NEWTON'S FORMULA AND A DIFFERENT VERSION OF LAGRANGE INTERPOLATION FORMULA

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ABSTRACT : Finding the n^{th} term of a sequence is one of the most common questions in Algebra. This article introduces an original and alternative formula to calculate any term of any degree sequence. The examples are solved by using this formula first, then the answers are confirmed by using either Newton's Formula or Lagrange Interpolation Formula. Both the formula and its proof take advantage of Pascal's triangle.

Keywords : Sequence, pattern, Newton's Interpolation formula, Lagrange Interpolation formula, explicit.

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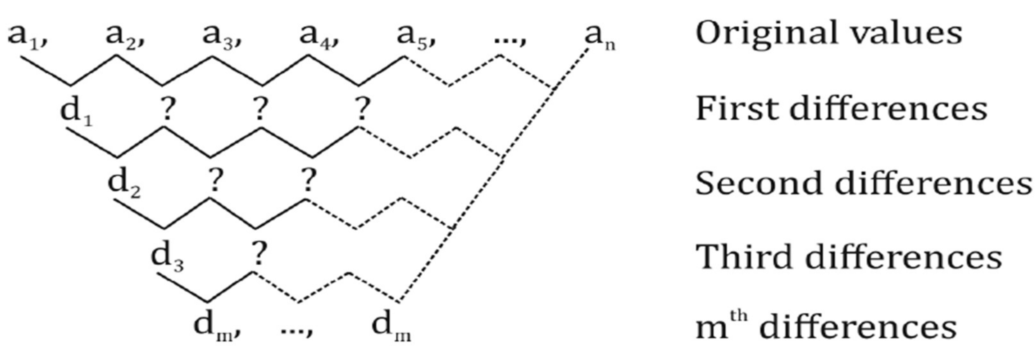
1 INTRODUCTION

I was a senior student in Erciyes University, Turkey in 2014. There was only one month left before I graduated and I was studying for Number Theory final exam in the university's library. While I was working on the sequences and series, I discovered a pattern between perfect squares. This pattern was $f(x)=2f(x-1)-f(x-2)+2$. For example, $5^2=2(4^2)-3^2+2$ and$6^2=2(5^2)-4^2+2$. I was so excited and immediately went to the professor's office to share my little discovery with him who taught me Number Theory. He advised me to search up formulas about sequences and series to see whether it was discovered before or not. I have continued studying and searching sequences and series for four years until I *came up with an explicit formula which allows to calculate any term of any degree sequences. Later on, I learned that my formula is a different version of Lagrange Interpolation formula that is discovered by an italian mathematician an astronomer, Joseph-Louis Lagrange.*

2 NEWTON'S FORMULA

Newton's formula allows us to calculate the n^{th} term of a sequence. The following diagram will be used for both presenting Newton's formula and developing proof of it. [1]

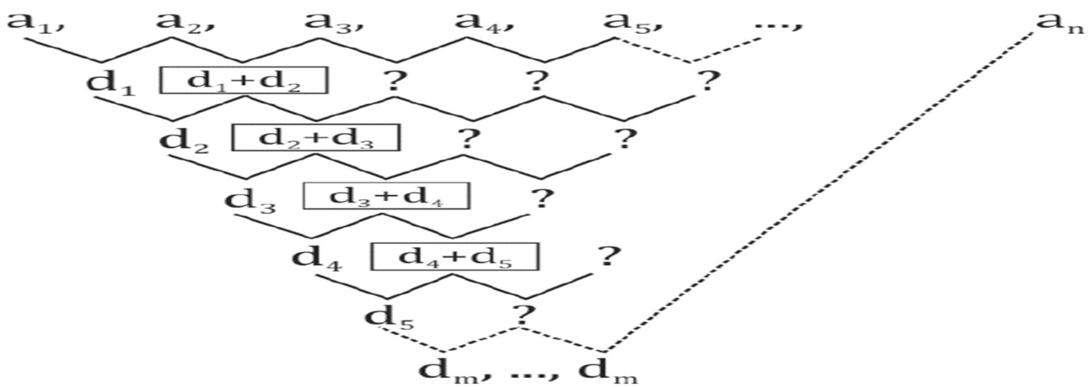


Newton's formula for the n^{th} term:

