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Analysis of Magnetohydrodynamics Stagnation point flow with Partial slip boundary conditions

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Abstract

In this article, the existence and uniqueness result for the solution of a singular third-order ordinary differential equation has been investigated on a semi-infinite domain $[0, \infty)$. Such differential equation arises in boundary layer flow near a stagnation point on a rough plate in the presence of a transverse magnetic field. A suitable similarity transformation is used to transform the governing partial differential equation into a nonlinear ordinary differential equation along with partial slip boundary conditions. The resulting equation with its boundary conditions contains two parameters: the magnetic parameter, M and the slip parameter, λ. Some properties of the velocity profiles such as monotonic behaviour and bounded are obtained before proceeding numerical results. Further, the asymptotic behaviour at the free boundary has also been discussed. The validation of the obtained solution has been done numerically by shifted Chebyshev collocation method. The velocity profiles are plotted to address the significance of the parameters. The results are also compared through the table with previous results and found remarkably good agreement.

KEYWORDS:
Stagnation point flow, Magnetic field, Partial slip, topological shooting argument, Shifting Chebyshev collocation method

1 | INTRODUCTION

The flow near the stagnation point appears in various applications in industry such as flow over submarines, aircraft etc. In aircraft, the stagnation point is important because its position on the airfoil has much to do with how the air flows around the wing and generates the forces. The stagnation flow model was first analyzed by Hiemenz [1] by considering a steady two-dimensional flow of viscous fluid impinging perpendicularly on a wall. He reduced the governing equations into a third-order ordinary differential equation with two-point boundary conditions. For this solution, the no-slip boundary condition was considered. The condition of no-slip not always exists in general. Instead, a certain degree of tangential slip may allow on the wall. However, the non-adherence of fluid to the wall is known as velocity slip. For example, rarefied gases and some non-Newtonian fluids exhibit slip boundary conditions. The slip boundary conditions have been considered by Wang [2], and he obtained an exact numerical solution of the Navier-Stokes equations. For its numerous practical applications, this classical stagnation problem has been extended in various ways [3,4,5,6].

We discuss the two-dimensional stagnation point flow in the presence of a magnetic field. In this case, a force is produced inside the fluid, which always opposes the flow. This force is called the Lorentz force. The Lorentz force is applied to stabilize the

boundary layer flow. In the presence of a magnetic field, the stagnation point flow first considered by Na [7] and solved it numerically. However, the reported missing values of the boundary value problem does not satisfy the asymptotic boundary condition at infinity. The numerical work has been extended by Ariel [8]. To learn more about the applications of this MHD stagnation point flow, the interested reader could read the works of Hashim et al. [9], Bilal et al. [10], Agbaje et al. [11] and the references therein.
The governing similarity equations, along with the boundary conditions, are generally complicated and highly nonlinear. In the absence of an analytical solution, the resulting boundary value problem is usually solved numerically or semi-analytical methods. Most of the numerical studies [4,12,13] on the boundary layer flows are solved by shooting technique or Keller-box finite difference method. Furthermore, there are other semi-analytical techniques such as Homotopy analysis method by Liao [14], Adomian's decomposition method by Hayat et al. [15] and numerical techniques such as Chebyshev collocation method by Uddin et al. [16] and many others. We adopted the shifted Chebyshev collocation method [17] to get the numerical results. In this method, the missing initial values directly define without any correction interpolation method.
Literature survey reveals that many numerical results have been discussed for different boundary layer flows, but there is always an open question on the existence and uniqueness results of solutions. To concern, this question, Weyl [18] done the first theoretical work by considering Falkner-Skan flow. After that, many researchers are trying to provide the existence and uniqueness results for different boundary layer flows. Ahmad et al. [19] gave the existence results for a coupled system of nonlinear fractional differential equations with three-point boundary conditions. Robert et al. [20,21] gave existence results of the solution for non-linear differential equation arising in stagnation point flow in porous media and MHD Falkner-skan flow. There are many general tools to prove the existence results of the solutions. Talay et al. [22] established existence and uniqueness results using the Schauder fixed point theorem. Paullet [23,24] have been demonstrated the results by topological shooting argument for many boundary layer flow problems. The shooting argument is a very useful technique to prove existence results of the solution as it also provides solution properties. We quote the comments of Kwong [25]: "in the special case of second-order equations, the shooting argument can be an effective tool, sometimes yielding better results than those obtainable via fixed point techniques". One may refer to Hastings et al. [26] and Rao et al. [27] for an introduction to the topological shooting arguments. Recently, Samanta et al. [28] discussed the existence result of the solution of modified Burger's equation.
It is widely known that the Lorentz force stabilizes fluid flows. Due to its effects, it is necessary to discuss far-field solution behaviour, and so, the present study also focuses on the asymptomatic approach to investigate the boundary layer. Sahoo et al. [29] discussed the far-field solution behaviour for non-Newtonian Bodewadt flow. The rate of decay for this asymptotic solution is also estimated.
Motivated by above litteraters, an attempt is made to discuss the existence, uniqueness and asymptotic results of MHD stagnation point flow with slip in the plate. Moreover, the salient features of the flow are discussed by numerical results in detail. The rest of the paper is arranged as follows: The formulation of the problem, along with its boundary conditions, is given in section 2. In section 3, the existence and uniqueness results of the resulting nonlinear boundary value problem are discussed thoroughly. The behaviour of the solution at the free boundary is given in section 4. Section 5 contains a brief description of the numerical scheme. In section 6, the theoretical and numerical results are compared, and the effects of the physical parameters on the flow domain are discussed. Lastly, section 7 summarizes a brief conclusion of our present works.

