

GLOBAL LARGE SOLUTIONS FOR THE NAVIER-STOKES EQUATIONS WITH THE CORIOLIS FORCE

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ABSTRACT. In this paper, we construct a class of global large solution to the three-dimensional Navier-Stokes equations with the Coriolis force in critical Fourier-Besov space $\dot{F}B_{p,r}^{2-\frac{3}{p}}(\mathbb{R}^3)$. In fact, our choice of special initial data u_0 can be arbitrarily large in $\dot{F}B_{p,r}^s(\mathbb{R}^3)$ for any $s \in \mathbb{R}$ and $1 \leq p, r \leq \infty$.

1. INTRODUCTION AND MAIN RESULT

Rotating fluid equations have important applications in meteorology and oceanography, particularly in the models describing large-scale ocean and atmosphere flows. The Coriolis force, arising from the rotation of the Earth, plays a significant role in such systems.

In 1868, Kelvin first observed that a sphere, moving along the axis of uniformly rotating water, takes with it a column of liquid as if this were a rigid mass, and pioneered the research on the motion of rotating fluid, see [10]. Later on, Taylor[28] and Proudman[25] strictly proved that high-speed rotation brings about a vertical rigidity in the fluid described by the Taylor-Proudman theorem: Under a fast rotation the velocity of all particles located on the same vertical line is horizontal and constant.

Mathematically, the Coriolis forces give rise to the so-called Poincaré waves, which are dispersive waves. Poincaré waves propagate in both directions with extremely fast speed in the propagation domain, and the waves with different wavenumbers move at different speeds. This makes that the nonlinear interactions between different modes are typically less significant.

On the other hand, Poincaré waves are a kind of high frequency wave, whose particles not only have vibrations parallel to the propagation direction, but also have vibrations perpendicular to the propagation direction. Therefore, one of the major difficulties encountered in understanding dynamics of rotating fluid is the influence of the oscillations generated by Coriolis forces.

In the paper, we consider the following Cauchy problem of three-dimensional incompressible Navier-Stokes equations with the Coriolis force:

$$\begin{cases} \partial_t u - \Delta u + \Omega e_3 \times u + u \cdot \nabla u + \nabla p = 0, \\ \operatorname{div} u = 0, \quad u(0, x) = u_0, \end{cases} \quad (1.1)$$

where the unknown functions $u = (u_1, u_2, u_3)$ and p denote velocity field and pressure, respectively; $\Omega \in \mathbb{R}$ is the Coriolis parameter, which is twice angular velocity of the rotation around the vertical unit vector $e_3 = (0, 0, 1)$, and $\Omega e_3 \times u$ represents the so-called Coriolis force; u_0 is the given initial velocity.

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The behavior of fluid flows in rapidly rotating environments is fundamentally different from that of non-rotating flows.

When $\Omega = 0$, (1.1) reduces to the problem of classical three-dimensional incompressible Navier-Stokes equations, which have been widely studied during the past seventy years. It has been proved that (1.1) with $\Omega = 0$ is globally well-posed for small initial data, see [3, 11, 16, 17, 20, 21]. For more results of large initial data with special structures in various scaling invariant spaces which generate unique global solutions to (1.1), we refer the reader to see [5, 6, 7, 8, 22, 23, 24] and the references therein. We note that the global regularity or global well-posedness issue of the three-dimensional incompressible Navier-Stokes equations for arbitrarily large initial data is still a challenging open problem.

When $\Omega \neq 0$, it is a very remarkable fact that (1.1) admits a global solution for arbitrary large initial data, provided that the speed of rotation is fast enough. More precisely, when Ω is large enough, by taking the full exploration of dispersive effects of the Coriolis forces, the existence and uniqueness of global solution has been proved for the periodic large data in [1, 2], for the spatially almost periodic large data in [29], and for the decay large data see [4, 14, 18, 26]. For any given and fixed Ω , we refer to [12, 13, 15, 19] for the global well-posedness of (1.1) with uniformly small initial data u_0 . Especially, it has been proved that (1.1) is globally well-posed for small initial data in $\dot{F}B_{p,r}^{2-\frac{3}{p}}(\mathbb{R}^3)$ ($1 < p, r \leq \infty$) and $\dot{F}B_{1,r}^{-1}(\mathbb{R}^3)$ ($1 \leq r \leq 2$), and is ill-posed in $\dot{F}B_{1,r}^{-1}(\mathbb{R}^3)$ ($r > 2$), see [15, 19].