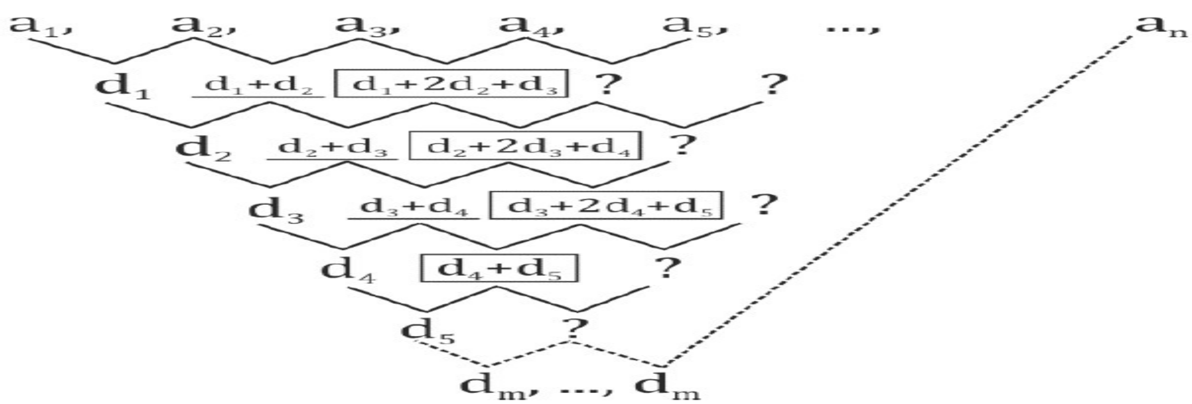
$$a_n = a_1\binom{n-1}{0} + d_1\binom{n-1}{1} + d_2\binom{n-1}{2} + \ldots + d_m\binom{n-1}{m}$$

3 DEVELOPING PROOOF OF NEWTON'S FORMULA

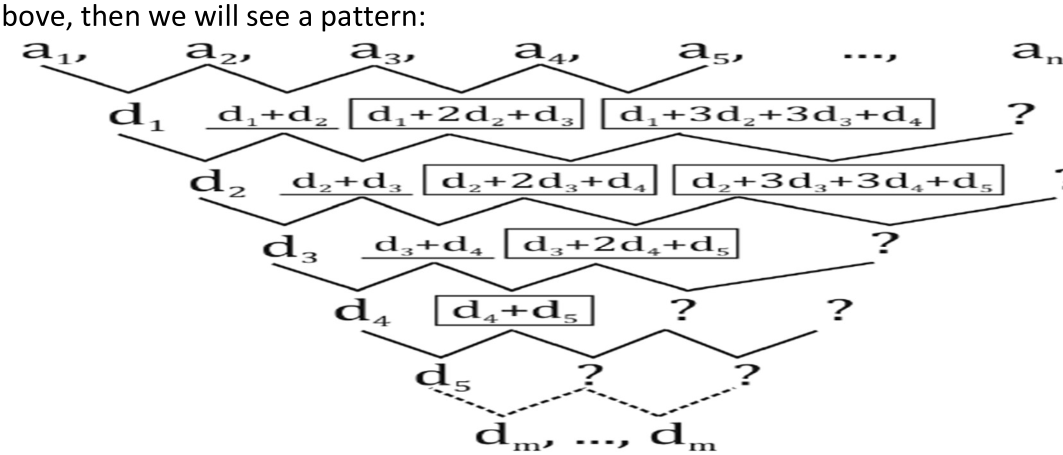
We can think of the diagram above as a puzzle. We begin solving the puzzle by finding the first missing term of each row by simply adding the first set of differences of two consecutive rows. So, they will be in the following order: d_1+d_2, d_2+d_3, d_3+d_4, d_4+d_5, …
It is also shown in the diagram below.



Now, we can solve the second missing term of each row by simply adding the second set of differences of two consecutive rows:



Finally, when we find the third missing term of each row by using the same way that is shown above, then we will see a pattern:



Using this final diagram, we can conclude that :

$$a_2 = a_1 + d_1,\qquad \begin{array}{rl} \cancel{a_2} &= a_1 + d_1 \\ + \; a_3 &= \cancel{a_2} + d_1 + d_2 \\ \hline a_3 &= a_1 + 2d_1 + d_2 \end{array},\qquad \begin{array}{rl} \cancel{a_2} &= a_1 + d_1 \\ \cancel{a_3} &= \cancel{a_2} + d_1 + d_2 \\ + \; a_4 &= \cancel{a_3} + d_1 + 2d_2 + d_3 \\ \hline a_4 &= a_1 + 3d_1 + 3d_2 + d_3 \end{array}$$

$$\begin{array}{rl} \cancel{a_2} &= a_1 + d_1 \\ \cancel{a_3} &= \cancel{a_2} + d_1 + d_2 \\ \cancel{a_4} &= \cancel{a_3} + d_1 + 2d_2 + d_3 \\ + \; a_5 &= \cancel{a_4} + d_1 + 3d_2 + 3d_3 + d_4 \\ \hline a_5 &= a_1 + 4d_1 + 6d_2 + 4d_3 + d_4 \end{array}$$

