

16 **Abstract**

17 The power input and gas-liquid mass transfer rank among the most important industrial
18 fermentation process parameters. The present study analyzes the power input and gas hold-up as a
19 function of the flow regime, impeller diameter, and rheological properties in a pilot scale reactor
20 (160 L) equipped with four Rushton impellers. This leads to four dimensionless numbers for
21 predicting measurements in pilot and industrial bioreactors (110 and 170 m³) with a standard
22 deviation of 7 % to 29 %. This is unparalleled for the underlying aerated and non-Newtonian
23 fermentation broths. Several existing correlation equations are discussed to be dissatisfying (up to
24 130 % deviation), and might be sufficiently valid only within scale or for small scaling factors.
25 The introduced approach predicts adequately accurate over three orders of magnitude. Based on
26 these encouraging results, we identified the Galilei number and the power concept as the central
27 elements in combination with the consequent dimensional analysis for an efficient scaling
28 between pilot and industrial scale.

29 **Keywords:** non-Newtonian fluid behavior, power input, scale-down, scale-up, viscous media

30

31 **Introduction**

32 The scaling of the power input and the oxygen mass-transfer of industrial bioreactors is important
33 for the development, process transfer and scale-up of industrial fermentation processes. The power
34 demand for agitation and aeration can be significant, even ranging above a megawatt for a single
35 fermenter. Thus, energy costs may contribute significantly to the manufacturing costs of a desired
36 product. The large power demand might also lead to higher investment costs (agitators, drives,
37 aerators, heat exchangers). However, effective agitation is critically important, since an
38 insufficient oxygen supply is undesired and may strongly influence the specific productivity of the
39 cultivated cells. Therefore, mixing and aeration are well-known targets for process development
40 at all scales (e.g. Garcia-Ochoa & Gomez, 2009; Gogate, Beenackers, & Pandit, 2000; Henzler,
41 1982; Oldshue, 1966; Paul, Atiemo-Obeng, & Kresta, 2004; Zlokarnik, 1999). For optimizing
42 large-scale operation parameters of industrial bioreactors, it is advisable to scale down and test
43 new conditions before implementing the setup at the production scale. Consequently, a reliable
44 scaling model is of utmost importance.

45 Many authors described the complex physical processes impacting the oxygen supply in the
46 bioreactor by applying dimensional and/or dimensionless numbers (Garcia-Ochoa, Castro, &
47 Santos, 2000; Garcia-Ochoa & Gomez, 1998; Liepe, Meusel, Möckel, Platzer, & Weißgärber,
48 1988; Linek, Moucha, & Sinkule, 1996; Loiseau, Midoux, & Charpentier, 1977; Mockel,
49 Wolleschensky, Drewas, & Rahner, 1990; Oldshue, 1966; Petricek et al., 2017; Saravanan,
50 Veerappan, & Kothandaraman, 2009; Van't Riet, Boom, & M. Smith, 1976; Xie et al., 2014b). The
51 most relevant properties proposed in biochemical engineering textbooks are the specific power
52 input (P/V), volumetric mass transfer coefficient ($k_L a$), vessel volume per minute (VVM), gas
53 superficial velocity (u_g), impeller tip speed (u_T) and dissolved oxygen concentration (c_{O_2})
54 (Oldshue, 1966). Keeping one or more of these variables constant represents the standard approach

55 for scaling fermentation processes (Figure S 1). However, it is also evident that the selection of
56 such criteria stems from practical considerations, not strictly following the principles of a
57 similitude model. This might limit the predictive power of those correlation equations.
58 Considerably fewer authors used the Flow number (Fl) and Froude number (Fr) for the prediction
59 of oxygen mass transfer, although these numbers originate from dimensional analysis and were
60 proven to reliably characterize the flow regime induced by the bottom impeller (John M. Smith,
61 Warmoeskerken, & Zeef, 1987).

62 Several authors applied laboratory-scale (up to 21 L; Garcia-Ochoa et al., 2000; Garcia-Ochoa &
63 Gomez, 1998; Liepe et al., 1988; Linek et al., 1996; Loiseau et al., 1977) and a few pilot-scale
64 experiments (up to 6.43 m³; Hoecker, Langer, & Werner, 1981; Mockel et al., 1990; Petricek et
65 al., 2017; Saravanan et al., 2009; Xie et al., 2014b). The largest scaling step applied in these studies
66 was a factor of 1 : 50 (Petricek et al., 2017). When scaling steps are small, the concepts yield
67 sufficiently accurate predictions. For large steps, scaling effects may occur (cf. Figure S 1) that
68 make predictions inaccurate (Garcia-Ochoa & Gomez, 2009). For application of scaling at a large
69 industrial scale, these concepts are questionable and validation data are lacking.

