ON THE ASYMPTOTIC BEHAVIOR AND APPROXIMATE SOLUTIONS OF VARICELLA ZOSTER VIRUS MODEL USING MODIFIED DIFFERENTIAL TRANSFORM METHOD

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Abstract

This article proposes a mathematical model describing the evolution and transmission of Varicella Zoster Virus (VZV) over large groups of individuals. The model was formulated to accommodate parameters and variables describing direct and indirect forms of transmission, re-activation of infectious shingles as well as treatment and vaccination of susceptible births and influx of immigrants. The model was analyzed to be positive, bounded and well posed. The controlled basic reproduction number R_{vzv}, obtained using the next generation matrix operator reveal that vaccination is effective as a control in creating a level herd immunity. Linearizing the model around the VZV - free equilibrium shows that the model is locally and globally asymptotically stable when R_{vzv} is less than unity. The approximate solutions of the model system equations was obtained using the modified differential transform which involves the Differential Tranform Method (DTM) and Laplace - Pade post-treatment technique (LPDTM). This technique was employed to enlarge the domain of convergence of the approximate solutions of the model. LPDTM was compared with the Fehlberg fourth order Runge - Kutta (RKF45) via the MAPLE computational software to show the accuracy of the results through simulations. Further simulations carried out on the model reveal that timely vaccination and treatment are effective strategies in containing VZV infection spread in human and environmental

host population.

1   Introduction

Infectious diseases pose a serious challenge to human existence, but the development of safe vaccines, drugs and medical equipments has led to the reduction of the prevalence of several diseases. Chicken pox, also known as Varicella Zoster Virus (VZV), is exclusively a human virus that belongs to the $\alpha_o - herpes$ virus family. VZV is present worldwide and it is highly infectious where primary infection leads to acute varicella or chicken pox, usually from exposure either through direct contact with skin or lesions, or indirect contact through airborne spread from respiratory droplets or infected environmental sources like towels, toiletries, etc [6]. After initial infection, VZV establishes lifelong latency in cranial nerves and can reactivate later as infectious Herpes Zoster (HZ) or shingles. Varicella vaccination is recommended for outbreak control, also a pregnant woman who has symptoms of varicella also needs to be vaccinated to prevent Foetal Varicella Syndrome (FVS), which causes birth defect such as congenital varicella syndrome. This can also cause shingles in the baby during the first $1-2$ years of life. Symptoms of VZV includes onset of slight fever, tiredness and weakness, followed by an itching blister-like rash which resolves within $7-10$ days [20, 21]. In Africa, mortality is low and morbidity is high, this has placed an endemic burden on the African society. However, this disease occurs mostly in children, pregnant women and other adults. Mathematical models are good predictive tools in analyzing the spread and control of epidemics. Several works have been done in using deterministic models to analyze the transmission of VZV. Edward, Kuznetsov and Mirau [1], examined the modeling and stability analysis for VZV with vaccination. They investigated the local and global asymptotic behavior of the model and bifurcation analysis to prove that the ba-

sic reproduction number of the model is supercritical, while sensitivity analysis of the reproduction number is performed to suggest control strategies to policy health maker. Garnett and Green follow [2], formulated a mathematical model to describe the epidemiology of VZV infections. A steady state age distribution of Zoster cases predicted by the model are compared to the observed distribution in the data involving VZV transmission. The results obtained showed that the likelihood of reactivation increases with the age. Corberian - Vallet *et al.* [3], used a discrete time Bayesian stochastic compartmental model approach to study the transmission of VZV in Valencia, Spain. The Bayesian analysis allows the computation of the posterior distribution of the model parameter and posterior predictive distribution of VZV incidence, which enables point forecasts and prediction interval. Gommel, Jaros and Luu [7], formulated and discuss the dynamics of VZV with United State as a case study. Korostil, Wood and Regan [4], investigated the impact of periodicity of VZV in the presence of immune boosting and chemical reinfection with VZV. Furthermore, Tang *et al.* [5], investigated the control strategies of VZV in china using mathematical modeling approach where a school based vaccination intervention scenario is compared with a baseline (no intervention) scenario. Moreover, a mathematical approach to characterize the transmission dynamics of the VZV in United States, these results reveals that 2 dose program and 90 percent converge allow the Zoster Medians to decrease for 27 years and then decrease after.

Several semi-analytical methods like DTM, Homotopy Perturbation Method (HPM), Homotopy Analysis Method (HAM) and Variational Iteration Method (VIM) are powerful techniques employed in approximating nonlinear and linear problems in engineering and physics. Among the analytical methods, DTM is very simple and effective in solving nonlinear differential equations. It does not depend on perturbation parameter like other analytical methods. DTM and other analytical methods have been used to solve various kind of models, see [8, 9] and [13 - 19]. The use of Lyapunov techniques to prove stability of models can also be seen in [10 - 12], which proved useful to this work. After careful consultation of the cited articles, previous works considered only the impact of

3

vaccination as well as fitting prevalence data on VZV transmision to models describing VZV transmission in human host population to show that vaccine is efficacious in boosting immunity against the disease.

In this work, we considered what is different from other cited authors, by formulating a theoretical model based on ordinary differential equations subdivided into seven compartments of human sub-populations and environmental sources. The model incorporated the direct human to human and indirect human to environment forms of transmission using the mass action and saturated non-linear incidence function. The parameters and variables were incorporated to describe the impact of vaccination of susceptible birth and immigrants, treatment of VZV infected individuals as well as reactivation of infectious shingles after recovery for some recovered human individuals. Furthermore, the modified LPDTM method was employed to obtain the approximate solutions of the model in comparison with RK45 to show the high degree of accuracy of the two methods with low error approximation. This work is partitioned into sections. Section 2 describes the model derivation and basic properties of the model. Also, the VZV - free equilibrium solutions and computation of R_{vzv} was carried out. Section 3 involve the analysis of the local and global asymptotic behavior of the VZV - free equilibrium solutions, while Section 4 involves using the modified DTM to obtain the approximate solutions of the model system equations. Section 5 discusses the numerical simulations using LPDTM in comparison with RK45 via MAPLE and graphical illustrations of the variations of some parameters involving the model. .

2   Mathematical Model Derivation

In this section, a mathematical model is derived describing the transmission of VZV in the total human and environmental host population. The total human host population is sub-divided into sub-population of susceptible individuals $S_h(t)$, Vaccinated individuals $V_h(t)$; Exposed individuals $E_h(t)$; Infected individuals $I_h(t)$; Treated individuals $T_h(t)$ and Recovered individuals $R_h(t)$, such

4

that $N(t) = S_h(t) + V_h(t) + E_h(t) + I_h(t) + T_h(t) + R_h(t)$. Also, the total environmental host population is considered and denoted by $E_v(t)$, where $t > 0$. The sub-population of the susceptible human individuals is generated by the rate at which newborns and immigrants who are recruited and vaccinated, denoted by ϕHN and $\rho\lambda$ respectively. Also the terms $(1 - \phi)HN$ and $(1 - \rho)\lambda$ denote the fractions of newborns and immigrants who are not vaccinated respectively. The sub-population of susceptible humans is further reduced by the quantities $\frac{\beta_1 c_1 S_h I_h}{N}$ and $\frac{\beta_2 c_2 S_h E_v}{A + E_v}$ which follows a mass action incidence and non linear incidence function respectively, where β_1 and β_2 denote the probability that the susceptible individuals will become infectious by coming directly or indirectly in contact with infected individuals and infected environmental sources respectively. Also c_1 and c_2 denote the per capita contact rate associated to direct and indirect transmission, while μ denote the natural mortality rate associated to all classes of human sub-population, and parameter A denote the concentration of varicella zoster virus in environmental sources. The exposed sub-population is increased by the quantities $\frac{\beta_1 c_1 S_h I_h}{N}$ and $\frac{\beta_2 c_2 S_h E_v}{A + E_v}$ and reduced by quantity $(\mu + \delta)E_h$, where δ is the progression rate from exposed to infected sub-population. In the infected sub - population, η_1 is denote the treatment rate while the rate of recovery through treatment is given by η_2. Individuals who have recovered, receive life long immunity denoted $K_o R$, and the reactivation of the virus in some fractions of recovered human individuals, known as shingles is denoted by $(1 - \alpha)K_o R$, where $\alpha K_o R$ is the rate at which some fractions of recovered individuals develop infectious shingles. Also, the virus increases in the environmental sources through human infectious contribution to the environment denoted by σ, and there is a natural degradation of the virus denoted by μ_v. Furthermore, the rate at which vaccination losses its potency overtime, is denoted by τ. The assumptions guiding the model formulations are listed below.