If we continue this pattern until the m^{th} difference and use binomial expansion, then the n^{th} term of the sequence is

$$a_n = a_1 \binom{n-1}{0} + d_1 \binom{n-1}{1} + d_2 \binom{n-1}{2} + \ldots + d_m \binom{n-1}{m}$$

4 LAGRANGE INTERPOLATION FORMULA

Lagrange polynomials are used for polynomial interpolation. For a given set of distinct points x_j and numbers y_j. Lagrange's interpolation is also an N^{th} degree polynomial approximation to *f*(*x*).

Find the **Lagrange Interpolation Formula** given below,

$$f(x) = \frac{(x-x_2)(x-x_3)\ldots(x-x_n)}{(x_1-x_2)(x_1-x_3)\ldots(x_1-x_n)}y_1 + \frac{(x-x_1)(x-x_3)\ldots(x-x_n)}{(x_2-x_1)(x_2-x_3)\ldots(x_2-x_n)}y_2 + \ldots.. + \frac{(x-x_1)(x-x_2)\ldots(x-x_{n-1})}{(x_n-x_1)(x_n-x_2)\ldots(x_n-x_{n-1})}y_n$$

[2].

Example 1 : What is the 10^{th} term of the sequence below.

1, 4, 9, …

Solution 1 : The Lagrange interpolation formula can be used to find the 10^{th}. term of the above sequence. If $f(x) = x^2$ is interpolated over the range $1 \le x \le 3$ using the consecutive first three terms which are 1,4, and 9, then

$x_0 = 1 \quad f(x_0) = 1$

$x_1 = 2 \quad f(x_1) = 4$

$x_2 = 3 \quad f(x_2) = 9$

The interpolating polynomial is :

$$L(x) = 1 \cdot \frac{x-2}{1-2} \cdot \frac{x-3}{1-3} + 4 \cdot \frac{x-1}{2-1} \cdot \frac{x-3}{2-3} + 9 \cdot \frac{x-1}{3-1} \cdot \frac{x-2}{3-2}$$

$$= x^2.$$

[3].

So, $10^2 = 100$.

5 A DIFFERENT VERSION OF LAGRANGE INTERPOLATION FORMULA

If $f(x)=ax^n+bx^{n-1}+\ldots+c$ is a n-degree polynomial function and (n+1) consecutive integer terms of f(x) are f(1), f(2), f(3), …,f(n), and f(n+1), then any integer term of f(x), greater than the consecutive(n+1) integer terms, can be calculated by the following equation:

$$f(x)=(n+1)\cdot\binom{x-1}{n+1}\cdot\left[\frac{\binom{n}{0}\cdot f(n+1)}{x-(n+1)}-\frac{\binom{n}{1}\cdot f(n)}{x-(n)}+\ldots+\frac{(-1)^{n+1}\cdot\binom{n}{n}f(1)}{x-1}\right]$$

Example 2: What is the 15th term of the sequence below?

4, 13, 34, 73, 136, …

Solution 2: First, the common difference between the consecutive terms is needed.

$$\begin{array}{ccccccccc|l}
4, & & 13, & & 34, & & 73, & & 136, \ldots & \text{Original values} \\
 & 9 & & 21 & & 39 & & 63 & & \text{First differences} \\
 & & 12 & & 18 & & 24 & & & \text{Second differences} \\
 & & & 6 & & 6 & & & & \text{Third differences}
\end{array}$$

Since the third differences are common, the sequence represents a cubic function. So, n=3 and the consecutive (n+1) terms of the sequence are 4, 13,34, and 73.

$$f(15)=(3+1)\cdot\binom{15-1}{3+1}\cdot\left[\frac{\binom{3}{0}\cdot(73)}{15-(3+1)}-\frac{\binom{3}{1}\cdot(34)}{15-(3)}+\frac{\binom{3}{2}\cdot(13)}{15-(2)}-\frac{\binom{3}{3}\cdot(4)}{15-(1)}\right]=$$

$$=4\times\frac{14\times13\times12\times11}{4\times3\times2\times1}\cdot\left[\frac{73}{11}-\frac{102}{12}+\frac{39}{13}-\frac{4}{14}\right]=3406$$

Second solution to example 2: Newton's formula can be used to calculate the 15th term of the sequence.

