HOMOMORPHSIM OF T-FUZZY SOFT NORMAL IDEALS OF RING

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Abstract

In this paper, we characterize $T-$soft ideals,$T-$fuzzy soft normal ideals over a ring and give a portion of their properties. Likewise, we present homomorphism, hostile to homomorphism, isomorphism and against isomorphism of $T-$fuzzy soft normal-ideals ordinary standards over the ring. In addition, we show that the image and pre-image of homomorphism, hostile to homomorphism, isomorphism

and against isomorphism of T-fuzzysoft normal ideals over the ring under certain conditions.

1 First section:INTRODUCTION

The regular techniques in science may plunge short to show vulnerability. To work out these vulnerability issues, numerous researchers try to create numerical instrument. Right off the bat, in 1965, the most proper hypothesis, for managing vulnerabilities is the hypothesis of fluffy sets created by Zadeh [20]. This hypothesis has been considered by numerous researchers until today and has advanced quickly. The hypothesis of delicate set, which is a totally new approach for displaying vulnerability, is presented by Molodtsov [13] in 1999. He gave fundamental properties of this hypothesis and demonstrated that this hypothesis has a rich potential for applications in a few fields, for example, examination, game hypothesis, likelihood hypothesis and so on. Arithmetical tasks, for example, soft subset, softest union, soft intersection and so forth among soft sets were examined in [3, 15] comprehensively. In [11], Maji et al. built up hypothesis of fuzzy soft set which is speculation of delicate set hypothesis. They examined set-hypothetical tasks of fuzzy soft sets. Kharal and Ahmad [8] fabricated the thought of mapping classes of fuzzy soft sets and concentrated the properties of fuzzy soft image and fuzzy inverse image of fuzzy soft sets. In [7], Kandemir and Tanay examined a few properties of fuzzy soft capacities in detail. The fuzzy soft sets are created to fuzzy soft semigroups by Yang [19] (2011).He characterized fuzzy soft [left, right] beliefs over semigroups and fuzzy soft semigroups, and concentrated adequate and essential conditions for α-level set, crossing point and association of fuzzy soft [left, right] ideals.

2 Second section

Definition 1 *A* $T-$ *norm is a binary operation* T: $[0,1]X[0,1] \to [0,1]$ *satisfying the following requirements:*
(i) $0Tx = 0, 1Tx = x$ *(boundary conditions)*
(ii) $xTy = yTx$ *(commutativity)*
(iii) $xT(yTz) = (xTy)Tz$ *(associativity)*
(iv) If $x \in y$ *and* $w \in z, then xTw \in yTz$ *(monotonicity).*

Definition 2 *Let* $(R, +, .)$ *be a ring . A* $T^{\smile}$ *fuzzy soft subset (F,A) of R is said to be a* $T^{\smile}$ *fuzzy soft ideal (TFSI) of R if the following conditions are satisfied:*
i) $\mu_{(F,A)}(x+y) \leq T(\mu_{(F,A)}(x), \mu_{(F,A)}(y))$,
ii) $\mu_{(F,A)}(\text{-}x) \leq \mu_{(F,A)}(x)$,
iii)$\mu_{(F,A)}(xy) \leq T(\mu_{(F,A)}(x), \mu_{(F,A)}(y))$, *for all x and y in R.*

Definition 3 *Let*$(F, A) and (G, B)$ *be any two* $T-$ *Fuzzy Soft subsets of sets* $RandH$, *respectively. The product of* $(F, A) and (G, B)$, *denoted by* $(F, A) \times (G, B)$, *is defined as* $\mu_{(F,A)\times(G,B)} = [(x, y), \mu_{(F,A)\times(G,B)}(x, y)$ *for all* $xinRandyinH]$, *where* $\mu_{(F,A)\times(G,B)}((x, y)) \leq T\mu_{(F,A)}(x), \mu_{(G,B)}(y)$.

