ARTICLE TEMPLATE

Joint ordering policy for a conditional trade credit model with two retailers

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ARTICLE HISTORY
Compiled February 18, 2020

ABSTRACT
This paper focuses on the cooperation mechanism between two retailers. To reduce the average processing cost, the supplier usually sets a threshold for trade credit to stimulate retailers' orders. Retailers can enjoy permissible delay in payments only when their order quantities are more than or equal to the given threshold. However, considering the diversity of retailers, the motivation effect of the threshold may be limited. To resolve the problem, the supplier can additionally provide retailers with a joint ordering policy under which two retailers can make delayed payments as long as their total order quantity meets the required threshold. Thus, the two retailers should decide whether to place a joint order or not and determine their respective order quantities simultaneously. We provide a mutually acceptable order-allocation scheme for retailers, and determine the optimal payment methods for them. In addition, an optimal threshold is identified for the supplier to maximize the total order quantity of retailers. Based on this, some managerial insights are obtained. A numerical experiment is performed to illustrate the validity of the model.

KEYWORDS
Inventory; Economic order quantity (EOQ); Supply chain management; Trade credit linked to order quantity; Joint ordering policy; Stackelberg game; Nash equilibrium; Tacit bargaining

1. Introduction

The permissible delay in payment is attractive to retailers because they can earn interest from sales revenue during the trade credit period. A supplier can stimulate retailers' orders by setting a threshold for trade credit. Specifically, if the order quantity of a retailer is more than or equal to the given threshold, the retailer can make a delayed payment for its order; otherwise, it must pay the supplier immediately. Thus, trade credit linked to order quantity (conditional trade credit) can effectively encourage retailers to place larger orders and, in turn, reduce the average processing cost for the supplier. On the other hand, retailers can unite to enhance their negotiating power with the

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‡ This work is supported by the National Natural Science Foundation of China (No, 11271175), the Social Science Plan Foundation of Shandong Province, China (No, 19BYSJ13).

supplier on, for example, quantity discounts and trade credits. This paper relates two areas, namely trade credit linked to order quantity, and cooperation mechanism between retailers.

As the main source of short-term financing, trade credit is crucial in supply chain management. Goyal [1] developed a basic economic order quantity (EOQ) model under permissible delay in payments. Aggarwal and Jaggi [2] generalized Goyal's model in [1] to allow for deteriorating items. Teng [3] extended Goyal's model to distinguish the selling price from the purchasing price. Chung [4] simplified the solution method for Goyal's model. Jaber and Osman [5] and Arkan and Hejazi [6] proposed joint decision policies between the supplier and the retailer. Esmaeili [7] and Teng *et al.* [8] investigated the cooperative and non-cooperative relationships between the supplier and the retailer. Zhou and Zhong [9] showed that trade credit increases each member's profit and brings more profits to the retailer than to the supplier. Many additional related studies can be found in the articles by Liao *et al.* [10], Shin *et al.* [11], Banu and Mondal [12], Tiwari *et al.* [13], Wu *et al.* [14], Srivastava *et al.* [15], Sarker *et al.* [16], and Chung *et al.* [17]. Recently, much research attention has been supply chains with conditional trade credit. Chang *et al.* [18] and Chung and Liao [19] developed inventory models for deteriorating items under conditional trade credit. Chun *et al.* [20] determined the retailer's optimal ordering strategy to minimize the total variable cost. Huang [21] later added a partial trade credit to Chun *et al.*'s model in [20]. Ouyang *et al.* [22] and Ting [23] generalized Chung and Liao's model in [19] to consider a conditional and partial trade credit. Zhong and Zhou [24] revealed that two-part conditional trade credit is superior to one-part conditional trade credit. Studies that have examined inventory models with trade credit linked to order quantity include, among many others, Wang *et al.* [25], Taleizadeh *et al.* [26], Vandana and Sharma [27], and Rajan and Uthayakumar [28]. In the inventory models reviewed above, retailers can only place orders with the supplier separately. Cooperation between retailers is quite common in commercial activities, and should not be ignored.

It is reasonable for retailers to unite to enhance their negotiating power with the supplier. Anand and Aron [29] provided a survey of the group buying consortia consisting of independent companies. Chen *et al.* [30] compared the group buying auction with the fixed price mechanism in terms of the seller's pricing strategy. Chen [31] developed an inventory model in which retailers place a joint order with the supplier to reduce their operating costs. Chen and Roma [32] revealed that group buying is always beneficial to symmetric retailers and the inefficient one of two retailers. Chen and Li [33] showed that when duopoly firms sell to a buyer group, their incentives to improve quality may be affected. Hu *et al.* [34] uncovered that a joint purchase from the supplier hurts the buyers when the information between them is asymmetric. Hsu *et al.* [35] developed a group buying mechanism under which the retailer acting as the follower pays the retailer acting as the leader a fixed price regardless of the wholesale price obtained from the manufacturer. Note that all of the aforementioned inventory models consider only how retailers can form an alliance to obtain quantity discounts from the supplier. However, few studies have considered how them can unite to enjoy permissible delay in payments. This motivates us to fill this research gap.

In this paper, we develop an inventory model that consists of one upstream supplier and two downstream retailers. The supplier first sets a threshold for trade credit and provides a joint ordering policy for retailers. In previous studies, retailers must pay the supplier immediately if their order quantities are less than the given threshold. However, under the joint ordering policy, as long as their total order quantity meets the required threshold, each of them can make a delayed payment. Under this agreement, the two retailers should decide whether to place a total order with the supplier or to place orders separately. Cooperation between them is crucial. However, in reality, each retailer would like the other to take on more responsibility, so that it, as a free rider, can enjoy a permissible delay in payment by placing a smaller order. Competition between retailers is therefore inevitable, and the game between them constitutes a static game. We discuss when the two retailers should place a joint order and when they should place orders separately. A mutually acceptable order-allocation scheme is provided for retailers for each given threshold, and an optimal threshold is identified for the supplier to maximize the total order quantity of retailers. We find that the joint ordering policy can substantially reduce the barriers to retailers benefiting from trade credits, and the two retailers unanimously give priority to placing a total order with the supplier. Moreover, we uncover that a sufficiently high threshold cannot motivate retailers, while a lower threshold may not effectively stimulate retailers facing identical demands or the retailer facing the smaller demand. Occasionally, for a relatively high threshold, the retailer facing the larger demand would rather place a larger order to meet the threshold than cooperate with the retailer facing the smaller demand.

2. Notation and assumptions

2.1. Notation

The following notation is used to model the problem.

Parameter	Description
d_i	the annual market demand faced by Retailer i, with $i = 1, 2$
p	the unit recommended retail price
c	the unit purchasing cost/price
c_o	the unit production cost
h	the unit inventory holding cost per year
A_i	the ordering cost of Retailer i per order
A_s	the processing cost of the supplier per order
M_i	the possible trade credit period of Retailer i
α	the fixed proportion of possible trade credit period to replenishment period
I_e	the interest earned from sales revenue per \$ per year
I_p	the interest charged to retailers for the stock in-hand per \$ per year
Decision variables	
Q_0	the threshold set for trade credit
t_i	the replenishment period of Retailer i
q_i	the order quantity of Retailer i, $q_i = d_i t_i$
*	an optimal value

2.2. Assumptions

The proposed model is based on the following assumptions:

(1) The model consists of one upstream supplier and two downstream retailers.