4, 13, 34, 73, 136, ...

9 21 39 63

12 18 24

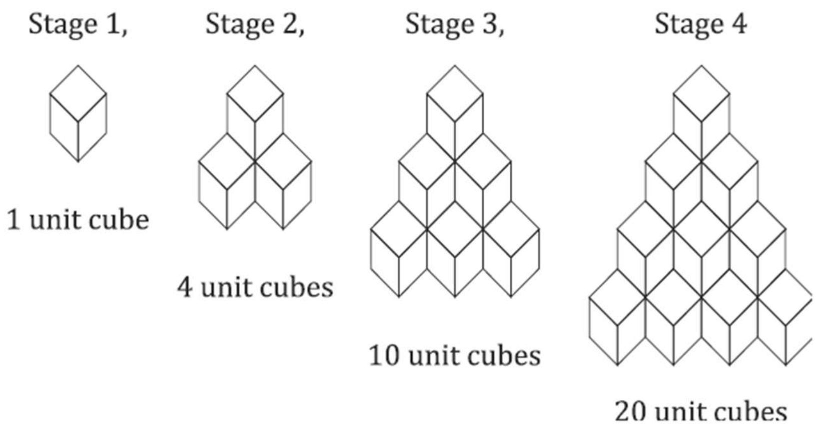
6 6

$$f(15)=4\cdot\binom{15-1}{0}+9\cdot\binom{15-1}{1}+12\cdot\binom{15-1}{2}+6\cdot\binom{15-1}{3}=$$

$$=4+9\times14+12\times\frac{14\times13}{2}+6\times\frac{14\times13\times12}{3\times2}=$$

$$=4+126+1092+2184=3406$$

Example 3: Each stage below is made of unit cubes. If the pattern shown below continues, then how many unit cubes will be used in the 25th stage.?



Solution 3 : If we count the number of unit cubes in each stage, starting from the top layer and continuing to count the number of cubes in each layer going down to the bottom layer, then we would see the pattern below.

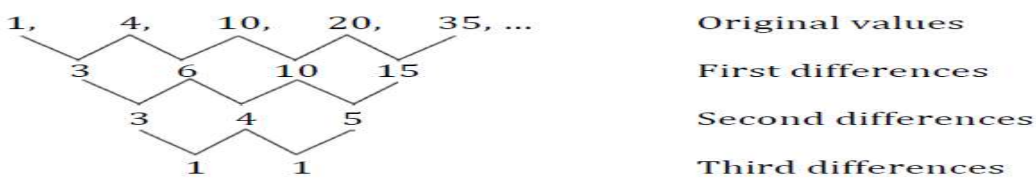
Stage 1 has 1unit cube,

Stage 2 has 1+(1+2) = 4 unit cubes,

Stage 3 has 1+(1+2) +(1+2+3) = 10 unit cubes,

Stage 4 has 1+(1+2) + (1+2+3) + (1+2+3+4) = 20 unit cubes.

If we continue to do this pattern, then stage 5 has 35 unit cubes. These numbers can be gathered to form a sequence.



Since the third differences are common, then the consecutive terms form a cubic function. So, it must be in the form of $f(x)=ax^3+bx^2+cx$. The degree of the function is 3 and the consecutive four terms of it are 1, 4, 10, and 20.

$$f(25)=(3+1)\cdot\binom{25-1}{3+1}\cdot\left[\frac{20\cdot\binom{3}{0}}{25-4}-\frac{10\cdot\binom{3}{1}}{25-3}+\frac{4\cdot\binom{3}{2}}{25-2}-\frac{1\cdot\binom{3}{3}}{25-1}\right]=$$

$$=4\cdot\frac{24\times23\times22\times21}{4\times3\times2\times1}\cdot\left[\frac{20}{21}-\frac{10\cdot(3)}{22}+\frac{4\cdot(3)}{23}-\frac{1}{24}\right]$$

= 2925 unit cubes.

The answer could be verified by Newton's formula as shown below. We need to see the diagram again in order to apply Newton's formula.