Definition 4 *Let* (F, A) *be a* $T-$*fuzzy soft subset in a set S, the strongest* $T-$ *fuzzy soft relation on* S, *that is a* $T-$*fuzzy soft relation* (G, V) *with respect to* (F, A) *given by* $\mu_{(G,V)}((x, y)) = T(\mu_{(F,A)}(x), \mu_{(F,A)}(y))$, *for all x and y in S.*

Definition 5 *Let* $(R, +, .)$*be a ring. A* $T-$*fuzzy soft ideal* (F, A) *of*R *is said to be a* $T-$ *fuzzy soft normal ideal (TFSNI) of* R *if* $\mu_{(F,A)}(xy) = \mu_{(F,A)}(yx)$, *for all x and y in R.*

Definition 6 *A* $T-$ *fuzzy soft subset* (F, A) *of a set* X *is said to be normalized if there exists an element* $xinX$ *such that* $\mu_{(F,A)}(x) = 1$.

Definition 7 *Let (F, A) be a $T-$fuzzy soft subset of X. For α in T, a level subset of (F, A) corresponding to α is the set $(F, A)_\alpha = [x \epsilon X \colon \mu_{(F,A)}(x) \geq \alpha]$.*

Definition 8 *Let $(R, +, ﬄ)$ and $(R^{'}, +, ﬄ)$ be any two rings. Let $f \colon R \to R^{'}$ be any function and (F,A) be a $T-$ fuzzy soft ideal in $R, (G, V)$ be a $T-$ fuzzy soft ideal in $f(R) = R^{'}$, defined by $\mu_{(G,V)}(y) = sup_{(x \epsilon f^{-1}(y))} \mu_{(F,A)}(x)$, for all $xinR$ and $yinR^{'}$.*
Then (F, A) is called a pre-image of (G, V) under f and is denoted by $f^{-1}(G, V)$.

3 SOME OF THE PROPERTIES BASED ON T-FUZZY SOFT NORMAL IDEALS OF RING

Theorem 1 *A $T-$fuzzy soft ideal (F, A) of a ring R is normalized if and only if $\mu_{(F,A)}(e) = 1$, where e is the identity element of R.*
Proof
If (F, A) is normalized, then there exists $xinR$ such that $(F, A)(x) =$ 1, but by properties of a T -fuzzy ideal $(F, A) of R$, $\mu_{(F,A)}(x) \leq \mu_{(F,A)}(e)$, for every $xinR$. Since$\mu_{(F,A)}(x) = 1$ and $\mu_{(F,A)}(x) \leq \mu_{(F,A)}(e), 1 \leq \mu_{(F,A)}(e)$. But $1 \geq \mu_{(F,A)}(e)$. Hence $\mu_{(F,A)}(e) = 1$, then by the definition of normalized $T-$ fuzzy subset, (F, A) is normalized.

Theorem 2 *Let $(F, A) and (G, B)$ be $T-$fuzzy soft subsets of the rings $RandH$, respectively. Suppose that $eande^{'}$ are the identity element of $RandH$, respectively. If $(F, A) \times (G, B)$ is a $T-$Fuzzy Soft normal ideal of $R \times H$, then at least one of the following two statements must hold.*
(i)$(G, B)(e_{'}) \geq (F, A)(x)$, for all $xinR$, (ii)$(F, A)(e) \geq (G, B)(y)$, for all$yinH$.
Proof
It is trivial

Theorem 3 *Let$(F, A) and (G, B)$ be $T-$fuzzy soft subsets of the rings $RandH$, respectively and $(F, A) \times (G, B)$ is a $T-$fuzzy soft normal ideal of $R \times H$. Then* *the following are true: 1.if $(F, A)(x) \leq (G, B)(e^{'})$, then (F, A) is a $T-$fuzzy soft normal ideal of R. 2.if$(G, B)(x) \leq (F, A)(e)$, then (G, B) is a $T-$fuzzy soft normal ideal of$H$. 3.Either (F, A) is a $T-$fuzzy soft normal ideal of $Ror(G, B)$ is a $T-$fuzzy soft normal ideal of H.*

Proof

Let $(F, A) \times (G, B)$ be a $T-$fuzzy soft normal ideal of$R \times H$ and $x, yinR$ and $e^{'}inH$. Then $(x, e^{'})and(y, e^{'})$ are in $R \times H$. Clearly $(F, A) \times (G, B)$ is a $T-$fuzzy soft ideal of $R \times H$. Now, using the property that $(F, A)(x) \geq (G, B)(e^{'})$, for all $xinR$, clearly (F, A) is a $T\breve{}$fuzzy soft ideal of R.