(2) As retail agents of the same product, retailers sell products at recommended retail price p[1].

(3) If Retailer i's order quantity q_i, $(i = 1, 2)$, is more than or equal to a given threshold Q_0, it is offered a credit period M_i, which is in proportion α to its replenishment period t_i, i.e., $M_i = \alpha t_i$; otherwise, Retailer i must pay the supplier immediately.

[1]The price at which the supplier suggests a product should be sold in the retail market, though this may be reduced by the retailer.

(4) The two retailers can unite to place a total order. If the total order quantity (i.e., $t_1 d_1 + t_2 d_2$) of them meets the required threshold, each retailer can enjoy a permissible delay in payment.

(5) When the two payment methods bring the same profit to a retailer, it will select the delayed payment for the consideration of capital turnover.

(6) The two retailers are rational and of equal status; they determine their order quantities and payment methods simultaneously.

(7) Shortages are not allowed, and replenishment is instantaneous.

The supplier usually offers retailers a fixed credit period M to stimulate their orders. However, considering the diversity of retailers, the fixed credit period may be inefficient. For example, if a retailer's replenishment period is much longer than M, its motivation to pursue the credit period is usually limited. In contrast, if the replenishment period is much shorter than M, the supplier will bear a higher opportunity cost for offering the credit period. Therefore, we adopt in our model a dynamic credit period linked to each retailer's replenishment period. In addition, without loss of generality, we may assume that $A_1 \leqslant A_2$ for convenience.

3. Model formulation

This paper considers a two-echelon supply chain consisting of one supplier and two retailers. The supplier, as the Stackelberg leader, sets a threshold Q_0 for trade credit and additionally offers a joint ordering policy to retailers. Under the joint ordering policy, retailers can enjoy permissible delay in payments as long as they unite to place a total order, and their total order quantity meets the required threshold. Hence, the two retailers, as followers, should decide whether to place a total order or to place orders separately. Moreover, their payment methods and respective order quantities should be determined simultaneously. This paper seeks to determine the optimal payment methods for retailers and provide a mutually acceptable order-allocation scheme for them. In addition, an optimal threshold Q_0^* will be identified for the supplier to maximize the total order quantity of retailers. To better illustrate this model, we consider a two-echelon supply chain in which Coca-Cola acts as the supplier; Costo and Wal-Mark are the two retail agents. The product is cola, which is produced by Coca-Cola and offered to Costco and Wal-Mark simultaneously. Note that after years of competition with Pepsi, the retail price of coke has tended to be stable.

3.1. Retailers' best responses

For given threshold Q_0 and Retailer j's order quantity q_j, Retailer i always gives priority to cooperating with Retailer j to place a total order because if Retailer i can enjoy a permissible delay in payment by placing an order separately, it can still enjoy the same credit period by cooperating with Retailer j, but not vice versa. Hence, for a given Q_0, we need determine Retailer i's best response to Retailer j's order decision t_j.

We first compute Retailer i's mean profit in each replenishment cycle, which consists of the sales revenue, the purchasing cost, the constant ordering cost, the inventory holding cost, the interest earned from sales revenue, and the interest charged for the stock in-hand.

For a given Q_0, if $t_1 d_1 + t_2 d_2 < Q_0$, Retailer i, $(i = 1, 2)$, must pay the supplier immediately for the items purchased and the stock in-hand, in which case, the mean profit is

$$\begin{aligned} f_{i1}(t_i) &= \frac{1}{t_i}\Big((p-c)q_i - A_i - \frac{1}{2}(h+cI_p)d_it_i^2\Big) \\ &= (p-c)d_i - \Big(\frac{A_i}{t_i} + \frac{(h+cI_p)d_it_i}{2}\Big); \end{aligned} \tag{1}$$

if $t_1d_1 + t_2d_2 \geqslant Q_0$, Retailer i can earn interest from sales revenue during the credit period $[0, M_i]$ and postpone paying the supplier until time M_i (see Figure 1), and the mean profit is

$$\begin{aligned} f_{i2}(t_i) =& \frac{1}{t_i}\Big((p-c)q_i - A_i - \frac{1}{2}hd_it_i^2 - \frac{1}{2}cI_pd_i(t_i-M_i)^2 + \frac{1}{2}pI_ed_iM_i^2\Big) \\ =& (p-c)d_i - \Big(\frac{A_i}{t_i} + \frac{Hd_it_i}{2}\Big), \end{aligned} \tag{2}$$

where $H = h + cI_p(1-\alpha)^2 - pI_e\alpha^2$.

If $H \leqslant 0$, $f_{i2}(t_i)$ is strictly increasing on $(0,\infty)$. Then, retailers will selfishly order as much as possible to maximize their profits. As a result, their credit periods are extended indefinitely and the supplier will not receive any payment from retailers. It is quite unrealistic. Alternatively, if $H \geqslant h + cI_p$, then $f_{i2}(t_i) \leqslant f_{i1}(t_i)$ for any $t_i > 0$; that is, trade credit cannot interest retailers in the slightest. Hence, it does make sense to have the value of α, with which the constant H satisfies $0 < H < h + cI_p$; see Figure 2.

From Eqs. (1) and (2), for given Q_0 and t_j, Retailer i's mean profit function is given by

$$f_i(t_i) = \begin{cases} f_{i1}(t_i), & \text{if } 0 < t_i < (Q_0 - t_jd_j)/d_i, \\ f_{i2}(t_i), & \text{if } t_i \geqslant (Q_0 - t_jd_j)/d_i, \end{cases} \tag{3}$$

where $j \neq i$. Note that $t_i = (Q_0 - t_jd_j)/d_i$ acts as a "boundary" line between the two sub-functions $f_{i1}(t_i)$ and $f_{i2}(t_i)$.

Property 1 shows the structural properties of $f_{i1}(t_i)$ and $f_{i2}(t_i)$. Note that t_{i1}^*, t_{i2}^*, t_{ai}, and t'_{ai} are fixed constants defined in Table 1.

Property 1. *(1)* $f_{i1}(t_i) < f_{i2}(t_i)$*, for any* $t_i > 0$.

(2) $f_{i1}(t_i)$ *and* $f_{i2}(t_i)$ *are strictly concave in* $(0,\infty)$*, and their respective maximizers are* t_{i1}^* *and* t_{i2}^*.

(3) $f_{i1}(t_{i1}^*) = f_{i2}(t'_{ai}) = f_{i2}(t_{ai}) < f_{i2}(t_{i2}^*)$ *and* $t'_{ai} < t_{i1}^* < t_{i2}^* < t_{ai}$.

Proof. From $H < h + cI_p$, the result is obtained by a straightforward computation. □

Next, we consider the best response of Retailer i, $(i = 1, 2)$, to the order decision t_j of Retailer j, $(j \neq i)$. For a given Q_0, let $\widetilde{t}_i(t_j)$, or $\widetilde{t}_i$ for short, be Retailer i's best response to t_j. In fact, for given Q_0 and t_j, $\widetilde{t}_i$ maximizes the mean profit function $f_i(t_i)$. The discussion is divided into four cases, based on the relationship between Q_0 and t_j; see Figure 3.

Case 1 $t_jd_j + t_{i2}^*d_i \geqslant Q_0$. Let $\widetilde{t}_i = t_{i2}^*$; then $\widetilde{t}_i$ maximizes $f_i(t_i)$, and Retailer i can enjoy a permissible delay in payment; see Figure 3. (a).

