Ajustes lineales
Cuadrados Mínimos

Laboratorio 1

Departamento de Física –FCEyN – UBA

(Adaptado de J. Sacanell)

Relaciones lineales entre variables

Posición/Velocidad: $pos = pos_0 + vel_0\, t$

Resorte: $F = - k\, \Delta X$

Resistencia eléctrica: $V = R\, I$

Densidad $Masa = Densidad\ Volumen$

Crecimiento: $Altura = Altura_0 + Ritmo\ Edad$

$$\boxed{\boldsymbol{y = m\, x + b}}$$

Tabla de valores

Posición/Velocidad: $pos = pos_0 + vel_0\, t$

$x\ (t)$	$y\ (pos)$
x_1	y_1
x_2	y_2
x_3	y_3
x_4	y_4

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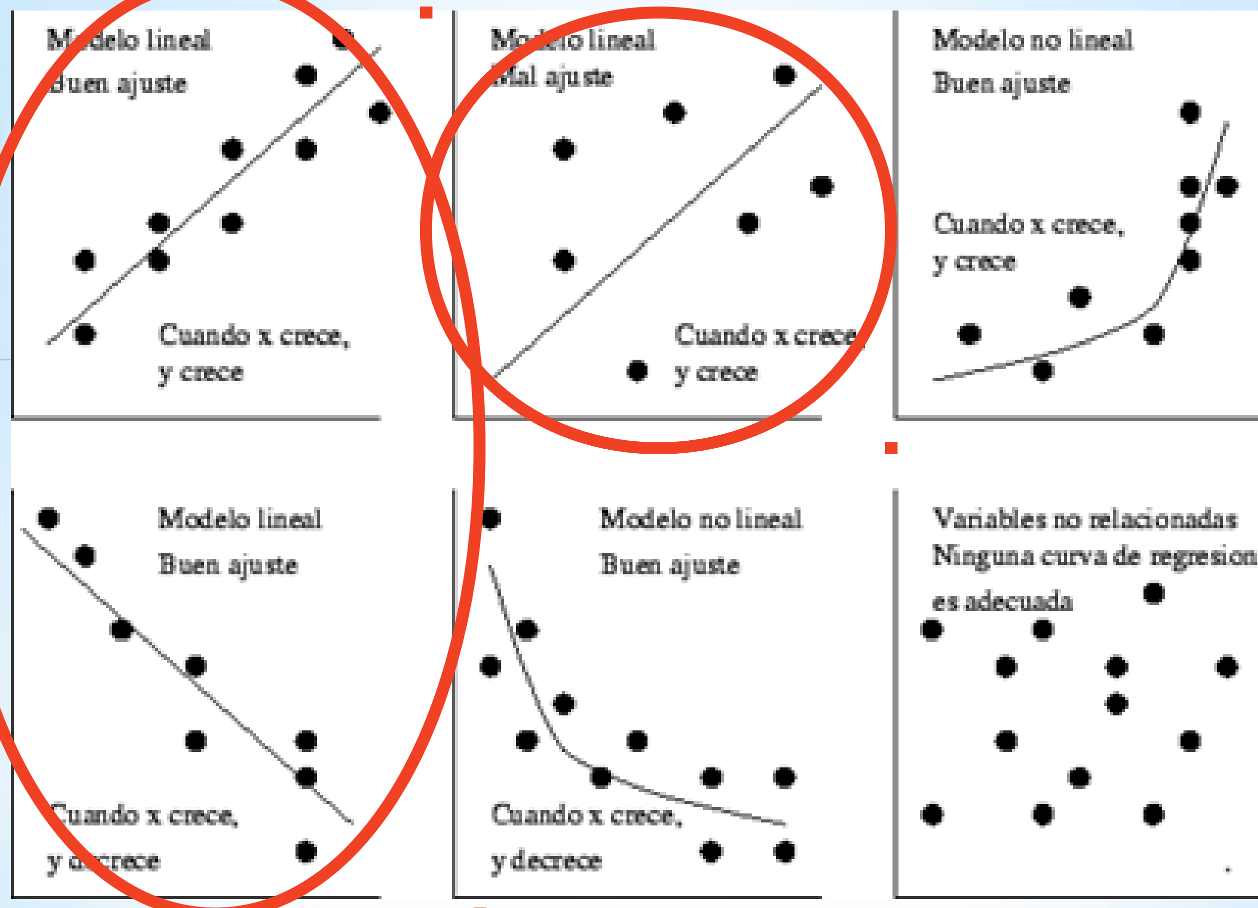
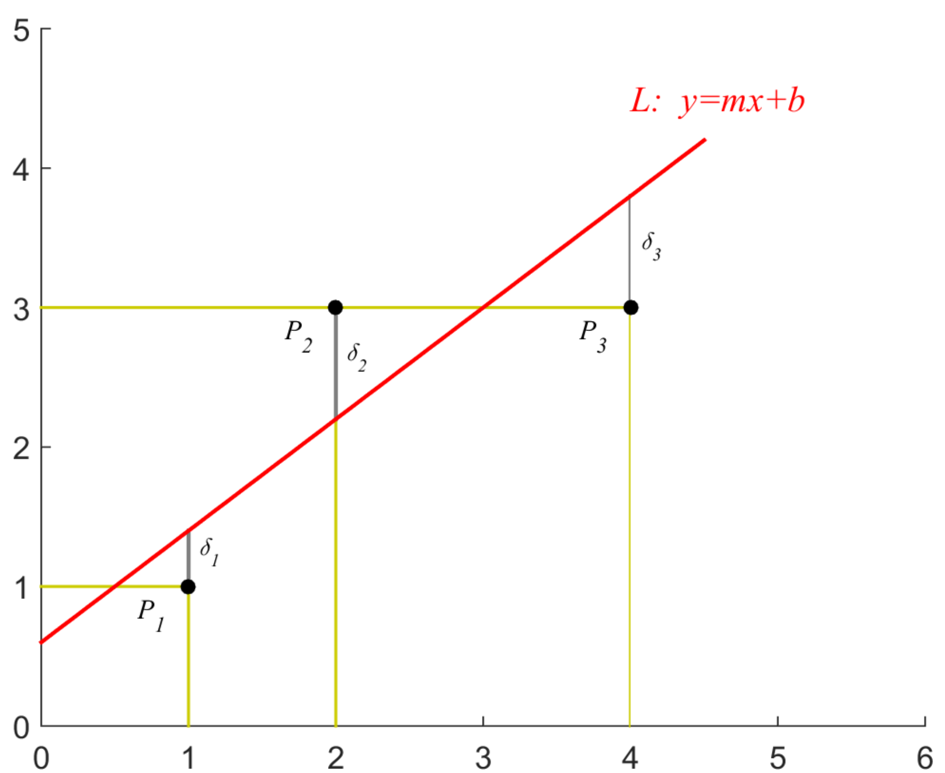
Graficamos: posibles resultados…



