manuscript No.
(will be inserted by the editor)

Government's Activities for Counterterrorism: Min-Max Differential Game Approach

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Received: date / Accepted: date

Abstract One of the main problems facing governments at present is terrorism. Most recent studies are striving to find an optimal solution to this problem that threatens the security and stability of peoples. To combat terrorism, government uses various means such as: education development, providing labor opportunities, seeking social justice, religious awareness, and security arrangements. The purpose of this research is to evaluate the optimum strategy for

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both government and terrorist organizations using Min-Max differential game approach. Also, a saddle point concept for this game was discussed.

Keywords Counterterrorism - Min-Max Differential Game - Saddle Point - Governmental Procedures.

1 Introduction

Nowadays, the term "terrorism" is a common word in the media. Terrorist acts of various forms have become a threat to the whole world and pose great danger to all countries. The term "terrorism" has no exact definition. One of the definitions developed by researchers: terrorism is the felonious acts planned to create a state of terror in a group of persons for specific purposes, may be a political, religious, ethnic or any other reason that may be an argument to justify their acts. The government seeks to eliminate terrorism through effective steps such as: improving education's quality, increasing labor opportunities, achieving social justice, raising religious awareness, and security arrangements. Researchers used various branches of science to help governments in fighting terrorism, such as the mathematical branch "Operations Research". In this research a min-max approach for differential game is used to study the optimal strategies for governments and terrorists organizations. In [1] terrorism was studied using a system which is complex and adaptive. While, in [2] a strategies called Nash and Stackelberg was introduced to solve a counterterrorism differential game. In [3] a game of multi stage with imperfect information was used to analyse the equilibrium responses to a prospective terrorist attack in a two country framework (Home and Foreign). [4] assumed that the success of counterterrorism relies on public opinion, where in [5] the efficiency of "water and fire" strategies are compared. In [6,7] Hsia introduced the fuzzy differential game to guard a territory movable and not movable. Youness presented a differential game in [8] called " Nash collative". A min-max fuzzy differential game with fuzzy on the objective and control, and the large-scale differential game are discussed in [9–12]. A min-max method was applied to get the optimum strategies of the government and international terror organization (ITO) by Megahed [13,14]. He studied two problems of view governments and ITO and showed that governmental procedures are important for combating terrorism. Megahed [15], applied the Stackelberg method to discuss the interaction strategies of governments and terrorist organization. In [16], to combat terrorism, the Stackelberg differential game of E-differentiable and E-convex function was applied, considering the government's proceedings.

2 The Model: Diffrential Game between Governmrent and ITO

Here, we will introduce a model of a differential game which has a state variable $x(t)$, that represents the resource of an ITO. These resources can be arms, monetary support, supporter's network. While, $E(t), L(t), J(t), R(t)\ and\ S(t)$ represents the procedures of government: education's efficiency, increasing employment opportunities, achieving social justice, raising religious awareness and security preparations, respectively and t refers to time. Consider a game consists of two players: the first player is "the government" which has a strategy $u(t)$ and the second one is "ITO" which has a strategy $\nu(t)$ where, these strategies are non negative. ITO's stock of resources grows in accordance with the growth of a linear function $g(x)$, i.e. $g(x) = rx, r > 0$, and government's procedures grows according to a linear functions: $A(E) = \mu_1 E, B(L) = \mu_2 L, D(J) = \mu_3 J, F(R) = \mu_4 R \; and \; G(S) = \mu_5 S$, where $\mu_i > 0$ $(i = 1, 2, 3, 4, 5)$ is the rate of growth of government's procedures. Resource stock growth declines as a result to attacks. Also, these attacks reduces number of terrorists, arms, financial support, and supporter's network. In addition to the intensity of attacks $\nu(t)$, the growth of the resource stock is also reduced due to the counter terror cost $u(t)$. Denote "harvest function" $h(u, \nu)$ to be the effect of the control variables of the two players on the growth of the resource. Hence, we can write dynamics of resource stock for ITO $x(t)$ and government's procedures $E(t), L(t), J(t), R(t) \; and \; S(t)$ as following

$$\dot{x} = rx(t) - h(u(t), \nu(t)), \qquad x(0) = x_0 > 0 \tag{1}$$

$$\dot{E} = \mu_1 E + a_1 u - b_1 \nu \qquad E(0) = E_0 > 0 \tag{2}$$

$$\dot{L} = \mu_2 L + a_2 u - b_2 \nu \qquad L(0) = L_0 > 0 \tag{3}$$

$$\dot{J} = \mu_3 J + a_3 u - b_3 \nu \qquad J(0) = J_0 > 0 \tag{4}$$

$$\dot{R} = \mu_4 R + a_4 u - b_4 \nu \qquad R(0) = R_0 > 0 \tag{5}$$

$$\dot{S} = \mu_5 S + a_5 u - b_5 \nu \qquad S(0) = S_0 > 0 \tag{6}$$

where, x_0 refers to initial stock of ITO's resources while, $E_0, L_0, J_0, R_0 \; and \; S_0$ indicates initial government's procedures and $a_i, b_i, \; i = 1, 2, 3, 4, 5$ are positive constants. Furthermore, we presume the following constraints

$$x(t) \geq 0, E(t) \geq 0, L(t) \geq 0, J(t) \geq 0, R(t) \geq 0, S(t) \geq 0, t \geq 0 \tag{7}$$

As the cost of fighting terror and attacks makes a decline in growth, so suppose that $h_u > 0$ and $h_\nu > 0$. Counterterrorism arrangements show a marginal decrease in efficiency $h_{uu} < 0$. Also, increasing attacks bring a decline in resources, i.e. $h_{\nu\nu} > 0$. Also, instruments support each other, i.e. $h_{u\nu} > 0$, and this is reasonable from the point of view of the economics. This means that the marginal efficiency of anti-terrorist acts increases with the intensity of terrorist attacks. Moreover, suppose satisfying "Inada conditions" in the economy as following

$$\lim_{u \to 0} h_u = \infty, \quad \lim_{u \to \infty} h_u = 0 \tag{8}$$

$$\lim_{\nu \to 0} h_\nu = 0, \quad \lim_{\nu \to \infty} h_\nu = \infty \tag{9}$$

These conditions act as an assurance that strategies $u(t)$ and $\nu(t)$ are non negative.