Now,$\mu_{(F,A)}(xy) \leq T\mu_{(F,A)}(xy), \mu_{(G,B)}(e^{'}e^{'}) = \mu_{(F,A)\times(G,B)}((xy), (e^{'}e^{'})) = \mu_{(F,A)\times(G,B)}((x, e^{'})(y, e^{'})) = mu_{(F,A)\times(G,B)}((y, e^{'})(x, e^{'})) = \mu_{(F,A)\times(G,B)}((yx), ((e^{'}e^{'}))) \leq T(\mu_{(F,A)}(yx), \mu_{(G,B)}((e^{'}e^{'}))) = \mu_{(F,A)}(yx)$.

Therefore, $\mu_{(F,A)}(xy) = \mu_{(F,A)}(yx)$, for all $xandyinR$. Hence (F, A) is a $T-$fuzzy soft normal ideal of R. Thus (i) is proved.

Now, using the property that $\mu_{(G,B)}(x) \leq \mu_{((F,A))}(e)$, for all$xinH$, let $xand\ yin\ H\ and\ einR$. Then $(e, x)and(e, y)$ are in $R \times H$. Clearly (G, B) is a $T-$fuzzy soft ideal of H. Now, $\mu_{(G,B)}(xy) \leq T(\mu_{(G,B)}(xy), \mu_{((F,A))}(ee)) \leq T(\mu_{((F,A))}(ee), \mu_{(G,B)}(xy)) = \mu_{(F,A)\times(G,B)}((ee), (xy)) = \mu_{(F,A)\times(G,B)}((e, x)(e, y)) = \mu_{(F,A)\times(G,B)}[((e, y)(e, x))] = \mu_{(F,A)\times(G,B)}((ee), (yx)) \leq T(\mu_{((F,A))}(ee), \mu_{(G,B)}(yx)) = \mu_{(G,B)}(yx)$.

Therefore,$\mu_{(G,B)}(xy) = \mu_{(G,B)}(yx)$, for all $xandyinH$. Hence (G, B) is a $T-$fuzzy soft normal ideal of H. Thus (ii) is proved. (iii) is clear.

Theorem 4 *Let (F, A) be a $T-$fuzzy soft subset of a ring R and (G, V) be the strongest $T-$Fuzzy Soft relation of R with respect to (F, A). Then (F, A) is a $T-$fuzzy soft normal ideal of R if and only if $(G, V)$is a $T-$fuzzy soft normal ideal of $R \times R$.*

Proof

Suppose that (F, A) is a $T-$fuzzy soft normal ideal of R. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in $R\times R$. Clearly (G, V) is a $T-$fuzzy soft ideal of $R\times R$. We have, $\mu_{(G,V)}(xy) = \mu_{(G,V)}[(x_1, x_2)(y_1, y_2)] = \mu_{(G,V)}((x_1y_1, x_2y_2)) = \mu_{(F,A)}((x_1y_1))\mu_{(F,A)}((x_2y_2)) = T(\mu_{(F,A)}((y_1x_1)), \mu_{(F,A)}((y_2x_2))) = \mu_{(G,V)}((y_1x_1, y_2x_2)) = \mu_{(G,V)}[(y_1, y_2)(x_1, x_2)] = \mu_{(G,V)}(yx)$. Therefore, $\mu_{(G,V)}(xy) = \mu_{(G,V)}(yx)$, for all x and y in $R \times R$. This proves that (G, V) is a $T-$fuzzy soft normal *ideal of $R \times R$.*

Conversely, assume that (G, V) is a$T-$Fuzzy Soft normal ideal of $R \times R$, then for any $x = (x_1, x_2) and y = (y_1, y_2)$ are in $R \times R$, we have $T(\mu_{((F,A))}(x_1y_1), \mu_{((F,A))}(x_2y_2)) = \mu_{(G,V)}((x_1y_1, x_2y_2)) = \mu_{(G,V)}[(x_1, x_2)(y_1, y_2)] = \mu_{(G,V)}(xy) = \mu_{(G,V)}(yx) = \mu_{(G,V)}[(y_1, y_2)(x_1, x_2)] = \mu_{(G,V)}((y_1x_1, y_2x_2)) = T\mu_{(F,A)}(y_1x_1)\mu_{(F,A)}(y_2x_2)$. If we put $x_2 = y_2 = e$, where e is the identity element of R.

