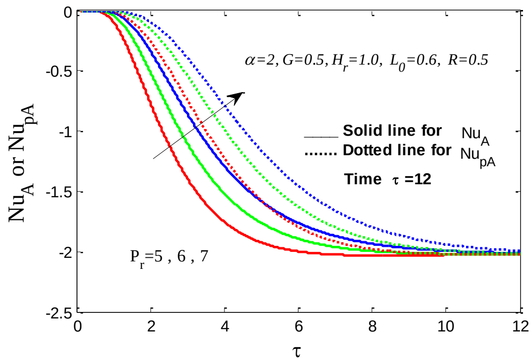

Fig.8(c): Effect of P_r on Nu_A (or Nu_{pA})

5. Conclusions

It is concluded that

- The velocity of the fluid particle and the dust particle are parallel to each other and the fluid particle is faster than the dust particle to reach a steady-state position.
- In the steady-state situation, there are no variations of velocities or temperature between clean fluid phase and dust particle phase.
- The velocities u and u_p are raised with increasing values of α and H_r, while it has decreasing effects with the increase of R and G.
- The fluid temperature θ decreases with increasing values of L_0 and P_r.
- Dust particle temperature θ_p increases with L_0, while it decreases with the increase of P_r.
- The velocity gradient at the plate for both fluid and dust particle has increased with the decreasing pressure gradient in the direction of motion and also the variation of the fluid temperature at the plate increases with the increases of Prandtl number and temperature relaxation time parameter .

6. Author contributions:

M. R. Islam and S. Nasrin have performed the literature review. M. R. Islam has developed the geometrical configuration and Mathematical model and illustrated the algorithm and written codes by using MATLAB R2015a tools. M. R. Islam and S. Nasrin has drawn the graphs and written the manuscript. Both authors have checked the code and the simulated data.

Funding: This research received no external or institutional funding.

Conflicts of Interest: The authors declare no conflict of interest.

7. References

[1] A Gailitis. and O.Leilausis, On a possibility to reduce the hydro dynamical resistance of a plate in an electrode.Appl. Magnetohydrodyn, Vol.12, pp. 143-146, 1961.

[2] A. Pantokratoras, E. Magyari , MHD free-convection boundary layer flow from a riga plate. J.Eng. Maths, Vol.64(3), pp. 303-315, 2009.

[3] A. Pantokratoras, The blasius and sakiadis flows along a Riga-plate. Progress in Computational Fluid Dynamics, An International Journal, Vol. 11 No. 5, pp. 329-333, 2011.

[4] L.Wahidunnisa, K.Subbarayudu, S.Suneetha, Effect of viscous dissipation over a Riga plate in a nano fluid with heat source/sink. IJTIMES, Vol.4, Issue.6, 2016.

[5] A. Anjum, N.A. Mir, M. Farooq, M. I. Khan and T. Hayat, Influence of thermal stratification and slip conditions on stagnation point flow towards variable thicked Riga plate. Results in Physics. Vol.9,pp.1021-1030, 2018.

[6] A. Ahmad, Flow Control of Non-Newtonain Fluid using Riga Plate:Reiner-Phillipoff and Powell-Eyring Viscosity Models. Journal of Applied Fluid Mechanics, Vol. 12, No. 1, pp. 127-133, 2019.

[7] T. Hayat, Tehseen Abbas, M. Ayub, M. Faroq, A. Alsaedi, Flow of nanofluid due to convectively heated Riga plate with variable thickness. Vol.222,pp.854-862, 2016.

[8] Z. Iqbal, Ehtsham Azhar, Zaffar Mehmood and E.N. Maraj, Melting heat transport of

nanofluidic problem over a Riga plate with erratic thickness: Use of Keller Box scheme, Results in Physics, vol.7, pp.3648–3658,2017.

[9] Z. Iqbal, E. Azhar, Z. MEhmood, E.N. Maraj, Unique outcomes of internal heat generation and thermal deposition on the viscous dissipative transport of viscoplastic fluid over a Riga plate. Communications in Theoretical Physics ,Vol.69, pp. 68-76,2018.

