Lower Bound of Blow Up Time for Solutions of a Class of Cross-coupled Porous Media Equations [1]

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Abstract: Blow-up phenomena of solutions of a class of parabolic equations for porous media with nonlocal source terms cross-coupled under Dirichlet and Neumann boundary conditions. The differential inequality technique is used to obtain the lower bounds on the blow up time of the equation set under two different boundary conditions.

Keywords: Porous media equations; Nonlocal source terms; Blow up time.

2010 MR Subject Classifications: 35K20; 35K55; 35K60; 35K65

§1 Introduction

In this paper, we study the lower bound of blow up time of solutions for nonlocal cross-coupled porous media equation

$$u_t = \Delta u^m + u^{p_1} \int v^{q_1} dx, \qquad (x,t) \in \Omega \times (0, t^\star), \tag{1}$$

$$v_t = \Delta v^n + v^{p_2} \int u^{q_2} dx, \qquad (x,t) \in \Omega \times (0, t^\star), \tag{2}$$

set and continuous bounded initial values

$$u(x,0) = u_0(x), v(x,0) = v_0(x), \qquad x \in \Omega, \tag{3}$$

under Dirichlet boundary condition

$$u(x,t) = v(x,t) = 0, \qquad (x,t) \in \partial\Omega \times (0, t^\star), \tag{4}$$

or Neumann boundary condition

$$\frac{\partial u^m}{\partial \upsilon} = lu, \frac{\partial v^n}{\partial \upsilon} = lv, \qquad (x,t) \in \partial\Omega \times (0, t^\star), \tag{5}$$

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Where $\Omega \in R^3$ is a bounded region of $\partial\Omega$ with a smooth boundary, $p_i, q_i > 0 (i = 1, 2)$, and satisfies that $p_1 + q_1 > m > 1, p_2 + q_2 > n > 1$, v is the unit external normal vector in the external normal direction of $\partial\Omega$.

Equations (1)-(5) can be used to describe reaction-diffusion phenomena in many fields such as fluid mechanics, population dynamics and bio-population mechanics etc. There are many research achievements on the lower bound estimation of blow up time for the solution of a single porous media equation, see literature [1-3]. Literature [1] studies a single equation composed of (1) and (3) with Dirichlet boundary conditions

$$u(x,t) = 0, \qquad (x,t) \in \partial\Omega \times (0, t^\star),$$

They have obtained the lower bound of the blow up time of the solution which

$$t^\star \geq C_7 [\int_\Omega u_0^\alpha (p+q-1) dx]^{-C_6}.$$

and homogeneous Neumann boundary conditions

$$\frac{\partial u}{\partial v} = 0, \qquad (x,t) \in \partial\Omega \times (0, t^\star),$$

the lower bound of the blow up time of the solution which

$$t^\star \geq \int_{\eta(0)}^{\infty} \frac{d\xi}{K_5 \xi^{\frac{\alpha+1}{\alpha}} + K_6 \xi^{\frac{(\alpha+1)(p+q-1)}{\alpha(p+q-1)-(p+q-m)}}}.$$

Literature [2] studies a single equation composed of (1) and (3) with Robin boundary conditions

$$\frac{\partial u}{\partial \eta} + ku = 0, \qquad (x,t) \in \partial\Omega \times (0, t^\star),$$

they have obtained the lower bound of the blow up time of the solution which

$$t^\star \geq \int_{\phi(0)}^{\infty} \frac{d\eta}{ms|\Omega| K_4 \eta^{\frac{ms+s}{ms+m-1}} + ms|\Omega| K_5 \eta^{\frac{ms+s}{n(m-1)(s+1)}}}.$$