Case 2 $t_jd_j + t_{i2}^*d_i < Q_0 < t_jd_j + t_{ai}d_i$. Let $\widetilde{t}_i = (Q_0 - t_jd_j)/d_i$; Retailer i can make a delayed payment for its order; see Figure 3. (b).

Case 3 $t_j d_j + t_{ai} d_i = Q_0$. From $f_{i2}(t_{ai}) = f_{i1}(t^*_{i1})$, each payment method brings the same profit to Retailer i. According to the assumptions, Retailer i will identify $\widetilde{t}_i = t_{ai}$ with its best response and make a delayed payment accordingly; see Figure 3. (c).

Case 4 $t_j d_j + t_{ai} d_i < Q_0$. In this case, Retailer i will identify $\widetilde{t}_i = t^*_{i1}$ with its best response and pay the supplier immediately; see Figure 3. (d).

In summary, for a given Q_0, the best response of Retailer i to t_j is given by

$$\widetilde{t}_i(t_j) = \begin{cases} t^*_{i2}, & \text{if } t_j d_j + t^*_{i2} d_i \geqslant Q_0, \\ (Q_0 - t_j d_j)/d_i, & \text{if } t_j d_j + t^*_{i2} d_i < Q_0 \leqslant t_j d_j + t_{ai} d_i, \\ t^*_{i1}, & \text{if } t_j d_j + t_{ai} d_i < Q_0, \end{cases} \tag{4}$$

from which we see that Retailer i can enjoy a permissible delay in payment only when $t_j \geqslant (Q_0 - t_{ai} d_i)/d_j$; see Figure 4.

We assume that retailers will place orders separately when they cannot reach a consensus on respective order quantities. Then, the optimal decision t^*_i of Retailer i can be determined by replacing Retailer j's order decision t_j with zero in Eq. (4). That is,

$$t^*_i = \begin{cases} t^*_{i2}, & \text{if } t^*_{i2} d_i \geqslant Q_0, \\ Q_0/d_i, & \text{if } t^*_{i2} d_i < Q_0 \leqslant t_{ai} d_i, \\ t^*_{i1}, & \text{if } t_{ai} d_i < Q_0. \end{cases} \tag{5}$$

3.2. Retailers' optimal order decisions

For a given Q_0, each retailer can determine its best response to the competing retailer's order decision. However, no retailer would like to expose its decision to the other retailer. Before revealing their decisions, each retailer always tries to speculate about the other's order decision. Thus, the game between them constitutes a static game.

For the static game, if there is a unique Nash equilibrium, retailers will unanimously accept it because neither one can benefit by altering its own decision when the other leaves its own decision unchanged; if there is no equilibrium point, retailers will separately place orders with the supplier. However, when the static game has multiple equilibrium points, retailers may be confused about them; further discussion is required.

To determine the Nash equilibrium in the static game, we need to find all intersections of the images of

$$\begin{aligned} t_1 &= \widetilde{t}_1(t_2), \\ t_2 &= \widetilde{t}_2(t_1). \end{aligned} \tag{6}$$

From Figure 4, the image of $\widetilde{t}_i$, $(i = 1, 2)$, consists of two parts (denoted by $\widetilde{t}_{i1}$ and $\widetilde{t}_{i2}$), each of which corresponds to a payment method. Without loss of generality, we may assume that $\widetilde{t}_{i1}$ and $\widetilde{t}_{i2}$ correspond to the immediate payment and the delayed payment, respectively. Lemma 1 shows the intersection of $\widetilde{t}_1$ and $\widetilde{t}_2$ to a certain extent.

Lemma 1. *(1) $\widetilde{t}_{11}$ and $\widetilde{t}_{22}$ ($\widetilde{t}_{21}$ and $\widetilde{t}_{12}$) have no intersection.*

(2) $\widetilde{t}_{11}$ and $\widetilde{t}_{21}$ have a unique intersection, i.e., (t^*_{11}, t^*_{21}), if and only if $t^*_{11} d_1 + t_{a2} d_2 < Q_0$ and $t^*_{21} d_2 + t_{a1} d_1 < Q_0$.

Proof. The result will be proven by using reduction to absurdity. Suppose that $\widetilde{t}_{11}$ and $\widetilde{t}_{22}$ have an intersection $(t_{11}^*, \widetilde{t}_2(t_{11}^*))$. From Eq. (4),

$$\widetilde{t}_2(t_{11}^*) = \begin{cases} t_{22}^*, & \text{if } t_{11}^* d_1 + t_{22}^* d_2 \geqslant Q_0, \\ (Q_0 - t_{11}^* d_1)/d_2, & \text{if } t_{11}^* d_1 + t_{22}^* d_2 < Q_0 \leqslant t_{11}^* d_1 + t_{a2} d_2. \end{cases} \tag{7}$$

Note that the intersection $(t_{11}^*, \widetilde{t}_2(t_{11}^*))$ satisfies $\widetilde{t}_2(t_{11}^*) \leqslant (Q_0 - t_{a1}d_1)/d_2$; see Figure 5. If $\widetilde{t}_2(t_{11}^*) = (Q_0 - t_{11}^* d_1)/d_2$, from $(Q_0 - t_{11}^* d_1)/d_2 \leqslant (Q_0 - t_{a1} d_1)/d_2$, $t_{11}^* d_1 \geqslant t_{a1} d_1$. This contradicts with $t_{11}^* < t_{a1}$. Alternatively, if $\widetilde{t}_2(t_{11}^*) = t_{22}^*$, then $t_{22}^* \leqslant (Q_0 - t_{a1} d_1)/d_2$ and $t_{11}^* \geqslant (Q_0 - t_{22}^* d_2)/d_1$. This implies $t_{a1} d_1 + t_{22}^* d_2 \leqslant t_{11}^* d_1 + t_{22}^* d_2$, which contradicts with $t_{a1} > t_{11}^*$. Hence, $\widetilde{t}_{11}$ and $\widetilde{t}_{22}$ have no intersection. Similarly, there is no intersection between $\widetilde{t}_{21}$ and $\widetilde{t}_{12}$.

In terms of the intersections of $\widetilde{t}_{11}$ and $\widetilde{t}_{21}$, the result follows directly from Figure 6. □

In the remainder part of this subsection, we need to further discuss the intersection of $\widetilde{t}_{12}$ and $\widetilde{t}_{22}$. However, the situation becomes more complex.

3.2.1. Optimal decisions for different market sizes

We will derive the optimal decisions for retailers when their own markets have different sizes, i.e., $d_1 \neq d_2$. The discussion will be divided into two cases based on the relationship between d_1 and d_2.

Case 1 $d_1 > d_2$. We will further divide the discussion into two subcases according to the relationship between Q_0 and $t_{12}^* d_1 + t_{a2} d_2$; see Figure 7.

Subcase 1.1 $t_{12}^* < (Q_0 - t_{a2} d_2)/d_1$. In this subcase, $\widetilde{t}_{12}$ and $\widetilde{t}_{22}$ have no intersection; see Figure 7. (a). In addition, from $t_{11}^* < t_{12}^* < (Q_0 - t_{a2} d_2)/d_1$ and Lemma 1, $\widetilde{t}_{11}$ and $\widetilde{t}_{21}$ have an intersection (t_{11}^*, t_{21}^*) if and only if $t_{21}^* d_2 + t_{a1} d_1 < Q_0$. Hence, the static game has no equilibrium (respectively, a unique equilibrium (t_{11}^*, t_{21}^*)) if and only if $t_{12}^* d_1 + t_{a2} d_2 < Q_0 \leqslant t_{21}^* d_2 + t_{a1} d_1$ (respectively, $Q_0 > t_{12}^* d_1 + t_{a2} d_2$ and $Q_0 > t_{21}^* d_2 + t_{a1} d_1$).