70 A further source of inaccurate scaling is that the number of impeller levels is often disregarded.
71 Most studies mentioned above applied stirred tanks equipped with one, two (Garcia-Ochoa &
72 Gomez, 1998; Hoecker et al., 1981; Loiseau et al., 1977; Saravanan et al., 2009), or three impellers
73 (Linek et al., 1996; Mockel et al., 1990; Petricek et al., 2017; J. M. Smith, 1992; Xie et al., 2014a).
74 In contrast, large industrial-scale reactors often exhibit four, five or more levels, depending on the
75 facility. These multi-stage reactors have been shown to behave differently regarding each impellers
76 flow regime (Bombac & Zun, 2006; John M. Smith et al., 1987), power input (Linek et al., 1996;
77 Middleton & Smith, 2004) and gas hold-up (Gogate et al., 2000; Linek et al., 1996) compared to

78 single impeller reactors. Adequate correlation equations are missing important information for
79 understanding and designing the large-scale cultivation of producer cells (Linek et al., 1996;
80 Nocentini, Fajner, Pasquali, & Magelli, 1993; Vrabel, van der Lans, Luyben, Boon, & Nienow,
81 2000; Zhang, Pan, & Rempel, 2006).

82 The non-ideal rheological behavior of the fermentation broth is also a known source of
83 inaccuracies of scaling concepts. Many industrial processes show viscous non-Newtonian
84 behavior, e.g., due to mycelial growth or production of metabolic polymers (Wucherpfennig, Kiep,
85 Driouch, Wittmann, & Krull, 2010). The apparent viscosity (Brown, Jones, Middleton,
86 Papadopoulos, & Arik, 2004; Henzler, 2007),

$$\eta_{app} = K\dot{\gamma}_{rep}^{n-1}, \quad 1$$

87 of these non-Newtonian fluids depends on the representative shear rate, $\dot{\gamma}_{rep}$ and the fluid-
88 characteristic properties (consistency index K, flow behavior index n). The Power-Law model by
89 Ostwald and de Waele,

$$\tau = K\dot{\gamma}_{rep}^n, \quad 2$$

90 represents the underlying physical relation between the shear stress, τ , and the representative shear
91 rate independent of scale. Sanchez Perez, Rodriguez Porcel, Casas Lopez, Fernandez Sevilla, and
92 Chisti (2006) summarized the available approaches for describing the dependency of shear rate
93 and impeller speed. Most authors apply the Metzner-Otto concept (Metzner & Otto, 1957) to
94 describe the representative shear rate, assuming a simple proportionality with the impeller speed
95 (Amanullah, Hjorth, & Nienow, 1998; Bohme & Stenger, 1988; Hoecker & Langer, 1977; Justen,
96 Paul, Nienow, & Thomas, 1996; Knoch, 1997; Sinevic, Kuboi, & Nienow, 1986).

97 The Metzner-Otto concept underestimates the representative shear rate for large industrial
98 bioreactors, and overestimates the apparent viscosity for shear thinning fluids (Equation 1)

99 (Henzler, 2007; Herbst, Schumpe, & Deckwer, 1992). It may be assumed that the Metzner-Otto
100 concept is applicable only for the same scale.

101 The so-called power concept by Henzler and Kauling (1985) is more advanced in that it is derived
102 from general physical principles (i.e., independent of scale). The representative shear rate is
103 expressed as a function of the specific power input (P):

$$\dot{\gamma}_{rep} = L^{\frac{2}{1+n}} \left(\frac{P/V}{K} \right)^{\frac{1}{1+n}} \quad 3$$

104 with the above fluid-characteristic properties (K, n) and a constant parameter (L). Remarkably, the
105 power concept reduces to the Metzner-Otto concept for laminar conditions, as shown by Giese et
106 al. (2014). Henzler (2007) demonstrated that the power concept, unlike the Metzner-Otto concept,
107 is valid over different scales.