2 | MATHEMATICAL FORMULATION

The problem considered here is a steady two-dimensional flow of viscous fluid impinging perpendicularly on a non-impregnable wall. It is assumed that the wall is subjected to obey the partial slip condition and a uniform magnetic field of strength B_0 is applied in the y-direction. The schematic diagram of the flow configuration is shown in Figure [1].

The governing boundary layer equations are (For details see Ariel [8])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \nu\frac{\partial^2 u}{\partial^2 x} + \frac{\sigma B_0^2}{\rho}(U - u) \tag{2}$$

where u, v are the velocity components along the $x - axis$ and $y - axis$ respectively, $U = ax$ is the free stream velocity, ν is kinematic viscosity, ρ is fluid density, σ is the electrical conductivity and B_0 is strength of the magnetic field.

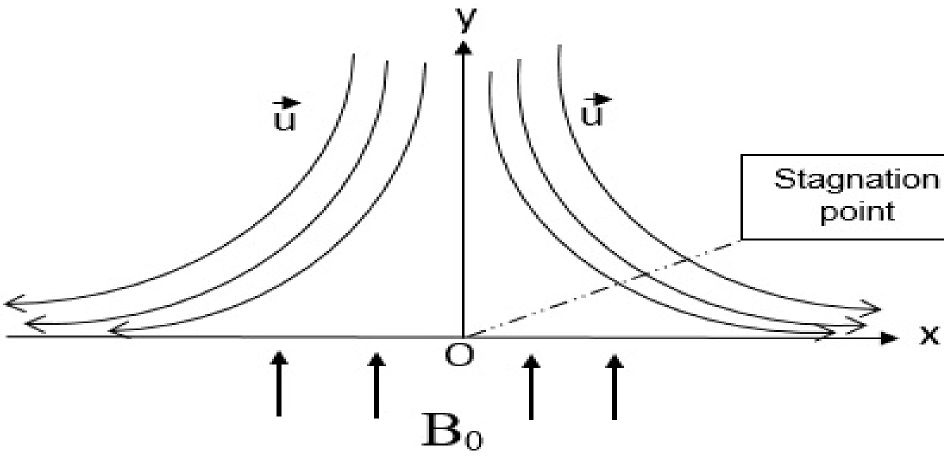

FIGURE 1 Schematic diagram of the flow

The similarity variable $\zeta = \sqrt{\frac{a}{\nu}}y$ which gives the similarity transform

$$u = axf'(\zeta), \quad v = \sqrt{a\nu} f(\zeta)$$

The equation (1) is satisfied automatically and equation (2) becomes

$$f''' + ff'' - f'^2 - M^2(f' - 1) + 1 = 0 \tag{3}$$

where $M = \sqrt{\frac{\sigma B_0^2}{a\rho}}$ is called Hartmann number and a is constant related to intensity of the potential fluid flow.

The general no-slip boundary conditions of the stagnation point flow has been changed to partial slip conditions and the appropriate boundary conditions for velocity components are (For details see Wang[2])

$$u = \lambda^* \frac{\partial u}{\partial y}, \quad v = 0 \qquad at\ y = 0$$

$$u \to ax \qquad as\ y \to \infty.$$

where λ^* is the slip parameter. Using similarity transforms the boundary conditions become

$$f(0) = 0, \quad f'(0) = \lambda f''(0), \quad f'(\infty) = 1 \tag{4}$$

3 | EXISTENCE AND UNIQUENESS RESULTS

In this section, the existence and uniqueness results of the solutions of the nonlinear boundary value problem (3)-(4) have been studied for all cases of $M,\ \lambda \geq 0$. The main focus of this section will be proving the following theorems:

Theorem 1 (Existence)**.** For any $M, \lambda \geq 0$ there exists at least one solution of the boundary value problem(BVP) (3)-(4). Further, the solution is monotonic.