Literature [3] studies a single equation composed of(1)with(3)and(5),the lower bound of the blow up time of the solution which

$$t^\star \geq \int_{\eta(0)}^{\eta(t)} \frac{d\xi}{C_1 \xi + C_2 \xi^{\frac{\alpha(p+q-1)-(m-1)}{\alpha(p+q-1)}} + C_2 \xi^{\frac{(\alpha+1)}{\alpha}} + C_3 \xi^{\frac{(\alpha+1)(p+q-1)}{\alpha(p+q-1)-(p+q-m)}}},$$

when $l > 0$ of(5). The lower bound of the blow up time of the solution which

$$t^\star \geq \int_{\eta(0)}^{\eta(t)} \frac{d\xi}{K_1 \xi^{\frac{(\alpha+1)}{\alpha}} + + K_2 \xi^{\frac{(\alpha+1)(p+q-1)}{\alpha(p+q-1)-(p+q-m)}}},$$

when$l < 0$ of equation (5).

For the blow up phenomenon of the solutions to the equation set (1)-(5) of porous media with nonlocal cross-coupling, only literature [4] studies the global existence of solutions to the equation set (1)-(4) of porous media and sufficient conditions for blowing up at finite time, in which, one of the sufficient conditions for the solution to blowing up at finite time is that one of $m < p_1$,$n < p_2$ and $q_1q_2 > (m-p_1)(n-p_2)$ holds.

The same method and technique can be used to obtain equations (1)-(3) and (5) to solve sufficient conditions for global existence and blow up at a finite time. However, studies on the lower bound of blow up time for equation set (1)-(5) have not been found. Blow-up at a finite time and lower bound of blowing up time of solution for the following parabolic equations are studied in literature [5] when m=1,n=1.

$$\begin{cases} u_t = \Delta u + k_1(t)f_1(v), \quad x = (x_1, \cdots, x_N) \in \Omega, t \in (0, t^\star), \\ v_t = \Delta v + k_2(t)f_2(u), \quad x \in \Omega, t \in (0, t^\star), \\ u(x,t) = v(x,t) = 0, \quad x \in \partial\Omega, t \in (0, t^\star), \\ u(x,0) = u_0(x) \geq 0, v(x,0) = v_0(x) \geq 0, \quad x \in \Omega. \end{cases}$$

Literature [6] established Blow-up at a finite time and lower bound of blowing up time of solution for the following parabolic equations when m=1,n=1, blowing up at a finite time and lower bound of blowing up time of solution.

$$\begin{cases} u_t = \Delta u + k_1(t)u^p v^q, \quad (x,t) \in \Omega \times (0, t^\star), \\ v_t = \Delta v + k_2(t)u^r v^s, \quad (x,t) \in \Omega \times (0, t^\star), \\ u(x,t) = v(x,t) = 0, \quad x \in \partial\Omega, t \in (0, t^\star), \\ u(x,0) = u_0(x), v(x,0) = v_0(x), \quad x \in \Omega, \end{cases}$$

The lower bound of blowing up time for solutions of other similar equations or equation set is shown in the literature [7-10].

Inspired by literature [5-6], this paper studies the lower bound estimation of blowing up time for the solutions of the porous media equation set (1)-(5) with non-local source cross-coupling and $m > 1, n > 1$ with relevant formulas and some basic inequalities in literature [10].

§2 Preliminary knowledge

This part introduces some important inequalities used in this paper.

Lemma 1: (Hölder inequality) Let $1 < p, q < \infty, \frac{1}{p} + \frac{1}{q} = 1$, if $u \in L^p(\Omega), v \in L^q(\Omega)$, then

$$\int_\Omega |uv| dx \leq \|u\|_{L^p(\Omega)} \|v\|_{L^q(\Omega)}.$$

Lemma 2: (Young inequality) 设$1 < p, q < \infty, \frac{1}{p} + \frac{1}{q} = 1$,则

$$ab \leq \frac{a^p}{p} + \frac{b^q}{q}, \quad (a, b > 0).$$

Lemma 3: (Membrane inequality)

$$\lambda \int_\Omega \omega^2 dx \leq \int_\Omega |\nabla \omega|^2 dx,$$

where λ is the first eigenvalue of $\Delta\omega + \lambda\omega = 0, \omega > 0 \ x \in \Omega, \omega = 0 \ x \in \partial\Omega$.