108 The objective of this study is to critically analyze the power input and gas hold-up for aerated
109 bioreactors and to provide novel correlations in a dimensionless form. It seems helpful choosing
110 correlation equations as outlined in Equations 1, 2 and 3 for deriving a new scaling concept for the
111 flow regime and rheology. Instead of impeller-speed-dependent models, we apply an extended
112 power concept considering the shear rate under aerated conditions and choosing corresponding
113 model fluids for scaling rheology. We furthermore extend the concept by a variable impeller
114 diameter, apply the Galilei number and consider pilot- and large-scale gas flow regime by
115 application of multi-impeller reactors with four impeller levels. In this approach we solely use
116 dimensional analysis and scale-independent representations for describing the relevant complex
117 physical processes efficiently and for minimizing scaling effects. In order to validate the novel
118 correlations that cover a large operating and scaling range, we test the prediction quality for two
119 industrial fermentation processes and compare results with existing correlations.

120 **Materials and Methods**

121 **Pilot-scale reactor (160 L)**

122 The experiments for the development of the scaling model and its validation were performed in a
123 Plexiglas reactor (*PGR*) with an inner diameter of $T = 0.44$ m and a fluid volume of 0.16 m³ (Witz,
124 Treffer, Hardiman, & Khinast, 2016). The vessel was equipped with four longitudinal baffles
125 (thickness = $T/10$) and six blade Rushton impellers (D/T of 0.36, 0.42, 0.46) arranged in four levels
126 with a spacing between the impellers of $Z/T = 0.6$ and a bottom clearance of $C/T = 0.36$.

127 **Industrial-scale reactors (data for model verification; > 100 m³)**

128 Further experiments for validation of the scaling concept were conducted in two large-scale
129 bioreactors. Fermenter 1 exhibits a D/T -ratio of 0.36 and a volume of 170 m³; fermenter 2 a D/T
130 -ratio of 0.43 and a volume of 110 m³. Both fermenters were equipped with four Rushton impeller
131 levels with a spacing of $Z/T = 0.6$. The fermentation broth in fermenter 1 represented a Newtonian
132 behavior with $K = 0.02$ Pas and $n = 1$ (invariant over process time). The broth in fermenter 2
133 displayed a non-Newtonian behavior with a process time variant consistency index ranging from
134 2.7 to 7.3 and a flow behavior index varying from 0.31 to 0.25.

135 **Model Fluids**

136 All measurements in pilot-scale were conducted applying model fluids. To mimic the fermentation
137 broth's non-coalescent behavior of the air bubbles, Na_2SO_4 was added to deionized water
138 (0.065 mol/L). Non-Newtonian fluids were made using xanthan in four concentrations (0.05 , 0.1 ,
139 0.3 and 0.4 % [w/w]; Cosphaderm X34, Alexmo Cosmetics, Germany). The density of the
140 solutions was 1005 to 1007 kg m⁻³. The rheological properties were characterized using a Kinexus
141 pro+ rotational rheometer (Malvern Instruments, UK) and the power-law model (Equation 2).

142 **Experimental range**

143 Impeller speed, aeration rate, impeller diameter and rheological properties of model fluids were
144 varied over a wide range to cover common fermentation conditions in industrial fermentation

145 processes (Table S 1). All experiments were carried out in triplicates, leading to 2889 data points
146 for determining the gas hold-up and 2916 for the power input estimation. For model verification
147 at the pilot-scale, a set of 726 experimentally determined values for the gas hold-up and Newton
148 number were applied.

149 **Measuring methods**

150 The large-scale reactors were equipped with filling level probes for measurement of the gas hold-
151 up. In the plexiglas vessel, the liquid level was measured using a scale. The gas hold-up,

$$\varepsilon = \frac{H_g}{(H_g + H_l)} \quad 4$$

152 was determined by measuring the aerated liquid height relative to the liquid level without aeration.
153 H_g represents the difference of the liquid height in the reactor, when aerated and non-aerated, and
154 H_l the liquid height, when not aerated. Temperature was set to $20 \pm 3^\circ$ C. The influence of the
155 temperature deviation on the air volume was corrected using

$$V_{g,20^\circ C} = \frac{293.15K}{Temperature} V_g(Temperature), \quad 5$$

156 where $V_{g,20^\circ C}$ stands for the representative gas volume at $20^\circ C$, and $V_g(Temperature)$ for the
157 measured gas hold-up at the observed temperature. A strain gauge with telemetry technique
158 (Trachsler Electronics GmbH, Switzerland) was used for determining the torque on the stirrer shaft
159 of the PGR. The strain gauges were applied above the top impeller and the determined torque
160 represents the sum of the four impellers. The power consumption in the large-scale reactors was
161 determined via the measured engine power input and the experimentally observed energy losses.