Theorem 2 (Uniqueness)**.** The solution is also unique for all $M, \lambda \geq 0$.

Begin with some propositions that are necessary to proof the theorems.

Proposition 1. For each positive solution $f'(\zeta)$ of (3) and (4), $\lambda f''(0) \leq f'(\zeta) < 1$ for all $0 \leq \zeta < L$.

Proof. For sake contradiction, suppose that $f'(\zeta) > 1$ for some $\zeta \in [0, L]$. Then, $f'(\zeta)$ has a positive maximum value in $[0, L)$, say, ζ_0. Then, at ζ_0, $f''(\zeta_0) = 0$ and $f'''(\zeta_0) \leq 0$. From (3),

$$0 < (f'(\zeta_0) - 1)(f'(\zeta_0) + 1 + M^2) = f'''(\zeta_0) + f(\zeta_0)f''(\zeta_0) \leq 0$$

which is a contradiction. Hence, $\lambda f''(0) \leq f'(\zeta) < 1$ on $\zeta \in [0, L)$. □

Proposition 2. If $f'(\zeta)$ be a positive solution of (3) with $\lambda\alpha \leq f'(\zeta) < 1$ on $\zeta \in [0, L)$ then $L \to \infty$.

Proof. The equation (3) can be rewrite as

$$(f''e^F)' = (f'(\zeta) - 1)(f'(\zeta) + 1 + M^2)e^F < 1,$$

where F=$\int_0^\zeta f(t)dt$. Then $f''(0) \leq f''(\zeta)e^F \leq 0$ implies f'' is bounded and therefore from (3) $f'''(\zeta)$ is bounded on $[0, L]$ if $L < \infty$. It also implies that f'', f', f have limits as $\zeta \to L$ and hence the solution of (3) will be continued, clearly a contradiction. Hence ˇ $\to \infty$. □

Now, proving the existence results of the solutions to the BVP (3)-(4), we will investigate a related initial value problem(IVP)(3) along with its initial conditions

$$f(0) = 0, \quad f'(0) = \lambda\alpha, \quad f''(0) = \alpha \tag{5}$$

where α is a free variable and hence, the solution of the IVP is dependent on both ζ and α. We denote the solution by $f(\zeta; \alpha)$, and the initial value problem has a solution for each α, although all the solutions don't satisfy the boundary conditions at infinity. Hence, we are looking for an α, for which corresponding solution $f(\zeta, \alpha)$ will satisfy the boundary condition at infinity i.e. $f'(\infty) = 1$. We use topological shooting argument to find at least one α, and define the two subsets A and B of a connected set $(0, \infty)$ as follows:

$$A = \{\alpha \in (0, \infty) \mid f''(\zeta; \alpha) = 0 \text{ whenever } \lambda\alpha < f'(\zeta; \alpha) < 1\}$$
$$B = \{\alpha \in (0, \infty) \mid f'(\zeta; \alpha) = 1 \text{ whenever } 0 < f''(\zeta; \alpha) < \alpha\}$$

The following lemmas will involve to proof theorem (1).

Lemma 1. If α is sufficiently small, then it is in A.

Proof. Let us first consider the case $\alpha = 0$ and we have from equation (3)

$$f'''(0; \alpha) = -(1 - \lambda\alpha)(1 + \lambda\alpha + M^2) \tag{6}$$

which implies, $f'''(0; 0) < 0$, then $\zeta \in (0; \epsilon]$ for some $\epsilon > 0$, we have $f''(\zeta; 0) < 0$ and $f'(\zeta; 0) < 1$. By neighborhood property of continuous solution of the initial value problem, $f'(\zeta; \alpha)$ will remain close to $f'(\zeta; 0)$, i.e. whenever $\zeta \in (0; \epsilon]$, then $f'(\zeta; \alpha) < 1$ with $f''(\epsilon; \alpha) < 0$. But $f''(0; \alpha) = \alpha > 0$. Hence there exists $\zeta_0 \in (0, \epsilon)$ such that $f''(\zeta_0; \alpha) = 0$ whenever $\lambda\alpha < f'(\zeta; \alpha) < 1$ on $\zeta \in (0, \zeta_0]$. Thus, sufficiently small $\alpha > 0$ is in A.

Therefore A is non-empty. □

Lemma 2. If α is sufficiently large, then it is in B.