Subcase 1.2 $t_{12}^* \geqslant (Q_0 - t_{a2} d_2)/d_1$. The images of $\widetilde{t}_{12}$ and $\widetilde{t}_{22}$ have a unique intersection $(t_{12}^*, \widetilde{t}_2(t_{12}^*))$; see Figure 7. (b)-(c). Specifically, using Eq. (4),

$$\widetilde{t}_2(t_{12}^*) = \begin{cases} (Q_0 - t_{12}^* d_1)/d_2, & \text{if } t_{12}^* d_1 + t_{22}^* d_2 < Q_0 \leqslant t_{12}^* d_1 + t_{a2} d_2, \\ t_{22}^*, & \text{if } t_{12}^* d_1 + t_{22}^* d_2 \geqslant Q_0. \end{cases} \tag{8}$$

Since $\widetilde{t}_2(t_{12}^*)$ is the maximizer of $f_2(t_2)$, then $f_2(\widetilde{t}_2(t_{12}^*)) > f_2(t_{21}^*)$. Furthermore, from $t_{a2} d_2 + t_{12}^* d_1 \geqslant Q_0$ and Eq. (4), $\widetilde{t}_1 = t_{12}^*$ and $f_1(t_{12}^*) > f_1(t_{11}^*)$. Then, even if $\widetilde{t}_{11}$ and $\widetilde{t}_{21}$ have an intersection (t_{11}^*, t_{21}^*), the two retailers' profits at (t_{11}^*, t_{21}^*) is lower than those at $(t_{12}^*, \widetilde{t}_2(t_{12}^*))$. Hence, (t_{11}^*, t_{21}^*) can be viewed as a noncredible threat in the static game [36]. We can conclude that the game has a unique subgame perfect Nash equilibrium $(t_{12}^*, (Q_0 - t_{12}^* d_1)/d_2))$ (respectively, (t_{12}^*, t_{22}^*)) if and only if $t_{12}^* d_1 + t_{22}^* d_2 < Q_0 \leqslant t_{12}^* d_1 + t_{a2} d_2$ (respectively, $Q_0 \leqslant t_{12}^* d_1 + t_{a2} d_2$).

As above, retailers' optimal decisions depend on the relationship among Q_0, $t_{21}^* d_2 + t_{a1} d_1$, $t_{12}^* d_1 + t_{a2} d_2$, and $t_{12}^* d_1 + t_{22}^* d_2$. Using $t_{22}^* < t_{a2}$, $t_{12}^* d_1 + t_{22}^* d_2 < t_{12}^* d_1 + t_{a2} d_2$. To determine the relationship between $t_{21}^* d_2 + t_{a1} d_1$ and $t_{12}^* d_1 + t_{a2} d_2$, we need only to compare $(t_{a1} - t_{12}^*) d_1$ and $(t_{a2} - t_{21}^*) d_2$. From Table 1,

$$\begin{aligned} (t_{a1} - t_{12}^*) d_1 &= \sqrt{\frac{2A_1 d_1}{H}} \left(\frac{\sqrt{h + cI_P} + \sqrt{\alpha(2-\alpha)cI_p + \alpha^2 pI_e}}{\sqrt{H}} - 1 \right), \\ (t_{a2} - t_{21}^*) d_2 &= \sqrt{\frac{2A_2 d_2}{H}} \left(\frac{\sqrt{h + cI_P} + \sqrt{\alpha(2-\alpha)cI_p + \alpha^2 pI_e}}{\sqrt{H}} - 1 \right). \end{aligned} \tag{9}$$

Using Eq. (9), $t_{21}^*d_2+t_{a1}d_1 > t_{12}^*d_1+t_{a2}d_2$ if and only if $(t_{a1}-t_{12}^*)d_1 > (t_{a2}-t_{21}^*)d_2$, which occurs of and only if $A_1d_1 > A_2d_2$. If $A_1d_1 > A_2d_2$, we have that $t_{12}^*d_1+t_{22}^*d_2 < t_{12}^*d_1+t_{a2}d_2 < t_{21}^*d_2+t_{a1}d_1$. Unfortunately, when $t_{12}^*d_1+t_{a2}d_2 < Q_0 \leqslant t_{21}^*d_2+t_{a1}d_1$, the static game has no Nash equilibrium and retailers will separately place orders with the supplier. Specifically, from Eq. (5) and $Q_0 > t_{a2}d_2$, Retailer 2 will identify $t_2^* = t_{21}^*$ with its optimal decision and pay the supplier immediately. In terms of Retailer 1's order decision, when $t_{12}^*d_1 + t_{a2}d_2 < t_{a1}d_1$, Retailer 1 will identify $t_1^* = Q_0/d_1$ (respectively, $t_1^* = t_{11}^*$) with the optimal order decision if $t_{12}^*d_1 + t_{a2}d_2 < Q_0 \leqslant t_{a1}d_1$ (respectively, $t_{a1}d_1 < Q_0 \leqslant t_{21}^*d_2 + t_{a1}d_1$); see Table 2. Alternatively, when $t_{12}^*d_1 + t_{a2}d_2 \geqslant t_{a1}d_1$, using $Q_0 > t_{12}^*d_1 + t_{a2}d_2 \geqslant t_{a1}d_1$ and Eq. (5), Retailer 1's optimal decision is $t_1^* = t_{11}^*$; see Table 3.

Alternatively, if $A_1d_1 \leqslant A_2d_2$, we have $t_{21}^*d_2 + t_{a1}d_1 \leqslant t_{12}^*d_1 + t_{a2}d_2$. Then, $Q_0 > t_{12}^*d_1 + t_{a2}d_2$ if and only if $Q_0 > t_{12}^*d_1 + t_{a2}d_2$ and $Q_0 > t_{21}^*d_2 + t_{a1}d_1$, which occurs if and only if the equilibrium point (t_{11}^*, t_{21}^*) exists. From $t_{12}^*d_1 + t_{22}^*d_2 < t_{12}^*d_1 + t_{a2}d_2$ and the above discussion, we have Table 3.

Case 2 $d_1 < d_2$. Similarly to the above, we can derive the closed-form optimal order decisions for the two retailers; see Tables 4 and 5. Note that $A_1d_1 < A_2d_2$ in our case.

Proposition 1 uncovers the corresponding core managerial insights.

Proposition 1. *When retailers' own markets have different sizes, we have the following:*

(a) A sufficiently high threshold cannot interest retailers, while a lower threshold cannot effectively stimulate the order of the retailer facing the smaller demand.

(b) When $d_i > d_j$, $(i \neq j)$, to maximize the total order quantity of retailers, the supplier should identify $Q_0^* = t_{i2}^*d_i + t_{aj}d_j$ with its optimal threshold; the corresponding order quantities of Retailer i and Retailer j are $t_{i2}^*d_i$ and $t_{aj}d_j$, respectively.