We get, $\mu_{(F,A)}((x_1y_1)) = \mu_{(F,A)}(y_1x_1)$, for all $x_1 and y_1$ in R. Hence (F, A) is a $T-$fuzzy soft normal ideal of R.

Theorem 5 *Let $(R, +, .)$ and $(R^{'}, +, .)$ be any two rings. The homomorphic image of a $T-$fuzzy soft normal ideal of R is a $T-$fuzzy soft normal ideal of$R^{'}$.*

Proof *Let $f\colon R \leftarrow R^{'}$ be a homomorphism. Let (F, A) be a $T-$fuzzy normal ideal of R. We have to prove that (G, V) is a $T-$fuzzy soft normal ideal of $f(R) = R^{'}$. Now, for $f(x) and f(y) in R^{'}$, we have clearly (G, V) is a $T-$fuzzy soft ideal of a ring $f(R) = R^{'}$, since $(F, A) is a T-$fuzzy soft ideal of a ring R. Now, $\mu_{(G,V)}(f(x)f(y)) = \mu_{(G,V)}(f(xy)) \geq \mu_{(F,A)}(xy) = \mu_{(F,A)}(yx) \leq \mu_{(G,V)}(f(yx)) = \mu_{(G,V)}(f(y)f(x))$, which implies that $\mu_{(G,V)}(f(x)f(y)) \leq \mu_{(G,V)}(f(y)f(x))$. Hence (G, V) is a $T-$fuzzy soft normal ideal of the ring $R^{'}$.*

Theorem 6 *Let $(R, +, .) and (R^{'} +, .)$ be any two rings . The homomorphic pre-image of a $T-$fuzzy soft normal ideal of $R^{'}$ is a $T-$fuzzy soft normal ideal of R.*

Proof *Let $f\colon R \leftarrow R^{'}$ be a homomorphism. Let (G, V) be a $T-$fuzzy soft normal ideal of $f(R) = R^{'}$. We have to prove that (F, A) is a $T-$fuzzy soft normal ideal of R. Let $x and y in R$. Then, clearly (F, A) is a $T-$Fuzzy Soft ideal of the ring R, since (G, V) is a $T\breve{}$fuzzy soft ideal of the ring $R^{'}$. Now, $\mu_{(F,A)}(xy) = \mu_{(G,V)}(f(xy)) = \mu_{(G,V)}(f(x)f(y)) = \mu_{(G,V)}(f(y)f(x)) = \mu_{(G,V)}(f(yx)) = \mu_{(F,A)}(yx)$, which implies that $\mu_{(F,A)}(xy) = \mu_{(F,A)}(yx)$, for $x and y in R$. Hence (F, A) is a $T\breve{}$fuzzy soft normal ideal of the ring R.*

Theorem 7 *Let $(R, +, .)$and $(R^{'} +, .)$ be any two rings . The anti-homomorphic image of a $T-$fuzzy soft normal ideal of $R^{'}$ is a $T-$fuzzy soft normal ideal of*

R.

Proof *Let $f\colon R \leftarrow R^{'}$ be anti-homomorphism. Let (F, A) be a $T-$fuzzy normal ideal of R. We have to prove that (G, V) is a $T-$fuzzy soft normal ideal of $f(R) = R^{'}$.*

Now, for $f(x) and f(y) in R^{'}$, we have clearly (G, V) is a $T-$fuzzy soft ideal of a ring $f(R) = R^{'}$, since $(F, A) isa T-$fuzzy soft ideal of a ring R. Now, $\mu_{(G,V)}(f(x)f(y)) = \mu_{(G,V)}(f(yx)) \geq \mu_{(F,A)}(f(x)f(y)) \leq \mu_{(G,V)}(f(xy)) = \mu_{(G,V)}(f(y)f(x)) = \mu_{(F,A)}(yx)$, which implies that $\mu_{(G,V)}(f(x)f(y)) \leq \mu_{(G,V)}(f(y)f(x))$, for $x and y in R$

. Hence (G, V) is a $T\breve{}$fuzzy soft normal ideal of the ring $R^{'}$.