Government receives benefit from its procedures, $E(t), L(t), J(t), R(t), S(t)$ and decline in the resources of ITO, but not benefit due to enormity of resource stock for ITO, attacks of terrorists and cost of combating terrorism procedures. To facilitate calculations we assume that the previous quantities are linear. Hence, government's objective is

$$\max_{u(t)} J_1 = \int_0^\infty e^{-\rho_1 t} \left[\omega_1 h(u(t), \nu(t)) + q_1 E(t) + q_2 L(t) + q_3 J(t) + q_4 R(t) + q_5 S(t) - cx(t) - k\nu(t) - \alpha u(t)\right] dt \tag{10}$$

where ω_1, c, k, α and q_i, $(i = 1, 2, 3, 4, 5)$ are positive quantities.

ITO benefits from the stock of resources $x(t)$ and intensity $\nu(t)$ of terrorist actions, while not benefiting from government's activities. Hence, the objective of the ITO

$$\max_{\nu(t)} J_2 = \int_0^{\infty} e^{-\rho_2 t} \left[\sigma x(t) + \beta \nu(t) - \omega_2 E(t) - \omega_3 L(t) - \omega_4 J(t) - \omega_5 R(t) - \omega_6 S(t)\right] \tag{11}$$

where σ, β and ω_j $(j = 2, 3, 4, 5, 6)$ are positive quantities.

Assume that rates of growth r and activity μ_i are both less than the rates of decline ρ_l, $l = 1, 2$ i.e.,

$$\rho_l > r, \quad \rho_l > \mu_i \quad \text{with} \quad l = 1, 2 \; and \; i = 1, 2, 3, 4, 5 \tag{12}$$

We use a min-max equilibrium method in this paper, to find the optimal solution for both players, taking in consideration Pontryagin's maximum principle [12].

3 Methodology: Min-Max Equilibrium

In the two persons zero sum differential game, the cost for anyone of the players is equivalent to the negative cost of the other. As we mentioned before, we have a game of two players. First player is the "Government", while second player is the "International Terror Organization" (ITO). The first player is interested in maximizing his cost, while the interest of the second player is minimizing his own cost. This problem has two perspectives:

3.1 **Perspective of the Government**

From this perspective, the maximizing player is the government. It has to find strategy $u(t)$ to maximize its payoff, while the ITO has to find strategy $\nu(t)$ to minimize that payoff, then we can formulate the game as following

$$\left.\begin{array}{ll}
\min\limits_{\nu(t)} \max\limits_{u(t)} J_1 = \int_0^\infty e^{-\rho_1 t}\left[w_1 h(u(t),\nu(t)) + q_1 E(t) + q_2 L(t) + q_3 J(t) + q_4 R(t) + q_5 S(t) - cx(t) - k\nu(t) - \alpha u(t)\right] dt & \\
\dot{x} = rx(t) - h(u(t),\nu(t)), & x(0) = x_0 > 0\ , x(t) \geq 0 \text{ for all } t \\
\dot{E} = \mu_1 E(t) + a_1 u(t) - b_1 \nu(t), & E(0) = E_0 > 0\ , E(t) \geq 0 \text{ for all } t \\
\dot{L} = \mu_2 L(t) + a_2 u(t) - b_2 \nu(t), & L(0) = L_0 > 0\ , L(t) \geq 0 \text{ for all } t \\
\dot{J} = \mu_3 J(t) + a_3 u(t) - b_3 \nu(t), & J(0) = J_0 > 0\ , J(t) \geq 0 \text{ for all } t \\
\dot{R} = \mu_4 R(t) + a_4 u(t) - b_4 \nu(t), & R(0) = R_0 > 0\ , R(t) \geq 0 \text{ for all } t \\
\dot{S} = \mu_5 S(t) + a_5 u(t) - b_5 \nu(t), & S(0) = S_0 > 0\ , S(t) \geq 0 \text{ for all } t
\end{array}\right\} \tag{13}$$

We can symbol some terms as following

$\Gamma_1(x(t), u(t), \nu(t), E(t), L(t), J(t), R(t), S(t)) = \omega_1 h(u(t), \nu(t)) + q_1 E(t) + q_2 L(t) + q_3 J(t) + q_4 R(t) + q_5 S(t) - cx(t) - k\nu(t) - \alpha u(t)$

and

$$f(x, u, \nu) = rx(t) - h(u(t), \nu(t))$$

Definition 3.1 In "min-max continuous differential game", the point (u^*, ν^*) is called "a saddle point" for game (13) if

$$J_1(u^*, \nu) \leq J_1(u^*, \nu^*) \leq J_1(u, \nu^*) \tag{14}$$

3.2 The Necessary Conditions of "An Open Saddle Point Solution"

Theorem 3.1 *Suppose that* $\Gamma_1(x(t), u(t), \nu(t), E(t), L(t), J(t), R(t), S(t))$ *and* $f(x, u, \nu)$ *are "continuous differentiable functions". If* (u^*, ν^*) *is "saddle point" with the state trajectories* $x^*(t),\ E^*(t), L^*(t), J^*(t), R^*(t),$ *and* $S^*(t)$ *for the game from the prespective of government, then there exists a costate vectors* $\lambda_1(t), P_1(t), P_2(t), P_3(t), P_4(t), P_5(t)$ *and the Hamiltonian function* H_1 *as following*

$$H_1(x(t), u(t), \nu(t), \lambda_1(t), P_1(t), P_2(t), P_3(t), P_4(t), P_5(t), E(t), L(t), J(t), R(t), S(t)) = \Gamma_1(x(t), u(t), \nu(t), E(t), L(t), J(t), R(t), S(t)) + \lambda_1(t) f(x, u, \nu) + P_1(t)\dot{E} + P_2(t)\dot{L} + P_3(t)\dot{J} + P_4(t)\dot{R} + P_5(t)\dot{S} \tag{15}$$

and the following conditions must be satisfied

$$\left.\begin{array}{c}
\dfrac{\partial H_1}{\partial u}=0, \qquad \dfrac{\partial H_1}{\partial \nu}=0 \\
\dfrac{\partial^2 H_1}{\partial u^2}\dfrac{\partial^2 H_1}{\partial \nu^2}-\left(\dfrac{\partial^2 H_1}{\partial u \partial \nu}\right)^2 \le 0, \qquad \dfrac{\partial^2 H_1}{\partial u^2}\le 0 \quad , \dfrac{\partial^2 H_1}{\partial \nu^2}\ge 0 \\
\dot{\lambda_1}=\rho_1\lambda_1-\dfrac{\partial H_1}{\partial x} \\
\dot{P_1}=\rho_1 P_1-\dfrac{\partial H_1}{\partial E} \\
\dot{P_2}=\rho_1 P_2-\dfrac{\partial H_1}{\partial L} \\
\dot{P_3}=\rho_1 P_3-\dfrac{\partial H_1}{\partial J} \\
\dot{P_4}=\rho_1 P_4-\dfrac{\partial H_1}{\partial R} \\
\dot{P_5}=\rho_1 P_5-\dfrac{\partial H_1}{\partial S} \\
\min\limits_{\nu(t)} H_1(x(t),u^*(t),\nu(t),\lambda_1(t),P_1(t),P_2(t),P_3(t),P_4(t),P_5(t),E(t),L(t),J(t),R(t),S(t)) = \\
H_1(x(t),u^*(t),\nu^*(t),\lambda_1(t),P_1(t),P_2(t),P_3(t),P_4(t),P_5(t),E(t),L(t),J(t),R(t),S(t)) \\
=\max\limits_{u(t)} H_1(x(t),u(t),\nu^*(t),\lambda_1(t),P_1(t),P_2(t),P_3(t),P_4(t),P_5(t),E(t),L(t),J(t),R(t),S(t))
\end{array}\right\} \tag{16}$$