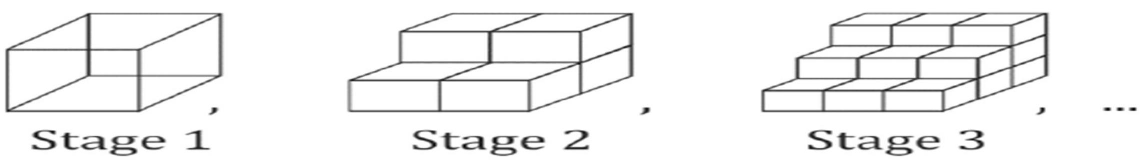
1,	4,	10,	20,	35, ...	Original values
3	6	10	15		First differences
3	4	5			Second differences
1	1				Third differences

Let the 25th term be a_{25}.

$$a_{25}=1\cdot\binom{25-1}{0}+3\cdot\binom{25-1}{1}+3\cdot\binom{25-1}{2}+1\cdot\binom{25-1}{3}=$$

$$=1+3\times(24)+3\times\frac{24\times23}{2\times1}+1\times\frac{24\times23\times22}{3\times2\times1}=2925$$

Example 4 : Each stage below is created by combining unit cubes in an orderly manner. So, how many unit cubes will be used in the 25th stage?



Solution 4 : First method:

Stage 1 has 1 unit cube.

Stage 2 has 2×2+2×1 unit cubes.

Stage 3 has 3×3+3×2+3×1 unit cubes.

If we continue to do this pattern, Stage 25 will have 25×25+25×24+25×23+ … + 25×1 unit cubes. Let the sum be S.

$$25\times25+25\times24+25\times23+\ldots+25\times1=S$$

$$25(25+24+23+\ldots+1)=S$$

$$25+24+23+\ldots+1=\frac{S}{25}$$

$$+\quad 1+2+3+\ldots+25=\frac{S}{25}$$

$$26+26+26+\ldots+26=\frac{2S}{25}$$

$$25\times26=\frac{2S}{25}$$

$$8125=S$$

Second method: The different version of Lagrange Interpolation formula or the Newton's formula can be used to find the 25th term of the sequence.

$$
\begin{array}{ccccccccc}
a_1, & & a_2, & & a_3, & & a_4, & & a_5, \ldots, a_{25}, \\
1, & & 6, & & 18, & & 40, & & 75, \ldots, ? \\
& 5 & & 12 & & 22 & & 35 & \\
& & 7 & & 10 & & 13 & & \\
& & & 3 & & 3 & & &
\end{array}
$$

$$a_{25} = \binom{25-1}{0}(1) + \binom{25-1}{1}(5) + \binom{25-1}{2}(7) + \binom{25-1}{3}(3)$$

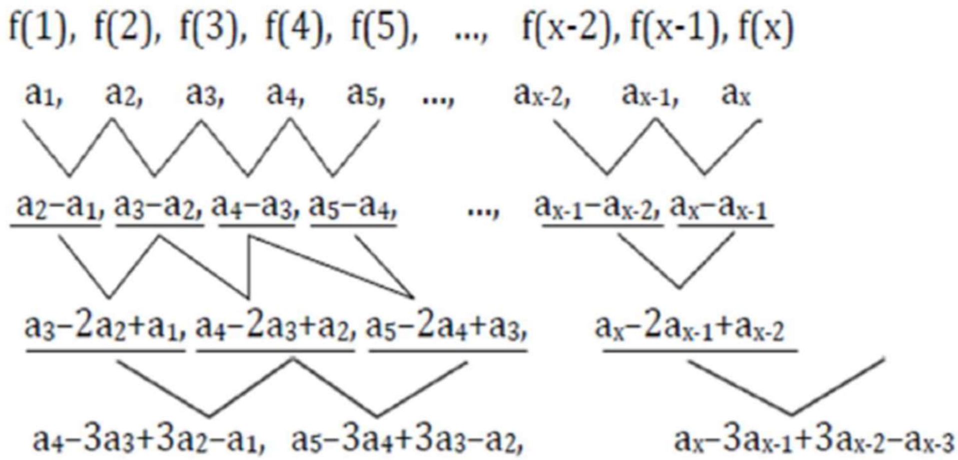
$$a_{25} = 1 + 24(5) + 276(7) + 2.024(3) = 8125$$