Theorem 8 *Let $(R, +, .) and (R^{'} +, .)$ be any two rings . The anti-homomorphic pre-image of a $T-$fuzzy soft normal ideal of $R^{'}$ is a $T-$fuzzy soft normal ideal of R.*

Proof *Let $f\colon R \leftarrow R^{'}$ be anti=homomorphism. Let (G, V) be a $T-$fuzzy normal ideal of R. We have to prove that (F, A) is a $T-$fuzzy soft normal ideal of R. Now, for $x and y in R$ we have clearly (F, A) is a $T-$fuzzy soft ideal of a ring R, since $(G, V) isa T-$fuzzy soft ideal of $R^{'}$.*

Now,$\mu_{(F,A)}(xy) = \mu_{(G,V)}(f(xy)) = \mu_{(G,V)}(f(y)f(x)) = \mu_{(G,V)}(f(x)f(y) = \mu_{(G,V)}(f(yx)) = \mu_{(F,A)}(yx)$, which implies that $\mu_{(F,A)}(xy) = \mu_{(F,A)}(yx)$, for $x and y in R$. Hence (F, A) is a $T-$fuzzy soft normal ideal of the ring R.

Theorem 9 *Let (F, A) be a $T-$ fuzzy soft normal ideal of a ring $H and f$ is a isomorphism from a ring $R onto H$. Then $((F, A) \circ f)$ is a $T-$fuzzy soft normal ideal of R.*

Proof *Let $x and y in R$ and (F, A) be a $T-$fuzzy soft normal ideal of a ring H. Then clearly $(F, A) \circ f)$ is a $T-$fuzzy soft ideal of the ring R.*

Then we have, $\mu_{(F,A)\circ f)}(xy) = \mu_{(F,A)}(f(xy)) = \mu_{(F,A)}(f(x)f(y)) = \mu_{(F,A)}(f(y)f(x)) = \mu_{((F,A))}(f(yx)) = \mu_{(F,A)\circ f)}(yx)$, which implies that $\mu_{(F,A)\circ f)}(xy) = \mu_{(F,A)\circ f)}(yx)$, for$x and y in R$.

Hence $(F, A) \circ f)$ is a $T-$ fuzzy soft normal ideal of the ring R.

Theorem 10 *Let (F, A) be a $T-$fuzzy soft normal ideal of a ring H and f is an anti-isomorphism from a ring$RontoH$. Then $(F, A) \circ f)$ is a $T-$fuzzy soft normal ideal of R.*

Proof *Let $xandyinR$ and (F, A) be a $T-$fuzzy soft normal ideal of a ring H. Then clearly $((F, A) \circ f)$ is a $T-$fuzzy soft ideal of the ring R. Then we have, $\mu_{(F,A)\circ f)}(xy) = \mu_{(F,A)}(f(xy)) = \mu_{(F,A)}(f(y)f(x)) = \mu_{(F,A)}(f(x)f(y)) = \mu_{(F,A)}(f(yx)) = \mu_{(F,A)\circ f)}(yx)$, which implies that $\mu_{(F,A)\circ f)}(xy) = \mu_{(F,A)\circ f)}(yx)$, for$xandyinR$. Hence $((F, A) \circ f)$ is a $T-$fuzzy soft normal ideal of the ring R.*

Theorem 11 *Let (F, A) be a $T-$fuzzy soft normal ideal of a ring R, then the pseudo $T-$fuzzy soft coset $(a(F, A))^p$ is a $T-$fuzzy soft normal ideal of the ring R, for $ainR$.*

Proof *Let (F, A) be a $T-$fuzzy soft normal ideal of a ring R. For every $xandyinR$, we have, clearly $(a(F, A))^p$ is a $T-$fuzzy soft ideal of the ring R and*
$((a\mu_{((F,A))})^p(xy) = p(a)\mu_((F, A))(xy) = p(a)\mu_((F, A))(yx) = ((a\mu_((F, A)))^p(yx)$. Therefore, $((a\mu_((F, A)))^p(xy) = ((a\mu_((F, A)))^p)(yx)$, for$xandyinR$. Hence $(a(F, A))^p$ is a $T-$fuzzy soft normal ideal of the ring R.