Lemma 4[10]**:** let Ω be the bounded star region in R^N,and$N \geq 2$, then

$$\int_{\partial\Omega} u^n ds \leq \frac{N}{\rho_0} \int_\Omega u^n dx + \frac{nd}{\rho_0} \int_\Omega u^{n-1} |\nabla u| dx.$$

Lemma 5: (Special Young inequality) let γbe an arbitrary constant, and $0 < x < 1$,then

$$a^x b^y = (\gamma a)^x \left(\frac{b^{\frac{y}{1-x}}}{\gamma^{\frac{x}{1-x}}}\right)^{1-x} \leq \gamma x a + (1-x) \gamma^{\frac{x}{1-x}} b^{\frac{y}{1-x}}, \quad (a, b > 0).$$

§3 Lower bound of blowing up time under Dirichlet boundary conditions

The lower bound of blowing up time for solutions of equations under Dirichlet boundary conditions is discussed below.

Theorem 1: Defines auxiliary functions

$$J(t) = \int_\Omega (u^k + v^k) dx, \tag{6}$$

for $k > max\{1, q_i, m-1, n-1, 2p_i + 3q_i - 2\}$. If (u, v) is a non-negative classical solution of equation set $(1) - (5)$ and blow up occurs in the sense of measure $J(t)$ at time $t^\star$, then the lower bound of $t^\star$ is

$$\int_{J(0)}^\infty \frac{d\eta}{H_1 \eta^{c_1} + H_2 \eta},$$

where $J(0) = \int_\Omega (u^0 + v^0) dx$, The normal number H_1, H_2, c_1 is given in the following proof.

Proof of theorem 1:

$$\begin{aligned}
J'(t) &= k\int_\Omega u^{k-1}u_t dx + k\int_\Omega v^{k-1}v_t dx\\
&= k\int_\Omega u^{k-1}\Delta u^m dx + k\int_\Omega u^{k-1}u^{p_1}\int_\Omega v^{q_1}dxdx+\\
&\quad k\int_\Omega v^{k-1}\Delta v^n dx + k\int_\Omega v^{k-1}v^{p_2}\int_\Omega u^{q_2}dxdx\\
&= -mk(k-1)\int_\Omega u^{k+m-3}|\nabla u|^2 dx + k\int_{\partial\Omega} u^{k-1}\tfrac{\partial u^m}{\partial v}ds + k\int_\Omega u^{k+p_1-1}dx\int_\Omega v^{q_1}dx\\
&\quad -nk(k-1)\int_\Omega v^{k+n-3}|\nabla v|^2 dx + k\int_{\partial\Omega} v^{k-1}\tfrac{\partial v^n}{\partial v}ds + k\int_\Omega v^{k+p_2-1}dx\int_\Omega u^{q_2}dx\\
&= -\tfrac{4mk(k-1)}{(k+m-1)^2}\int_\Omega |\nabla u^{\frac{k+m-1}{2}}|^2 dx + k\int_{\partial\Omega} u^{k-1}\tfrac{\partial u^m}{\partial v}ds + k\int_\Omega u^{k+p_1-1}dx\int_\Omega v^{q_1}dx\\
&\quad -\tfrac{4nk(k-1)}{(k+n-1)^2}\int_\Omega |\nabla u^{\frac{k+n-1}{2}}|^2 dx + k\int_{\partial\Omega} v^{k-1}\tfrac{\partial v^n}{\partial v}ds + k\int_\Omega v^{k+p_2-1}dx\int_\Omega u^{q_2}dx. \quad (7)
\end{aligned}$$