It is now a natural question to ask whether there exists a unique global solution to (1.1) for any given and fixed Ω , if the initial data is not small in $\dot{F}B_{p,r}^{2-\frac{3}{p}}(\mathbb{R}^3)$ ($1 \leq p, r \leq \infty$). Based on a full understanding of the structure of the equation (1.1), we shall prove that (1.1) is globally well-posed for some special initial data u_0 whose $\dot{F}B_{p,r}^{2-\frac{3}{p}}(\mathbb{R}^3)$ -norm can be arbitrarily large, namely, $\|u_0\|_{\dot{F}B_{p,r}^{2-\frac{3}{p}}} \gg 1$, for any $1 \leq p, r \leq \infty$.

We first recall the definition of the Fourier-Besov spaces $\dot{F}B_{p,r}^s(\mathbb{R}^3)$. As usual we denote by $\mathcal{S}(\mathbb{R}^3)$ the space of Schwartz functions on \mathbb{R}^3 , and by $\mathcal{S}'(\mathbb{R}^3)$ the space of tempered distributions on \mathbb{R}^3 . Choose radial function $\psi \in \mathcal{S}(\mathbb{R}^3)$ such that its Fourier transform $\hat{\psi}$ satisfies the following properties:

$$\text{supp } \hat{\psi} \subset \mathcal{C} := \{\xi \in \mathbb{R}^3 : \frac{3}{4} \leq |\xi| \leq \frac{8}{3}\},$$

and

$$\sum_{j \in \mathbb{Z}} \hat{\psi}(2^{-j}\xi) = 1 \quad \text{for all } \xi \in \mathbb{R}^3 \setminus \{0\}.$$

Let $\psi_j(x) := 2^{3j}\psi(2^jx)$ for $j \in \mathbb{Z}$ and $\mathcal{S}'_h(\mathbb{R}^3) := \mathcal{S}'(\mathbb{R}^3)/\mathcal{P}[\mathbb{R}^3]$, where $\mathcal{P}[\mathbb{R}^3]$ denotes the linear space of polynomials on \mathbb{R}^3 . The homogeneous dyadic blocks Δ_j is defined by

$$\Delta_j f := \psi_j * f$$

for $j \in \mathbb{Z}$ and $f \in \mathcal{S}'(\mathbb{R}^3)$. Then the Fourier-Besov spaces $\dot{F}B_{p,r}^s(\mathbb{R}^3)$ are defined as follows:

Definition 1.1. For $s \in \mathbb{R}$ and $1 \leq p, r \leq \infty$, the Fourier-Besov space $\dot{F}B_{p,r}^s(\mathbb{R}^3)$ is defined to be the set of all tempered distributions $u \in \mathcal{S}'_h(\mathbb{R}^3)$ such that

$$\|u\|_{\dot{F}B_{p,r}^s} := \left\| \left\{ 2^{js} \|\widehat{\Delta_j u}\|_{L^p} \right\}_{j \in \mathbb{Z}} \right\|_{\ell^r(\mathbb{Z})} < \infty.$$

Remark 1.2. It is easy to show that $\|u\|_{\dot{F}B_{1,1}^0} = \|\hat{u}\|_{L^1}$.

Let U satisfy the following linear system:

$$\begin{cases} \partial_t U - \Delta U + \Omega e_3 \times U + \nabla p' = 0, \\ \operatorname{div} U = 0, \quad U(0, x) = u_0. \end{cases}$$

According to [13], we can show that U have the following explicit form:

$$\hat{U} = \cos\left(\Omega \frac{\xi_3}{|\xi|} t\right) e^{-|\xi|^2 t} \hat{u}_0 + \sin\left(\Omega \frac{\xi_3}{|\xi|} t\right) \frac{1}{|\xi|} e^{-|\xi|^2 t} \begin{pmatrix} \xi_3 \hat{u}_0^2 - \xi_2 \hat{u}_0^3 \\ -\xi_3 \hat{u}_0^1 + \xi_1 \hat{u}_0^3 \\ \xi_2 \hat{u}_0^1 - \xi_1 \hat{u}_0^2 \end{pmatrix}, \quad (1.2)$$

and it is easy to check that

$$\|U\|_{L^\infty(0, \infty; \dot{F}B_{1,1}^{-1})} + \|U\|_{L^1(0, \infty; \dot{F}B_{1,1}^1)} \leq C \|u_0\|_{\dot{F}B_{1,1}^{-1}}.$$