¿Cómo encontramos la mejor recta que ajuste nuestros datos?



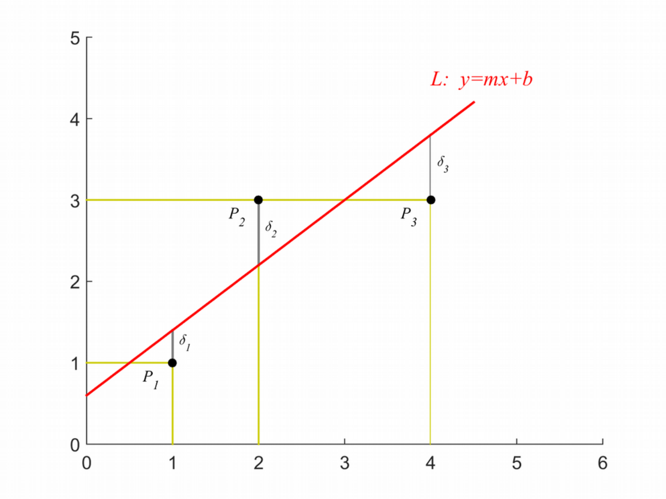
$$y = mx + b$$

$$\delta y_i = y_i - (mx_i + b)$$

$$(\delta y_i)^2 = [y_i - (mx_i + b)\ ]^2$$

$$\begin{aligned} M &= \sum (\delta y_i)^2 \\ &= \sum {y_i}^2 + m^2 \sum {x_i}^2 \\ &\quad + nb^2 + 2mb \sum x_i \\ &\quad - 2m \sum x_i y_i \\ &\quad - 2b \sum y_i \end{aligned}$$

¿Cómo encontramos la mejor recta que ajuste nuestros datos?