Proof The proof is like the method of proving theorem 3.1 in [12] □

As the optimum strategies for both government and ITO require increasing and decreasing H_1, then

$$\left.\begin{array}{c}
\dfrac{\partial H_1}{\partial u}=(\omega_1-\lambda_1)h_u-\alpha+P_1a_1+P_2a_2+P_3a_3+P_4a_4+P_5a_5=0 \\
h_u=\dfrac{\alpha-P_1a_1-P_2a_2-P_3a_3-P_4a_4-P_5a_5}{\omega_1-\lambda_1} \\
\dfrac{\partial H_1}{\partial \nu}=(\omega_1-\lambda_1)h_\nu-k-P_1b_1-P_2b_2-P_3b_3-P_4b_4-P_5b_5=0 \\
h_\nu=\dfrac{k+P_1b_1+P_2b_2+P_3b_3+P_4b_4+P_5b_5}{\omega_1-\lambda_1}
\end{array}\right\} \tag{17}$$

where, the adjoint variables satisfy the differential equations

$$\dot{\lambda_1}=\rho_1\lambda_1-\frac{\partial H_1}{\partial x}=\lambda_1(\rho_1-r)+c \tag{18}$$

$$\dot{P}_1 = \rho_1 P_1 - \frac{\partial H_1}{\partial E} = (\rho_1 - \mu_1) P_1 - q_1 \tag{19}$$

$$\dot{P}_2 = \rho_1 P_2 - \frac{\partial H_1}{\partial L} = (\rho_1 - \mu_2) P_2 - q_2 \tag{20}$$

$$\dot{P}_3 = \rho_1 P_3 - \frac{\partial H_1}{\partial J} = (\rho_1 - \mu_3) P_3 - q_3 \tag{21}$$

$$\dot{P}_4 = \rho_1 P_4 - \frac{\partial H_1}{\partial R} = (\rho_1 - \mu_4) P_4 - q_4 \tag{22}$$

$$\dot{P}_5 = \rho_1 P_5 - \frac{\partial H_1}{\partial S} = (\rho_1 - \mu_5) P_5 - q_5 \tag{23}$$

and the limiting transversality conditions

$$\lim_{t \to \infty} e^{-\rho_1 t} x(t) \lambda_1(t) = 0 \tag{24}$$

$$\lim_{t \to \infty} e^{-\rho_1 t} E(t) P_1(t) = 0 \tag{25}$$

$$\lim_{t \to \infty} e^{-\rho_1 t} L(t) P_2(t) = 0 \tag{26}$$

$$\lim_{t \to \infty} e^{-\rho_1 t} J(t) P_3(t) = 0 \tag{27}$$

$$\lim_{t \to \infty} e^{-\rho_1 t} R(t) P_4(t) = 0 \tag{28}$$

$$\lim_{t \to \infty} e^{-\rho_1 t} S(t) P_5(t) = 0 \tag{29}$$

hence, solutions for the adjoint equations are

$$\lambda_1(t) = (\lambda_0 + \frac{c}{(\rho_1 - r)}) e^{(\rho_1 - r)t} - \frac{c}{\rho_1 - r} \tag{30}$$

$$P_1(t) = (P_{10} - \frac{q_1}{\rho_1 - \mu_1}) e^{(\rho_1 - \mu_1)t} + \frac{q_1}{\rho_1 - \mu_1} \tag{31}$$

$$P_2(t) = (P_{20} - \frac{q_2}{\rho_1 - \mu_2}) e^{(\rho_1 - \mu_2)t} + \frac{q_2}{\rho_1 - \mu_2} \tag{32}$$

$$P_3(t) = (P_{30} - \frac{q_3}{\rho_1 - \mu_3}) e^{(\rho_1 - \mu_3)t} + \frac{q_3}{\rho_1 - \mu_3} \tag{33}$$

$$P_4(t) = (P_{40} - \frac{q_4}{\rho_1 - \mu_4}) e^{(\rho_1 - \mu_4)t} + \frac{q_4}{\rho_1 - \mu_4} \tag{34}$$

$$P_5(t) = (P_{50} - \frac{q_5}{\rho_1 - \mu_5}) e^{(\rho_1 - \mu_5)t} + \frac{q_5}{\rho_1 - \mu_5} \tag{35}$$

where $\lambda_1(0) = \lambda_0$, $P_1(0) = P_{10}$, $P_2(0) = P_{20}$, $P_3(0) = P_{30}$, $P_4(0) = P_{40}$ and $P_5(0) = P_{50}$, since $\rho_1 > r$ and $\rho_1 > \mu_i (i = 1, 2, 3, 4, 5)$, then $\lambda_1(t) \to \infty$,$P_1(t) \to \infty$, $P_2(t) \to \infty$, $P_3(t) \to \infty$, $P_4(t) \to \infty$ and $P_5(t) \to \infty$ as $t \to \infty$, but this dissenting the transversality conditions unless we choose the constant steady state values

$$\begin{aligned}
\lambda_1 &= \lambda_0 = -\frac{c}{\rho_1 - r} \\
P_1 &= P_{10} = \frac{q_1}{\rho_1 - \mu_1} \\
P_2 &= P_{20} = \frac{q_2}{\rho_1 - \mu_2} \\
P_3 &= P_{30} = \frac{q_3}{\rho_1 - \mu_3} \\
P_4 &= P_{40} = \frac{q_4}{\rho_1 - \mu_4} \\
P_5 &= P_{50} = \frac{q_5}{\rho_1 - \mu_5}
\end{aligned}$$

H_1 is "concave" relative to $u(t)$, but "convex" relative to $\nu(t)$ hence, we find the maximization of H_1 relative to $u(t)$ and the minimization of H_1 relative to $\nu(t)$. Define $h(u(t), \nu(t))$ as following

$$h(u(t), \nu(t)) = u^\tau \nu^\delta, \ with \ 0 < \tau < 1 < \delta$$

Remark 1 Since $H_{1uu} = (\omega_1 - \lambda_1)\tau(\tau - 1)u^{\tau-2}\nu^\delta < 0$ and $H_{1\nu\nu} = (\omega_1 - \lambda_1)\delta(\delta - 1)u^\tau \nu^{\delta-2} > 0$, hence, H_1 is "concave" relative to $u(t)$ and "convex" relative to $\nu(t)$.