The main result of this paper reads as follows:

Theorem 1.3. Then there exist two constants $\delta, C > 0$ such that for any $u_0 \in \dot{F}B_{1,1}^{-1}(\mathbb{R}^3)$ satisfying the condition

$$\int_0^\infty \|U \cdot \nabla U\|_{\dot{F}B_{1,1}^{-1}} dt \cdot e^{C\|u_0\|_{\dot{F}B_{1,1}^{-1}}^2} \leq \delta, \quad (1.3)$$

then (1.1) admits a unique global solution

$$u \in L^\infty(0, \infty; \dot{F}B_{1,1}^{-1}(\mathbb{R}^3)) \cap L^1(0, \infty; \dot{F}B_{1,1}^1(\mathbb{R}^3)).$$

Corollary 1.4. Assume that the initial data fulfills

$$\operatorname{supp} \hat{u}_0(\xi) \subset \tilde{\mathcal{C}} \triangleq \{\xi \in \mathbb{R}^3 : |\xi| \geq 1\}, \quad (1.4)$$

then there exist a sufficiently small positive constant δ and a universal constant C such that if

$$\|u_0\|_{\dot{F}B_{1,1}^{-1}} \left(\|u_0^1 + u_0^2, u_0^3\|_{\dot{F}B_{\frac{3}{2},1}^1} + \|\partial_3 u_0\|_{\dot{F}B_{\frac{3}{2},1}^1} \right) \cdot e^{C\|u_0\|_{\dot{F}B_{1,1}^{-1}}^2} \leq \delta, \quad (1.5)$$

then the system (1.1) has a unique global solution.

Remark 1.5. Let two functions $a(x_1, x_2)$ with $\hat{a}(x_1, x_2) = \hat{a}(-x_1, -x_2)$ and $b(x_3)$ with $\hat{b}(x_3) = \hat{b}(-x_3)$ satisfying $\hat{a}, \hat{b} \in [0, 1]$,

$$\operatorname{supp} \hat{b} \subset \{\xi_3 \in \mathbb{R} \mid \frac{1}{2}\varepsilon < |\xi_3| < \varepsilon\},$$

$$\hat{b} = 1 \quad \text{on} \quad \{\xi_3 \in \mathbb{R} \mid \frac{5}{8}\varepsilon < |\xi_3| < \frac{7}{8}\varepsilon\},$$

$$\operatorname{supp} \hat{a} \subset \{\xi \in \mathbb{R}^2 \mid |\xi_1 - \xi_2| \leq \varepsilon, \frac{11}{8} \leq |\xi| \leq \frac{35}{24}\},$$

and

$$\hat{a}(\xi) = 1 \quad \text{on} \quad \{\xi \in \mathbb{R}^2 \mid |\xi_1 - \xi_2| \leq \frac{1}{2}\varepsilon, \frac{67}{48} \leq |\xi| \leq \frac{69}{48}\}.$$

Then, we have for all $p \in [1, \infty]$,

$$\|\hat{a}_0\|_{L^p} \approx \varepsilon^{\frac{1}{p}}, \quad \|\hat{b}_0\|_{L^p} \approx \varepsilon^{\frac{1}{p}}.$$

Let us consider the initial data $u_0 = (u_0^1, u_0^2, 0)$ with

$$u_0^1 = \varepsilon^{-2}(\log \log \frac{1}{\varepsilon})^{\frac{1}{2}} \partial_2 a(x_1, x_2) b(x_3), \quad u_0^2 = -\varepsilon^{-2}(\log \log \frac{1}{\varepsilon})^{\frac{1}{2}} \partial_1 a(x_1, x_2) b(x_3).$$

Direct calculation shows that

$$\begin{aligned} \|u_0\|_{FB_{1,1}^1} &\approx \|u_0\|_{FB_{1,1}^{-1}} \approx (\log \log \frac{1}{\varepsilon})^{\frac{1}{2}}, \\ \|u_0^1 + u_0^2\|_{FB_{\frac{3}{2},1}^1} + \|\partial_3 u_0\|_{FB_{\frac{3}{2},1}^1} + \|u_0^3\|_{FB_{\frac{3}{2},1}^1} &\approx \varepsilon^{\frac{1}{3}} (\log \log \frac{1}{\varepsilon})^{\frac{1}{2}}. \end{aligned}$$