(c) Occasionally, the retailer facing the larger demand would rather place a larger order by itself to meet a relatively high threshold than cooperate with the supplier facing the smaller demand.

Proof. When $d_1 > d_2$, we see from Tables 2 and 3 that the two retailers can place a joint order only when $Q_0 \leqslant t_{12}^*d_1 + t_{a2}d_2$. Specifically, if $Q_0 \leqslant t_{12}^*d_1 + t_{22}^*d_2$, the total order quantity of retailers are $t_{12}^*d_1 + t_{22}^*d_2$. Alternatively, if $t_{12}^*d_1 + t_{22}^*d_2 < Q_0 \leqslant t_{12}^*d_1 + t_{a2}d_2$, Retailer 1's order is unchanged, while Retailer 2's order quantity increases to $Q_0 - t_{12}^*d_1$. Then, their total order quantity increases to Q_0. When $d_1 < d_2$, we can obtain the similar result. □

3.2.2. Optimal decisions for approximate market sizes

The closed-form optimal decisions will be derived for retailers when their market sizes are approximately equal. For convenience, we may assume that $d_1 = d_2 = d$.

Lemma 2. *$\widetilde{t}_{11}$ and $\widetilde{t}_{21}$ have a unique intersection, i.e., (t_{11}^*, t_{21}^*), if and only if $(t_{11}^* + t_{a2})d < Q_0$.*

Proof. The result will be proven by verifying the inequality $t_{21}^* + t_{a1} \leqslant t_{11}^* + t_{a2}$. Using $A_1 \leqslant A_2$, $t_{11}^* < t_{a1}$, $t_{21}^* = \sqrt{A_2/A_1}t_{11}^*$, and $t_{a2} = \sqrt{A_2/A_1}t_{a1}$,

$$(t_{21}^* + t_{a1}) - (t_{11}^* + t_{a2}) = (1 - \sqrt{A_2/A_1})(t_{a1} - t_{11}^*) \leqslant 0. \tag{10}$$

From Eq. (10), $t_{21}^* + t_{a1} \leqslant t_{11}^* + t_{a2}$. Then, $(t_{11}^* + t_{a2})d < Q_0$ if and only if $(t_{11}^* + t_{a2})d < Q_0$ and $(t_{21}^* + t_{a1})d < Q_0$. The result follows by Lemma 1. □

In terms of the intersections of $\widetilde{t}_{12}$ and $\widetilde{t}_{22}$, since $d_1 = d_2$, $\widetilde{t}_{12}$ and $\widetilde{t}_{22}$ have intersections if and only if $Q_0/d - t_{a1} \leqslant t_{a2}$, i.e., $(t_{a1} + t_{a2})d \geqslant Q_0$; see Figure 4. The discussion will be divided into two cases based on the relationship between Q_0 and $[t_{12}^* + t_{22}^*, t_{a1} + t_{a2}]$; see Figure 8.

Case 1 $Q_0 \leqslant (t_{12}^* + t_{22}^*)d$. In this case, $t_{12}^* \geqslant Q_0/d - t_{22}^*$. Then, the static game has a unique equilibrium point (t_{12}^*, t_{22}^*); see Figure 8. (a).

Case 2 $(t_{12}^*+t_{22}^*)d < Q_0 \leqslant (t_{a1}+t_{a2})d$. From $t_{12}^* \geqslant Q_0/d-t_{22}^*$, there are multiple Nash equilibria $(Q_0/d - t, t)$, where $t \in [a, b]$ and

$$\begin{aligned} a =& \max\left\{t_{22}^*, Q_0/d - t_{a1}\right\}, \\ b =& \min\left\{t_{a2}, Q_0/d - t_{12}^*\right\}; \end{aligned} \tag{11}$$

see Figure 8. (b)-(d). In particular, $Q_0 = (t_{a1} + t_{a2})d$ if and only if $a = b = t_{a2}$; the corresponding equilibrium point is (t_{a1}, t_{a2}).

Based on Lemma 2 and the above discussion, the Nash equilibrium of the static game can be determined according to the relationship among Q_0 $t_{12}^*+t_{22}^*$, $t_{11}^*+t_{a2}$, and $t_{a1}+t_{a2}$. Using $t_{11}^* < t_{a1}$, $t_{11}^* + t_{a2} < t_{a1} + t_{a2}$. From $H < h + cI_p$ and $t_{12}^* = \sqrt{2A_1/d}$,

$$\frac{t_{a1} + t'_{a1}}{2} = \frac{\sqrt{2A_1(h + cI_p)}}{\sqrt{d}H} > \frac{\sqrt{2A_1H}}{\sqrt{d}H} = t_{12}^*. \tag{12}$$

Using Eq. (12), $t_{12}^* - t'_{a1} < t_{a1} - t_{12}^*$. From $t'_{a1} < t_{11}^*$ and $A_1 \leqslant A_2$,

$$t_{12}^* - t_{11}^* < t_{12}^* - t'_{a1} < t_{a1} - t_{12}^* \leqslant \sqrt{A_2/A_1}(t_{a1} - t_{12}^*) = t_{a2} - t_{22}^*. \tag{13}$$

From Eq. (13), $t_{12}^* + t_{22}^* < t_{11}^* + t_{a2} < t_{a1} + t_{a2}$. Table 6 summarizes all Nash equilibria in the static game.

From Table 6, when $(t_{12}^*+t_{22}^*)d < Q_0 \leqslant (t_{a1}+t_{a2})d$, the static game has multiple Nash equilibria $(Q_0/d - t, t)$, with $t \in [a, b]$. In particular, when $(t_{a2} + t_{11}^*)d < Q_0 \leqslant (t_{a1} + t_{a2})d$, there exists an additional equilibrium point (t_{11}^*, t_{21}^*). Lemma 3 uncovers that (t_{11}^*, t_{21}^*) is not considered by rational retailers.

Lemma 3. *When $(t_{12}^* + t_{22}^*)d < Q_0 \leqslant (t_{a1} + t_{a2})d$, we have the following:*
(1) $f_{12}(Q_0/d - t)$ and $f_{22}(t)$ are strictly increasing and decreasing on $[a, b]$, respectively.
(2) The subgame perfect Nash equilibria of the static game are $(Q_0/d - t, t)$, with $t \in [a, b]$.

Proof. Let $t \in [a, b]$, from Eq. (11),

$$t_{12}^* \leqslant \frac{Q_0}{d} - b \leqslant \frac{Q_0}{d} - t \leqslant \frac{Q_0}{d} - a \leqslant t_{a1}, \tag{14}$$

$$t_{22}^* \leqslant a \leqslant t \leqslant b \leqslant t_{a2}. \tag{15}$$

According to Property 1, $f_{i2}(t)$, $(i = 1, 2)$, is strictly decreasing on $[t_{i2}^*, t_{ai}]$. Using Eqs. (14) and (15), and the monotonicity of $f_{i2}(t)$ on $[t_{i2}^*, t_{ai}]$, $f_{12}(Q_0/d - t)$ and $f_{22}(t)$ are increasing and decreasing on $[a, b]$, respectively. Then, for any $t \in [a, b]$, we have

$$f_{11}(t_{11}^*) = f_{12}(t_{a1}) \leqslant f_{12}(Q_0/d - a) \leqslant f_{12}(Q_0/d - t) \leqslant f_{12}(Q_0/d - b) \leqslant f_{12}(t_{12}^*), \tag{16}$$

$$f_{21}(t_{21}^*) = f_{22}(t_{a2}) \leqslant f_{22}(a) \leqslant f_{22}(t) \leqslant f_{22}(b) \leqslant f_{22}(t_{22}^*). \tag{17}$$

From Eqs. (16) and (17), $f_{11}(t_{11}^*) \leqslant f_{12}(Q_0/d - t)$ and $f_{21}(t_{21}^*) \leqslant f_{22}(t)$ for any $t \in [a, b]$. This implies that retailers' mean profits at (t_{11}^*, t_{21}^*) are never higher than those at $(Q_0/d - t, t)$, with $t \in [a, b]$.