When the equations take *Dirichlet* boundary conditions of equation (3) , equation (7) becomes

$$\begin{aligned}
J'(t) &= [-\tfrac{4mk(k-1)}{(k+m-1)^2}\int_\Omega |\nabla u^{\frac{k+m-1}{2}}|^2 dx + k\int_\Omega u^{k+p_1-1}dx\int_\Omega v^{q_1}dx]\\
&\quad +[-\tfrac{4nk(k-1)}{(k+n-1)^2}\int_\Omega |\nabla v^{\frac{k+n-1}{2}}|^2 dx + k\int_\Omega v^{k+p_2-1}dx\int_\Omega u^{q_2}dx]\\
&= J_1'(t)+J_2'(t). \qquad (8)
\end{aligned}$$

where

$$J_1'(t) = [a_1\int_\Omega |\nabla u^\alpha|^2 dx + k\int_\Omega u^{k+p_1-1}dx\int_\Omega v^{q_1}dx],$$

$$J_2'(t) = [b_1\int_\Omega |\nabla v^\beta|^2 dx + k\int_\Omega v^{k+p_2-1}dx\int_\Omega u^{q_2}dx]$$

$$a_1 = -\frac{4mk(k-1)}{(k+m-1)^2}, b_1 = -\frac{4nk(k-1)}{(k+n-1)^2}, \alpha = \frac{k+m-1}{2}, \beta = \frac{k+n-1}{2}.$$

First, lemma 1 is used to estimate the second term of $J_1'(t)$ in equation (8), and it is obtained that

$$\int_\Omega u^{k+p_1-1}dx \le (\int_\Omega (u^{k+p_1-1})^{\frac{k}{k-q_1}}dx)^{\frac{k-q_1}{k}}|\Omega|^{\frac{q_1}{k}},$$

$$\int_\Omega v^{q_1}dx \le (\int_\Omega (v^{q_1})^{\frac{k}{q_1}}dx)^{\frac{q_1}{k}}|\Omega|^{\frac{k-q_1}{k}},$$

$$\int_\Omega u^{k+p_1-1}dx\int_\Omega v^{q_1}dx \le (\int_\Omega (u^{k+p_1-1})^{\frac{k}{k-q_1}}dx)^{\frac{k-q_1}{k}}(\int_\Omega (v^{q_1})^{\frac{k}{q_1}}dx)^{\frac{q_1}{k}}|\Omega|,$$

according to lemma 2 and equation above, it is obtained that

$$\int_\Omega u^{k+p_1-1}dx\int_\Omega v^{q_1}dx \le \frac{|\Omega|(k-q_1)}{k}\int_\Omega u^{\frac{k(k+p_1-1)}{k-q_1}}dx + \frac{|\Omega|q_1}{k}\int_\Omega v^k dx. \qquad (9)$$

Second, lemma 1 is used to estimate the first term on the right side of equation(9), and it is obtained that

$$\int_\Omega u^{\frac{k(k+p_1-1)}{k-q_1}}dx = \int_\Omega u^\alpha u^{\frac{k(k+p_1-1)}{k-q_1}-\alpha}dx \le (\int_\Omega u^{4\alpha}dx)^{\frac14}(\int_\Omega u^{\frac{2[\frac{2k(k+p_1-1)}{k-q_1}-(k+m-1)]}{3}}dx)^{\frac34}, \qquad (10)$$

from the second term on the right side of inequality sign of(10) and lemma 1, we can know

$$\int_\Omega u^{\frac{2[\frac{2k(k+p_1-1)}{k-q_1}-(k+m-1)]}{3}}dx \le (\int_\Omega u^k dx)^{\frac{2[1+\frac{2(p_1+q_1-1)}{k-q_1}-\frac{m-1}{k}]}{3}}dx|\Omega|^{\frac{2[\frac{1}{2}-\frac{2(p_1+q_1-1)}{k-q_1}+\frac{m-1}{k}]}{3}}, \tag{11}$$

from the first term on the right side of inequality sign of(10) and lemma 1, we can know

$$\int_\Omega u^{4\alpha}dx = \int_\Omega u^\alpha u^{3\alpha}dx \le (\int_\Omega u^{2\alpha}dx)^{\frac{1}{2}}(\int_\Omega (u^\alpha)^6 dx)^{\frac{1}{2}}, \tag{12}$$

using the following Sobolev inequality[11]