Proof. We want to claim that if positive α is large enough then $f'(\zeta; \alpha) = 1$ before $f''(\zeta; \alpha) > 0$ in $[0, 1]$. Suppose, our claim is false then one of the following assertion must hold:

(i) $f''(\zeta; \alpha) = 0$ at a point in $[0, 1]$ with $f'(\zeta; \alpha) < 1$. Or,

(ii) $f'(\zeta; \alpha) < 1$ and $f''(\zeta; \alpha) > 0$ for all $\zeta \in [0, 1]$. Or,

(iii) $f'(\zeta; \alpha) = 1$ and $f''(\zeta; \alpha) = 0$ happen simultaneously.

To prove our claim, we have to discard each of assertion and begin with (i). Let us consider there exists $\zeta_1 \in [0,1]$ such that

$$f''(\zeta_1;\alpha) = 0 \tag{7}$$

with $f'(\zeta;\alpha) < 1$ for $\zeta \in [0,1]$. Since $f''(\zeta;\alpha)$ is decreasing and positive on $(0,\zeta_1)$, $f'(\zeta;\alpha)$ is monotonic increasing and concave down. Thus $\lambda\alpha < f'(\zeta;\alpha) < 1$ on $[0,1]$. Integrating the inequality in $[0,\zeta]$, we have $\lambda\alpha\zeta < f(\zeta;\alpha) < \zeta$. Again if we integrate the ODE (3) in between 0 to ζ, we have

$$f''(\zeta) - f''(0) = -f(\zeta)f'(\zeta) + 2\int_0^\zeta f'^2 dt + M^2(f(\zeta) - \zeta) - \zeta$$

$$\Rightarrow \; f''(\zeta) = \alpha - (1+M^2)\zeta - f(\zeta)f'(\zeta) + 2\int_0^\zeta f'^2 dt + M^2 f(\zeta) \tag{8}$$

Using above bounds on $f(\zeta)$ and $f'(\zeta)$ and avoiding all the positive term except α we get the following

$$f''(\zeta) \geq \alpha - 2 - M^2, \;\; \text{for all } \; \zeta \in [0,\zeta_1]$$

Thus, if $\alpha > 2 + M^2$ then $f''(\zeta_1) > 0$, contradicting the fact (7). So, for large α case (i) does no happen. Similarly, we can eliminate case (ii) when $\alpha > 2 + M^2 + M$. Thus, only one case (iii) remain i.e. $f'(\zeta;\alpha) = 1$ and $f''(\zeta;\alpha) = 0$ happen simultaneously. However, substituting it into ODE (3) gives $f'''(\zeta) \equiv 0$. Also we have $f^k(\zeta) \equiv 0, \;\; \forall\, k \geq 3$ implying that $f'(\zeta) \equiv 1$. It is clear a contradiction the basic existence theorem of initial value problem, $f'(0) = \lambda\alpha \neq 1$. Thus, if $\alpha > 2 + M^2 + M$, large enough then $f'(\zeta;\alpha) = 1$ whenever $0 < f''(\zeta;\alpha) < \alpha$.

Therefore the set B is non-empty. □

Lemma 3. The set A and B are mutually disjoint and both are open.

Proof. The sets A and B are clearly disjoint subset of $(0,\infty)$. To prove A is open, suppose that if α^* is in A then α sufficiently nearby to α^* is in A and corresponding solution of the IVP satisfy at ζ_0 that $f''(\zeta_0;\alpha^*) = 0$ with $\lambda\alpha^* < f'(\zeta_0;\alpha^*) < 1$. From equation (3),

$$f'''(\zeta_0;\alpha^*) = f'^2(\zeta_0;\alpha^*) - 1 + M^2(f'(\zeta_0;\alpha^*) - 1) \neq 0$$

Thus, by neighborhood property of continuous solutions, $f''(\zeta;\alpha)$ has a root near ζ_0 for α sufficiently nearby to α^* with $\lambda\alpha < f'(\zeta;\alpha) < 1$. Thus A is open. Using similar argument, B is open.

□

Remark 1 : Thus by previous lemmas (1), (2) and (3), the non-empty sets A and B are both mutually disjoint open subset of connected set $(0,\infty)$.