162 **Statistical evaluation and parameter identification**

163 The experimental data were subjected to a principal component analysis using the software
164 SIMCA (V14.1, Sartorius Stedim, Germany). The unknown parameters of the correlation
165 equations for the new scaling concept were identified using the Minitab Software (V 17.2, Minitab

166 Inc. USA), via a non-linear regression and a Gauss-Newton algorithm. The quality of the
 167 estimation was assessed by the relative standard deviation (SD) of the experimental ($x_{experimental}$)
 168 and predicted values ($x_{predicted}$):

$$SD = \sqrt{\frac{1}{1-N} \sum_{j=1}^N \left(\frac{x_{experimental,j} - x_{predicted,j}}{x_{experimental,j}} \right)^2}. \quad 6$$

169 **Theoretical Aspects**

170 A full dimensional analysis of the power input into an aerated stirred tank reactor by Bieseker
 171 (1972) led to a set of 24 dimensionless numbers and was reduced further by Zlokarnik (1973, 1999)
 172 to the Froude number ($Fr = \frac{N^2 D}{g}$), Reynolds number ($Re = \frac{ND^2 \rho}{\eta_{app}}$), Gas Flow number ($Fl = \frac{Q}{ND^3}$) and
 173 the Weber number ($We = \frac{\rho N^2 D^3}{\sigma}$). One shortcoming of selecting a combination of the Reynolds,
 174 Froude and Flow numbers is that these depend on the impeller speed. Moreover, the term ND^2 of
 175 the Reynolds number is also part of the Flow number. Although the Reynolds number contains
 176 information on the rheology of the fluid, a further aspect of its application is that the Newton
 177 number is independent of the Reynolds number in the turbulent flow regime (Zlokarnik, 1973).
 178 Typically, industrial microbial fermentation processes work in the turbulent regime. The approach
 179 proposed by Hoecker et al. (1981) uses the Galilei number (Ga) instead of the Reynolds number:

$$Ga = \frac{Re^2}{Fr} = \frac{D^3 g \rho^2}{\eta_{app}^2} \quad 7$$

180 Besides substance properties, such as the dynamic viscosity (η), the Galilei number contains only
 181 the impeller diameter, D , as an operating variable, and thus, appears to be a good choice to
 182 efficiently scale rheology. Furthermore, the density, ρ , and surface tension, σ , of different
 183 fermentation broths were analyzed (data not shown) and the influence was found to be negligible
 184 and therefore the We number was excluded. The resulting dimensionless relationship is given by

$$Ne = f(Fr, Fl, Ga). \quad 8$$

185 Hoecker et al. (1981) demonstrated that this concept adequately describes the power input in
186 experiments with model fluids of different rheological properties. However, these authors did not
187 include geometric aspects, as experiments were done only at a single scale (50 L), with one
188 Rushton turbine and an untypical H/T ratio. The present work extends this concept through a
189 dimensionless number representing the reactor geometry

$$Ne = A * Fr^\alpha Fl^\beta \left(\frac{D}{T}\right)^\gamma Ga^\delta. \quad 9$$

190 The variation of the impeller versus tank diameter (D/T) seems to be important, since the impeller
191 diameter is part of all dimensionless numbers, and otherwise the tank diameter would not be
192 represented. A further extension is the aforementioned application of the power concept (Equation
193 3), considering aerated conditions ($P = P_g$ with P_g , power input under aerated conditions).

194 In addition to scaling the power input (Ne), the correlation was used for describing the
195 dimensionless gas hold-up (ε , Equation 10)

$$\varepsilon = B * Fr^\zeta Fl^\theta \left(\frac{D}{T}\right)^\iota Ga^k \quad 10$$

196 For Newtonian fluids with low viscosity the Galilei number depends solely on the impeller
197 diameter. Equations 9 and 10, however, formulate the impeller diameter as separate dimensionless
198 number. Thus, for Newtonian fluids, the Galilei number does not contain additional information
199 on the variation in the data set and was excluded from the Equations 9 and 10. The resulting set of
200 Equations 1, 2, 3 (using $P = P_g$), and Equations 7 to 10 represent the new concept.