- Birth and death rate are constant.

- The population is homogeneously mixed.

- The vaccinations of immigrants and newborns are considered.

- There is a natural degradation of the virus in the environment.

- Infectious human individuals contribute to environmental sources infection.

The model derived after the incorporation of the assumptions, parameters and state variables is given by

$$
\begin{aligned}
\frac{dS_h}{dt} &= (1-\phi)HN + (1-\rho)\lambda - \frac{\beta_1 c_1 S_h I_h}{N} - \frac{\beta_2 c_2 S_h E_v}{A + E_v} - \mu S_h + \tau V_h, \\
\frac{dV_h}{dt} &= \rho\lambda + \phi HN - (\mu + \tau)V_h, \\
\frac{dE_h}{dt} &= \frac{\beta_1 c_1 S_h I_h}{N} + \frac{\beta_2 c_2 S_h E_v}{A + E_v} - (\mu + \delta)E_h, \\
\frac{dI_h}{dt} &= \delta E_h - (\eta_1 + \mu)I_h + \alpha K_o R_h, \\
\frac{dT_h}{dt} &= \eta_1 I_h - (\eta_2 + \mu)T_h, \\
\frac{dR_h}{dt} &= \eta_2 T_h + (1-\alpha)K_o R_h - \mu R_h, \\
\frac{dE_v}{dt} &= \sigma I_h - \mu_v E_v.
\end{aligned}
\tag{1}
$$

Subject to the initial conditions $S_h(t) \geq 0, V_h(t) \geq 0, E_h(t) \geq 0, I_h(t) \geq 0, T_h \geq 0, R_h(t) \geq 0, E_v(t) \geq 0$.
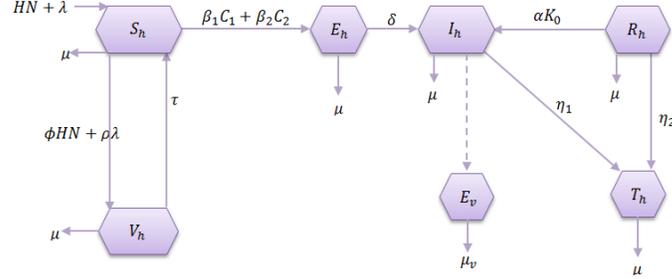
Figure (1)   Flow Diagram Describing the VZV Infectious Interactions Among Compartments of Human Individuals

Table (1)   Parameter Description

Description	Parameter	Value(s)	Source(s)
Vaccinated newborn rate	ϕ	0.0052/per day	[1]
Birth rate	H	0.0352/per day	[1]
Immigration influx rate	λ	0.00356/per day	[1]
Vaccinated immigrant rate	ρ	0.0121/per day	[4]
Direct contact rate	β_1	0.0325/per day	[4]
Indirect contact rate	β_2	0.0241/per day	[4]
Per capita direct contact rate	c_1	0.11/per day	[4]
Per capita indirect contact rate	c_2	0.019/per day	[4]
Natural mortality rate	μ	0.0096/per day	[1]
Progression rate	η_1	0.52/per day	[2]
Concentration of virus rate	A	0.05/ per day	[6]
Lifelong immunity rate	K_o	0.46/per day	[4]
Progression rate	η_2	0.14/ per day	[4]
Natural degradation rate	μ_v	0.014/per day	[6]
Vaccination waning rate	τ	0.00033/per day	[2]
Human infectious contribution rate	σ	0.009/per day	[4]
Development of shingles rate	α	0.00002/per day	[4]

3 Basic Analysis of the Model

3.1 Positivity and Boundedness of the Model Solutions

For the model system (1) to be epidemiologically reasonable in the sense of VZV transmission, it is pertinent to show that all the solutions of the model system (1) and its non-negative initial conditions remain non-negative at time $t > 0$.

Theorem 1 Let the initial conditions $S_h(0) \geq 0, V_h(0) \geq 0, E_h(0) \geq 0, I_h(0) \geq 0, T_h(0) \geq 0, R_h(0) \geq 0, E_v(0) \geq 0$. Then the solutions $(S_h, V_h, E_h, I_h, T_h, R_h, E_v)$ of model (1) are non-negative for $t > 0$. Also, $\lim_{t \to \infty} sup N \leq 1$ with $N = S_h, +V_h + E_h + I_h + T_h + R_h + E_v$, where sup is supremum.

Proof. Let

$$t_0 = sup\,\{t > 0 : S_h(t) > 0, V_h(t) > 0, E_h(t) > 0, I_h(t) > 0, T_h(t) > 0, R_h(t) > 0, E_v(t) > 0\}$$

. Since $S_h(0) > 0, V_h(0) > 0, E_h(0) > 0, I_h(0) > 0, T_h(0) > 0, R_h(0) > 0, E_v(0) > 0$ then $t_0 > 0$, if $t_0 < \infty$, then $S_h, V_h, E_h, I_h, T_h, R_h, E_v$ is equal to zero at t_0. From the first equation of model system (1), given by

$$\frac{dS_h}{dt} = (1 - \phi)HN + (1 - \rho)\lambda - \frac{\beta_1 c_1 S_h I_h}{N} - \frac{\beta_2 c_2 S_h E_v}{A + E_v} - \mu S_h + \tau V_h. \quad (2)$$

Thus,

$$\frac{d}{dt} S_h(t) exp\left\{\left[\frac{\beta_1 c_1 S_h I_h}{N} + \frac{\beta_2 c_2 S_h E_v}{A + E_v}\right]\right\} = (1-\phi)HN + (1-\rho)\lambda\, exp\left[\frac{\beta_1 c_1 I_h}{N} - \frac{\beta_2 c_2 E_v}{A + E_v}\right]t,$$

$$(3)$$

Hence,

$$S_h(t_0) exp\left[\frac{\beta_1 c_1 I_h}{N} + \frac{\beta_2 c_2 E_v}{A + E_v}\right] - S_h(0) = \int_0^{t_0} (1-\phi)HN + (1-\rho)\lambda exp\left[-\left(\frac{\beta_1 c_1 I_h}{N} + \frac{\beta_2 c_2 E_v}{A + E_v}\right)P\right]dP \geq 0,$$

$$(4)$$

The same procedure can be shown for $S_h > 0, V_h > 0, E_h > 0, I_h > 0, T_h > 0, R_h > 0, E_v > 0$ for time $t > 0$, which implies that the solutions of (1) are positive.

In addition, adding the total human host population in model system equations in (1) in the absence of VZV infections yields

$$\frac{dN(t)}{dt} = (1 - \phi)HN + (1 - \rho)\lambda - \mu N \tag{5}$$

So that,

$$\frac{dN(t)}{dt} \leq (1 - \phi)HN + (1 - \rho)\lambda - \mu N. \tag{6}$$

and

$$0 \leq \lim_{t \to \infty} inf N(t) \leq \lim_{t \to \infty} sup N(t) \leq \frac{(1 - \phi)HN + (1 - \rho)\lambda}{\mu}. \tag{7}$$

The invariant region of (1) is the feasible domain where the solution of (1) is contained, given by,

$$\Omega_1 = \left\{ (S_h, V_h, E_h, I_h, T_h, R_h) \in R_+^7 : S_h, V_h, E_h, I_h, T_h, R_h \leq \frac{(1 - \phi)HN + (1 - \rho)\lambda}{\mu} \right\} \tag{8}$$

Hence, (8) is well-posed and reasonable in the sense of VZV dynamics, so that model (1) solution starts and remain in Ω_1.