Mejor recta: la que minimice M

$$M = \sum(\delta y_i)^2 = \sum {y_i}^2 + m^2 \sum {x_i}^2 + nb^2 + 2mb\sum x_i - 2m\sum x_i y_i - 2b\sum y_i$$

$$\frac{\partial M}{\partial m} = 0 \qquad \frac{\partial M}{\partial b} = 0$$

$$\left\{\begin{array}{l} 2m\sum {x_i}^2 + 2b\sum x_i - 2\sum(x_i y_i) = 0 \\ 2nb + 2m\sum x_i - 2\sum y_i = 0 \end{array}\right.$$

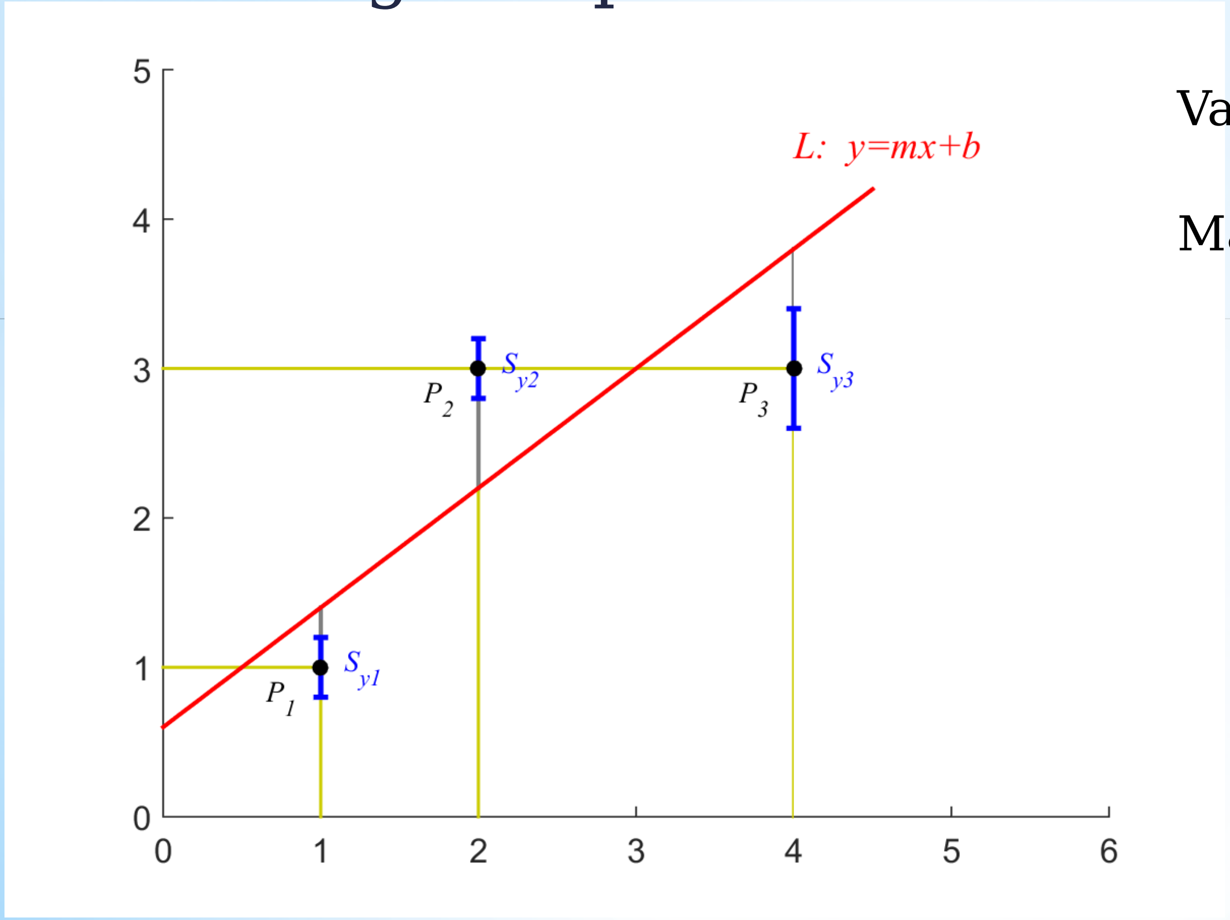
$$m = \frac{n\sum(x_i y_i) - \sum x_i \sum y_i}{n\sum {x_i}^2 - (\sum x_i)^2}$$