201 Successfully correlating the gas hold-up with the flow regime (Fr, Fl), rheology (Ga) and
202 geometry (D/T) according to this dimensionless concept should support the idea of appropriately
203 characterizing bioreactor conditions with regard to the general oxygen supply as an important
204 biotechnological operations. As a representative control for a dimensional approach, the $k_L a$

205 (volumetric mass transfer coefficient) value was determined additionally and correlated analogously
206 to Equation 10 (see supplemental material).

207 **Results and Discussion**

208 **Derivation of correlation parameters**

209 Initially, the large data set generated in the pilot-scale fermenter was statistically analyzed. A
210 principal component analysis revealed data clusters over two principal components (Figure 1, left).
211 Interestingly, these clusters seem to agree with the bottom-impeller flow regimes (Figure 1, right)
212 flooding, large and vortex cavities, with and without recirculation, as described by Middleton and
213 Smith (2004). These results correspond to our study (Bernauer et al., 2020), which indicates that
214 the bottom impellers flow regime influences the gas-liquid flow of the whole reactor.

215 Furthermore, the principal component analysis of the experimental dataset indicated that a
216 separation of experiments applying the Newtonian and the non-Newtonian model fluids is
217 beneficial. The data fit after these classifications led to a significant improvement of the “goodness
218 of fit” (Equation 6). We thus conclude that it is beneficial to use this *a priori* knowledge, when
219 facing fundamentally different liquid and gas flow conditions and statistical analysis supports the
220 corresponding classification.

221 The parameters identified for Equations 9 and 10 are listed in Tables S2 and S3 including the
222 determined SDs, ranging between 1.5 % and 10.6 % and < 7 % for most conditions (70%). The
223 predicted and measured values are available in Figure S 2 of the supplemental material.

224 **Model verification in pilot-scale fermenters**

225 The correlations’ accuracies were challenged applying (i) a fluid with rheological properties that
226 were not used for identifying their parameters (0.2 % xanthan test fluid, with the rheological
227 parameters $K = 0.46$, $n = 0.35$) and (ii) the full experimental range (i.e., the large cavities, vortex
228 cavities and flooded flow regime). The predicted values of the gas hold-up and Newton number

229 lie close to the experimental values (Figure 2). The SD is 7 % for the Newton number and 8 % for
230 gas hold-up (Table S 2, Table S 3).

231 Furthermore, a comparison of the experimental values with predictions from literature correlations
232 (listed in Table S 5) was carried out. The prediction of the Newton number (Figure 2 A) by the
233 dimensionless approach of Nienow, Wisdom, and Middleton (1977) shows a very good fit with a
234 SD of 11 %. The approach also includes Froude and Flow number but has no term for describing
235 viscosity. The dimensional correlation of Mockel et al. (1990) shows a large SD of 36 %, which
236 may be due to the application of a dimensional concept and missing viscosity term. The most
237 promising study by Xie et al. (2014a) is a similar approach to ours and based on a dimensionless
238 form including Froude-, Flow- and Reynolds number and was established applying non-
239 Newtonian fluids. However, it exhibits the highest SD of 58% to the experimental data. The large
240 error may be explained by the authors' application of the Metzner-Otto concept for calculation of
241 the shear rate, which leads to inaccurate predictions as expected. Furthermore, their experiments
242 were conducted in the transition range from laminar to turbulent flow, which makes the application
243 of their correlation for turbulent conditions in bioreactors difficult.

244 The most precise correlation of the gas hold-up (Figure 2 B) with a SD of 21 % is obtained with
245 the correlation by Garcia-Ochoa and Gomez (2004) based on a theoretical approach. Zhang et al.
246 (2006) also shows a good correlation with a SD of 29 %. The study, however, was conducted in a
247 multi-stage reactor with a large d/D ratio of 0.5 and is therefore only comparable to a limited
248 extent with our data. Other studies (Loiseau et al., 1977; J. M. Smith, 1992) exhibit large SDs of
249 34% and 62%. All predictions show an underprediction of the actual values. It has to be noted that
250 viscosity dependent models of the mentioned gas hold-up studies were obtained in Newtonian
251 liquids.

252 The $k_L a$ (volumetric mass transfer coefficient) value was determined and correlated analogously
253 to Equation 10 (supplemental material) as a representative control for an established dimensional
254 approach. However, the predictions were not satisfactory with SDs 17 to 34 % (Figure S 3) and in
255 line with dimensional or mixed approaches (39 to 70 % SD) by other authors (Table S 5). This
256 result was expected due to the dimensional form of the $k_L a$ value in Equation 10, and, the
257 application of the Metzner-Otto concept by some authors.