3.2 VZV-Free Equilibrium Solution and Basic Transmission Threshold R_{vzv}

The equilibrium solutions of model system (1) is obtained by fixing the right hand side of model system (1) to zero. The time independent solutions in the absence of VZV infection is given by,

$$W^0 = (S_h^o, V_h^o, E_h^o, I_h^o, T_h^o, R_h^o, E_v^o) = $$
$$\left(\frac{(1 - \phi)HN + (1 - \rho)\lambda}{\mu}, \frac{\rho\lambda + \phi HN}{(\mu + \tau)}, 0, 0, 0, 0, 0 \right). \tag{9}$$

The basic reproduction number threshold R_{vzv} of model system (1) is the rate of average secondary cases of VZV generated when a primary VZV infected individual is introduced into a large susceptible human population during his or her infection period. The next generation matrix operator approach [1, 7], was used to obtain the value of R_{vzv}. Therefore, R_{vzv} of the model system (1)

9

is given by,

$$R_{vzv} = \frac{\beta_1 c_1((1-\phi)HN + (1-\rho)\lambda)\eta_1\alpha K}{N\mu(\alpha K(\mu(\mu + \eta_1 + \eta_2)) - K(\mu^2 + \mu(\eta_1 + \eta_2) + \eta_1 + \eta_2) + \mu^3 + \mu^2(\eta_1 + \eta_2) + \mu\eta_1\eta_2)} \quad (10)$$

The R_{vzv} obtained in (10) is called a vaccination controlled basic reproduction number, where the susceptible birth and influx of immigrant are vaccinated, such that $\phi > 1 - \frac{1}{R_{vzv}}$ and $\rho > 1 - \frac{1}{R_{vzv}}$ leads to a herd immunity level. If $R_{vzv} < 1$, VZV is minimized in the host community and if $R_{vzv} > 1$, VZV infections continues in the population and becomes endemic.

(a)

(b)
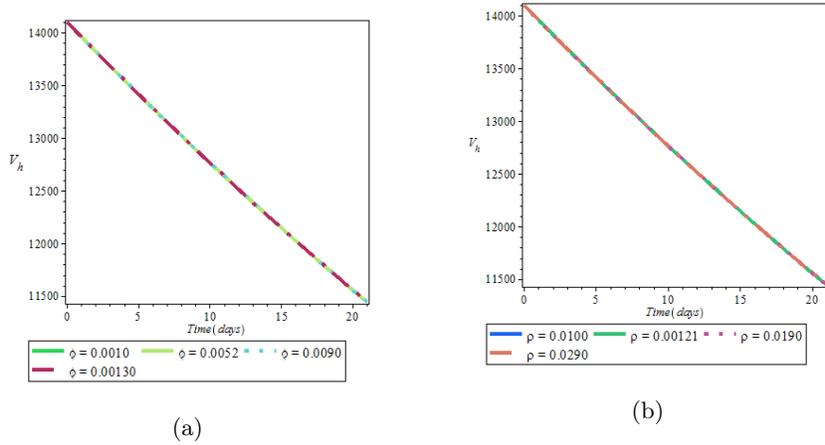
Figure (2)   The impact of the variation of vaccination parameters on the sub-population of vaccinated newborns and immigrants.

3.3   Local and Global Asymptotic Behavior of the Model

Theorem 2. The VZV-free equilibrium (9) of model system (1) solution is locally asymptotically stable when $R_{vzv} < 1$.

Proof. Linearizing the model around the VZV - free equilibrium solutions in

(9), one obtains the Jacobian given by

$$J_c = \begin{pmatrix} -\mu & \tau & 0 & -\frac{\beta_1 c_1 S_h}{N} & 0 & 0 & 0 \\ 0 & -(\mu + \tau) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -(\mu + \delta) & \frac{\beta_1 c_1 S_h}{N} & 0 & 0 & 0 \\ 0 & 0 & 0 & -(\eta_1 + \mu) & 0 & 0 & 0 \\ 0 & 0 & 0 & \eta_1 & -(\eta_2 + \mu) & 0 & 0 \\ 0 & 0 & 0 & 0 & \eta_2 & -\mu + (1-\alpha)K_o & 0 \\ 0 & 0 & 0 & \sigma & 0 & 0 & -\mu_v \end{pmatrix} \tag{11}$$

It is observed that the real part of (11) is negative. The eigenvalues of (11) are given by $\mu, -\mu_v, -(\mu + \tau), -(\mu + \delta)$ which reduces (11) to another 3×3 matrix given by

$$\begin{pmatrix} -q_1 & 0 & 0 \\ \eta_1 & -q_2 & 0 \\ 0 & \eta_2 & -q_3 \end{pmatrix}, \tag{12}$$

where $q_1 = -(\eta_1 + \mu), q_2 = -(\eta_2 + \mu)$ and $q_3 = -\mu + (1-\alpha)K_o$. The characteristic equations of (12) yields

$$\lambda^3 + A_1 \lambda^2 + A_2 \lambda + A_3, \tag{13}$$

where

$$A_1 = (q_1 + q_2 - q_3),$$
$$A_2 = (q_2 q_3 + q_1 q_3 - q_1 q_3), \tag{14}$$
$$A_3 = q_3 q_2 q_1 (1 - R_{vzv})$$

Going by the Routh - Hurwitz conditions, $A_i > 0$ for $i = 1, 2, 3$ and $A_1 A_2 - A_3 > 0$ could be verified easily. Also, $(1 - R_{vzv}) > 0 \Leftrightarrow R_{vzv} < 1$. Thus, the VZV - free equilibrium (9) of model system (1) is locally asymptotically stable.

Theorem 2. The VZV - free equilibrium (9) of model system (1) is globally asymptotically stable when $R_{vzv} < 1$.

Proof. Let $\frac{dX_1}{dt} = F_1(X_1, Z_1)$, $\frac{dZ_1}{dt} = G_1(X_1, Z_1)$, where $X_1 = (S_h, V_h, T_h, R_h)$ denote the sub-populations without VZV infections, i.e., $X \in R^{+4}$. and $Z_1 =$

(E_h, I_h, E_v) denote the compartment with VZV infections. The VZV - free equilibrium solution is denoted as $W^0 = (X_1^*, Z_1^*) = (X_1^*, 0)$, where $X_1^* = \left(\frac{(1-\phi)HN+(1-\rho)\lambda}{\mu}, \frac{\rho\lambda+\phi HN}{(\mu+\tau)}\right)$, the following two conditions are guaranteed

- $X_1^* = F_1(X_1, 0), X_1^*$ is globally asymptotically stable

- $G_1(X_1, Z_1) = LZ_1 - G_1(X_1, Z_1) \geq 0$ for all $(X_1, Z_1) \in \Omega_1$, which is a domain of relevance of model system (1).