Proposition 3.1 *For game (13), the "optimum strategies" are given by*

$$\begin{aligned} u &= \left[\left(\frac{\alpha - P_1a_1 - P_2a_2 - P_3a_3 - P_4a_4 - P_5a_5}{\tau(\omega_1-\lambda_1)}\right)^{\delta-1}\left(\frac{k + b_1P_1 + b_2P_2 + b_3P_3 + b_4P_4 + b_5P_5}{\delta(\omega_1-\lambda_1)}\right)^{-\delta}\right]^{\frac{1}{1-\tau-\delta}} \\ \nu &= \left[\left(\frac{k + b_1P_1 + b_2P_2 + b_3P_3 + b_4P_4 + b_5P_5}{\delta(\omega_1-\lambda_1)}\right)^{\tau-1}\left(\frac{\alpha - P_1a_1 - P_2a_2 - P_3a_3 - P_4a_4 - P_5a_5}{\tau(\omega_1-\lambda_1)}\right)^{-\tau}\right]^{\frac{1}{1-\tau-\delta}} \end{aligned} \tag{36}$$

while the" harvest function" is

$$h(u,\nu) = \left(\frac{\alpha - P_1a_1 - P_2a_2 - P_3a_3 - P_4a_4 - P_5a_5}{\tau(\omega_1-\lambda_1)}\right)^{\frac{-\tau}{1-\tau-\delta}}\left(\frac{k + b_1P_1 + b_2P_2 + b_3P_3 + b_4P_4 + b_5P_5}{\delta(\omega_1-\lambda_1)}\right)^{\frac{-\delta}{1-\tau-\delta}} \tag{37}$$

Proof Since

$$h_u = \tau u^{\tau-1}\nu^{\delta} = \frac{\alpha - P_1a_1 - P_2a_2 - P_3a_3 - P_4a_4 - P_5a_5}{(\omega_1-\lambda_1)}$$

then

$$u = \left(\frac{\alpha - P_1a_1 - P_2a_2 - P_3a_3 - P_4a_4 - P_5a_5}{\tau(\omega_1-\lambda_1)}\right)^{\frac{1}{\tau-1}}\nu^{\frac{-\delta}{\tau-1}} \tag{38}$$

and

$$h_\nu = \delta u^{\tau}\nu^{\delta-1} = \frac{k + b_1P_1 + b_2P_2 + b_3P_3 + b_4P_4 + b_5P_5}{(\omega_1-\lambda_1)}$$

then

$$\nu = \left(\frac{k + b_1P_1 + b_2P_2 + b_3P_3 + b_4P_4 + b_5P_5}{\delta(\omega_1-\lambda_1)}\right)^{\frac{1}{\delta-1}}u^{\frac{-\tau}{\delta-1}} \tag{39}$$

and thus

$$\left.\begin{aligned} u &= \left[\left(\frac{\alpha - P_1a_1 - P_2a_2 - P_3a_3 - P_4a_4 - P_5a_5}{\tau(\omega_1-\lambda_1)}\right)^{\delta-1}\left(\frac{k + b_1P_1 + b_2P_2 + b_3P_3 + b_4P_4 + b_5P_5}{\delta(\omega_1-\lambda_1)}\right)^{-\delta}\right]^{\frac{1}{1-\tau-\delta}} \\ \nu &= \left[\left(\frac{k + b_1P_1 + b_2P_2 + b_3P_3 + b_4P_4 + b_5P_5}{\delta(\omega_1-\lambda_1)}\right)^{\tau-1}\left(\frac{\alpha - P_1a_1 - P_2a_2 - P_3a_3 - P_4a_4 - P_5a_5}{\tau(\omega_1-\lambda_1)}\right)^{-\tau}\right]^{\frac{1}{1-\tau-\delta}} \\ h(u,\nu) &= \left(\frac{\alpha - P_1a_1 - P_2a_2 - P_3a_3 - P_4a_4 - P_5a_5}{\tau(\omega_1-\lambda_1)}\right)^{\frac{-\tau}{1-\tau-\delta}}\left(\frac{k + b_1P_1 + b_2P_2 + b_3P_3 + b_4P_4 + b_5P_5}{\delta(\omega_1-\lambda_1)}\right)^{\frac{-\delta}{1-\tau-\delta}} \end{aligned}\right\} \tag{40}$$

and

$$\left.\begin{aligned}\begin{vmatrix} H_{1uu} & H_{1u\nu} \\ H_{1\nu u} & H_{1\nu\nu} \end{vmatrix} = (\omega_1-\lambda_1)^2 \begin{vmatrix} \tau(\tau-1)u^{\tau-2}\nu^{\delta} & \tau\delta u^{\tau-1}\nu^{\delta-1} \\ \tau\delta u^{\tau-1}\nu^{\delta-1} & \delta(\delta-1)u^{\tau}\nu^{\delta-2} \end{vmatrix} \\ = (\omega_1-\lambda_1)^2\tau\delta(1-\tau-\delta)u^{2(\tau-1)}\nu^{2(\delta-1)} < 0\end{aligned}\right\} \tag{41}$$

hence, $(u(t), \nu(t))$ is "a saddle point" for game (13) □

Lemma 3.1 *For the government, the objective J_1 with constant strategies u, ν is*

$$\mathrm{J}_1 = \frac{h\omega_1}{\rho_1} + q_1 I_1 + q_2 I_2 + q_3 I_3 + q_4 I_4 + q_5 I_5 - c I_6 - \frac{k\nu}{\rho_1} - \frac{\alpha u}{\rho_1} \tag{42}$$

where,

$$I_1 = \frac{1}{(\rho_1-\mu_1)}(E_0 + \frac{(a_1u-b_1\nu)}{\mu_1}) - \frac{(a_1u-b_1\nu)}{\rho_1\mu_1}$$
$$I_2 = \frac{1}{(\rho_1-\mu_2)}(L_0 + \frac{(a_2u-b_2\nu)}{\mu_2}) - \frac{(a_2u-b_2\nu)}{\rho_1\mu_2}$$
$$I_3 = \frac{1}{(\rho_1-\mu_3)}(J_0 + \frac{(a_3u-b_3\nu)}{\mu_3}) - \frac{(a_3u-b_3\nu)}{\rho_1\mu_3}$$
$$I_4 = \frac{1}{(\rho_1-\mu_4)}(R_0 + \frac{(a_4u-b_4\nu)}{\mu_4}) - \frac{(a_4u-b_4\nu)}{\rho_1\mu_4}$$
$$I_5 = \frac{1}{(\rho_1-\mu_5)}(S_0 + \frac{(a_5u-b_5\nu)}{\mu_5}) - \frac{(a_5u-b_5\nu)}{\rho_1\mu_5}$$
$$I_6 = \frac{(\rho_1x_0-h)}{\rho_1(\rho_1-r)}$$