258 **Model verification in industrial-scale fermenters**

259 After verification in the pilot-scale, the predictions of the Newton number and gas hold-up were
260 critically assessed using measured data from two large-scale processes. Figure 3 A and B illustrate
261 the predictions and experimentally measured values for a 170 m³ scale fermentation process. The
262 fermentation broth had a low-viscosity Newtonian rheology and was invariant over the process
263 time. The flow regime was defined by large cavities without recirculation.

264 Our model predicts the Newton number extraordinarily well with a SD of 7 %. A comparison to
265 other authors shows that the dimensionless correlation proposed by Nienow et al. (1977) represents
266 the data with a SD of 14 %. The works of Xie et al. (2014a) and Mockel et al. (1990) show a lower
267 prediction capability with 26 % and 29 % SD. Again, the measurements in the transition range
268 between laminar and turbulent flow (Xie et al., 2014a) and the dimensional approach by Mockel
269 et al. (1990) may be the reason for these inaccuracies. Generally, the correlations of all authors do
270 not predict an increase of the Newton number at this large-scale. A possible reason for this
271 underestimation are fluctuations of the large-scale reactors engine power.

272 The predicted gas hold-up at the large scale exhibits a similar accuracy at the large-scale compared
273 to the prediction of the Newton number (SD 8%). However, other existing correlations display a
274 much lower prediction capability (Liepe et al. (1988): 63 %, Xie et al. (2014b): 85 %, Saravanan
275 et al. (2009): 86 % and Linek et al. (1996): 91 % SD). All cited correlations overpredict the actual

276 gas hold-up. This may be explained by the correlations' dimensional structure of $\varepsilon = u_g^x (P/V)^y$.
277 Scaling with such concepts changes the process conditions significantly, as illustrated in Figure S
278 1. Again, the use of a similitude model is beneficial, as it aims at scaling (keeping) the flow regime
279 of the aerated reactor.

280 Furthermore, measured data from a reactor scale of 110 m³ (Figure 3, C and D) were used for
281 validating the proposed correlations for Newton number and gas hold-up. The fermentation broth
282 had a non-Newtonian rheology, the rheological parameters varied over the process time, and the
283 flow regime represented the large-cavities regime without recirculation.

284 The predicted values of the Newton number by our proposed correlation show a SD of 12%. The
285 relation provided by Nienow et al. (1977) presents a comparable accuracy to our data with a SD
286 of 10%. This can be explained by the similar dimensionless forms of the applied equations with
287 similar exponents for the Froude and Flow number, although Nienow et al. (1977) did not use a
288 viscosity term. Furthermore, in our prediction, the exponent of the Galilei number is small and
289 therefore the influence of viscosity on Newton number was found to be also minor. However, the
290 influence depends on the flow regime and increases, e.g., in the recirculation regime (Table S 2).

291 The correlation proposed by Mockel et al. (1990) shows a better fit for the non-Newtonian process
292 as for the Newtonian process (SD 13%). Xie et al. (2014a) proposed two correlations, one
293 excluding viscosity and one including viscosity by means of the Reynolds number. Interestingly,
294 the equation including the Reynolds number shows a very poor fit with a SD of 65% (as mentioned
295 above, not based on turbulent conditions). The equation without the Reynolds number is a better
296 fit with a SD of 23%.

297 Our prediction of gas hold-up for non-Newtonian behavior with a SD of 29 % was the closest to
298 the experimental data compared to other studies. The concept of Loiseau et al. (1977) exhibits a

299 SD of 39 %, but fails at low apparent viscosities. The remaining correlations (Middleton & Smith,
300 2004; J. M. Smith, 1992; Zhang et al., 2006) show poor predictions of the experimental values (SD
301 of 73%, 80% and 78%). All equations underestimate the gas hold-up of the large-scale bioreactor.
302 A straightforward explanation is the application of the Metzner-Otto concept, which overestimates
303 the viscosity when applying for scale-up. Interestingly, for the data points at the experimentally
304 highest gas hold-up (Figure 3 D) the existing correlations predict a constantly low gas hold-up,
305 whereas the real hold-up increases. A general explanation for this behavior is difficult due to the
306 variety of the dimensional approaches. However, our dimensionless approach and using the Galilei
307 number is capable of describing the high gas hold-up values (Figure 3 D). The large SD of the
308 correlation by Zhang et al. (2006) might be caused by the different system and a large
309 impeller/reactor diameter-ratio. Additionally, Zhang et al. (2006) directly used the impeller speed
310 in the correlation, and therefore, underestimated significantly the gas hold-up in large-scale
311 fermenters, as impeller speed is lower in large-scale reactors for similar volumetric power input.
312 The overall underprediction of gas hold-up by the studied literature correlations may be attributed
313 to an insufficient description of the rheological behavior by applying the viscosity itself or the
314 Reynolds number and paired with the application of the Metzner-Otto concept for non-Newtonian
315 fluids. Our approach still underestimates the hold-up but shows a significantly better prediction of
316 the experimental values.