$$b = \frac{\sum {x_i}^2 \sum y_i - \sum x_i \sum(x_i y_i)}{n\sum {x_i}^2 - (\sum x_i)^2}$$

Δm

Δb

¿Todos los puntos son equivalentes? ¿Confiamos más en alguno que en otro?



Valores ponderados: s_{yi}

Más incerteza → menos peso

¿Todos los puntos son equivalentes? ¿Confiamos más en alguno que en otro?

Cuadrados mínimos ponderados

$$b = \frac{\sum \frac{1}{(S_{yi})^2} y_i \sum \frac{1}{(S_{yi})^2} {x_i}^2 - \sum \frac{1}{(S_{yi})^2} x_i \sum \frac{1}{(S_{yi})^2} x_i y_i}{\sum \frac{1}{(S_{yi})^2} \sum \frac{1}{(S_{yi})^2} {x_i}^2 - \left(\sum \frac{1}{(S_{yi})^2} x_i \right)^2}$$

$$m = \frac{\sum \frac{1}{(S_{yi})^2} \sum \frac{1}{(S_{yi})^2} (x_i y_i) - \sum \frac{1}{(S_{yi})^2} x_i \sum \frac{1}{(S_{yi})^2} y_i}{\sum \frac{1}{(S_{yi})^2} \sum \frac{1}{(S_{yi})^2} {x_i}^2 - \left(\sum \frac{1}{(S_{yi})^2} x_i \right)^2}$$

Δm
Δb

[https://www.che.udel.edu/pdf/FittingData.pdf]

Pero, ¿qué hacemos si la relación no es lineal?

Si podemos, cambiamos de variables, ¡LINEALIZAMOS!

Ejemplo 1: Período de un péndulo

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\boxed{T = \frac{2\pi}{\sqrt{g}}\sqrt{l}}$$

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Leyes de escala – Leyes alométricas

$$y = y_0\, M^b$$

Transformamos, linealizamos:

$$log_{10}(y) = log_{10}(y_0\, M^b)$$

$$log_{10}(y) = log_{10}(y_0) + log_{10}(M^b)$$

$$\underbrace{log_{10}(y)}_{y'} = log_{10}(y_0) + b\; \underbrace{log_{10}(M)}_{x'}$$

(cuidado con M^b vs $y=mx+b$)