Proof Solving the equations corresponding to $x^{\cdot}, E^{\cdot}, L^{\cdot}, J^{\cdot}, R^{\cdot}, S^{\cdot}$ in (13), we get

$$\begin{aligned} x(t)e^{-rt} &= \frac{1}{r}e^{-rt}h(u,\nu) + c_1 \\ E(t)e^{-\mu_1 t} &= -\frac{a_1u-b_1\nu}{\mu_1}e^{-\mu_1 t} + c_2 \\ L(t)e^{-\mu_2 t} &= -\frac{a_2u-b_2\nu}{\mu_2}e^{-\mu_2 t} + c_3 \\ J(t)e^{-\mu_3 t} &= -\frac{a_3u-b_3\nu}{\mu_3}e^{-\mu_3 t} + c_4 \end{aligned}$$

$$R(t)e^{-\mu_4 t} = -\frac{a_4 u - b_4 \nu}{\mu_4} e^{-\mu_4 t} + c_5$$
$$S(t)e^{-\mu_5 t} = -\frac{a_5 u - b_5 \nu}{\mu_5} e^{-\mu_5 t} + c_6$$

where c_1, c_2, c_3, c_4, c_5 and c_6 are constants, for $t \to 0, c_1 = x_0 - \frac{1}{r} h(u, \nu)$, $c_2 = E_0 + \frac{a_1 u - b_1 \nu}{\mu_1}$, $c_3 = L_0 + \frac{a_2 u - b_2 \nu}{\mu_2}$, $c_4 = J_0 + \frac{a_3 u - b_3 \nu}{\mu_3}$, $c_5 = R_0 + \frac{a_4 u - b_4 \nu}{\mu_4}$ and $c_6 = S_0 + \frac{a_5 u - b_5 \nu}{\mu_5}$

then

$$x(t) = (x_0 - \frac{1}{r} h(u, \nu))e^{rt} + \frac{h}{r} \tag{43}$$

$$\left.\begin{aligned}
E(t) &= (E_0 + \frac{a_1 u - b_1 \nu}{\mu_1})e^{\mu_1 t} - \frac{a_1 u - b_1 \nu}{\mu_1} \\
L(t) &= (L_0 + \frac{a_2 u - b_2 \nu}{\mu_2})e^{\mu_2 t} - \frac{a_2 u - b_2 \nu}{\mu_2} \\
J(t) &= (J_0 + \frac{a_3 u - b_3 \nu}{\mu_3})e^{\mu_3 t} - \frac{a_3 u - b_3 \nu}{\mu_3} \\
R(t) &= (R_0 + \frac{a_4 u - b_4 \nu}{\mu_4})e^{\mu_4 t} - \frac{a_4 u - b_4 \nu}{\mu_4} \\
S(t) &= (S_0 + \frac{a_5 u - b_5 \nu}{\mu_5})e^{\mu_5 t} - \frac{a_5 u - b_5 \nu}{\mu_5}
\end{aligned}\right\} \tag{44}$$

and thus

$$\mathrm{J}_1 = \frac{h\omega_1}{\rho_1} + q_1 I_1 + q_2 I_2 + q_3 I_3 + q_4 I_4 + q_5 I_5 - c I_6 - \frac{k\nu}{\rho_1} - \frac{\alpha u}{\rho_1} \tag{45}$$

where u, ν and $h(u, \nu)$ are defined in (36) and (37) . □

3.3 **Graphical Interpretation from Government Perspective for Relations Between Terrorism Activity and Government's Activities**

From equation (38), we denote it by $u = A\nu^B$ where, $A = (\frac{\alpha - \sum_i a_i P_i}{\tau(\omega_1 - \lambda_1)})^{\frac{1}{\tau - 1}}$, $B = \frac{-\delta}{\tau - 1}$. Since $0 < \tau < 1 < \delta$, then $B > 1$. Due to $u \geq 0$, hence

$\alpha \geq \sum_i a_i P_i$ and $\tau(\omega_1 - \lambda_1) > 0$. In figure (1), we note an increase in ν for a while, and over time the curve begins to take a near-constant shape, while increasing in u, indicating a decrease in terrorist activity as a result of increasing cost of resistance. But from the equations (44), since $u = A\nu^B$, then we can see that any of these equations can be symbolized by $Y = D - E\nu + F\nu^B$, where Y denote the activities of the government at a specific time t. In figure (2), we find that ν is increasing for a while, and over time the curve begins to take a near-constant shape, while increasing in Y. We conclude from the foregoing that the government can success in combating terrorism by paying attention to the five activities E, L, J, R and S.

3.4 Perspective of the ITO

Here, ITO is the maximizing player. It has to find strategy $\nu(t)$ to maximize its payoff, while government has to find strategy $u(t)$ to minimize that payoff, then we can formulate the game as following

$$\left.\begin{array}{l}
\min\limits_{u(t)} \max\limits_{\nu(t)} \left\{ J_2 = \displaystyle\int_0^\infty e^{-\rho_2 t} \left[\sigma x(t) + \beta\nu(t) - \omega_2 E(t) - \omega_3 L(t) - \omega_4 J(t) - \omega_5 R(t) - \omega_6 S(t)\right] dt \right\} \\
\qquad \dot{x} = r x(t) - h(u(t), \nu(t)), \qquad x(0) = x_0 > 0 \ , x(t) \geq 0 \text{ for all } t \\
\qquad \dot{E} = \mu_1 E(t) + a_1 u(t) - b_1 \nu(t), \qquad E(0) = E_0 > 0 \ , E(t) \geq 0 \text{ for all } t \\
\qquad \dot{L} = \mu_2 L(t) + a_2 u(t) - b_2 \nu(t), \qquad L(0) = L_0 > 0 \ , L(t) \geq 0 \text{ for all } t \\
\qquad \dot{J} = \mu_3 J(t) + a_3 u(t) - b_3 \nu(t), \qquad J(0) = J_0 > 0 \ , J(t) \geq 0 \text{ for all } t \\
\qquad \dot{R} = \mu_4 R(t) + a_4 u(t) - b_4 \nu(t), \qquad R(0) = R_0 > 0 \ , R(t) \geq 0 \text{ for all } t \\
\qquad \dot{S} = \mu_5 S(t) + a_5 u(t) - b_5 \nu(t), \qquad S(0) = S_0 > 0 \ , S(t) \geq 0 \text{ for all } t
\end{array}\right\} \tag{46}$$