317 Furthermore, our study applied four impeller stages and all other referred publications only three
318 or less stages. Since the upper impellers have been shown to yield a larger gas hold-up and power
319 draw (Gogate et al., 2000; Linek et al., 1996; Nocentini, Magelli, Pasquali, & Fajner, 1988; John
320 M. Smith et al., 1987), predominantly due to predispersion of the gas by the bottom impeller
321 (Bernauer et al., 2020), we may expect an underestimation of both values by the correlations of

322 Table S 5 (derived from data using reactors with less than four impeller levels). However, the
323 results showed variations in the Newton number (Figure 3, A and C) and gas hold-up (Figure 3, B
324 and D) in both directions and to a larger extent than expected from the influence of the number of
325 impeller levels of the reactor. These variations were mainly associated with the previously
326 described influence of the dimensional approaches. Nevertheless, the influence of the number of
327 impellers should be considered for scaling between pilot and large scale and this should be
328 objective of further studies.

329 **Conclusions**

330 Within the scope of the present work, we derived a set of equations constituting a new bioreactor
331 scaling concept for stirring and aeration of Newtonian and non-Newtonian fluids, based on
332 dimensional analysis and scale-independent representations of the relevant physical processes. The
333 scaling of the flow regime by means of the Froude and Flow number leads to significantly better
334 predictions, especially for large scaling steps, as compared with other approaches. The separation
335 of the operating ranges according to the bottom impeller flow regime classification significantly
336 further improves the prediction quality. The underlying fundamentally different overall gas and
337 liquid flow conditions in the reactor impact the power input and gas hold-up strongly, and this
338 might not be adequately described by a single equation. Applying the Galilei number combined
339 with the power concept for aerated conditions enables improved scaling of the shear rate and
340 apparent viscosity, and thus, of both the power input and gas hold-up for non-Newtonian fluids.

341 The derived correlation equations were tested with pilot-scale data for a non-Newtonian model
342 fluid, and showed good accuracy for the Newton number and gas hold-up ($SD < 10\%$). Predictions
343 of the gas hold-up and Newton number for production-scale reactors with non-Newtonian and
344 Newtonian fermentation broths (scaling factor 650 and 1000, respectively) demonstrated good
345 agreement with the measured data. Our approach showed no loss in accuracy compared to pilot-

346 scale for Newtonian ($SD < 10\%$) and only moderate inaccuracies for non-Newtonian broths (SD
347 of 12% for Ne number and 29% for gas hold-up).

348 A wealth of different approaches is available in text books and many important fundamental
349 contributions to the field that yield sufficiently accurate predictions of power input and oxygen
350 supply, when applied for similar scales as they were derived for. This work comprehensively
351 analyzes these scaling concepts for large scaling steps. Remarkable results are generated for
352 predicting the Ne number using Nienow's correlation ($SD < 15\%$), which is based solely on two
353 dimensionless numbers (Fl, Fr), however, does not consider rheology or prediction of gas hold-up.
354 One reason for the discouraging results for the other approaches ($SD > 20\%$ and up to 130% for
355 predicting Ne and gas hold-up) may be a change of the flow regimes in the reactors, which is
356 caused by most correlations' dimensional scaling approaches. The presence of an equal number of
357 impeller levels in the reactors might be another important basis for ensuring a comparable flow
358 regime during scale-up/-down. This should be objective for further studies. Another cause of
359 inaccuracy is the common application of the Metzner-Otto concept for scaling the apparent
360 viscosity. Such models yield unsatisfactory predictions due to the assumption of a linear
361 dependence of the shear rate on the stirrer speed. The present work supports the assumption that
362 the application of the Metzner-Otto concept leads to inadequate estimations, during scaling up and
363 potentially even within scale.