Proof of Theorem 1. Using the definition of connected set, $A \cup B \neq (0,\infty)$. Thus there exists $\bar{\alpha} \in (0,\infty)$ which is neither in A nor in B (i.e., we can't have $f''(\zeta;\bar{\alpha}) = 0$ whenever $\lambda\bar{\alpha} < f'(\zeta;\bar{\alpha}) < 1$ and $f'(\zeta;\bar{\alpha}) = 1$ whenever $0 < f''(\zeta;\bar{\alpha}) < \bar{\alpha}$). Also, $f'(\zeta;\bar{\alpha}) = 1$ and $f''(\zeta;\alpha) = 0$ can't happen simultaneously by lemma (2). Thus, only one possibility is $f''(\zeta,\bar{\alpha}) > 0$ with $f'(\zeta,\bar{\alpha}) < 1$ for all $\zeta > 0$. From equation (3), we get $f'(\infty;\bar{\alpha}) \to 1$ giving the existence results of the solution of BVP (3)-(4).

□

Proof of Theorem 2. To claim unique solution of the BVP, let us first consider that there exists two positive α_1 and α_2 with $\alpha_2 > \alpha_1$ (without loss of generality) such that corresponding solutions of IVP, $f(\zeta;\alpha_1)$ and $f(\zeta;\alpha_2)$ are both the solution of BVP (3)-(4). Next, applying the mean value theorem in the interval $[\alpha_1,\alpha_2]$ we get:

$$f'(\zeta;\alpha_2) - f'(\zeta;\alpha_1) = \left(\frac{\partial f'}{\partial \alpha}\right)_{\alpha=\alpha^*} (\alpha_2 - \alpha_1) \tag{9}$$

where $\alpha^* \in (\alpha_1,\alpha_2)$.
Now, at $\zeta \to \infty$, equation (9) implies

$$\left(\frac{\partial f'}{\partial \alpha}\right)_{\alpha=\alpha^*} = \frac{f'(\infty;\alpha_2) - f'(\infty;\alpha_1)}{(\alpha_2 - \alpha_1)} = 0 \tag{10}$$

Considering the function $w(\zeta;\alpha) = \frac{\partial f'}{\partial \alpha}$, we differentiate ODE (3) with respect to α along with its initial conditions (5) gives

$$w''' + f w'' - 2f'w' + wf'' - M^2 w' = 0 \tag{11}$$

subject to

$$w(0;\alpha) = 0, \quad w'(0;\alpha) = \lambda, \quad w''(0;\alpha) = 1 \tag{12}$$

where primes are denoting differentiation with respect to ζ which implies

$$\begin{aligned} w'''(0;\alpha) &= 2\lambda^2\alpha + M^2\lambda \\ w^{iv}(0;\alpha) &= 2\lambda\alpha + M^2 > 0 \end{aligned} \tag{13}$$

Thus, from the initial condition (12) on w, we get $w'(\zeta;\alpha) > 0,\ w''(\zeta;\alpha) > 1$ and $w'''(\zeta;\alpha) > 0$ when $0 < \zeta < \epsilon$ for $\epsilon > 0$. Particularly, positive function $w'(\zeta;\alpha)$ is concave up increasing initially and to become zero, it has to change first from concave up to concave down. Thus there exists a first ζ_2 such that $w'''(\zeta_2;\alpha) = 0$ and $w^{iv}(\zeta_2;\alpha) \leq 0$. But until this point ζ_2, $w(\zeta;\alpha)$ and all its derivatives through $w'''(\zeta;\alpha)$ are positive and increasing. Therefore, $f(\zeta;\alpha)$ and all its derivative through $f'''(\zeta;\alpha)$ are increasing function with respect to α. Thus, for $\alpha \in (\alpha_1, \alpha_2)$ we have

$$w^{iv}(\zeta_2;\alpha) = -w(\zeta_2;\alpha)f'''(\zeta_2;\alpha) + w'(\zeta_2;\alpha)f''(\zeta_2;\alpha) + f'(\zeta_2;\alpha)w''(\zeta_2;\alpha) + M^2 w''(\zeta_2;\alpha) \tag{14}$$

which implies $w^{iv}(\zeta_2;\alpha) > 0$ for all α. Thus $w'(\zeta;\alpha) = \frac{\partial f'}{\partial \alpha}$ can never be zero, contradicting the fact (10). Hence, we claim that there exists a unique α which must satisfy all boundary conditions. □

Remark 2: The BPV problem (3) with its boundary conditions (4) has a unique monotonic solution and further it is unique for all values of $M,\ \lambda > 0$.