We can symbol some terms as following

$$\Gamma_2(x(t),\nu(t),E(t),L(t),J(t),R(t),S(t)) = \sigma x(t)+\beta\nu(t)-\omega_2 E(t)-\omega_2 L(t)-\omega_2 J(t)-\omega_2 R(t)-\omega_2 S(t)$$

Definition 3.2 In "min-max continuous differential game", the point (u^*,ν^*) is called "a saddle point" for game (46) if

$$J_2(u^*,\nu) \leq J_2(u^*,\nu^*) \leq J_2(u,\nu^*) \tag{47}$$

3.5 The Necessary Conditions of "An Open Saddle Point Solution"

Theorem 3.2 *Suppose that $\Gamma_2(x(t),\nu(t),E(t),L(t),J(t),R(t),S(t))$ and $f(x,u,v)$ are "continuous differentiable functions". If (u^*,ν^*) is "saddle point" with the state trajectory $x^*(t)$ for game from the prespective of ITO, then there exists a costate vector $\lambda_2(t)$ and the Hamiltonian function H_2 as following*

$$\begin{aligned} &H_2(x(t),u(t),\nu(t),\lambda_2(t),P_6(t),P_7(t),P_8(t),P_9(t),P_{10}(t),E(t),L(t),J(t),R(t),S(t)) \\ &\quad = \Gamma_2(x(t),\nu(t),E(t),L(t),J(t),R(t),S(t)) + \lambda_2(t)f(x,u,\nu) + P_6(t)\dot{E} \\ &\qquad + P_7(t)\dot{L} + P_8(t)\dot{J} + P_9(t)\dot{R} + P_{10}(t)\dot{S} \end{aligned} \tag{48}$$

and the following conditions must be satisfied

$$\left.\begin{array}{c}
\dfrac{\partial H_2}{\partial u}=0, \qquad \dfrac{\partial H_2}{\partial \nu}=0 \\
\dfrac{\partial^2 H_2}{\partial u^2}\dfrac{\partial^2 H_2}{\partial \nu^2}-\left(\dfrac{\partial^2 H_2}{\partial u \partial \nu}\right)^2 \leq 0,\ \dfrac{\partial^2 H_2}{\partial u^2}\geq 0 \quad , \dfrac{\partial^2 H_2}{\partial \nu^2}\leq 0 \\
\dot{\lambda_2}=\rho_2\lambda_2-\dfrac{\partial H_2}{\partial x} \\
\dot{P_6}=\rho_2 P_6-\dfrac{\partial H_2}{\partial E} \\
\dot{P_7}=\rho_2 P_7-\dfrac{\partial H_2}{\partial L} \\
\dot{P_8}=\rho_2 P_8-\dfrac{\partial H_2}{\partial J} \\
\dot{P_9}=\rho_2 P_9-\dfrac{\partial H_2}{\partial R} \\
\dot{P_{10}}=\rho_2 P_{10}-\dfrac{\partial H_2}{\partial S} \\
\max\limits_{\nu(t)} H_2(x(t),u^*(t),\nu(t),\lambda_2(t),P_6(t),P_7(t),P_8(t),P_9(t),P_{10}(t),E(t),L(t),J(t),R(t),S(t)) \\
=H_2(x(t),u^*(t),\nu^*(t),\lambda_2(t),P_6(t),P_7(t),P_8(t),P_9(t),P_{10}(t),E(t),L(t),J(t),R(t),S(t))= \\
\min\limits_{u(t)} H_2(x(t),u(t),\nu^*(t),\lambda_2(t),P_6(t),P_7(t),P_8(t),P_9(t),P_{10}(t),E(t),L(t),J(t),R(t),S(t))
\end{array}\right\} \tag{49}$$

Proof The proof is like the method of proving Theorem 3.1 in [12]. □

As the optimum strategies for both government and ITO require increasing and decreasing H_2, then

$$\left.\begin{array}{c}
\dfrac{\partial H_2}{\partial u}=-\lambda_2 h_u+a_1P_6+a_2P_7+a_3P_8+a_4P_9+a_5P_{10}=0 \\
h_u=\dfrac{a_1P_6+a_2P_7+a_3P_8+a_4P_9+a_5P_{10}}{\lambda_2} \\
\dfrac{\partial H_2}{\partial \nu}=\beta-\lambda_2 h_\nu-b_1P_6-b_2P_7-b_3P_8-b_4P_9-b_5P_{10}=0 \\
h_\nu=\dfrac{1}{\lambda_2}[\beta-b_1P_6-b_2P_7-b_3P_8-b_4P_9-b_5P_{10}]
\end{array}\right\} \tag{50}$$

where, the adjoint variables satisfy the differential equations

$$\left.\begin{aligned}
\dot{\lambda_2} &= \rho_2\lambda_2 - \frac{\partial H_2}{\partial x} = \lambda_2(\rho_2 - r) - \sigma \\
\dot{P_6} &= \rho_2 P_6 - \frac{\partial H_2}{\partial E} = P_6(\rho_2 - \mu_1) + \omega_2 \\
\dot{P_7} &= \rho_2 P_7 - \frac{\partial H_2}{\partial L} = P_7(\rho_2 - \mu_2) + \omega_3 \\
\dot{P_8} &= \rho_2 P_8 - \frac{\partial H_2}{\partial J} = P_8(\rho_2 - \mu_3) + \omega_4 \\
\dot{P_9} &= \rho_2 P_9 - \frac{\partial H_2}{\partial R} = P_9(\rho_2 - \mu_4) + \omega_5 \\
\dot{P_{10}} &= \rho_2 P_{10} - \frac{\partial H_2}{\partial S} = P_{10}(\rho_2 - \mu_5) + \omega_6
\end{aligned}\right\} \tag{51}$$