6 PROOF OF THE DIFFERENT VERSION OF THE LAGRANGE INTERPOLATION FORMULA

The formula states that if f(x) is a n-degree polynomial function and the consecutive (n+1) terms of f(x) are f(1), f(2), f(3), …, f(n), and f(n+1), then any terms of f(x), greater than the consecutive (n+1) terms, should be as follows:

$$f(x) = (n+1)\cdot\binom{x-1}{n+1}\cdot\left[\frac{\binom{n}{0}\cdot f(n+1)}{x-(n+1)} - \frac{\binom{n}{1}\cdot f(n)}{x-(n)} + \ldots + \frac{(-1)^{n+1}\cdot\binom{n}{n} f(1)}{x-1}\right]$$

if f(x) is a quadratic function and the consecutive five terms of f(x) are $f(1)=a_1$, $f(2)=a_2$, …, and $f(5)=a_5$, then using the following diagram, it will be obvious that all terms of f(x) can be written in terms of the first consecutive three terms of f(x).



Since f(x) is a quadratic function, the second differences between the consecutive terms must be common. So, the third differences are obviously all equal to zero. Therefore, the following equations can be concluded:

$a_4 = 3a_3 - 3a_2 + a_1$

$a_5 = 3a_4 - 3a_3 + a_2$

$a_6 = 3a_5 - 3a_4 + a_3$

The value of a_4 can be substituted in the second equation and then simplified as much as possible as, shown below.

$3a_4 = 9a_3 - 9a_2 + 3a_1$

$-3a_3 = -3a_3$

$+a_2 = \quad +a_2$

$a_5 = 6a_3 - 8a_2 + 3a_1$

It can also be concluded from the diagram that $a_6 = 3a_5 - 3a_4 + a_3$. The similar steps can be followed to substitue the value of a_5 and a_4 in that equation.

$3a_5 = 18a_3 - 24a_2 + 9a_1$

$-3a_4 = -9a_3 + 9a_2 - 3a_1$

$+a_3 = +a_3$

$a_6 = 10a_3 - 15a_2 + 6a_1$

A pattern will be seen among the coefficients of the sequence if a_4 , a_5, a_6 , and a_7 are written in an orderly manner as shown below.

$a_4=3a_3 - 3a_2+a_1$, $a_5=6a_3 - 8a_2+3a_1$, and $a_6 =10a_3 - 15a_2+6a_1$.

Every term of f(x) can be written in terms of a_1, a_2, and a_3. There is a beautiful relationship among the coefficients of a_1, a_2, and a_3.

$$a_4=\frac{(4-1)\cdot(4-2)\cdot\binom{2}{0}\cdot a_3}{2\times 1}-\frac{(4-1)\cdot(4-3)\cdot\binom{2}{1}\cdot a_2}{2\times 1}+\frac{(4-2)\cdot(4-3)\cdot\binom{2}{2}\cdot a_1}{2\times 1}$$

.

$$a_5=\frac{(5-1)\cdot(5-2)\cdot\binom{2}{0}\cdot a_3}{2\times 1}-\frac{(5-1)\cdot(5-3)\cdot\binom{2}{1}\cdot a_2}{2\times 1}+\frac{(5-2)\cdot(5-3)\cdot\binom{2}{2}\cdot a_1}{2\times 1}$$

$$a_6=\frac{(6-1)\cdot(6-2)\cdot\binom{2}{0}\cdot a_3}{2\times 1}-\frac{(6-1)\cdot(6-3)\cdot\binom{2}{1}\cdot a_2}{2\times 1}+\frac{(6-2)\cdot(6-3)\cdot\binom{2}{2}\cdot a_1}{2\times 1}$$

$$\vdots$$

$$a_x=\frac{(x-1)\cdot(x-2)\cdot\binom{2}{0}\cdot a_3}{2\times 1}-\frac{(x-1)\cdot(x-3)\cdot\binom{2}{1}\cdot a_2}{2\times 1}+\frac{(x-2)\cdot(x-3)\cdot\binom{2}{2}\cdot a_1}{2\times 1}$$

The coefficients can be converted into $\binom{x-1}{3}$ by multiplying and then dividing the first fraction by $(x-3)\times 3$, the second fraction by $(x-2)\times 3$, and the third fraction by $(x-1)\times 3$.