4 | ASYMPTOTIC BEHAVIOUR OF THE SOLUTION

In this section, we discuss the behavior of the solution at $\zeta \to \infty$(i.e, far field behaviour of the solution). To get the asymptotic behaviour of (7)-(9), we convert it into a three-dimensional linear dynamical system as following:

$$\begin{aligned} u_1' &= u_2 \\ u_2' &= u_3 \\ u_3' &= -u_1u_3 + u_2^2 + M^2(u_2 - 1) - 1 \end{aligned} \tag{15}$$

and find an exact solution at $\zeta \to \infty$ which satisfy the asymptotic boundary conditions. Since, $f'(\zeta) \to 1$ as $\zeta \to \infty$, we can assume that $f(\zeta) \to f_\infty$ as $\zeta \to \infty$, where f_∞ is a constant. Then, for the system, we discuss system at the point $(u_1^*, u_2^*, u_3^*) = (f_\infty, 1, 0)$. Now, Jacobian ($J$) of the non-linear system (15)

$$J = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -u_3 & 2u_2 + M^2 & -u_1 \end{pmatrix}$$

gives three eigen values and corresponding eigen values are 0, $\frac{-f_\infty + \sqrt{f_\infty^2 + 8 + 4M^2}}{2}$, $\frac{-f_\infty - \sqrt{f_\infty^2 + 8 + 4M^2}}{2}$. Using the boundary conditions (4), we eliminate the solutions which blow up as $\zeta \to \infty$. Thus only one solution remains which satisfy all the boundary conditions and at $\zeta \to \infty$ solution behave like

$$f(\zeta) \equiv f_\infty + A_1 e^{-r\zeta}, \quad f'(\zeta) \equiv 1 + A_2 e^{-r\zeta}$$

where $r = r(f_\infty) = \frac{\sqrt{f_\infty^2 + 8 + 4M^2} + f_\infty}{2}$, is rate of decay of the solution and A_i, $i = 1, 2$ are constant. It is observed that r increase as f_∞ increases or M increases. In other words, when the magnetic parameter M increases then $f'(\zeta)$ tends to approach their far-field values more rapidly. Also, the rate of decay is independent on the slip parameter, λ.

5 | NUMERICAL METHOD

5.1 | Shifted Chebyshev polynomials and collocation points

The Shifted Chebyshev polynomials (ShCP) on the interval $[0, \zeta_\infty]$ are the affine transformation of the chebyshev 1st kind polynomial $T_r(\zeta)$ and defined as

$$\mathbb{T}_r(\zeta) = T_r(\frac{2\zeta}{\zeta_\infty} - 1) \tag{16}$$

The shifted collocation points, $0 = \zeta_0 < \zeta_1 < \cdots < \zeta_N = \zeta_\infty$, are obtained by

$$\zeta_i = \frac{\zeta_\infty}{2}(1 - cos(\frac{i\pi}{N})), \quad i = 0, 1, 2 \ldots, N \tag{17}$$

5.2 | Function approximation

To find the stable numerical solution of the resulting boundary equation (3), we 1st approximate $f''(\zeta)$ as

$$f''(\zeta) \approx \sum_{r=0}^{N} a_r \mathbb{T}_r(\zeta), \quad 0 \leq \zeta \leq \zeta_\infty \tag{18}$$

Now, from (18)

$$f'(\zeta) = \int_0^\zeta f''(t)dt + f'(0) \approx \sum_{r=0}^{N} a_r \int_0^\zeta \mathbb{T}_r(t)dt + \lambda \sum_{r=0}^{N} a_r \mathbb{T}_r(0),$$

where $f'(0) = \lambda f''(0)$. Using formulae for integral chebyshev polynomial of 1st kind we have,

$$f'(\zeta) \approx \sum_{r=0}^{N+1} b_r \mathbb{T}_r(t)dt + \lambda \sum_{r=0}^{N} a_r \mathbb{T}_r(0), \tag{19}$$

where

$$b_r = \begin{cases} \frac{\zeta_\infty}{2}(a_0 - \frac{a_1}{4} + \sum_{i=2}^{N-1} \frac{(-1)^i}{i^2-1} a_i), & r = 0 \\ \frac{\zeta_\infty}{2}(a_0 - \frac{a_2}{2}), & r = 1 \\ \frac{\zeta_\infty}{2}(\frac{a_{r-1}}{2r} - \frac{a_{r+1}}{2r}), & r = 2, 3, \ldots, N-1 \\ \frac{\zeta_\infty}{2}(\frac{a_{r-1}}{2r}), & r = N, N+1 \end{cases}$$