Next, we will prove that the mean profits of retailers at (t_{11}^*, t_{21}^*) are equal to those at $(Q_0/d - t_0, t_0)$ for some $t_0 \in [a, b]$ if and only if $Q_0 = (t_{a1} + t_{a2})d$. If $f_{11}(t_{11}^*) = f_{12}(Q_0/d - t_0)$ and $f_{21}(t_{21}^*) = f_{22}(t_0)$ for some $t_0 \in [a, b]$, then $f_{12}(t_{a1}) = f_{12}(Q_0/d - t_0)$ and $f_{22}(t_{a2}) = f_{22}(t_0)$. Since $f_{12}(Q_0/d - t)$ and $f_{22}(t)$ are respectively increasing and decreasing on $[a, b]$, $t_{a1} = Q_0/d - t_0$ and $t_{a2} = t_0$. This implies $Q_0 = (t_{a1} + t_{a2})d$. Conversely, if $Q_0 = (t_{a1} + t_{a2})d$, let $t_0 = t_{a2}$; then $f_{12}(t_{a1}) = f_{12}(Q_0/d - t_0)$ and $f_{22}(t_{a2}) = f_{22}(t_0)$. Note that $Q_0 = (t_{a1} + t_{a2})d$ if and only if $a = b$.

Based on the above, when $Q_0 < (t_{a1} + t_{a2})d$, $a < b$ and at least one retailer's profit at (t_{11}^*, t_{21}^*) is lower than that at $(Q_0/d - t_0, t_0)$ for some $t_0 \in [a, b]$. Thus, (t_{11}^*, t_{21}^*) can be seen as a noncredible threat in the static game and will be excluded by rational retailers. Alternatively, when $Q_0 = (t_{a1} + t_{a2})d$, using Table 6, there are exactly two equilibrium points (t_{11}^*, t_{21}^*) and (t_{a1}, t_{a2}) corresponding to the immediate payment and the delayed payment, respectively. Since $f_{i1}(t_{i1}^*) = f_{i2}(t_{ai})$, each retailer can earn the same profit at each equilibrium point. According to the assumptions, Retailer i will identify $t_i^* = t_{ai}$ with its optimal decisions. Hence, even if (t_{11}^*, t_{21}^*) is an equilibrium of the game, it will be excluded by rational retailers.

In terms of equilibrium points $(Q_0/d - t, t)$, with $t \in [a, b]$, following the monotonicity of $f_{12}(Q_0/d - t)$ and $f_{22}(t)$ on $[a, b]$, an increase in one retailer's profit will surely lead to a decrease in the other's profit. Thus, $(Q_0/d - t, t)$, with $t \in [a, b]$, are all the subgame perfect Nash equilibria of the static game. $\square$

From Lemma 3, when $(t_{12}^* + t_{22}^*)d < Q_0 \leqslant (t_{a1} + t_{a2})d$, retailers are still confused about multiple equilibria $(Q_0/d - t, t)$, with $t \in [a, b]$. Since private communication between the two retailers is not allowed, they have to confront a tacit bargaining with divergent profits, and their overriding interest is to coordinate their decisions. If a particular equilibrium point commands attentions as the "focal" point, such an equilibrium is the only extant offer, and no counterproposal can be made. The conflict is reconciled as a by-product of the dominant need for coordination [37].

Let $g(t) = f_{12}(Q_0/d - t) - f_{22}(t)$; then, $|g(t)|$ denotes the gap between the two retailers' mean profits at $(Q_0/d - t, t)$. Note that $g(t)$ is strictly increasing on $[a, b]$. Let $t_g(Q_0)$ lead to a minimum value of $|g(t)|$ on $[a, b]$; then, the particular equilibrium point $(Q_0/d - t_g(Q_0), t_g(Q_0))$ attracts significant attention for two reasons:

(1) $t_g(Q_0)$ is unique and remarkable for the two retailers.

(2) $t_g(Q_0)$ leads to the minimum gap between the mean profits of the two retailers, and contributes to a long-term cooperative relationship between them.

Hence, the equilibrium point $(Q_0/d - t_g(Q_0), t_g(Q_0))$ can be viewed as a "focal" point, and the two retailers tacitly accept it simultaneously. Table 7 summarizes retailers' optimal decisions for each given threshold Q_0.

The closed-form expression for $t_g(Q_0)$ can be determined as follows. If $Q_0 = (t_{a1} + t_{a2})d$, we have $a = b = t_{a2}$ and $t_g(Q_0) = t_{a2}$. Alternatively, if $Q_0 < (t_{a1} + t_{a2})d$, we have $a < b$. Since $g(t)$ is strictly increasing on $[a, b]$, $t_g(Q_0) = a$ if $g(a) \geqslant 0$, and $t_g(Q_0) = b$ if $g(b) \leqslant 0$. When $g(a) < 0$ and $g(b) > 0$, using the well-known intermediate value theorem, there exists a unique $t_g(Q_0) \in (a, b)$ satisfying $g(t_g(Q_0)) = 0$. In fact, $t_g(Q_0)$ is the unique solution of the following equation

$$t^3 - \frac{3Q_0}{2d}t^2 + \Big(\frac{A_1 + A_2}{Hd} + \frac{Q_0^2}{2d^2}\Big)t - \frac{A_2 Q_0}{Hd^2} = 0, \tag{18}$$

located in (a, b), which can be found algebraically using the Cardano formula [38].

We have Proposition 2, which summarizes the major managerial insights.

Proposition 2. *When retailers' market sizes have approximate sizes, we have the following:*

(a) A sufficiently high threshold cannot interest retailers, while a lower threshold may not effectively encourage retailers to place larger orders.

(b) To maximize the total order quantity of retailers, the supplier should identify $Q_0^* = (t_{a1}+t_{a2})d$ *with the optimal threshold; the corresponding order quantities of Retailer* 1 *and Retailer* 2 *are* $t_{a1}d$ *and* $t_{a2}d$*, respectively.*

Proof. From Table 7, retailers can place a total order only when $Q_0 \leqslant (t_{a1} + t_{a2})d$. When $Q_0 \leqslant (t_{12}^* + t_{22}^*)d$, their total order quantity is $(t_{12}^* + t_{22}^*)d$. Alternatively, when $(t_{12}^* + t_{22}^*)d < Q_0 \leqslant (t_{a1} + t_{a2})d$, we have $q_1^* = Q_0 - t_g(Q_0)d$, $q_2^* = t_g(Q_0)d$ and $q_1^* + q_2^* = Q_0$. Using $t_{22}^* \leqslant a \leqslant t_g(Q_0) \leqslant b \leqslant Q_0/d - t_{12}^*$, $q_1^* \geqslant t_{12}^*$ and $q_2^* \geqslant t_{22}^*$. □

3.3. Supplier's optimal decision

As the Stackelberg leader, the supplier knows the optimal decisions of the two retailers for each given Q_0. Based on Tables 2, 3, 4, 5 and 7, the supplier can identify an optimal threshold Q_0^* to maximize its own profit. For convenience, we ignore the inventory holding cost of the supplier and the opportunity cost charged for offering trade credits.