Theorem 12 *Let (F, A) be a $T-$ Fuzzy Soft ideal of a ring R. Then for α in S such that $\alpha \geq (F, A)(e), (F, A)\alpha isaidealofR$*
*. **Proof** For all $xandyin(F, A)_\alpha$, we have, $\mu_((F, A))(x) \leq \alpha and \mu_((F, A))(y) \leq \alpha$. Now, $\mu_((F, A))(x\text{-}y) \leq T\mu_{((F,A))}(x), \mu_{((F,A))}(y) \leq T\alpha, \alpha = \alpha$, which implies that, $\mu_{((F,A))}(x\text{-}y) \leq \alpha$. And,$\mu_{((F,A))}(xy) \leq T\mu_{((F,A))}(x), \mu_{((F,A))}(y) \leq T\alpha, \alpha = \alpha$ which implies that, $\mu_{((F,A))}(xy) \leq \alpha$.*
Therefore, $\mu_{((F,A))}(x\text{-}y) \leq \alpha, \mu_{((F,A))}(xy) \leq \alpha$
we get x$-$y and xy in $(F, A)_\alpha$. Hence $(F, A)_\alpha$ is a ideal of R.

Theorem 13 *Let (F, A) be a $T-$fuzzy ideal of a ring R. Then two ideals $(F, A)_{\alpha_1}, (F, A)_{\alpha_2}$ and $\alpha_1 and \alpha_2$in S and $\alpha_1 \leq \mu_{((F,A))}(e), \alpha_2 \leq \mu_{((F,A))}(e)$ with $\alpha 2 < \alpha 1$ of (F, A) are equal if and only if there is no x in R such that$\alpha_1 < \mu_{((F,A))}(x) < \alpha_2$. **Proof** Assume that $(F, A)_{\alpha_1} = (F, A)_{\alpha_2}$. Suppose there exists an $x\epsilon R$ such that $\alpha_1 < \mu_{(F,A)}(x) < \alpha_2$*

Then $(F,A)_{\alpha_1 \subseteq (F,A)_{\alpha_2}}$, which implies that x belongs to$(F,A)_{\alpha_2}$, but not in $(F,A)_{\alpha_1}$. This is a contradiction to $(F,A)_{\alpha_1} = (F,A)_{\alpha_2}$.
Therefore, there is no $x \epsilon R$ such that $\alpha_1 < \mu_{((F,A))}(x) < \alpha_2$. Conversely, if there is no $x \epsilon R$ such that $\alpha_1 < \mu_{((F,A))}(x) \alpha_2$, then $(F,A)_{\alpha_1} = (F,A)_{\alpha_2}$.

Theorem 14 *The homomorphic image of a ideal of a $T-$fuzzy soft ideal of the ring R is a ideal of a $T-$fuzzy soft ideal of the ring R'.* ***Proof*** *Let $f\colon R \leftarrow R'$ be a homomorphism. Let $(G,V) = f((F,A), where (F,A)$ is a $T-$fuzzy soft ideal of the ring R. Clearly (G,V) is a $T-$fuzzy soft ideal of the ring R'. Let$x and y in R$, implies $f(x) and f(y) in R'$. Let $(F,A)_\alpha$ is a ideal of (F,A). That is,$\mu_{((F,A))}(x) \leq \alpha and \mu_{((F,A))}(y) \leq \alpha$; $\mu_{((F,A))}(x\text{-}y) \leq \alpha, \mu_{((F,A))}(xy) \leq \alpha$.We have to prove that $f((F,A)_\alpha$ is a ideal of (G,V). Now,$\mu_{(G,V)}(f(x)) \leq \mu_{((F,A))}(x) \leq \alpha$,which implies that $\mu_(G,V)(f(x)) \leq \alpha$; and $\mu_{(G,V)}(f(y)) \geq \mu_{((F,A))}(y) \leq \alpha$, which implies that $\mu_{(G,V)}(f(y)) \geq \alpha$ and $\mu_{(G,V)}(f(x)\text{-}f(y)) = \mu_{(G,V)}(f(x\text{-}y)) \geq \mu_{((F,A))}(x\text{-}y) \leq \alpha$ which implies that $\mu_{(G,V)}(f(x)\text{-}f(y) \geq \alpha$. Also,$\mu_{(G,V)}(f(x)f(y)) = \mu_{(G,V)}(f(xy)) \geq \mu_{((F,A))}(xy) \leq \alpha$, which implies that $\mu_{(G,V)}(f(x)f(y) \leq \alpha$. Therefore,$\mu_{(G,V)}(f(x)\text{-}f(y) \leq \alpha$ and $\mu_{(G,V)}(f(x)f(y)) \leq \alpha$. Hence $f((F,A)_\alpha)$ is a ideal of a $T-$fuzzy soft ideal (G,V) of the ring R'.*