Therefore,

$$F_1(X_1, 0) = \begin{pmatrix} (1-\phi)HN + (1-\rho)\lambda - \mu S_h + \tau V_h \\ \rho\lambda + \phi HN - (\mu+\tau)V_h - (\eta_2+\mu)T_h \\ \eta_2 T_h + (1-\alpha)KR_h - \mu R_h \end{pmatrix} \tag{15}$$

and

$$G_1(X_1, Z_1) = \begin{pmatrix} \frac{\beta_1 c_1 S_h I_h}{N} + \frac{\beta_2 c_2 S_h E_v}{(A+E_v)} - (\mu+\delta)E_h \\ \delta E_h - (\eta_1+\mu)I_h + \alpha KR_h \\ \sigma I_h - \mu_v E_v \end{pmatrix} \tag{16}$$

In the absence VZV infections, $G_1(X_1, 0) = 0$. Also, let

$$L = \begin{pmatrix} -\mu & \tau & 0 & -\frac{\beta_1 c_1 S_h^o}{N} & 0 & 0 & 0 \\ 0 & -(\mu+\tau) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -(\mu+\delta) & -\frac{\beta_1 c_1 S_h^o}{N} & 0 & 0 & 0 \\ 0 & 0 & \delta & -(\eta_1+\mu) & 0 & \alpha K_o & 0 \\ 0 & 0 & 0 & \eta_1 & -(\eta_2+\mu) & 0 & 0 \\ 0 & 0 & 0 & 0 & \eta_2 & -(\mu+(1-\alpha)K_o) & 0 \\ 0 & 0 & 0 & \sigma & 0 & 0 & -\mu_v \end{pmatrix}, \tag{17}$$

then $G_1(X_1, Z_1)$ can be re-written as $G_1(X_1, Z_1) = LZ_1 - \hat{G}_1(X_1, Z_1)$, where

$$\hat{G}_1(X_1,\ Z_1) = \begin{pmatrix} \frac{\beta_1 c_1 (S_h - S_h^o)(I_h - I_h^o)}{N\mu} + \frac{\beta_2 c_2 (S_h - S_h^o)(E_v - E_v^o)}{(A+(E_v - E_v^o))} - (\mu + \delta)(E_h - E_h^o) \\ \delta(E_h - E_h^o) - (\eta_1 + \mu)(I_h - I_h^o) + \alpha K(R_h - R_h^o) \\ \sigma(I_h - I_h^o) - \mu_v(E_v - E_v^o) \end{pmatrix}$$

$$(18)$$

Conclusively, $X_1^* = \left(\frac{(1-\phi)HN+(1-\rho)\lambda}{\mu},\ \frac{\rho\lambda+\phi HN}{(\mu+\tau)} \right)$ is a global asymptotic stable equilibrium point of (15), where the solution is given by $S_h = \frac{(1-\phi)HN+(1-\rho)\lambda}{N\mu} + \left(S_h(0) - \frac{(1-\phi)HN+(1-\rho)\lambda}{N\mu} \right) e^{-\mu t}$ converges to X_1^* as $t \to \infty$. This implies the global convergence of (15) in Ω_1. From (15) and (17), the two conditions are satisfied, that is, $G_1(X_1,\ 0)$ and $G_1(X_1,\ Z_1) = LZ_1 - \hat{G}_1(X_1,\ Z_1) \geq 0$, thus the model system (1) at the VZV - free equilibrium (9) is globally asymptotically stable.
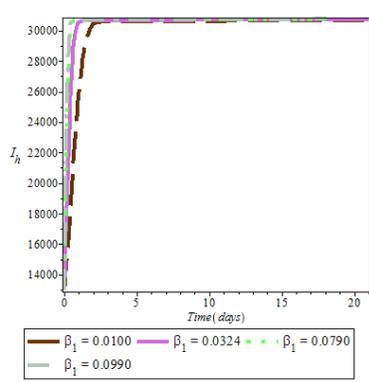
(a)
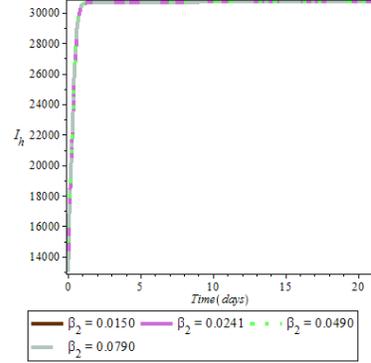
(b)
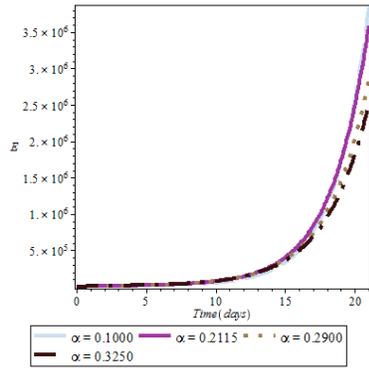
(c)
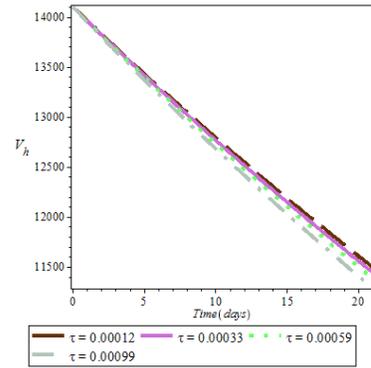
(d)

Figure (3)   The impact of the variations of parameter values of contact rate, human infectious contribution and loss of immunity

4   Method of Solution

Here, a modified differential transform method (LPDTM) is employed to obtain the approximate solutions of the model system equations (1). The DTM is used to obtain the truncated series solution. Therefore the LPDTM is applied to enlarge the convergence domain of the truncated series solution. The use of this method is highlighted step by step.

14

4.1 Differential Transform Method (DTM)

Definition 1. If a function $u(t)$ is differentiable with respect to t, then

$$U(k) = \frac{1}{k!}\left[\frac{d^k u(t)}{dt^k}\right]_{t=0}. \tag{19}$$

Definition 2. The differential inverse of set $\{U(k)\}_{k=0}^n$ is given by

$$u(t) = \sum_{k=0}^{\infty} U(k)t^k. \tag{20}$$

Substituting (19) into (20) yields

$$u(t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{d^k u(t)}{dt^k}\right]_{t=0} t^k \tag{21}$$

From (19) to (21), the concept of DTM is derived. Further examples on application of DTM to models and the derivation of the basic properties involving DTM can be seen in [13, 14, 16, 17].

<div align="center">

Table (2) Some Basic Operations of DTM

</div>

Original Function	Transformed Function
$\alpha_1 u(t) \pm \alpha_2 v(t)$	$\alpha_1 U(t) \pm \alpha_2 V(t)$
$u(t)v(t)$	$\sum_{l=0}^{k} U(k)V(k-r)$
$\frac{d}{dt}[u(t)]$	$(k+1)U(k+1)$
t^n	$\delta(k-n) = 1$, when $k = n$ and 0, when $k \neq n$
$t^n u(t)$	$U(k-n)$
$e^{\lambda t}$	$\frac{\lambda^k}{k!}$
$\sin(\omega t + \alpha_1)$	$\frac{\omega^k}{k!}\sin\left(\frac{\pi k}{2} + \alpha_1\right)$
$\cos(\omega t + \alpha_1)$	$\frac{\omega^k}{k!}\cos\left(\frac{\pi k}{2} + \alpha_1\right)$