Then, we can show that the left side of (1.5) becomes

$$C\varepsilon^{\frac{1}{3}} \left(\log \log \frac{1}{\varepsilon} \right) \exp \left(C \log \log \frac{1}{\varepsilon} \right),$$

which implies (1.1) have a global solution for ε sufficiently small. For small enough ε , we can deduce that $\text{supp } \hat{u}_0 \in \{\xi \in \mathbb{R}^3 \mid \frac{4}{3} \leq |\xi| \leq \frac{3}{2}\}$ and

$$\Delta_j u_0 = 0, j \neq 0; \quad \Delta_0 u_0 = u_0.$$

Therefore, we can conclude that for any $s \in \mathbb{R}$ and $1 \leq p, r \leq \infty$

$$\|u_0\|_{FB_{p,r}^s} \gtrsim \|u_0\|_{FB_{1,\infty}^{-1}} \approx \|\hat{u}_0\|_{L^1} \approx \log \log \frac{1}{\varepsilon}.$$

2. PROOF OF THE MAIN RESULTS

Proof of Theorem 1.3 Introduce the quantity $u = U + v$, we can show that v satisfies the following Cauchy problem:

$$\begin{cases} \partial_t v - \Delta v + \Omega e_3 \times v + v \cdot \nabla v + \nabla p'' = -U \cdot \nabla U - v \cdot \nabla U - U \cdot \nabla v, \\ \text{div } v = 0, \quad v(0, x) = 0. \end{cases}$$

By the Duhamel principle, this problem is equivalent to the integral equation

$$v(t) = - \int_0^t T_\Omega(t - \tau) \mathbb{P} [U \cdot \nabla U + v \cdot \nabla v - v \cdot \nabla U - U \cdot \nabla v] d\tau,$$

where $\mathbb{P} = (\delta_{ij} + R_i R_j)_{1 \leq i, j \leq 3}$ denotes the Helmholtz projection onto the divergence free vector fields, and $\{T_\Omega(t)\}_{t \geq 0}$ denotes the Stokes-Coriolis semigroup given explicitly in [13].

By the similar argument of Lemma 2.2 in [27], we have for all $t \in [0, T]$ that

$$\begin{aligned} &\|v\|_{L_t^\infty(FB_{1,1}^{-1})} + \|v\|_{L_t^1(FB_{1,1}^1)} \\ &\lesssim \int_0^t \|U \cdot \nabla U\|_{FB_{1,1}^{-1}} + \|v \cdot \nabla v\|_{FB_{1,1}^{-1}} + \|U \cdot \nabla v\|_{FB_{1,1}^{-1}} + \|v \cdot \nabla U\|_{FB_{1,1}^{-1}} d\tau \\ &\lesssim \int_0^t \|U \cdot \nabla U\|_{FB_{1,1}^{-1}} d\tau + \|v\|_{L_t^2(FB_{1,1}^0)} \|v\|_{L_t^2(FB_{1,1}^0)} + \int_0^t \|v\|_{FB_{1,1}^0} \|U\|_{FB_{1,1}^0} d\tau, \end{aligned}$$

where we have used Remark 1.2 and the fact $\|\widehat{ab}\|_{L^1} \leq \|\widehat{a}\|_{L^1} \|\widehat{b}\|_{L^1}$ in the last inequality. Now, we define

$$\Gamma \triangleq \sup \{t \in [0, T^*) : \|v\|_{L_t^\infty(FB_{1,1}^{-1})} + \|v\|_{L_t^1(FB_{1,1}^1)} \leq \eta \ll 1\},$$

where η is a small enough positive constant which will be determined later on. Then, it yields

$$\|v\|_{L_t^\infty(FB_{1,1}^{-1})} + \|v\|_{L_t^1(FB_{1,1}^1)} \leq C \int_0^t \|U \cdot \nabla U\|_{FB_{1,1}^{-1}} d\tau + C \int_0^t \|v\|_{FB_{1,1}^{-1}} \|U\|_{FB_{1,1}^0}^2 d\tau.$$