$$(\int_\Omega |\phi|^{\gamma_1}dx)^{\frac{1}{\gamma_1}} \le C(\int_\Omega |\nabla\phi|^{\gamma_2}dx)^{\frac{1}{\gamma_2}}$$

where $\gamma_1 = 6, \gamma_2 = 2, C = 4^{\frac{1}{3}}3^{-\frac{1}{2}}\pi^{-\frac{2}{3}}$,the second term of equation (12) can be simplified to

$$(\int_\Omega (u^\alpha)^6 dx)^{\frac{1}{2}} \le C^3(\int_\Omega |\nabla u^\alpha|^2 dx)^{\frac{3}{2}}, \tag{13}$$

by synthesizing equations (12) and (13), we have

$$\int_\Omega u^{4\alpha}dx \le C^3(\int_\Omega u^{2\alpha}dx)^{\frac{1}{2}}(\int_\Omega |\nabla u^\alpha|^2 dx)^{\frac{3}{2}},$$

$$(\int_\Omega u^{4\alpha}dx)^{\frac{1}{4}} \le C^{\frac{3}{4}}(\int_\Omega u^{2\alpha}dx)^{\frac{1}{8}}(\int_\Omega |\nabla u^\alpha|^2 dx)^{\frac{3}{8}}, \tag{14}$$

based on lemma 3, equation (14) becomes

$$(\int_\Omega u^{4\alpha}dx)^{\frac{1}{4}} \le C^{\frac{3}{4}}\lambda^{-\frac{1}{8}}(\int_\Omega |\nabla u^\alpha|^2 dx)^{\frac{1}{2}}. \tag{15}$$

Combining equations (11) and (15), equation (10) becomes

$$\int_\Omega u^{\frac{k(k+p_1-1)}{k-q_1}}dx \le a_2(\int_\Omega |\nabla u^\alpha|^2 dx)^{\frac{1}{2}}(\int_\Omega u^k dx)^{e_1}, \tag{16}$$

where $a_2 = C^{\frac{3}{4}}\lambda^{-\frac{1}{8}}|\Omega|^{\frac{2[\frac{1}{2}-\frac{2(p_1+q_1-1)}{k-q_1}+\frac{m-1}{k}]}{3}}, e_1 = \frac{2[1+\frac{2(p_1+q_1-1)}{k-q_1}-\frac{m-1}{k}]}{3}$.

By synthesizing equations (9)-(16),$J_1^{'}(t)$ of equation (8) becomes

$$J_1^{'}(t) \le a_1\int_\Omega |\nabla u^\alpha|^2 dx + a_2(k-q_1)(\theta_1[\int_\Omega |\nabla u^\alpha|^2 dx]^{\frac{1}{2}}\theta_1^{-1}[(\int_\Omega u^k dx)^{2e_1}]^{\frac{1}{2}}) + q_1\int_\Omega v^k dx, \tag{17}$$

we use the fundamental inequality

$$a^q b^p \le qa + pb \qquad (a, b > 0, p, q \ge 0, p+q = 1),$$

equation (17) becomes

$$J_1^{'}(t) \leq (a_1 + \frac{a_2(k-q_1)\theta_1}{2}) \int_\Omega |\nabla u^\alpha|^2 dx + \frac{a_2(k-q_1)\theta_1^{-1}}{2} (\int_\Omega u^k dx)^{2e_1} + q_1 \int_\Omega v^k dx. \quad (18)$$

The same derivation method is used to estimate the $J_2^{'}(t)$ term in equation (8),

$$J_2^{'}(t) \leq (b_1 + \frac{b_2(k-q_2)\theta_2}{2}) \int_\Omega |\nabla v^\beta|^2 dx + \frac{b_2(k-q_2)\theta_2^{-1}}{2} (\int_\Omega v^k dx)^{2g_1} + q_2 \int_\Omega u^k dx, \quad (19)$$

where $b_2 = C^{\frac{3}{4}} \lambda^{-\frac{1}{8}} |\Omega|^{\frac{2[\frac{1}{2} - \frac{2(p_2+q_2-1)}{k-q_2} + \frac{n-1}{k}]}{3}}, g_1 = \frac{2[1 + \frac{2(p_2+q_2-1)}{k-q_1} - \frac{n-1}{k}]}{3}$.