Analogously, the approximate of $f(\zeta)$ is given by,

$$f(\zeta) - f(0) = \int_0^\zeta f'(t)dt \approx \sum_{r=0}^{N+1} b_r \int_0^\zeta \mathbb{T}_r(t)dt + \zeta\lambda \sum_{r=0}^{N} a_r \mathbb{T}_r(0),$$

and since $f(0) = 0$,

$$f(\zeta) \approx \sum_{r=0}^{N+2} c_r \mathbb{T}_r(t)dt + \zeta\lambda \sum_{r=0}^{N} a_r \mathbb{T}_r(0), \tag{20}$$

where

$$c_r = \begin{cases} \frac{\zeta_\infty}{2}(b_0 - \frac{b_1}{4} + \sum_{i=2}^{N-1} \frac{(-1)^i}{i^2-1} b_i), & r = 0 \\ \frac{\zeta_\infty}{2}(b_0 - \frac{b_2}{2}), & r = 1 \\ \frac{\zeta_\infty}{2}(\frac{b_{r-1}}{2r} - \frac{b_{r+1}}{2r}), & r = 2, 3, \ldots, N-1 \\ \frac{\zeta_\infty}{2}(\frac{b_{r-1}}{2r}), & r = N, N+1 \end{cases}$$

It is noticed that the coefficients b_i and c_i are the linear combination of the unknown coefficients a_i. Now the approximate $f'''(\zeta)$ is obtained by

$$f'''(\zeta) = \frac{df''}{d\zeta} \approx \sum_{r=0}^{N} a_r \mathbb{T}'_r(\zeta) \tag{21}$$

We use all the approximate function to find the solution.

5.3 | Collocation method for solving resulting boundary value problem

Substituting all the approximate solution (18)-(21) in the resulting equation (3) gives

$$\sum_{r=1}^{N} a_r \mathbb{T}_r'(\zeta) + \left(\sum_{r=0}^{N+2} c_i \mathbb{T}_r(\zeta) + \zeta\lambda \sum_{r=0}^{N} a_r \mathbb{T}_r(0) \right) \sum_{0}^{N} a_r \mathbb{T}_r(\zeta) - \left(\sum_{0}^{N+1} b_r \mathbb{T}_r(\zeta) + \lambda \sum_{r=0}^{N} a_r \mathbb{T}_r(0) \right)^2 - M^2 \left(\sum_{0}^{N+1} b_r \mathbb{T}_r(\zeta) + \lambda \sum_{r=0}^{N} a_r \mathbb{T}_r(0) - 1 \right) + 1 = 0 \tag{22}$$

Our technique is based on domain truncation, the boundary condition at ζ_∞ is replaced by approximation which yields

$$\sum_{r=0}^{N+1} b_r \mathbb{T}_r(\zeta_\infty) = 1 \tag{23}$$

At the collocating point ζ_r, $r = 0, 2, \dots, N-1$ the equation (22) gives

$$\sum_{r=1}^{N} a_r \mathbb{T}_r'(\zeta_r) + \left(\sum_{r=0}^{N+2} c_i \mathbb{T}_r(\zeta_r) + \zeta_r\lambda \sum_{r=0}^{N} a_r \mathbb{T}_r(0) \right) \sum_{0}^{N} a_r \mathbb{T}_r(\zeta_r) - \left(\sum_{0}^{N+1} b_r \mathbb{T}_r(\zeta_r) + \lambda \sum_{r=0}^{N} a_r \mathbb{T}_r(0) \right)^2 - M^2 \left(\sum_{0}^{N+1} b_r \mathbb{T}_r(\zeta_r) + \lambda \sum_{r=0}^{N} a_r \mathbb{T}_r(0) - 1 \right) + 1 = 0, \quad r = 0, 1, 2 \dots, N-1 \tag{24}$$

The system (24) together with (23) is an algebraic system of $(N+1)$ equation with a_r, $r = 0, 2, \dots, N$ unknowns. This system is solved using Newton's iteration technique. It is worth mentioning that the initial condition $\zeta = 0$ is already taken in the approximation of solutions.

6 | NUMERICAL RESULTS AND DISCUSSION

For several values of governing parameters M and λ, the equation (3), with its boundary conditions (4) have been solved numerically by the Shifted Chebyshev Collocation technique. The semi-infinite domain $[0, \infty)$ is approximated by $[0, \zeta_\infty]$ for numerical computation where the value of ζ_∞ is assumed sufficiently large and fixed. The values of skin-friction coefficient $f''(0)$ are given in the following tables, and it is verified with Wang's[2] results for $M = 0$, and it shows a good comparison. Therefore, the present results are accurate enough. For any fixed M, values of $f''(0)$ decreases continuously as λ increases.

Current results for N=25		Wang[2]
λ	$f''(0)$	$f''(0)$
0	1.232588	1.2325
0.4	0.886346	0.8863
1	0.593462	0.5934
2	0.375886	0.3758
5	0.177260	0.1772

TABLE 1 Comparison of the numerical results for $M = 0$ and N=25

Table [2] gives the values of $f''(0)$ for different values of λ and M. It is observed that $f''(0)$ is increasing as increases M for fixed λ. The results also agree with the reported values of Ariel[8]. The Table [3] is given for different values of N to verify the rapid convergence of the present numerical method. It is noticed that the convergence of the proposed method is very fast.