From Gronwall's inequality, we have

$$\begin{aligned} \|v\|_{L_t^\infty(FB_{1,1}^{-1})} + \|v\|_{L_t^1(FB_{1,1}^1)} &\leq C \int_0^t \|U \cdot \nabla U\|_{FB_{1,1}^{-1}} d\tau \cdot e^{C \int_0^t \|U\|_{FB_{1,1}^0}^2 d\tau} \\ &\leq C\delta. \end{aligned}$$

Choosing $\eta = 2C\delta$, thus we can get

$$\|v\|_{L_t^\infty(FB_{1,1}^{-1})} + \|v\|_{L_t^1(FB_{1,1}^1)} \leq \frac{\eta}{2} \quad \text{for } t \leq \Gamma.$$

So if $\Gamma < T^*$, due to the continuity of the solutions, we can obtain that there exists $0 < \epsilon \ll 1$ such that

$$\|v\|_{L_t^\infty(FB_{1,1}^{-1})} + \|v\|_{L_t^1(FB_{1,1}^1)} \leq \eta \quad \text{for } t \leq \Gamma + \epsilon < T^*,$$

which is contradiction with the definition of Γ .

Thus, we can conclude $\Gamma = T^*$ and

$$\|v\|_{L_t^\infty(FB_{1,1}^{-1})} \leq C < \infty \quad \text{for all } t \in (0, T^*),$$

which implies that $T^* = +\infty$.

Proof of Corollary 1.4 Notice that $\operatorname{div} U = 0$, we have

$$\begin{aligned} U \cdot \nabla U^1 &= (U^1 + U^2) \partial_1 U^1 + U^2 \partial_2 (U^1 + U^2) + U^2 \partial_3 U^3 + U^3 \partial_3 U^1, \\ U \cdot \nabla U^2 &= (U^1 + U^2) \partial_2 U^2 + U^1 \partial_1 (U^1 + U^2) + U^1 \partial_3 U^3 + U^3 \partial_3 U^2, \\ U \cdot \nabla U^3 &= U^1 \partial_1 U^3 + U^2 \partial_2 U^3 - U^3 (\partial_1 U^1 + \partial_2 U^2). \end{aligned}$$

Using the fact $\|ab\|_{FB_{\frac{3}{2},1}^0} \leq \|a\|_{FB_{\frac{3}{2},1}^0} \|b\|_{FB_{1,1}^0}$, we have

$$\begin{aligned} \int_0^t \|U \cdot \nabla U\|_{FB_{1,1}^{-1}} d\tau &\lesssim \int_0^t \|U \cdot \nabla U\|_{FB_{\frac{3}{2},1}^0} d\tau \\ &\lesssim \int_0^t \|U^1 + U^2, U^3\|_{FB_{\frac{3}{2},1}^0 \cap FB_{\frac{3}{2},1}^1} \|U^1, U^2\|_{FB_{1,1}^0 \cap FB_{1,1}^1} d\tau \end{aligned}$$

From (1.2), the direct calculation shows that

$$\begin{aligned} |\hat{U}^1(\xi)| + |\hat{U}^2(\xi)| &\leq e^{-t|\xi|^2} |\hat{u}_0(\xi)|, \\ |\hat{U}^3(\xi)| &\leq \Omega t e^{-t|\xi|^2} \frac{|\xi_3|}{|\xi|} |\hat{u}_0^h(\xi)| + e^{-t|\xi|^2} |\hat{u}_0^3(\xi)|, \\ |\hat{U}^1(\xi) + \hat{U}^2(\xi)| &\leq t e^{-t|\xi|^2} |\hat{u}_0^1 + \hat{u}_0^2| + \Omega t e^{-t|\xi|^2} \frac{|\xi_3|}{|\xi|} |\hat{u}_0|. \end{aligned}$$

This along with the property (1.4) yield

$$\begin{aligned} & \int_0^\infty \|U^1 + U^2, U^3\|_{FB_{\frac{3}{2},1}^0 \cap FB_{\frac{3}{2},1}^1} \|U^1, U^2\|_{FB_{1,1}^0 \cap FB_{1,1}^1} dt \\ & \lesssim \|u_0\|_{FB_{1,1}^{-1}} (\|u_0^1 + u_0^2\|_{FB_{\frac{3}{2},1}^1} + \|\partial_3 u_0\|_{FB_{\frac{3}{2},1}^1} + \|u_0^3\|_{FB_{\frac{3}{2},1}^1}). \end{aligned} \quad (2.1)$$

Thus, (2.1) is ensured whenever (1.5) holds. We complete the proof of Corollary 1.4.

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