In order to deal with the gradient terms in (18)and (19),set $\theta_1 = -\frac{2a_1}{a_2(k-q_1)}, \theta_2 = -\frac{2b_1}{b_2(k-q_2)}$.

Finally, by synthesizing equations (18) and (19), we obtain

$$J^{'}(t) \leq -\frac{a_2^2(k-q_1)^2}{4a_1} (\int_\Omega u^k dx)^{2e_1} - \frac{b_2^2(k-q_2)^2}{4b_1} (\int_\Omega v^k dx)^{2g_1} + q_1 \int_\Omega v^k dx + q_2 \int_\Omega u^k dx, \quad (20)$$

take $H_1 = -\frac{a_2^2(k-q_1)^2}{4a_1} - \frac{b_2^2(k-q_2)^2}{4b_1}, H_2 = q_1 + q_2, c_1 = max\{2e_1, 2g_1\} > 1$, equation (19) becomes

$$J^{'}(t) \leq H_1 J^c(t) + H_2 J(t), \quad (21)$$

integrating (21) from 0 to $t^\star$, we obtain

$$t^\star \geq \int_{J(0)}^\infty \frac{d\eta}{H_1 \eta^{c_1} + H_2 \eta}. \quad (22)$$

§4 Lower bound of blowing up time under Neumann boundary conditions

The lower bound of blowing up time for solutions of equations under Neumann boundary conditions is discussed below.

3.1 when $l > 0$

Theorem 2: Define the same measure as (6) and the same condition as k. If$(u,v)$is a non-negative classical solution to the equation set (1) with (2) and (4), then the lower bound of $t^\star$ is

$$\int_{J(0)}^\infty \frac{d\eta}{H_1 \eta^{c_1} + H_3 \eta^{c_2} + (H_2 + H_4)\eta}.$$

where $J(0) = \int_\Omega (u^0 + v^0) dx$,The normal number $H_1, H_2, H_3, H_4, c_1, c_2$ is given in the following proof.

Proof of theorem 2:

The lemma 4 is used to estimate two boundary terms in equation (7), then

$$\int_{\partial\Omega} u^{k-1}\frac{\partial u^m}{\partial \upsilon}ds = l\int_{\Omega} u^k ds \le \frac{3l}{\rho_0}\int_{\Omega} u^k dx + \frac{kd}{\rho_0}\int_{\partial\Omega} u^{k-1}|\nabla u|dx, \tag{23}$$

where $\rho_0 = \min\limits_{\partial\Omega}(\mathbf{x}\cdot\mathbf{n}) > 0, d = \max\limits_{\partial\Omega}|\mathbf{x}|$.

From the second term on the right side of equation(23) and lemma 1 and lemma 5, we can know

$$\begin{aligned}\int_\Omega u^{k-1}|\nabla u|dx &\le (\int_\Omega u^{k+m-3}|\nabla u|^2dx)^{\frac{1}{2}}(\int_\Omega u^{k-(m-1)}dx)^{\frac{1}{2}}\\ &\le \tfrac{r_1}{2}(\int_\Omega u^{k+m-3}|\nabla u|^2dx) + \tfrac{1}{2r_1}\int_\Omega u^{k-(m-1)}dx\\ &= -\tfrac{2mkr_1(k-1)}{(k+m-1)^2}\int_\Omega |\nabla u^{\frac{k+m-1}{2}}|^2dx + \tfrac{1}{2r_1}\int_\Omega u^{k-(m-1)}dx,\end{aligned} \tag{24}$$

where r_1 is an arbitrary constant.