For validating our theoretical results, figure [2 -3] are figured, and it also describes the significance of physical parameters M and λ on velocity profiles. Figure (2) displays the velocity component $f'(\zeta)$ with varies of slip parameter λ for fixed $M = 1$.

	M			
λ	0	0.1	0.5	1
0	1.232588	1.236603	1.329407	1.585331
0.4	0.886346	0.887939	0.923970	1.015985
1	0.593462	0.594053	0.607312	0.640276
2	0.375886	0.376095	0.380765	0.392275
5	0.177260	0.177301	0.178233	0.180520

TABLE 2 Values of $f''(0)$ for several values of M and λ when N=25

N	5	10	15	20	25
$f''(0)$	0.60975587	0.60731194	0.6073118	0.6073118	0.6073118

TABLE 3 Values of $f''(0)$ for different N when $\lambda = 1$ and $M = 0.5$

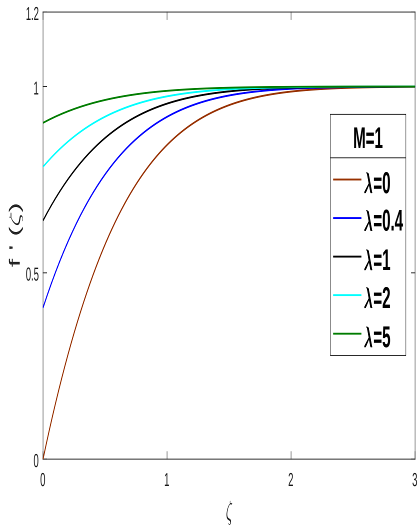

FIGURE 2 Variation of $f'(\zeta)$ with λ for $M = 1$.

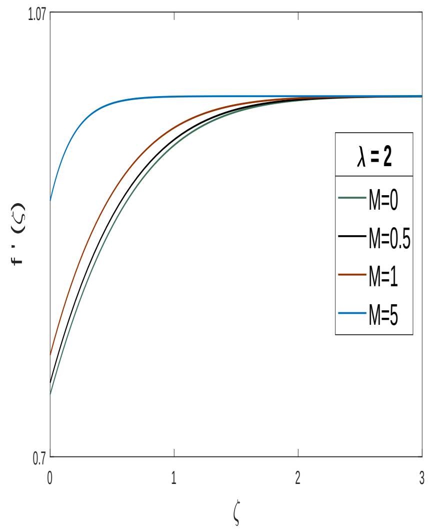

FIGURE 3 Variation of $f'(\zeta)$ with M for $\lambda = 2$.

It is noticed that $f'(\zeta)$ increases as increase of λ. Physically, when slip acts, the velocity of fluid near the wall in not equal to zero. So, the increase in λ means slip velocity increases.
Figure (3), the velocity component $f'(\zeta)$ is plotted against ζ for varies of M with fixed for $\lambda = 1$. It is observed that $f'(\zeta)$ increases as increase of M. It is worth mentioning that our discussion on asymptotic behaviour at free boundary also agrees with the numerical results, i.e., $f'(\zeta)$, mainstream velocity is approaching quickly to the value of the velocity of the potential flow. This similar trend of velocity profile has been found in the article of Ariel[8] for without slip, i.e. $\lambda = 0$.

7 | CONCLUSION

The existence result of the solution of third-order nonlinear boundary value problem related to MHD stagnation point flow with partial slip has been proved. Notably, there is a unique $f(\zeta)$ of the resulting boundary value problem which satisfies the following properties.

1. $f(\zeta)$ is increasing and non-negative.
2. $f'(\zeta)$ is increasing, non-negative and $\lambda f''(0) < f'(\zeta) < 1$.
3. $f''(\zeta)$ is decreasing , non-negative and $0 < f''(\zeta) < f''(0)$.

Therefore, shooting arguments becomes a very powerful tool to prove existence results. The asymptotic analysis shows that the solution of the boundary value problem is approaching more quickly to free stream velocity with the increasing value of M for any value of λ. Further, all the properties of $f(\zeta)$ are validated by numerical results as presented in section [6]. Note that the results are obtained for all relevant parameters, i.e. magnetic parameter $M \geq 0$ and slip parameter $\lambda \geq 0$. It is worth mentioning that the numerical solution and question of the existence, uniqueness and far-field behaviour of the solution are the pillars of our manuscript, and all are discussed in detail.

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