$$a_x=\frac{(x-1)\cdot(x-2)\cdot\binom{2}{0}\cdot a_3\cdot(x-3)\cdot 3}{2\times 1\times(x-3)\times 3}-\frac{(x-1)\cdot(x-3)\cdot\binom{2}{1}\cdot a_2\cdot(x-2)\times 3}{2\times 1\times(x-2)\times 3}+$$
$$+\frac{(x-2)\cdot(x-3)\cdot\binom{2}{2}\cdot a_1\cdot(x-1)\cdot 3}{2\times 1\times(x-3)\times 3}$$

If the the last equation is simplified, then it can be written as follows:

$$a_x=\frac{3\cdot\binom{x-1}{3}\cdot\binom{2}{0}\cdot a_3}{(x-3)}-\frac{3\cdot\binom{x-1}{3}\cdot\binom{2}{1}\cdot a_2}{(x-2)}+\frac{3\cdot\binom{x-1}{3}\cdot\binom{2}{2}\cdot a_1}{(x-1)}$$

Since $\binom{x-1}{3}\times 3$ is a common factor, we can simplify the equation.

$$a_x = 3\cdot\binom{x-1}{3}\cdot\left[\frac{\binom{2}{0}\cdot a_3}{(x-3)}-\frac{\binom{2}{1}\cdot a_2}{(x-2)}+\frac{\binom{2}{2}\cdot a_1}{(x-1)}\right]$$

Similarly, if the sequence is cubic in the form of $f(x)=ax^3+bx^2+cx+d$ and the four consecutive terms of f(x) are in the form, $f(1)=a_1$, $f(2)=a_2$, …, $f(4)=a_4$. Then, the following equations can be written using the same strategy used for quadratic sequences as explained in the previous pages.

$f(1),\ f(2),\ f(3),\ f(4),\ f(5),\ \ldots,\ f(x-2),\ f(x-1),\ f(x)$

$a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ \ldots,\ a_{x-2},\ a_{x-1},\ a_x$

$a_2-a_1,\ a_3-a_2,\ a_4-a_3,\ a_5-a_4,\ \ldots,\ a_{x-1}-a_{x-2},\ a_x-a_{x-1}$

$a_3-2a_2+a_1,\ a_4-2a_3+a_2,\ a_5-2a_4+a_3,\ a_x-2a_{x-1}+a_{x-2}$

$a_4-3a_3+3a_2-a_1,\ a_5-3a_4+3a_3-a_2,\ a_x-3a_{x-1}+3a_{x-2}-a_{x-3}$

$a_5-4a_4+6a_3-4a_2+a_1,\ a_6-4a_5+6a_3-4a_2+a_1,\ \ldots,\ a_x-4a_{x-1}+6a_{x-2}-4a_{x-3}+a_{x-4}$

Since f(x) is a cubic function, the third differences between the consecutive terms must be common. So, the fourth differences are obviously all equal to zero. Therefore, the following equations can be written:

$a_5 = 4a_4 - 6a_3 + 4a_2 - a_1$

$a_6 = 4a_5 - 6a_4 + 4a_3 - a_2$

The value of a_5 can be substituted in the second equation and then simplified as much as possible as, shown below.

$4a_5 = 16a_4 - 24a_3 + 16a_2 - 4a_1$

$-6a_4 = -6a_4$

$+4a_3 = \qquad +4a_3$

$-a_2 = \qquad\qquad -a_1$

$a_6 = 10a_4 - 20a_3 + 15a_2 - 4a_1$

It can be drawn from the diagram that $a_7 = 4a_6 – 6a_5 + 4a_4 – a_3$. The similar steps can be followed to substitute the value of a_5 and a_6 in that equation.

$4a_6 = 40a_4 – 80a_3 + 60a_2 – 16a_1$

$-6a_5 = -24a_4 + 36a_3 - 24a_2 + 6a_1$

$+4a_4 = +4a_4$
$-a_3 = -a_3$

$a_7 = 20a_4 – 45a_3 + 36a_2 – 10 a_1$

Using the diagram, the value of a_8 can also be concluded and then be written in terms of a_4 , a_3 , a_2 , and a_1 .

$a_8= 4a_7 – 6a_6 + 4a_5 – a_4$

If similar steps are followed, then the value of a_5 , a_6 , and a_7 can be substituted in the final equation.

$4a_7 = 80a_4 – 180a_3 + 144a_2 – 40a_1$
$-6a_6 = -60a_4 + 120a_3 - 90a_2 + 24a_1$
$4a_5 = 16a_4 – 24a_3 + 16a_2 – 4a_1$
$– a_4 = – a_4$

$a_8 = 35a_4 – 84a_3 + 70a_2 – 20a_1$

A pattern will be seen among the coefficients of the sequence if a_4 , a_5, a_6 , and a_7 are written in an orderly manner as shown below.