364 The encouraging results of this work for remarkably large scaling factors of three orders of
365 magnitude are a significant progress as underlined by the comprehensive analysis of the
366 correlations available from previous work. This strongly supports the application of the new
367 concept and provides an important step forward in our understanding of industrial fermentation
368 processes.

369 **Nomenclature**

370	A, B	empirical constants [-]
371	C	reactor bottom clearance [m]
372	D	impeller diameter [m]
373	D_L	diffusivity of gas in liquid [$\text{m}^2 \text{s}^{-1}$]
374	Fl	Flow number [-]
375	Fr	Froude number [-]
376	g	gravitational constant [$\text{m}^3 \text{kg}^{-1} \text{s}^{-2}$]
377	Ga	Galilei number [-]
378	H_g	aerated liquid height [m]
379	H_L	unaerated liquid height [m]
380	K	consistency index [Pa s]
381	K_s	proportionality constant [-]
382	L	power concept constant [-]
383	n	flow behavior index [-]
384	N	impeller frequency [s^{-1}]
385	Ne	Newton number [-]
386	P	Power input [W]
387	PCA	Principal component analysis
388	P_g	specific power input under aerated condition [W]
389	P_0	specific power input without aeration [W]
390	PGR	Plexiglas reactor
391	Re	Reynolds number [-]
392	SD	standard deviation
393	T	tank diameter [m]
394	u_g	gas superficial velocity [m s^{-1}]
395	u_T	impeller tip speed [m s^{-1}]
396	V	volume of the liquid [m^3]

397	$V_g, 20^\circ\text{C}$	gas volume at 20°C
398	$V_g(\text{Temperature})$	gas volume at the observed temperature
399	VVM	vessel volume per minute [min^{-1}]
400	We	Weber number [-]
401	c_{O_2}	dissolved oxygen concentration [%]
402	Q	aeration rate [$\text{m}^3 \text{h}^{-1}$]
403	Z	spacing between impellers [m]
404		
405	<i>Greek letters</i>	
406	$\alpha, \beta, \gamma, \delta, \zeta, \theta, \iota, \kappa$	empirical constants [-]
407	ρ	liquid density [kg m^{-3}]
408	ρ_g	density of gas phase [kg m^{-3}]
409	σ	surface tension [kg s^{-2}]
410	ε	gas hold-up [-]
411	τ	shear stress [N m^{-2}]
412	η_{app}	apparent dynamic viscosity of the liquid [Pa s]
413	η_w	dynamic viscosity of water [Pa s]
414	$\dot{\gamma}_{rep}$	representative shear rate [s^{-1}]
415		

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546

547 **Figure Legends**

548 **Figure 1:** Principal component analysis (left) of all Newton number and gas hold-up experimental data and the
549 corresponding flow regime map (right). Flooded (green), large cavities (dark blue), large cavities with recirculation
550 (red), vortex cavities (light blue) and vortex cavities with recirculation (orange).

551 **Figure 2:** Newton numbers (A) and gas hold-up- (B) observed in pilot-scale in a 0.2% xanthan test fluid with non-
552 Newtonian flow behavior and predictions using correlation equations of other authors (pilot-scale). Graph A: □,
553 Nienow et al. (1977); ◇, Xie et al. (2014a); x, Mockel et al. (1990); Graph B: □, Loiseau et al. (1977); ◇, J. M. Smith
554 (1992); x, Zhang et al. (2006); Δ, Garcia-Ochoa and Gomez (2004). The solid line represents the correlation equation,
555 and dashed lines the ± 20 % error (full data set in Figure S 2).

556 **Figure 3:** Newton number and gas hold-up in a 170 m³ scale fermenter with Newtonian fluid (Graph A and B) and in
557 a 110 m³ scale fermenter with non-Newtonian fluid (Graph C and D). The measured values are compared to predicted
558 values using various correlations. ○ represents data of this work. Graph A: □, Nienow et al. (1977); ◇, Xie et al. (2014a);
559 x, Mockel et al. (1990); Graph B: □, Linek et al. (1996); ◇, Xie et al. (2014b); x, Liepe et al. (1988); Graph C: □,
560 Nienow et al. (1977); ◇, Xie et al. (2014a); x, Mockel et al. (1990); Graph D: □, Loiseau et al. (1977); ◇, J. M. Smith
561 (1992); x, Zhang et al. (2006), Δ, Garcia-Ochoa and Gomez (2004). The solid line represents the correlation equation,
562 and dashed lines the respective standard deviation (Tables S2 to S3) of the present work.