4.2 Pade Approximant

Given an analytical function $u(t)$ with Mclaurin series expansion

$$u(t) = \sum_{n=0}^{\infty} u_n t^n, \ 0 \le t \le T \tag{22}$$

The Pade approximant to $u(t)$ of order $[q_1, \ q_2]$, which is represented by $\left[\frac{q_1}{q_2}\right]_u(t)$, so that

$$\left[\frac{q_1}{q_2}\right]_u(t) = \frac{r_0 + r_1 t + .. + r_{q_1} t^{q_1}}{1 + s_1 t + .. + s_{q_2} t^{q_2}}, \tag{23}$$

where $s_1 = 1$ and the numerator and denominator possess no common factors. The numerator and denominator in (23) are developed so that $u(t)$ and $\left[\frac{q_1}{q_2}\right]_u (t)$ and their derivatives agree at $t = 0$ up to $q_1 + q_2$, that is,

$$u(t) - \left[\frac{q_1}{q_2}\right]_u (t) = 0(t^{q_1+q_2+1}). \tag{24}$$

From (24), we obtain the following algebraic system given by

$$u_{q_1} s_1 + ... + u_{q_1-q_2+1} s_n = -u_{q_1+1}$$

$$u_{q_1+1} s_1 + ... + u_{q_1-q_2+2} s_n = -u_{q_1+2}$$

$$. \tag{25}$$

$$.$$

$$.$$

$$u_{q_1+q_2-1} s_1 + ... + u_{q_1} s_n = -u q_1 + q_2$$

and

$$r_o = u_o$$

$$r_1 = u_1 + u_o s_1$$

$$. \tag{26}$$

$$.$$

$$.$$

$$r_{q_1} = u_n + u_{q_1-1} s_1 + ... + u_o s_{q_1}$$

From (25), we compute for all the coefficients s_n, $1 \leq n \leq q_1$. Then, the coefficients r_n can be determined, so that $0 \leq n \leq q_2$ from (26) Note that for a fixed value of $q_1 + q_2 + 1$, error in (24) is smallest when the numerator and denominator of (23) possess the same degree or when the numerator possess a degree one higher than the denominator.

4.3 Laplace - Pade Posttreatment Technique

Laplace - Pade posttreatment technique is used to widen the domain of convergence of solutions or to obtain exact solutions. The procedure governing the Laplace - Pade technique is outlined below.

- Apply DTM to the given model system (1)

- Perform desirable number of iterations and obtain the solutions in power series form.

- The Laplace transform is applied to the power series solutions obtained using DTM

- Next, s is substituted by $\frac{1}{z}$ in the resulting equation.

- The transformed series obtained is converted into a meromorphic function by forming its Pade approximant of order $\frac{G}{H}$, where G and H are arbitrarily chosen, but they should be smaller than the order of the power series. In this process, the Pade approximant extends the domain of the truncated series solution to obtain better accuracy and convergence.

- Then, z is substituted by 1/s.

- Finally, by using the inverse Laplace transformation, the exact or approximate solution is obtained.

Therefore, applying DTM to the formulated model system (1) yields following equations

$$S_h(k+1) = \frac{1}{(k+1)}\Big((1-\phi)HN + (1-\epsilon)\lambda - \frac{\beta_1 c_1}{N}\Big(\sum_{l=0}^{k} S_h(k)I_h(k-l)\Big)$$

$$- \beta_2 c_2\Big(\sum_{l=0}^{k} S_h(k)\frac{E_v(k-l)}{A+E_v(k)}\Big) - \mu S_h(k) + \tau V_h(k)\Big),$$

$$V_h(k+1) = \frac{1}{(k+1)}(\rho\lambda + \phi HN - (\mu+\tau)V_h(k)),$$

$$E_h(k+1) = \frac{1}{(k+1)}\Big(\frac{\beta_1 c_1}{N}(\sum_{l=0}^{k} S_h(k)I_h(k-l))+$$

$$\beta_2 c_2\Big(\sum_{l=0}^{k} S_h(k)\frac{E_v(k-l)}{A+E_v}\Big) - (\mu+\delta)E_h(k)\Big),$$

$$I_h(k+1) = \frac{1}{(k+1)}(\delta E_h(k) - (\eta_1+\mu)I_h(k) + \alpha K_o R_h(k)),$$

$$T_h(k+1) = \frac{1}{(k+1)}(\eta_1 I_h(k) - (\eta_2+\mu)T_h(k)),$$

$$R_h(k+1) = \frac{1}{(k+1)}(\eta_2 T_h(k) + (1-\alpha)K_o R_h(k) - \mu R_h(k)),$$

$$E_v(k+1) = \frac{1}{(k+1)}(\sigma I_h(k) - \mu_v E_v(k)).$$

$$(27)$$

Using the following assumed initial conditions $S_h = 0.1750, V_h = 0.950, E_h = 0.1000, I_h = 0.1200, T_h = 0.850, R_h = 0.1500, E_v = 0.1000$ and the parameter values given in Table 1, with the aid of computational software MAPLE, yields the tenth order series solution approximations for each sub-population of system of equations in (27) as follows;

$$S_h(t) = \sum_{k=0}^{10} S_h(k)t^k = 0.1750 + 0.000620103333t + 0.002202535436t^2+$$

$$0.001443985993t^3 + 0.001091823929t^4 + 0.0008767035118t^5 \qquad (28)$$

$$+ 0.0007322445605t^6 + 0.000628592248t^7 + 0.0005506172475t^8$$

$$+ 0.0004898381148t^9 + 0.0004411353066t^{10},$$

$$V_h(t) = \sum_{k=0}^{10} V_h(k)t^k = 0.950 - 0.009424616400t + 0.00005123502040t^2$$

$$+ 0.000002791612082t^3 + 0.000002213969823t^4 + 0.000001772323056t^5 \quad (29)$$

$$+ 0.000001477666805t^6 + 0.000001266989538t^7 + 0.000001108877349t^8$$

$$+ 9.858432053 \ 10^{-7}t^9 + 8.873810577 \ 10^{-7}t^{10},$$

$$E_h(t) = \sum_{k=0}^{10} E_h(k)t^k = 0.1000 - 0.01994310333t + 0.002216145340t^2$$

$$- 0.0001363775366t^3 + 0.00002075219132t^4 + 0.000007082305302t^5 \quad (30)$$

$$0.000005091756240t^6 + 0.000003671273394t^7 + 0.000002776977319t^8 +$$

$$0.000002173005673t^9 + 0.000001746403645t^{10},$$

$$I_h(t) = \sum_{k=0}^{10} I_h(k)t^k = 0.1200 - 0.0278085000t + 0.01362001802t^2$$

$$- 0.001410535693t^3 + 0.0002507779532t^4 - 0.00001896508281t^5 \quad (31)$$

$$+ 0.000002193728463t^6 + 1.263436314 \ 10^{-8}t^7 + 9.670781112 \ 10^{-8}t^8$$

$$+ 5.962821041 \ 10^{-8}t^9 + 4.280278913 \ 10^{-8}t^{10},$$

$$T_h(t) = \sum_{k=0}^{10} T_h(k)t^k = 0.850 - 0.0.0647600t - 0.002386162000t^2 +$$

$$0.002479793068t^3 - 0.0002761139008t^4 + 0.00003434223506t^5 - \quad (32)$$

$$0.000002499906905t^6 + 2.163892677 \ 10^{-7}t^7 - 3.225245702 \ 10^{-9}t^8$$

$$+ 5.641173171 \ 10^{-10}t^9 + 3.016274990 \ 10^{-9}t^{10},$$

$$R_h(t) = \sum_{k=0}^{10} R_h(k)t^k = 0.1500 + 0.1719665000t + 0.02582834541t^2$$

$$+ 0.002928728123t^3 + 0.0003453335540t^4 + 0.0001665695703t^5 \quad (33)$$

$$+ 0.00001781608500t^6 + 3.987383010 \ 10^{-8}t^7 + 5.546793204 \ 10^{-9}t^8$$

$$+ 1.674548611 \ 10^{-10}t^9 + 8.488942299 \ 10^{-11}t^{10},$$

$$E_v(t) = \sum_{k=0}^{10} E_v(k)t^k = 0.1000 - 0.0003200t - 0.0001228982500t^2$$

$$+ 0.00004143357923t^3 - 0.000003318722838t^4 + 4.606927398 \ 10^{-7}t^5 \quad (34)$$

$$+ 2.95225739510^{-8}t^6 + 2.879553173 \ 10^{-9}t^7 + 9.174440485 \ 10^{-12}t^8$$