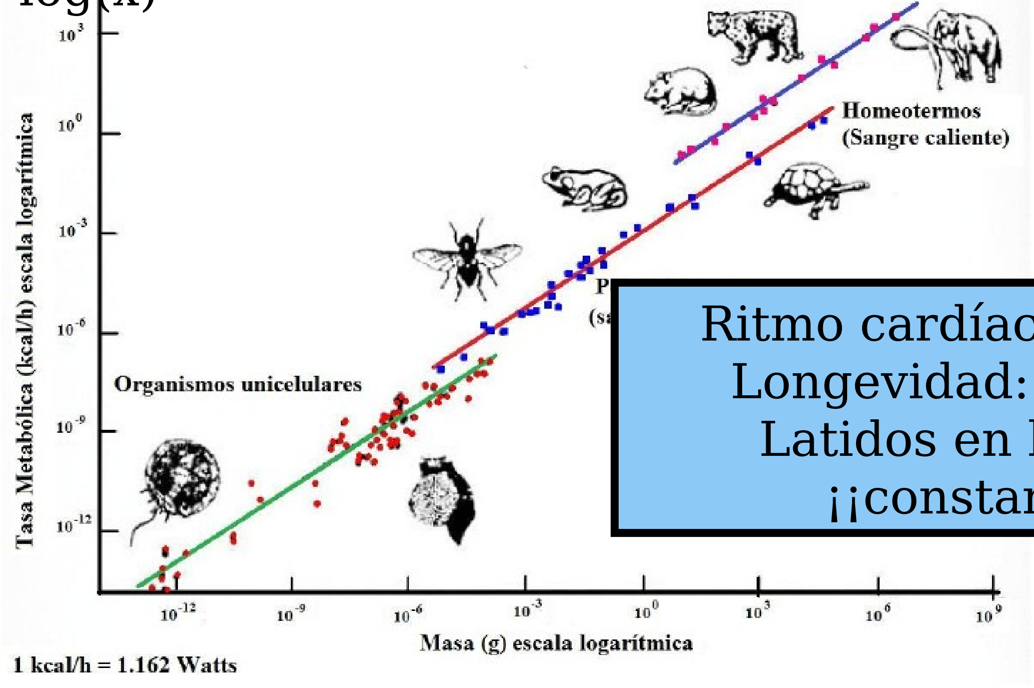
“m” pendiente

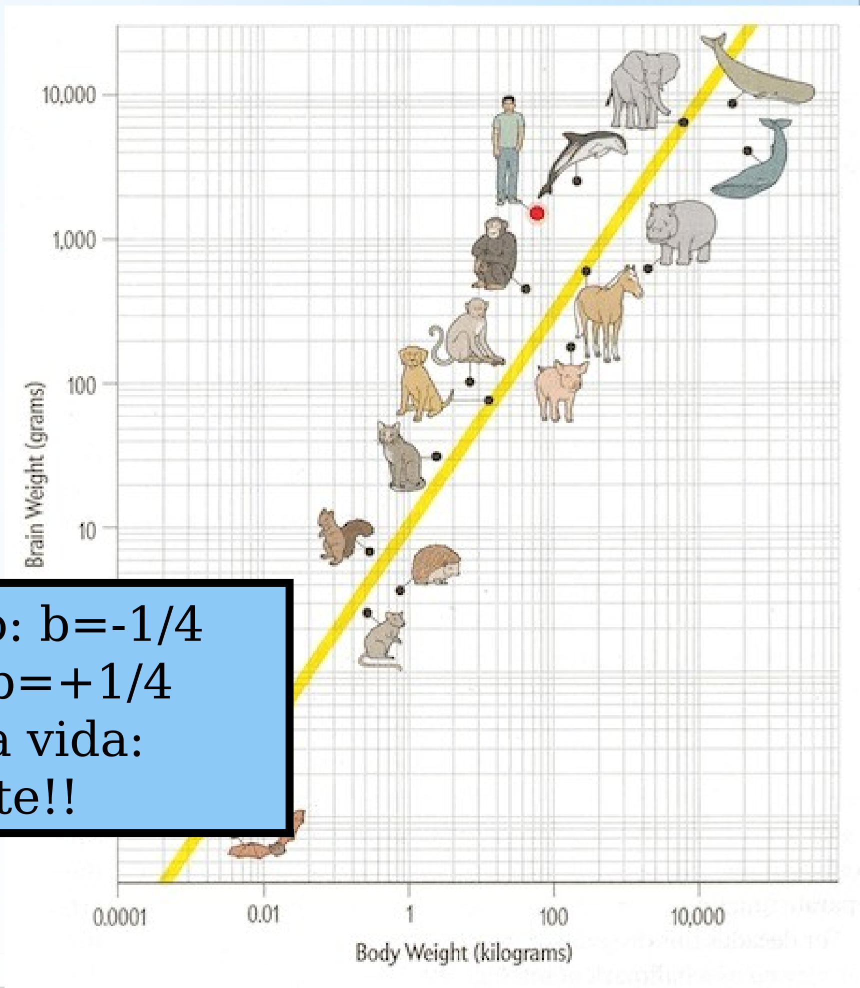
“b” → $y_0 = 10^{\text{"}b\text{"}}$

Leyes de escala – Leyes alométricas

$$y = y_0\, M^b$$

Graficar en escala logarítmica
es parecido a graficar log(y) vs log(x)