and the limiting transversality conditions

$$\lim_{t\to\infty} e^{-\rho_2 t}x(t)\lambda_2(t) = 0$$

$$\lim_{t\to\infty} e^{-\rho_2 t}E(t)P_6(t) = 0$$

$$\lim_{t\to\infty} e^{-\rho_2 t}L(t)P_7(t) = 0$$

$$\lim_{t\to\infty} e^{-\rho_2 t}J(t)P_8(t) = 0$$

$$\lim_{t\to\infty} e^{-\rho_2 t}R(t)P_9(t) = 0$$

$$\lim_{t\to\infty} e^{-\rho_2 t}S(t)P_{10}(t) = 0$$

hence, solutions for the adjoint equations are

$$\left.\begin{aligned}
\lambda_2(t) &= (\lambda_{20} - \frac{\sigma}{(\rho_2 - r)})e^{(\rho_2 - r)t} + \frac{\sigma}{\rho_2 - r} \\
P_6(t) &= (P_{60} + \frac{\omega_2}{\rho_2 - \mu_1})e^{(\rho_2 - \mu_1)t} - \frac{\omega_2}{\rho_2 - \mu_1} \\
P_7(t) &= (P_{70} + \frac{\omega_3}{\rho_2 - \mu_2})e^{(\rho_2 - \mu_2)t} - \frac{\omega_3}{\rho_2 - \mu_2} \\
P_8(t) &= (P_{80} + \frac{\omega_4}{\rho_2 - \mu_3})e^{(\rho_2 - \mu_3)t} - \frac{\omega_4}{\rho_2 - \mu_3} \\
P_9(t) &= (P_{90} + \frac{\omega_5}{\rho_2 - \mu_4})e^{(\rho_2 - \mu_4)t} - \frac{\omega_5}{\rho_2 - \mu_4} \\
P_{10}(t) &= (P_{100} + \frac{\omega_6}{\rho_2 - \mu_5})e^{(\rho_2 - \mu_5)t} - \frac{\omega_6}{\rho_2 - \mu_5}
\end{aligned}\right\} \tag{52}$$

where $\lambda_2(0) = \lambda_{20}, P_6(0) = P_{60}, P_7(0) = P_{70}, P_8(0) = P_{80}, P_9(0) = P_{90}$ and $P_{10}(0) = P_{100}$, since $\rho_1 > r$ and $\rho_1 > \mu_i$ $(i = 1, 2, 3, 4, 5)$, then $\lambda_2(t) \to \infty$,$P_6(t) \to \infty$, $P_7(t) \to \infty$, $P_8(t) \to \infty$, $P_9(t) \to \infty$ and $P_{10}(t) \to \infty$ as $t \to \infty$, but this dissenting the transversality conditions unless we choose the constant steady state values

$$\begin{aligned}
\lambda_2 = \lambda_{20} &= \frac{\sigma}{\rho_2 - r} \\
P_6 = P_{60} &= -\frac{\omega_2}{\rho_2 - \mu_1} \\
P_7 = P_{70} &= -\frac{\omega_3}{\rho_2 - \mu_2} \\
P_8 = P_{80} &= -\frac{\omega_4}{\rho_2 - \mu_3} \\
P_9 = P_{90} &= -\frac{\omega_5}{\rho_2 - \mu_4} \\
P_{10} = P_{100} &= -\frac{\omega_6}{\rho_2 - \mu_5}
\end{aligned}$$

H_2 is "concave" relative to $\nu(t)$, and "convex" relative to $u(t)$ hence, we find the maximization of H_2 relative to ν and the minimization of H_2 relative to u.

Remark 2. Since $h(u(t), \nu(t)) = u^\tau \nu^\delta$, with $0 < \tau < 1 < \delta$, $H_{2uu} = -\lambda_2 \tau(\tau - 1)u^{\tau-2}\nu^\delta > 0$ and $H_{2\nu\nu} = -\lambda_2 \delta(\delta - 1)u^\tau \nu^{\delta-2} < 0$, hence H_2 is "convex" relative to $u(t)$ and "concave" relative to $\nu(t)$.

Proposition 3.2 *For game (46), the optimum strategies are given by*

$$\left.\begin{aligned}
u &= \left(\frac{a_1P_6 + a_2P_7 + a_3P_8 + a_4P_9 + a_5P_{10}}{\tau\lambda_2}\right)^{\frac{\delta-1}{1-\tau-\delta}} \left(\frac{\beta - b_1P_6 - b_2P_7 - b_3P_8 - b_4P_9 - b_5P_{10}}{\delta\lambda_2}\right)^{\frac{-\delta}{1-\tau-\delta}} \\
\nu &= \left(\frac{a_1P_6 + a_2P_7 + a_3P_8 + a_4P_9 + a_5P_{10}}{\tau\lambda_2}\right)^{\frac{-\tau}{1-\tau-\delta}} \left(\frac{\beta - b_1P_6 - b_2P_7 - b_3P_8 - b_4P_9 - b_5P_{10}}{\delta\lambda_2}\right)^{\frac{1-\tau}{1-\tau-\delta}} \\
h(u,\nu) &= \left(\frac{a_1P_6 + a_2P_7 + a_3P_8 + a_4P_9 + a_5P_{10}}{\tau\lambda_2}\right)^{\frac{-\tau}{1-\tau-\delta}} \left(\frac{\beta - b_1P_6 - b_2P_7 - b_3P_8 - b_4P_9 - b_5P_{10}}{\delta\lambda_2}\right)^{\delta(1-2\tau)}
\end{aligned}\right\} \tag{53}$$

Proof Proof of this proposition is like the method of proving proposition 3.1.

Note the following

$$\left.\begin{array}{l} \begin{vmatrix} H_{2uu} & H_{2u\nu} \\ H_{2\nu u} & H_{2\nu\nu} \end{vmatrix} = (-\lambda_2)^2 \begin{vmatrix} \tau(\tau-1)u^{\tau-2}\nu^{\delta} & \tau\delta u^{\tau-1}\nu^{\delta-1} \\ \tau\delta u^{\tau-1}\nu^{\delta-1} & \delta(\delta-1)u^{\tau}\nu^{\delta-2} \end{vmatrix} \\ \qquad = (\lambda_2)^2\tau\delta(1-\tau-\delta)u^{2(\tau-1)}\nu^{2(\delta-1)} < 0 \end{array}\right\} \tag{54}$$

hence, from (53) and (54), we find that $(u(t), \nu(t))$ is saddle point of game (46).