For the supplier, if retailers place a joint order, the processing cost paying for each retailer's order can be seen as $A_s/2$, in which case, the mean profit is

$$\begin{aligned} F(Q_0) =& \frac{d_1}{q_1^*}\Big((c-c_0)q_1^* - \frac{1}{2}A_s\Big) + \frac{d_2}{q_2^*}\Big((c-c_0)q_2^* - \frac{1}{2}A_s\Big) \\ =& (c-c_0)(d_1+d_2) - \frac{A_s}{2}\Big(\frac{1}{q_1^*} + \frac{1}{q_2^*}\Big); \end{aligned} \tag{19}$$

if retailers place orders separately, the supplier has to process their orders separately, and the mean profit is

$$F(Q_0) = (c-c_0)(d_1+d_2) - A_s\Big(\frac{1}{q_1^*} + \frac{1}{q_2^*}\Big). \tag{20}$$

From Tables 2, 3, 4, 5, and 7, q_1^* and q_2^* are fixed constants when either $Q_0 \leqslant t_{12}^*d_1 + t_{22}^*d_2$, or $Q_0 > t_{a1}d_1 + t_{a2}d_2$. Thus, we need only to solve the following optimization problem

$$\max_{t_{11}^*d_1 + t_{12}^*d_2 \leqslant Q_0 \leqslant t_{a1}d_1 + t_{a2}d_2 + 1} F(Q_0). \tag{21}$$

3.4. Algorithm

Based on the previous discussion, the following algorithm is developed.

Algorithm 1 Optimal decisions of the supplier and the two retailers

1: Input parameters d_1, d_2, p, c, c_0, h, α, A_1, A_2, A_s, I_e, and I_p.
2: Compute the constants H, t_{11}^*, t_{12}^*, t_{21}^*, t_{22}^*, t_{a1}, and t_{a2}.
3: Compute q_1^* and q_2^* for each $Q_0 \in [t_{12}^*d_1 + t_{22}^*d_2, t_{a1}d_1 + t_{a2}d_2 + 1]$.
4: Substitute q_1^* and q_2^* into Eqs. (19) and (20) to obtain $F(Q_0)$.
5: Solve optimization problem (21) to obtain Q_0^*.
6: Based on Q_0^*, determine the optimal order quantities and payment methods of the two retailers.

4. Numerical experiments

In this section, a numerical experiment is presented to illustrate the validity of the proposed model.

Example 1. *Consider the following parameters:* $d_1 = 140$, $d_2 = 80$, $p = 0.9$, $c = 0.7$, $c_0 = 0.4$, $h = 0.1$, $A_1 = 3$, $A_2 = 4$, $A_s = 12$, $I_e = 0.02$, $I_p = 0.03$, *and* $\alpha = 0.3$.

For this model, it holds that $H = 0.10867$, $t^*_{11} = 0.59514$, $t^*_{12} = 0.628$, $t^*_{21} = 0.90909$, $t^*_{22} = 0.95928$, $t_{a1} = 0.8742$, and $t_{a2} = 1.0094$. Since $d_1 > d_2$, $A_1d_1 > A_2d_2$, and $t^*_{12}d_1 + t_{a2}d_2 > t_{a1}d_1$, using Table 3, we can derive the optimal decisions of the two retailers. The numerical results are presented in Table 8, from which we see that the optimal threshold of the supplier is $Q^*_0 = 168.674$.

From Figures 9, 10 and 11, a relatively lower threshold (i.e., $Q_0 < 164.662$) is favoured by retailers, but is not earthly use for the supplier. Moreover, a relatively high threshold (i.e., $164.662 \leqslant Q_0 \leqslant 168.674$) can increase the supplier's profit, while reducing the profit of the retailer who faces the smaller demand. However, a sufficiently high threshold (i.e., $Q_0 > 168.674$) substantially reduce all supply chain agents's profits, which is useless for the supply chain.

5. Conclusions

Trade credit linked to order quantity is crucial in supply chain finance, but it is seldom designed within the framework of the cooperation mechanism between retailers. In our model, two retailers can enjoy permissible delay in payment as long as their total order quantity meets the given threshold. Although cooperation can bring more profits to retailers, neither of them would like to take on more responsibility. We provided a mutually acceptable order-allocation scheme for retailers and identify an optimal threshold for the supplier to maximize retailers' total order quantity. Based on this, we find that two retailers unanimously give priority to placing a total order with the supplier. In addition, we uncovered that a sufficiently high threshold cannot interest retailers, while a lower threshold may not effectively stimulate retailers facing identical demands or the retailer facing the smaller demand.

No research is perfect, the paper has a few limitations. For example, the two retailers, as retail agents of the same product, are assumed to sell the product at the recommended retail price. In reality, any product is fully or partially substitutable in the market and the market demand each retailer faces is therefore price dependent [39, 40]. The Bertrand competition between retailers within the framework of the EOQ model is left for the future study [41].

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Tables

Table 1. Definitions of the notations for Property 1.

Notations	Definitions	Notations	Definitions
t^*_{i1}	$\frac{\sqrt{2A_i}}{\sqrt{d_i(h+cI_p)}}$	t_{ai}	$\frac{\sqrt{2A_i}(\sqrt{h+cI_p}+\sqrt{\alpha(2-\alpha)cI_p+\alpha^2pI_e})}{\sqrt{d_iH}}$
t^*_{i2}	$\frac{\sqrt{2A_i}}{\sqrt{d_iH}}$	t'_{ai}	$\frac{\sqrt{2A_i}(\sqrt{h+cI_p}-\sqrt{\alpha(2-\alpha)cI_p+\alpha^2pI_e})}{\sqrt{d_iH}}$

†: $i = 1, 2$.

Table 2. Optimal order decisions of retailers when $d_1 > d_2$, $A_1d_1 > A_2d_2$, and $t^*_{12}d_1 + t_{a2}d_2 < t_{a1}d_1$.

Range of Q_0	q^*_1	q^*_2	Retailer 1's PM	Retailer 2's PM	Joint ordering
$Q_0 > t_{a1}d_1$	$t^*_{11}d_1$	$t^*_{21}d_2$	Immediate payment	Immediate payment	N
$t^*_{12}d_1 + t_{a2}d_2 < Q_0 \leqslant t_{a1}d_1$	Q_0	$t^*_{21}d_2$	Delayed payment	Immediate payment	N
$t^*_{12}d_1 + t^*_{22}d_2 < Q_0 \leqslant t^*_{12}d_1 + t_{a2}d_2$	$t^*_{12}d_1$	$Q_0 - t^*_{12}d_1$	Delayed payment	Delayed payment	Y
$Q_0 \leqslant t^*_{12}d_1 + t^*_{22}d_2$	$t^*_{12}d_1$	$t^*_{22}d_2$	Delayed payment	Delayed payment	Y

†: $q^*_1 = t^*_1d_1$, $q^*_2 = t^*_2d_2$, PM=Payment method, Y=Yes, N=No.