The lemma 1 is used to estimate the second term on the right side of equation(24) , we have

$$\int_\Omega u^{k-(m-1)}dx \le (\int_\Omega u^k dx)^{\frac{k-(m-1)}{k}}|\Omega|^{\frac{m-1}{k}}. \tag{25}$$

Substituting (24) and (25) into equation (23),we get

$$\int_{\partial\Omega} u^{k-1}\frac{\partial u^m}{\partial \upsilon}ds \le \frac{3l}{\rho_0}\int_\Omega u^k dx - a_3\int_\Omega |\nabla u^\alpha|^2dx + a_4(\int_\Omega u^k dx)^{e_2}, \tag{26}$$

where $a_3 = \frac{2mkr_1(k-1)}{(k+m-1)^2}, a_4 = \frac{kd}{2r_1\rho_0}|\Omega|^{\frac{m-1}{k}}, e_2 = \frac{k-(m-1)}{k}$.

Similarly, another boundary term in equation (7) is estimated as follows

$$\int_{\partial\Omega} \upsilon^{k-1}\frac{\partial u^m}{\partial \upsilon}ds \le \frac{3l}{\rho_0}\int_\Omega \upsilon^k dx - b_3\int_\Omega |\nabla u^\beta|^2dx + b_4(\int_\Omega \upsilon^k dx)^{g_2}, \tag{27}$$

where $b_3 = -\frac{2nkr_2(k-1)}{(k+n-1)^2}, b_4 = \frac{kd}{2r_2\rho_0}|\Omega|^{\frac{n-1}{k}}, g_2 = \frac{k-(n-1)}{k}$, r_2 is an arbitrary constant.

Substituting (21), (26) and (27) into equation (7), we get

$$\begin{aligned}J_1'(t) &\le (a_1 - a_3 + \tfrac{a_2(k-q_1)\theta_3}{2})\int_\Omega |\nabla u^\alpha|^2dx + (b_1 - b_3 + \tfrac{b_2(k-q_2)\theta_4}{2})\int_\Omega |\nabla \upsilon^\beta|^2dx\\ &\quad + a_4(\int_\Omega u^k dx)^{e_2} + a_5(\int_\Omega u^k dx)^{2e_1} + (\tfrac{3l}{\rho_0} + q_2)\int_\Omega u^k dx\\ &\quad + b_4(\int_\Omega \upsilon^k dx)^{g_2} + b_5(\int_\Omega \upsilon^k dx)^{2g_1} + (\tfrac{3l}{\rho_0} + q_1)\int_\Omega \upsilon^k dx.\end{aligned} \tag{28}$$

令

$$a_5 = -\frac{a_2^2(k-q_1)^2}{4a_1}, b_5 = -\frac{b_2^2(k-q_2)^2}{4b_1},$$

$$a_1 - a_3 + \frac{a_2(k-q_1)\theta_3}{2} = 0, b_1 - b_3 + \frac{b_2(k-q_2)\theta_4}{2} = 0,$$

$$H_1 = a_5 + b_5, H_2 = q_1 + q_2, H_3 = a_4 + b_4, H_4 = \frac{3l}{\rho_0},$$

$$c_1 = max\{2e_1, 2g_1\} > 1, c_2 = max\{e_1, g_1\} > 0,$$

equation (28) becomes

$$J^{'}(t) \leq H_1 J^{c_1}(t) + H_3 J^{c_2}(t) + (H_2 + H_4) J(t), \tag{29}$$

integrating (29) from 0 to $t^\star$ we obtain

$$t^\star \geq \int_{J(0)}^{\infty} \frac{d\eta}{H_1\eta^{c_1} + H_3\eta^{c_2} + (H_2 + H_4)\eta},$$

where$J(0) = \int_\Omega (u^0 + v^0) dx$.

3.2 When $l \leq 0$

If $l \leq 0$,then$\frac{\partial u^m}{\partial v} \leq 0$, $\frac{\partial v^n}{\partial v} \leq 0$,according to equation (7),we obtain

$$J^{'}(t) \leq J_1^{'}(t) + J_2^{'}(t), \tag{30}$$

that is, the same measure relation is obtained with equation (8). Therefore, when $l \leq 0$, the lower bound of blow up time of the equation set (1)with(2)and(4) is consistent with that of equation (22).

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