Theorem 15 *The homomorphic pre- image of a ideal of a $T-$ fuzzy soft ideal of the ring R' is a ideal of a $T-$fuzzy soft ideal of the ring R.*
Proof *Let $f\colon R \leftarrow R'$ be a homomorphism. Let $(G,V) = f(F,A)$, where (G,V) is a $T-$fuzzy soft ideal of the ring R'. Clearly (F,A) is a $T-$fuzzy soft ideal of the ring R. Let $f(x) and f(y) in R'$, implies $x and y in R$. Let $f((F,A)_\alpha)$ is a ideal of (G,V). That is,$\mu_{(G,V)}(f(x)) \leq \alpha$ and $\mu_{(G,V)}(f(y)) \leq \alpha; \mu_{(G,V)}(f(x)\text{-}f(y)) \leq \alpha, \mu_{(G,V)}(f(x)f(y)) \leq \alpha$. We have to prove that $(F,A)_\alpha$ is a ideal of (F,A). Now,$mu_{((F,A))}(x) = \mu_{(G,V)}(f(x)) \leq \alpha$, implies that $\mu_{((F,A))}(x) \leq \alpha$; $\mu_{((F,A))}(y) = \mu_{(G,V)}(f(y)) \leq \alpha$, implies that $\mu_{((F,A))}(y) \leq \alpha$ and $\mu_{((F,A))}(x\text{-}y) = \mu_{((G,V))}(f(x\text{-}y)) = \mu_{(G,V)}(f(x)\text{-}f(y) \leq \alpha$, which implies that $\mu_{((F,A))}(x\text{-}y) \leq \alpha$. Also, $\mu_{((F,A))}(xy) = \mu_{(G,V)}(f(xy)) = \mu_(G,V)(f(x)f(y)) \leq \alpha$, which implies that $\mu_{((F,A))}(xy) \leq \alpha$. Therefore, $\mu_{((F,A))}(x\text{-}y) \leq \alpha$, $\mu_{((F,A))}(xy) \leq \alpha$. Hence, $(F,A)_\alpha$ is a ideal of a $T-$fuzzy soft ideal $(F,A) of R$.*

Theorem 16 *The anti-homomorphic image of a ideal of a $T-$fuzzy soft ideal of a ring R is a ideal of a $T-$fuzzy soft ideal of a ring R'.*

Proof *Let* $f\colon R \leftarrow R'$ *be an anti- homomorphism. Let* $(G,V) = f((F,A)$*, where*(F,A) *is a* $T-$ *fuzzy soft ideal of R. Clearly* (G,V) *is a* $T-$*fuzzy soft ideal of* R'*. Let* $xandyinR$ *, implies*$f(x)andf(y)inR'$*. Let* $(F,A)_\alpha$ *is a ideal of*(F,A)*. That is,*$\mu_{((F,A))}(x) \leq \alpha$ *and* $\mu_{((F,A))}(y) \leq \alpha$*.*$\mu_{((F,A))}(y\text{-}x) \leq \alpha, \mu_{((F,A))}(yx) \leq \alpha$*. We have to prove that* $f((F,A)_{alpha})$ *is a ideal of* (G,V)*. Now,* $\mu_{(G,V)}(f(x)) \geq \mu_{((F,A))}(x) \leq \alpha$*, which implies that* $\mu_{(G,V)}(f(x)) \leq \alpha; \mu_{(G,V)}(f(y)) \geq \mu_{((F,A))}(y) \leq \alpha$*, which implies that* $\mu_{(G,V)}(f(y)) \leq \alpha$*. Now,* $\mu_{(G,V)}(f(x)\text{-}f(y) = \mu_{(G,V)}(f(x)\text{-}f(y)) = \mu_{(G,V)}(f(y\text{-}x)) \geq \mu_{((F,A))}(y\text{-}x) \leq \alpha$*, which implies that* $\mu_{(G,V)}(f(x)\text{-}f(y)) \leq \alpha$*. Also,*$\mu_{(G,V)}(f(x)f(y)) = \mu_{(G,V)}(f(yx)) \geq \mu_{((F,A))}(yx) \leq \alpha$*, which implies that* $\mu_{(G,V)}(f(x)f(y)) \leq \alpha$*. Therefore,*$\mu_{(G,V)}(f(x)\text{-}f(y) \leq \alpha$ *and* $\mu_{(G,V)}(f(x)f(y)) \leq \alpha$*. Hence* $f((F,A)_\alpha)$ *is a ideal of a* $T-$*fuzzy soft ideal* (G,V) *of* R'*.*