Table 3. Optimal order decisions of retailers when either $d_1 > d_2$, $A_1d_1 > A_2d_2$, and $t^*_{12}d_1 + t_{a2}d_2 \geqslant t_{a1}d_1$, or $d_1 > d_2$ and $A_1d_1 \leqslant A_2d_2$.

Range of Q_0	q^*_1	q^*_2	Retailer 1's PM	Retailer 2's PM	Joint ordering
$Q_0 > t^*_{12}d_1 + t_{a2}d_2$	$t^*_{11}d_1$	$t^*_{21}d_2$	Immediate payment	Immediate payment	N
$t^*_{12}d_1 + t^*_{22}d_2 < Q_0 \leqslant t^*_{12}d_1 + t_{a2}d_2$	$t^*_{12}d_1$	$Q_0 - t^*_{12}d_1$	Delayed payment	Delayed payment	Y
$Q_0 \leqslant t^*_{12}d_1 + t^*_{22}d_2$	$t^*_{12}d_1$	$t^*_{22}d_2$	Delayed payment	Delayed payment	Y

Table 4. Optimal order decisions of retailers when $d_1 < d_2$ and $t^*_{22}d_2 + t_{a1}d_1 < t_{a2}d_2$.

Range of Q_0	q^*_1	q^*_2	Retailer 1's PM	Retailer 2's PM	Joint ordering
$Q_0 > t_{a2}d_2$	$t^*_{11}d_1$	$t^*_{21}d_2$	Immediate payment	Immediate payment	N
$t^*_{22}d_2 + t_{a1}d_1 < Q_0 \leqslant t_{a2}d_2$	$t^*_{11}d_1$	Q_0	Immediate payment	Delayed payment	N
$t^*_{22}d_2 + t^*_{12}d_1 < Q_0 \leqslant t^*_{22}d_2 + t_{a1}d_1$	$Q_0 - t^*_{22}d_2$	$t^*_{22}d_2$	Delayed payment	Delayed payment	Y
$Q_0 \leqslant t^*_{22}d_2 + t^*_{12}d_1$	$t^*_{12}d_1$	$t^*_{22}d_2$	Delayed payment	Delayed payment	Y

Table 5. Optimal order decisions of retailers when $d_1 < d_2$ and $t^*_{22}d_2 + t_{a1}d_1 \geqslant t_{a2}d_2$.

Range of Q_0	q^*_1	q^*_2	Retailer 1's PM	Retailer 2's PM	Joint ordering
$Q_0 > t^*_{22}d_2 + t_{a1}d_1$	$t^*_{11}d_1$	$t^*_{21}d_2$	Immediate payment	Immediate payment	N
$t^*_{22}d_2 + t^*_{12}d_1 < Q_0 \leqslant t^*_{22}d_2 + t_{a1}d_1$	$Q_0 - t^*_{22}d_2$	$t^*_{22}d_2$	Delayed payment	Delayed payment	Y
$Q_0 \leqslant t^*_{22}d_2 + t^*_{12}d_1$	$t^*_{12}d_1$	$t^*_{22}d_2$	Delayed payment	Delayed payment	Y

Table 6. Nash equilibria in the static game when $d_1 = d_2 = d$.

Range of Q_0	Nash equilibria	Retailer 1's PM	Retailer 2's PM	Joint ordering
$Q_0 > (t_{a1} + t_{a2})d$	(t^*_{11}, t^*_{21})	Immediate payment	Immediate payment	N
$Q_0 = (t_{a1} + t_{a2})d$	(t^*_{11}, t^*_{21})	Immediate payment	Immediate payment	N
	(t_{a1}, t_{a2})	Delayed payment	Delayed payment	Y
$(t_{a2} + t^*_{11})d < Q_0 < (t_{a1} + t_{a2})d$	(t^*_{11}, t^*_{21})	Immediate payment	Immediate payment	N
	$(Q_0/d - t, t)$, with $t \in [a, b]$	Delayed payment	Delayed payment	Y
$(t^*_{12} + t^*_{22})d < Q_0 \leqslant (t_{a2} + t^*_{11})d$	$(Q_0/d - t, t)$, with $t \in [a, b]$	Delayed payment	Delayed payment	Y
$0 \leqslant Q_0 \leqslant (t^*_{12} + t^*_{22})d$	(t^*_{21}, t^*_{22})	Delayed payment	Delayed payment	Y

Table 7. Optimal order decisions of the two retailers when $d_1 = d_2 = d$.

Range of Q_0	q_1^*	q_2^*	Retailer 1's PM	Retailer 2's PM	Joint ordering
$Q_0 > (t_{a1} + t_{a2})d$	$t_{11}^* d$	$t_{21}^* d$	Immediate payment	Immediate payment	N
$(t_{12}^* + t_{22}^*)d < Q_0 \leqslant (t_{a1} + t_{a2})d$	$Q_0 - t_g(Q_0)d$	$t_g(Q_0)d$	Delayed payment	Delayed payment	Y
$0 \leqslant Q_0 \leqslant (t_{12}^* + t_{22}^*)d$	$t_{12}^* d$	$t_{22}^* d$	Delayed payment	Delayed payment	Y

Table 8. Numerical result on Example 1.

Q_0	q_1^*	q_2^*	$f_1(q_1^*)$	$f_2(q_2^*)$	$F(Q_0)$	Overall profit	Payment method	Joint ordering
⩽164.661	87.919	76.742	18.446	7.6604	65.8536	91.96	Delayed payment	Y
164.662	87.919	76.743	18.446	7.6604	65.8536	91.96	Delayed payment	Y
165.1	87.919	77.181	18.446	7.6603	65.854	91.96	Delayed payment	Y
165.7	87.919	77.781	18.446	7.6597	65.8546	91.96	Delayed payment	Y
166.3	87.919	78.381	18.446	7.6585	65.8552	91.96	Delayed payment	Y
166.9	87.919	78.981	18.446	7.657	65.8558	91.959	Delayed payment	Y
167.5	87.919	79.581	18.446	7.6549	65.8564	91.957	Delayed payment	Y
168.1	87.919	80.181	18.446	7.6524	65.8569	91.955	Delayed payment	Y
168.674	**87.919**	**80.755**	**18.446**	**7.6496**	**65.8574**	**91.953**	**Delayed payment**	Y
⩾168.675	83.32	72.727	17.918	7.2	65.691	90.809	Immediate payment	N

Figure legends.

Figure 1. Total amount of interest earned and payable.
Figure 2. Sub-functions $f_{i1}(t_i)$ and $f_{i2}(t_i)$, with $i = 1, 2$.
Figure 3. The maximizer of the mean profit function $f_i(t_i)$, with $i = 1, 2$.
Figure 4. The best response functions of Retailer 1 and Retailer 2.
Figure 5. The intersection of $\widetilde{t}_{11}$ and $\widetilde{t}_{22}$.
Figure 6. The intersection of $\widetilde{t}_{11}$ and $\widetilde{t}_{21}$.
Figure 7. The intersection of $\widetilde{t}_{12}$ and $\widetilde{t}_{22}$ when $d_1 > d_2$.
Figure 8. The intersection of $\widetilde{t}_{12}$ and $\widetilde{t}_{22}$ when $d_1 = d_2$.
Figure 9. The effect of Q_0 on the supplier's mean profit.
Figure 10. The effect of Q_0 on Retailer 1's mean profit.
Figure 11. The effect of Q_0 on Retailer 2's mean profit.