$$+ 9.669353977 \ 10^{-11}t^9 + 5.353001841 \ 10^{-11}t^{10}$$

The approximate series solutions obtained from (28) - (34) may have restricted domains of convergence. The accuracy is enhanced by applying the Laplace - Pade posttreatment technique earlier described. On applying the Laplace transformations to (28) - (34), yields the following;

$$L[S_h(t)] = \frac{1}{s^{11}}(1.000000000 \ 10^{-12}(1.750000000 \ 10^{11}s^{10} + 6.20103333 \ 10^8 s^9$$

$$+ 4.405070872 \ 10^9 s^8 + 8.663915958 \ 10^9 s^7 + 2.620377430 \ 10^{11}s^6$$

$$+ 1.052044214 \ 10^{11}s^5 + 5.272160836 \ 10^{11}s^4 + 3.168104933 \ 10^{12}s^3$$

$$+ 2.220088741 \ 10^{13}s^2 + 1.777524551 \ 10^{14}s + 1.600791801 \ 10^{15})),$$

$$(35)$$

$$L[V_h(t)] = \frac{1}{s^{11}}(4.000000000 \ 10^{-15}(2.375000000 \ 10^{14}s^{10} - 2.356154100 \ 10^12s^9$$

$$+ 2.561751020 \ 10^{10}s^8 + 4.187418123 \ 10^9 s^7 + 1.328381894 \ 10^{10}s^6$$

$$+ 5.316969168 \ 10^{10}s^5 + 2.659800249 \ 10^{11}s^4 + 1.596406818 \ 10^{12}s^3$$

$$+ 1.117748368 \ 10^{13}s^2 + 8.943569558 \ 10^{13}s + 8.050320955 \ 10^{14})),$$

$$(36)$$

$$L[E_h(t)] = \frac{1}{s^{11}}(8.000000000 \ 10^{-14}(1.250000000 \ 10^{12}s^{10} - 2.492887916 \ 10^{11}s^9$$

$$+ 5.540363350 \ 10^{10}s^8 - 1.022831524 \ 10^{10}s^7 + 6.225657396 \ 10^9 s^6$$

$$+ 1.062345795 \ 10^{10}s^5 + 4.582580616 \ 10^{10}s^4 + 2.312902238 \ 10^{11}s^3$$

$$+ 1.399596569 \ 10^{12}s^2 + 9.856753733 \ 10^{12}s + 7.921686934 \ 10^{13})),$$

$$(37)$$

$$L[I_h(t)] = \frac{1}{s^{11}}(1.600000000 \ 10^{-15}(7.500000000 \ 10^{13}s^{10} - 1.738031250 \ 10^{13}s^9$$

$$+ 1.702502252 \ 10^{13}s^8 - 5.28950849 \ 10^{12}s^7 + 3.761669298 \ 10^{12}s^6$$

$$- 1.422381211 \ 10^{12}s^5 + 9.871778084 \ 10^{11}s^4 + 3.979824389 \ 10^{10}s^3$$

$$+ 2.437036840 \ 10^{12}s^2 + 1.35236812 \ 10^{13}s + 9.707672575 \ 10^{13})),$$

$$(38)$$

$$L[T_h(t)] = \frac{1}{s^{11}}(3.200000000\ 10^{-16}(2.656250000\ 10^{15}s^{10} - 2.023750000\ 10^{14}s^9$$

$$- 1.491351250\ 10^{13}s^8 + 4.649612002\ 10^{13}s^7 - 2.070854256\ 10^{13}s^6$$

$$+ 1.287833815\ 10^{13}s^5 - 5.624790536\ 10^{12}s^4 + 3.408130966\ 10^{12}s^3$$

$$- 4.063809585\ 10^{11}s^2 + 6.397090376\ 10^{12}s + 3.420455839\ 10^{13})),$$

$$(39)$$

$$L[R_h(t)] = \frac{1}{s^{11}}(6.400000000\ 10^{-17}(2.343750000\ 10^{15}s^{10} + 2.686976562\ 10^{15}s^9$$

$$+ 8.071357941\ 10^{14}s^8 + 2.74568261510^{14}s^7 + 1.29500082810^{14}s^6$$

$$+ 3.123179443\ 10^{13}s^5 + 2.004309562\ 10^{13}s^4 + 3.140064120\ 10^{12}s^3$$

$$+ 3.494479719\ 10^{12}s^2 + 9.494690624\ 10^{11}s + 4.813230284\ 10^{12})),$$

$$(40)$$

$$L[E_v(t)] = \frac{1}{s^{11}}(3.200000000\ 10^{-18}(3.125000000\ 10^{16}s^{10} - 1.000000000\ 10^{14}s^9$$

$$- 7.681140625\ 10^{13}s^8 + 7.768796106\ 10^{13}s^7 - 2.489042128\ 10^{13}s^6$$

$$+ 1.72759774\ 10^{13}s^5 - 6.642579139\ 10^{12}s^4 + 4.535296247\ 10^{12}s^3$$

$$+ 1.155979501\ 10^{11}s^2 + 1.096504741\ 10^{13}s + 6.070304088\ 10^{13})),$$

$$(41)$$

Substituting $s = \frac{1}{z}$ in (35) to (41) yields the following;

$$L[S_h(t)] = 0.1750000000z + 0.0006201033330z^2 + 0.004405070872z^3$$

$$+ 0.008663915958z^4 + 0.02620377430z^5 + 0.1052044214z^6$$

$$+ 0.5272160838z^7 + 3.168104933z^8 + 22.20088741z^9$$

$$+ 177.7524551z^{10} + 1600.791801z^{11},$$

$$(42)$$

$$L[V_h(t)] = 0.9500000000z - 0.009424616400z^2 + 0.0001024700408z^3$$

$$+ 0.00001674967249z^4 + 0.00005313527576z^5 + 0.0002126787667z^6$$

$$+ 0.001063920100z^7 + 0.006385627272z^8 + 0.04470993472z^9$$

$$+ 0.3577427823z^{10} + 3.220128382z^{11},$$

$$(43)$$

21

$$L[E_h(t)] = 0.1000000000z - 0.01994310333z^2 + 0.004432290680z^3$$
$$+ 0.0008182652192z^4 + 0.0004980525917z^5 + 0.0008498766360z^6$$
$$+ 0.003666064493z^7 + 0.01850321790z^8 + 0.1119677255z^9$$
$$+ 0.7885402986z^10 + 6.337349547z^{11}, \tag{44}$$

$$L[I_h(t)] = 0.1553227612z^{11} + 0.02163788499z^10 + 0.003899258944z^9$$
$$+ 0.00006367719022z^8 + 0.001579484493z^7 - 0.002275809938z^6$$
$$+ 0.006018670877z^5 - 0.008463214158z^4 + 0.02724003603z^3$$
$$- 0.02780850000z^2 + 0.1200000000z, \tag{45}$$

$$L[T_h(t)] = 0.8500000000z - 0.06476000000z^2 + 0.004772324000z^3$$
$$+ 0.01487875841z^4 - 0.006626733619z^5 + 0.004121068208z^6 -$$
$$0.001799932972z^7 + 0.001090601909z^8 - 0.0001300419067z^9$$
$$+ 0.002047068920z^{10} + 0.01094545868z^{11}, \tag{46}$$

$$L[R_h(t)] = 0.1500000000z - 0.1719665000z^2 + 0.05165669082z^3$$
$$+ 0.01757236874z^4 + 0.008288005299z^5 + 0.001998834844z^6 +$$
$$0.001282758120z^7 + 0.0002009641037z^8 + 0.0002236467020z^9$$
$$+ 0.00006076601999z^{10} + 0.000308046382z^{11}, \tag{47}$$

$$L[E_v(t)] = 0.0001942497308z^{11} + 0.00003508815171z^{10} + 3.699134403 \ 10^{-7}z^9$$
$$+ 0.00001451294799z^8 - 0.00002125625324z^7 + 0.0000528312877z^6$$
$$- 0.00007964934810z^5 + 0.0002486014754z^4 - 0.000245796500z^3$$
$$- 0.0003200000000z^2 + 0.1000000000z. \tag{48}$$