Theorem 17 *The anti-homomorphic pre-image of a level ideal of a* $T\breve{}$*fuzzy soft ideal of a ring*R' *is a level ideal of a* $T-$*fuzzy soft ideal of a ring R.*
Proof *Let*$f\colon R \leftarrow R'$ *be an anti-homomorphism. Let* $(G,V) = f(F,A)$*, where* (G,V) *is a* $T-$ *fuzzy soft ideal of the ring* R'*. Clearly* (F,A) *is a* $T-$*fuzzy soft ideal of the ring R. Let* $f(x)andf(y)inR'$*, implies* $xandyinR$ *. Let*$f((F,A)_\alpha)$ *is a ideal of* (G,V)*. That is,* $\mu_{(G,V)}(f(x)) \leq \alpha$ *and* $\mu_{(G,V)}(f(y)) \leq \alpha; \mu_{(G,V)}(f(y)\text{-}f(x)) \leq \alpha, \mu_{(G,V)}(f(y)f(x)) \leq \alpha$*. We have to prove that* $(F,A)_\alpha$ *is a ideal of* (F,A)*. Now,*$\mu_{((F,A))}(x) = \mu_{(G,V)}(f(x)) \leq \alpha$*, which implies that*$\mu_{((F,A))}(x) \leq \alpha; \mu_{((F,A))}(y) = \mu_{(G,V)}(f(y)) \leq \alpha$*, which implies that* $\mu_{((F,A))}(y) \leq \alpha$*.Now,*$\mu_{((F,A))}(\text{-}y) = \mu_{(G,V)}(f(x\text{-}y)) = \mu_{(G,V)}(f(y)\text{-}f(x)) = \mu_{(G,V)}(f(y)\text{-}f(x)) \leq \alpha$*, which implies that*$\mu_{((F,A))}(x\text{-}y) \leq \alpha$*.Also,*$\mu_{((F,A))}(xy) = \mu_{(G,V)}(f(xy)) = \mu_{(G,V)}(f(y)f(x)) \leq \alpha$*, which implies that*$\mu_{((F,A))}(xy) \leq \alpha$*.Therefore,*$\mu_{((F,A))}(x\text{-}y) \leq \alpha and \mu_{((F,A))}(xy) \leq \alpha$*. Hence* $(F,A)_\alpha$ *is a ideal of a* $T-$*fuzzy soft ideal (F,A) of R.*

Theorem 18 *Let* (F,A)*be a*$T-$*fuzzy soft ideal of a ring R. Then* $a+(F,A)_\alpha = (a+(F,A))_\alpha$ *, for every* $ainR, \alpha inT$*.*
Proof *Let* (F,A) *be a* $T-$*fuzzy soft ideal of a ring R and let x in R. Now,* $x\epsilon(a+(F,A)_\alpha$ *if and only if* $(a+(F,A))(x) \leq \alpha$ *if and only if* $\mu_{((F,A))}(x\text{-}a) \leq \alpha$*if and only if* $x\text{-}a\epsilon(F,A)_\alpha$ *if and only if*$x\epsilon a+(F,A)_\alpha$ *. Therefore,* $a+(F,A)_\alpha = (a+(F,A))_\alpha$ *, for every* $xinR$*.*

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