Ritmo cardíaco: b=-1/4
Longevidad: b=+1/4
Latidos en la vida:
¡¡constante!!

Leyes de escala – Leyes alométricas

$$y = y_0\, M^b$$

Con b=±n/4
(modelo WBE)

Table 1 Predicted values of scaling exponents for physiological and anatomical variables of plant vascular systems.

Variable	Plant mass: Exponent predicted	Plant mass: Symbol	Branch radius: Symbol	Branch radius: Exponent Predicted	Branch radius: Exponent Observed
Number of leaves	$\frac{3}{4}$ (0.75)	n_0^L	n_k^L	2 (2.00)	2.007 (ref. 12)
Number of branches	$\frac{3}{4}$ (0.75)	N_0	N_k	−2 (−2.00)	−2.00 (ref. 6)
Number of tubes	$\frac{3}{4}$ (0.75)	n_0	n_k	2 (2.00)	n.d.
Branch length	$\frac{1}{4}$ (0.25)	l_0	l_k	$\frac{2}{3}$ (0.67)	0.652 (ref. 6)
Branch radius	$\frac{3}{8}$ (0.375)	r_0			
Area of conductive tissue	$\frac{7}{8}$ (0.875)	A_0^{CT}	A_k^{CT}	$\frac{7}{3}$ (2.33)	2.13 (ref. 8)
Tube radius	$\frac{1}{16}$ (0.0625)	a_0	a_k	$\frac{1}{6}$ (0.167)	n.d.
Conductivity	1 (1.00)	K_0	K_k	$\frac{8}{3}$ (2.67)	2.63 (ref. 12)
Leaf-specific conductivity	$\frac{1}{4}$ (0.25)	L_0	L_k	$\frac{2}{3}$ (0.67)	0.727 (ref. 17)
Fluid flow rate			$\dot{Q}_k$	2 (2.00)	n.d.
Metabolic rate	$\frac{3}{4}$ (0.75)	$\dot{Q}_0$			
Pressure gradient	$-\frac{1}{4}$ (−0.25)	$\Delta P_0/l_0$	$\Delta P_k/l_k$	$-\frac{2}{3}$ (−0.67)	n.d.
Fluid velocity	$-\frac{1}{8}$ (−0.125)	u_0	u_k	$-\frac{1}{3}$ (−0.33)	n.d.
Branch resistance	$-\frac{3}{4}$ (−0.75)	Z_0	Z_k	$-\frac{1}{3}$ (−0.33)	n.d.
Tree height	$\frac{1}{4}$ (0.25)	h			
Reproductive biomass	$\frac{3}{4}$ (0.75)				
Total fluid volume	$\frac{25}{24}$ (1.0415)				

Values are given as a function of total plant mass, M, and branch radius, r_k. For the latter case, predictions are compared with measured values in the last column. References cited do not quote confidence levels, except for branch length, where they are given as ±0.036. Because botanists rarely report allometric scaling with mass, no values for observed exponents are quoted. n.d., no data available.

West, G. B.; Brown, J.H.; Enquist, B. J. (1997). "A general model for the origin of allometric scaling laws in biology". *Science*. **276** (5309): 122–126.

ID	Masa [g]	Error Masa [g]	Largo [cm]	Error Largo [cm]	Ancho [cm]	Error Ancho [cm]	Área [cm²]	Error Área [cm²]
1								
2								
3								
4								
5								

- Graficar variables originales.
- Transformar logaritmos.
- Transformar los errores (propagar errores).
- Graficar variables transformadas.
- Ajustar.
- Graficar variables originales con la función ajustada.