[10] Ayub M, Abbas T, Bhatti MM., Inspiration of slip effects on electromagneto hydrodynamics (EMHD) nano fluid flow through a horizontal Riga plate. Eur Phys J Plus 16193, 2016.

[11] Adeel Ahmed, Saleem Asghar and Sumaira Afzal, Flow of nanofluid past a Riga plate. Journal of Magnetisn and Magnetic Materials.Vol.420, pp. 44-48, 2016.

[12] Parul Saxena, Manju Agarwal , Unsteady flow of a dusty fluid between two parallel plates bounded above by porous medium. International Journal of Engineering, Science and Technology, Vol. 6, pp. 27-33, 2014.

[13] Hazem Ali Attia, W. Abbas, Amira M. D. , Salama, Mostafa , A. M. Abdeen , Transient Couette Flow of a Dusty Fluid between Parallel Porous Plates with the Ion slip Effect and Heat Transfer under Exponential Decaying Pressure Gradient. IJIRSET,Vol. 4, Issue 8, 2015.

[14] Hazem Ali Attia and Karem Mahmoud Ewis , Magnetohydrodynamic flow of continuous dusty particles and non-Newtonian Darcy fluids between parallel plates. Advances in Mechanical Engineering, Vol. 11(6) , pp.1–11,2019.

[15] H.A. Attia, A.L. Aboul-Hassan, M.A.M. Abdeen, A.El-Din Abdin, MHD flow of a dusty fluid between two infinite parallel plates with temperature dependent physical properties under exponentially decaying pressure gradient. Bulgarian Chemical Communications, Vol. 46, pp. 320 – 329, 2014.

[16] Rajesh Kumar, Unsteady Flow of a Dusty Conducting Fluid between Parallel Porous Plates through Porous Medium with Temperature Dependent Viscosity. IJETR, ISSN: 2321-0869, 2014.

[17] P. Eguia , J. Zueco, E. Granada , D. Patino , NSM solution for unsteady MHD Couette flow of a dusty conducting fluid with variable viscosity and electric conductivity. Applied Mathematical Modelling Vol.35, 303–316, 2011.

[18] G. C. Hazarika and Santana Hazarika, Effects of Variable Viscosity and Thermal Conductivity on the Flow of Dusty Fluid over a Continuously Moving Plate. International Journal of Computer Applications, pp.0975–8887, Vol. 122, 2015.

[19] O.D. Makinde , T. Chinyoka , MHD transient flows and heat transfer of dusty fluid in a channel with variable physical properties and Navier slip condition. Computers and Mathematics with Applications,Vol. 60, pp.660–669, 2010.

[20] B. J. Gireesha, G. S. Roopa, Channabasappa Shanthappa Bagewadi, Unsteady Flow and Heat Transfer of a Dusty Fluid through a Rectangular Channel. Mathematical Problems in Engineering , 2010, (1024-123X). DOI: 10.1155/2010/898720.

[21] Rebhi A. Damseh, On Boundary Layer Flow of a Dusty Gas from a Horizontal Circular Cylinder. Brazilian Journal of Chemical Engineering, Vol. 27, No. 04, pp. 653 - 662, 2010.

[22] Mudassar Jalil, Saleem Asghar, and Shagufta Yasmeen , An Exact Solution of MHD Boundary Layer Flow of Dusty Fluid over a Stretching Surface .Mathematical Problems in Engineering ,Article ID 2307469, 2017. https://doi.org/10.1155/2017/2307469.

[23] Sharena Mohamad Isa and Nurul Farahain Mohammad , Boundary Layer Flow of Dusty Fluid on a Stretching Sheet of Another Quiescent Fluid. J. Phys.: Conf. Ser. 819 012027, 2017.

[24] Ramesh G K and B. J. Gireesha, Flow Over a Stretching Sheet in a Dusty Fluid With Radiation Effect. Journal of Heat Transfer, 135(10):102702, 2013. DOI: 10.1115/1.4024587.

[25] Isah Bala Yabo, Basant Kumar Jha, Jeng-Eng Lin, On a Couette Flow of Conducting Fluid. International Journal of Theoretical and Applied Mathematics, Vol.4(1), pp. 8-21, 2018.