$a_5 = 4a_4 – 6a_3 + 4a_2 – a_1$

$a_6 = 10a_4 – 20a_3 + 15a_2 – 4a_1$

$a_7 = 20a_4 – 45a_3 + 36a_2 – 10 a_1$

$a_8 = 35a_4 – 84a_3 + 70a_2$

All terms can be written in terms of a_1, a_2, a_3, and a_4. There is a great relationship among the coefficients, x, which are similar to the quadratic equations.

$$a_5 = \frac{(5-1)\cdot(5-2)\cdot(5-3)\cdot\binom{3}{0}\cdot a_4}{3\times2\times1} - \frac{(5-1)\cdot(5-2)\cdot(5-4)\cdot\binom{3}{1}\cdot a_3}{3\times2\times1} + \\ + \frac{(5-1)\cdot(5-3)\cdot(5-4)\cdot\binom{3}{2}\cdot a_2}{3\times2\times1} - \frac{(5-2)\cdot(5-3)\cdot(5-4)\cdot\binom{3}{3}\cdot a_1}{3\times2\times1}$$

$$a_6 = \frac{(6-1)\cdot(6-2)\cdot(6-3)\cdot\binom{3}{0}\cdot a_4}{3\times2\times1} - \frac{(6-1)\cdot(6-2)\cdot(6-4)\cdot\binom{3}{1}\cdot a_3}{3\times2\times1} + \frac{(6-1)\cdot(6-3)\cdot(6-4)\cdot\binom{3}{2}\cdot a_2}{3\times2\times1} - \frac{(6-2)\cdot(6-3)\cdot(6-4)\cdot\binom{3}{3}\cdot a_1}{3\times2\times1}$$

$$\vdots$$

$$a_x = \frac{(x-1)\cdot(x-2)\cdot(x-3)\cdot\binom{3}{0}\cdot a_4}{3\times2\times1} - \frac{(x-1)\cdot(x-2)\cdot(x-4)\cdot\binom{3}{1}\cdot a_3}{3\times2\times1} + \frac{(x-1)\cdot(x-3)\cdot(x-4)\cdot\binom{3}{2}\cdot a_2}{3\times2\times1} - \frac{(x-2)\cdot(x-3)\cdot(x-4)\cdot\binom{3}{3}\cdot a_1}{3\times2\times1}$$

The coefficients can be converted into $\binom{x-1}{4}\times4$ by multiplying and then dividing the first fraction by (x–4)·4, the second fraction by (x–3)·4, the third fraction by (x–2)·4, and the fourth fraction by (x–1)·4. Thus, every term will have $\binom{x-1}{4}\times4$ in it.

$$a_x = \frac{\binom{x-1}{4}\times4\times\binom{3}{0}\cdot a_4}{(x-4)} - \frac{\binom{x-1}{4}\times4\times\binom{3}{1}\cdot a_3}{(x-3)} + \frac{\binom{x-1}{4}\times4\times\binom{3}{2}\cdot a_2}{(x-2)} - \frac{\binom{x-1}{4}\times4\times\binom{3}{3}\cdot a_1}{(x-1)}$$

Since $\binom{x-1}{4}\times4$ is a common factor, we can write the equation in a more simplified way.

$$a_x = 4\times\binom{x-1}{4}\times\left[\frac{\binom{3}{0}\cdot a_4}{x-4} - \frac{\binom{3}{1}\cdot a_3}{x-3} + \frac{\binom{3}{2}\cdot a_2}{x-2} - \frac{\binom{3}{3}\cdot a_1}{x-1}\right]$$

If that pattern continues, then the following formula can be written. If $a_x=ax^n+bx^{n+1}+...+c$, and the consecutive (n+1) terms of a_x are $a_1, a_2, ..., a_n, a_{n+1}$, then any term of a_x, greater than the consecutive (n+1) terms, is

$$a_x=(n+1)\cdot\binom{x-1}{n+1}\cdot\left[\frac{\binom{n}{0}\cdot f_{(n+1)}}{x-(n+1)}-\frac{\binom{n}{1}\cdot f_{(n)}}{x-n}+...+\frac{(-1)^{n+1}\cdot\binom{n}{n}\cdot f_{(1)}}{x-1}\right]$$

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