Similar to lemma 3.1 we get

$$x(t) = (x_0 - \frac{1}{r}h(u,\nu))e^{rt} + \frac{h}{r} \tag{55}$$

$$\left.\begin{array}{l} E(t) = (E_0 + \dfrac{a_1u - b_1\nu}{\mu_1})e^{\mu_1 t} - \dfrac{a_1u - b_1\nu}{\mu_1} \\ L(t) = (L_0 + \dfrac{a_2u - b_2\nu}{\mu_2})e^{\mu_2 t} - \dfrac{a_2u - b_2\nu}{\mu_2} \\ J(t) = (J_0 + \dfrac{a_3u - b_3\nu}{\mu_3})e^{\mu_3 t} - \dfrac{a_3u - b_3\nu}{\mu_3} \\ R(t) = (R_0 + \dfrac{a_4u - b_4\nu}{\mu_4})e^{\mu_4 t} - \dfrac{a_4u - b_4\nu}{\mu_4} \\ S(t) = (S_0 + \dfrac{a_5u - b_5\nu}{\mu_5})e^{\mu_5 t} - \dfrac{a_5u - b_5\nu}{\mu_5} \end{array}\right\} \tag{56}$$

$$J_2 = \sigma I_7 + \frac{\beta\nu}{\rho_2} - \omega_2 I_8 - \omega_3 I_9 - \omega_4 I_{10} - \omega_5 I_{11} - \omega_6 I_{12} \tag{57}$$

where

$$\left.\begin{array}{r}
I_7 = \frac{1}{(\rho_2-\mu_1)}(E_0 + \frac{(a_1u-b_1\nu)}{\mu_1}) - \frac{(a_1u-b_1\nu)}{\rho_2\mu_1} \\
I_8 = \frac{1}{(\rho_2-\mu_2)}(L_0 + \frac{(a_2u-b_2\nu)}{\mu_2}) - \frac{(a_2u-b_2\nu)}{\rho_2\mu_2} \\
I_9 = \frac{1}{(\rho_2-\mu_3)}(J_0 + \frac{(a_3u-b_3\nu)}{\mu_3}) - \frac{(a_3u-b_3\nu)}{\rho_2\mu_3} \\
I_{10} = \frac{1}{(\rho_2-\mu_4)}(R_0 + \frac{(a_4u-b_4\nu)}{\mu_4}) - \frac{(a_4u-b_4\nu)}{\rho_2\mu_4} \\
I_{11} = \frac{1}{(\rho_2-\mu_5)}(S_0 + \frac{(a_5u-b_5\nu)}{\mu_5}) - \frac{(a_5u-b_5\nu)}{\rho_2\mu_5} \\
I_{12} = \frac{(\rho_2x_0-h)}{\rho_2(\rho_2-r)} \\
u = \left(\frac{a_1P_6+a_2P_7+a_3P_8+a_4P_9+a_5P_{10}}{\tau\lambda_2}\right)^{\frac{\delta-1}{1-\tau-\delta}} \left(\frac{\beta-b_1P_6-b_2P_7-b_3P_8-b_4P_9-b_5P_{10}}{\delta\lambda_2}\right)^{\frac{-\delta}{1-\tau-\delta}} \\
\nu = \left(\frac{a_1P_6+a_2P_7+a_3P_8+a_4P_9+a_5P_{10}}{\tau\lambda_2}\right)^{\frac{-\tau}{1-\tau-\delta}} \left(\frac{\beta-b_1P_6-b_2P_7-b_3P_8-b_4P_9-b_5P_{10}}{\delta\lambda_2}\right)^{\frac{1-\tau}{1-\tau-\delta}} \\
h(u,\nu) = \left(\frac{a_1P_6+a_2P_7+a_3P_8+a_4P_9+a_5P_{10}}{\tau\lambda_2}\right)^{\frac{-\tau}{1-\tau-\delta}} \left(\frac{\beta-b_1P_6-b_2P_7-b_3P_8-b_4P_9-b_5P_{10}}{\delta\lambda_2}\right)^{\delta(1-2\tau)}
\end{array}\right\} \tag{58}$$

□

3.6 **Graphical Interpretation from ITO Perspective for Relations Between Terrorism Activity and Government's Activities**

Since, $u = (\frac{a_1P_6+a_2P_7+a_3P_8+a_4P_9+a_5P_{10}}{\tau}\lambda_2)^{\frac{1}{\tau-1}}\nu^{\frac{-\delta}{\tau-1}}$, where $\frac{\partial H_2}{\partial u} = 0$.
Denote this equation by $u = C\nu^B$ where, $C = (\frac{a_1P_6+a_2P_7+a_3P_8+a_4P_9+a_5P_{10}}{\tau\lambda_2})^{\frac{1}{\tau-1}}$ and $B = \frac{-\delta}{\tau-1}$, where $C > 0$ and $B > 1$. In figure (1), we note an increase in ν for a while, and over time the curve begins to take a near-constant shape, while increasing in u, indicating a decrease in terrorist activity as a result of increasing cost of resistance. But from the equations (56), since $u = A\nu^B$, then we can see that any of these equations can be symbolized by $Y = D - E\nu + F\nu^B$,

where Y denote the activities of the government at a specific time t. In figure (2), we find that ν is increasing for a while, and over time the curve begins to take a near-constant shape, while increasing in Y.

4 Conclusions

The dynamics of the governmental activities E, L, J, R and S were investigated in this study, and min-max equilibrium approach was employed to solve the game between government and ITO. It is clear from the relation between the activities Y of the government at a specific time t and the intensity of terrorist attacks ν, that terrorist activity decreases as the value of Y increases as in figure (2). So we conclude that the government can success in combating terrorism by paying attention to the activities E, L, J, R and S.

5 Conflicting Intrest

On behalf of all authors, the corresponding authors states that there is no conflict of interest.

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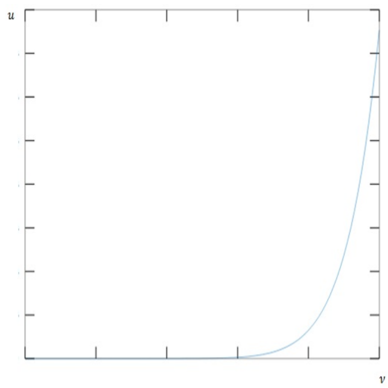

Fig. 1: The relation between u and ν

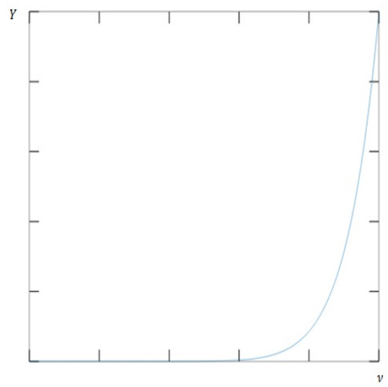

Fig. 2: The relation between Y and ν