Again, substituting $z = \frac{1}{s}$, we compute the $\left[\frac{4}{4}\right]$ Pade approximant of (42) to (48) to obtain the following

$$\left[\frac{4}{4}\right]_{S_h} =$$

$$\frac{0.1750000z}{1.000000000 - 14.51158393z + 54.13271792z^2 - 42.36828055z^3 - 11.79047117z^4}$$

$$- \frac{2.538907085z^2}{1.000000000 - 14.51158393z + 54.13271792z^2 - 42.36828055z^3 - 11.79047117z^4}$$

$$+ \frac{9.468632026z^3}{1.000000000 - 14.51158393z + 54.13271792z^2 - 42.36828055z^3 - 11.79047117z^4}$$

$$- \frac{7.436176858z^4}{1.000000000 - 14.51158393z + 54.13271792z^2 - 42.36828055z^3 - 11.79047117z^4}$$

$$- \frac{1.950670201z^5}{1.000000000 - 14.51158393z + 54.13271792z^2 - 42.36828055z^3 - 11.79047117z^4},$$

$$(49)$$

$$\left[\frac{4}{4}\right]_{V_h}$$

$$= \frac{0.9499999999z}{0.9999999998 - 14.97506834z + 59.70082036z^2 - 59.10246230z^3 - 0.5983573139z^4}$$

$$- \frac{14.23573954z^2}{0.9999999998 - 14.97506834z + 59.70082036z^2 - 59.10246230z^3 - 0.5983573139z^4}$$

$$+ \frac{56.85701609z^3}{0.9999999998 - 14.97506834z + 59.70082036z^2 - 59.10246230z^3 - 0.5983573139z^4}$$

$$- \frac{56.71151428z^4}{0.9999999998 - 14.97506834z + 59.70082036z^2 - 59.10246230z^3 - 0.5983573139z^4}$$

$$- \frac{0.005501559544z^5}{0.9999999998 - 14.97506834z + 59.70082036z^2 - 59.10246230z^3 - 0.5983573139z^4},$$

$$(50)$$

$$\left[\frac{4}{4}\right]_{E_h} =$$

$$\frac{0.1000000000z}{1.000000000 - 14.33716523z + 52.81052051z^2 - 40.65776383z^3 - 11.51830894z^4}$$

$$- \frac{1.453659627z^2}{1.000000000 - 14.33716523z + 52.81052051z^2 - 40.65776383z^3 - 11.51830894z^4}$$

$$+ \frac{5.571411910z^3}{1.000000000 - 14.33716523z + 52.81052051z^2 - 40.65776383z^3 - 11.51830894z^4}$$

$$- \frac{5.183346800z^4}{1.000000000 - 14.33716523z + 52.81052051z^2 - 40.65776383z^3 - 11.51830894z^4}$$

$$- \frac{0.09468767487z^5}{1.000000000 - 14.33716523z + 52.81052051z^2 - 40.65776383z^3 - 11.51830894z^4},$$

$$(51)$$

$$\left[\frac{4}{4}\right]_{I_h}$$

$$= \frac{0.1200000000z}{1.000000000 - 2.616947449z - 9.254596881z^2 - 0.5982149832z^3 + 1.582317721z^4}$$

$$- \frac{0.3418421940z^2}{1.000000000 - 2.616947449z - 9.254596881z^2 - 0.5982149832z^3 + 1.582317721z^4}$$

$$- \frac{1.010538206z^3}{1.000000000 - 2.616947449z - 9.254596881z^2 - 0.5982149832z^3 + 1.582317721z^4}$$

$$+ \frac{0.1058217024z^4}{1.000000000 - 2.616947449z - 9.254596881z^2 - 0.5982149832z^3 + 1.582317721z^4}$$

$$- \frac{0.01741550710z^5}{1.000000000 - 2.616947449z - 9.254596881z^2 - 0.5982149832z^3 + 1.582317721z^4}, \tag{52}$$

$$\left[\frac{4}{4}\right]_{T_h}$$

$$= \frac{0.8500000000z}{0.9999999999 - 3.294825574z - 1.163710935z^2 - 0.5451756029z^3 + 0.09324692916z^4}$$

$$- \frac{2.865361738z^2}{0.9999999999 - 3.294825574z - 1.163710935z^2 - 0.5451756029z^3 + 0.09324692916z^4}$$

$$- \frac{0.7805537144z^3}{0.9999999999 - 3.294825574z - 1.163710935z^2 - 0.5451756029z^3 + 0.09324692916z^4}$$

$$+ \frac{0.5693639163z^4}{0.9999999999 - 3.294825574z - 1.163710935z^2 - 0.5451756029z^3 + 0.09324692916z^4}$$

$$- \frac{0.006141723955z^5}{0.9999999999 - 3.294825574z - 1.163710935z^2 - 0.5451756029z^3 + 0.09324692916z^4}, \tag{53}$$

$$\left[\frac{4}{4}\right]_{R_h} =$$

$$\frac{0.1500000000z}{1.000000000 - 10.61523753z - 3.153979011z^2 + 1.751414950z^3 + 0.2961675194z^4}$$

$$- \frac{1.420319130z^2}{1.000000000 - 10.61523753z - 3.153979011z^2 + 1.751414950z^3 + 0.2961675194z^4}$$

$$- \frac{2.246905406z^3}{1.000000000 - 10.61523753z - 3.153979011z^2 + 1.751414950z^3 + 0.2961675194z^4}$$

$$+ \frac{0.8104421638z^4}{1.000000000 - 10.61523753z - 3.153979011z^2 + 1.751414950z^3 + 0.2961675194z^4}$$

$$- \frac{0.004438845357z^5}{1.000000000 - 10.61523753z - 3.153979011z^2 + 1.751414950z^3 + 0.2961675194z^4}, \tag{54}$$

$$\left[\frac{4}{4}\right]_{E_v}$$

$$= \frac{0.1000000000z}{1.000000000 - 3.335431054z - 0.6790590667z^2 + 0.6136657858z^3 + 0.004050155030z^4}$$

$$- \frac{0.3338631053z^2}{1.000000000 - 3.335431054z - 0.6790590667z^2 + 0.6136657858z^3 + 0.004050155030z^4}$$

$$- \frac{0.06708436522z^3}{1.000000000 - 3.335431054z - 0.6790590667z^2 + 0.6136657858z^3 + 0.004050155030z^4}$$

$$+ \frac{0.06265231624z^4}{1.000000000 - 3.335431054z - 0.6790590667z^2 + 0.6136657858z^3 + 0.004050155030z^4}$$

$$- \frac{0.0005332896357z^5}{1.000000000 - 3.335431054z - 0.6790590667z^2 + 0.6136657858z^3 + 0.004050155030z^4}.$$
(55)

Finally, applying the inverse Laplace transforms to the Padé approximants solutions in (49) - (55) yields the following approximate solutions given by the following;

$$S_h(t) = 0.008021267759e^{-0.2154814030t} + 0.0014782625e^{1.428139387t}$$
$$+ 0.0000554751e^{4.220069807t} + 3.593 \ 10^{-7}e^{9.078856139} + 0.1654446352,$$

$$V_h(t) = 0.9408028175e^{-0.01002234779t} + 0.000002642e^{1.512421411t}$$
$$+ 1.00 \ 10^{-7}e^{4.306612083t} + 1.0 \ 10^{-9}e^{9.166057196t} + 0.009194438534,$$

$$E_h(t) = 0.09174529798e^{-0.2179141521t} + 0.00003359144e^{1.412673919t}$$
$$+ 4.861 \ 10^{-7}e^{4.170294470t} + 1.610 \ 10^{-9}e^{8.972110993t} + 0.008220622955,$$

$$I_h(t) = 0.00000102380e^{-1.886000729t} + 0.08826015413e^{-0.4930913575t}$$
$$+ 0.04274513347e^{0.3676131923t} + 1.622 \ 10^{-8}e^{4.628426343t} - 0.01100632753,$$

$$T_h(t) = -0.1289348104e^{-0.4967191290t} + 0.9993346520e^{-0.1453263101t}$$
$$+ 0.0454653058e^{0.2612685710t} + .27 \ 10^{-8}e^{3.575602442t} - 0.06586516050,$$

$$R_h(t) = 0.0219625305e^{-0.4938390223t} - 0.2724635881e^{-0.1494023376t}$$
$$+ 0.3855134406e^{0.3686142756t} - 1.0 \ 10^{-10}e^{10.88986461t} - 0.01498761703,$$

$$E_v(t) = 0.00161998552e^{-0.4959164711t} + 0.2321791195e^{-0.006553937959t}$$
$$+ 0.00111227874e^{0.3581018392} + 3.1 \ 10^{-10}e^{3.479799624t} - 0.1316714130.$$

$$(56)$$

5 Discussion of Results and Graphical Illustrations

In this work, a mathematical model describing VZV dynamics have been derived. The analytical findings reveal that the model solutions are positive, bounded and realistic epidemiologically. The controlled R_{vzv} showed that vaccination of susceptible births and immigrants, treatment are effective in minimizing VZV infections in human and environmental host community. Linearizing the model around the VZV - free equilibrium by obtaining the model Jacobian showed that the model system is locally and globally asymptotically stable when R_{vzv} is less than unity.

However, we use the DTM to solve the model system equations, yielding a recursive system of power series solutions. In order to enhance the convergence of the DTM solutions, an hybrid post-treatment Laplace Pade DTM (LPDTM) is employed to widen the domain of convergence of the solutions. Furthermore, we compared the results using the LPDTM technique with embedded Fehlberg Runge - Kutta fourth order method via Maple computational software.

Tables 1 and 2 displays the definition of parameters, values and their respective sources and basic operations of DTM. Figure 1 displays the diagrammatic representation of the epidemic interactions among sub - population of human individuals and the environment. Furthermore, Figures 4(a) - 4(d) and 5(a) - 5(c) reveals the favorable agreement between the two methods in obtaining the approximate solutions of the model equations while producing a very low approximation error for the approximate solutions of the model system equations. The simulations further reveal that DTM and the Laplace Pade post-treatment technique reduces the volume of computation and enhances the efficiency of the technique.

Figures 2(a) and 2(b) shows the impact of timely vaccination of susceptible newborns and influx varying ϕ and ρ. The drastic decline in both figures showed that drastic and timely vaccination is effective in creating a level herd immunity

and minimizing VZV infection in the host community. Also, Figures 3(a) - 3(c) displays the effect of the varying the direct and indirect infectious contact rate between human to human β_1 and human to environment β_2 as well as infectious contribution of infected humans to environmental sources. The gradual rise in the curve depict that further interventions strategies are needed to be forestalled in order to curtail the spread of VZV infections, while the gradual decline in Figure 3(c) describing the variation of vaccination rate τ in vaccinated individuals reveal that vaccination wanes overtime in vaccinated individuals.
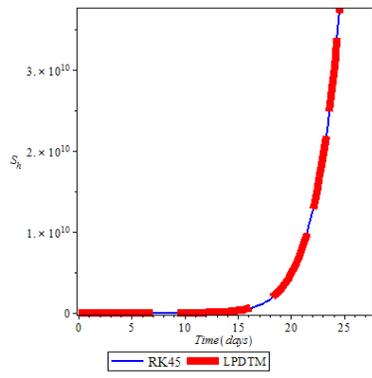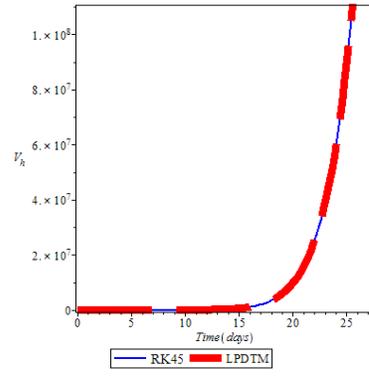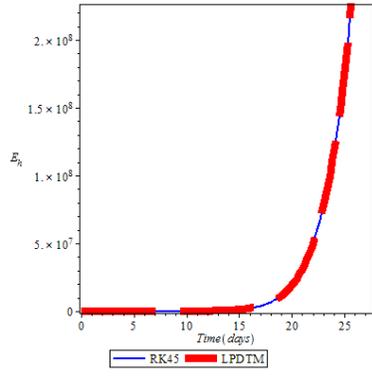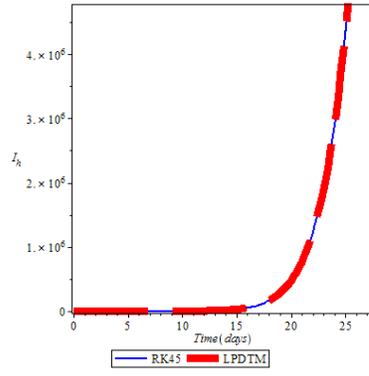
Figure (4)   Approximate solutions using LPDTM and RK45
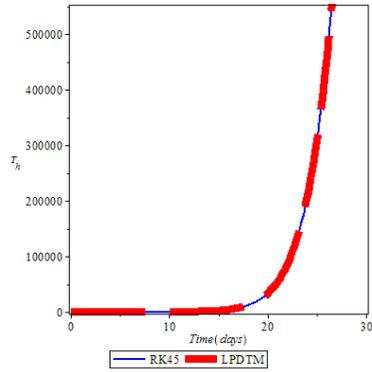
(a)
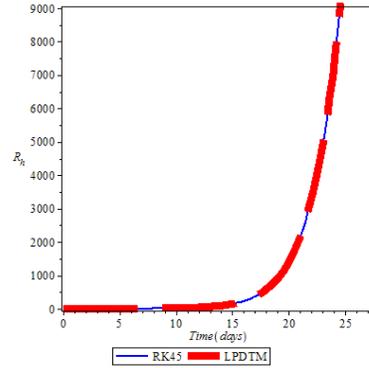
(b)
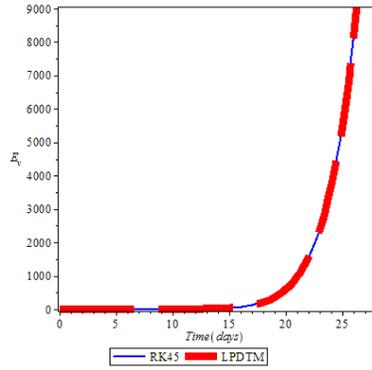
(c)

Figure (5)   Approximate solutions using LPDTM and RK45

5.1   Conclusion and Recommendations

This work proposes a mathematical model formulation of VZV dynamics incorporating essential variables and parameters. The model was shown to be positive, bounded and exist in the reasonable domain of transmission. The model is asymptotically stable locally and globally when R_{vzv} is less than unity. DTM, being an efficient tool was employed to solve the model equations, since it doesn't need a perturbation parameter to work or derive noise terms like HPM, HAM, VIM etc. The Laplace Pade post-treatment technique further enhances the convergence of the model solutions using DTM. This method is recom-

mended to scientist and engineers to solve highly nonlinear models. Also, forms of interventions like timely vaccination and treatment are to be forestalled by public health practitioners to minimize VZV infections. This work also suggest application of further controls to minimize reactivation of infectious shingles and concentration of the virus in environmental sources.

6   References

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