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TUTORIAL

Inside GNSS: An open source SDR receiver for GPS users

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Summary

Navigation based satellite system, in the image of GPS system, occupy a crucial place in our daily activities: finding its way, a place or tracking a craft.... The increasingly prevalent applications of such systems are driving the development of increasingly sophisticated GPS receivers. This research work falls within this perspective and aims at the development and design of algorithms for future generations of satellite location receivers based SDR technology. The proposed approach for this research work is based on the satellite collection signal in RAW format and their digital processing, all by improving and elaborating localization algorithms. The work presented aim to design and integrate a system able of processing the signals recorded in RAW format on a CPU unit based on correlation, FFT, PLL and DLL modules. Different algorithms and advanced signal processing techniques are implemented and validated. The work carried out aims at identifying, modeling and correcting the errors introduced on the signals initially emitted by the satellites as the Ionospheric and Tropospheric errors, the supreme objectives relate to the design of a real-time GPS receiver.

KEYWORDS:
Global Navigation Satellite System, SDR technology, MATLAB, Real time GPS, Channel effect, Troposphere effect, Ionosphere effect.

1 | INTRODUCTION

Satellite navigation becomes an essential technology in our daily life. The number of applications that use this type of system is increasing every day because of the large areas that require this service, whether for navigation, positioning, public security, surveillance, geographic searches, cartography and meteorological and atmospheric information [1]. The systems that provide this service are known as GNSS (Global Navigation Satellite System) global satellite navigation systems. This is the set of systems which use a constellation of satellites to offer the possibility to an unlimited number of receivers the calculation of the exact position in three dimensions: latitude, longitude and altitude, the precision can achieve few centimeter using signals from satellites located in orbits distributed to provide full coverage of the earth. The most known and most widely used positioning system in the world today is the American GPS system (Global Positioning System) and the Russian system GLONASS, two others systems are being developed such as the European Galileo system and the Chinese Beidou system [1,2].
In this work we used SDR technology to develop a GPS receiver. This technology can generate significant interest for the GNSS receivers industry. The only limitation of this type of receiver is that it cannot process analog signals for this reason we have placed an analog-digital converter (A/D) near the antenna before our receiver. So the analog signal collected by the antenna will be transferred to a digital signal, the samples of this signal will be used as an input to our program to calculate positions. To

test our algorithms, we chose MATLAB as the coding and calculation language, it is a flexible language and easy to learn. In addition, it provides excellent facilities for the presentation of graphical results.

2 | GPS SYSTEM

The GPS system has a constellation of 28 satellites distributed in six different MEO orbits with an inclination of 55° to the equator and an average altitude of 20,200 km, the orbits are separated by 60° between them to cover 360°. Each satellite travels the orbit in 11h58m02s with a speed of 3874 m/s, so each one orbits the earth twice a day. Currently several GPS signals are available for different categories of users, these signals are transmitted in the UHF band (500 MHz to 3 GHz), on three different frequencies. These carrier frequencies constructed from a base frequency F0 = 10.23 MHz, are given by, the frequency L1 equal to 154.F0 (1575.42 MHz), which contains the two former services of the GPS system; The PPS and SPS service as well as the civil L1C and military signals M. The frequency L2 equal to 120.F0 (1227.60 MHz) currently carries the PPS service and the civil signal L2C. The frequency L5 equal to 115.F0 (1176.45 MHz) reserved for the civil signal L5 as well as the military signal M [4,5,6].

3 | GPS SIGNALS

In general, GPS signals are constructed in the same way. Each signal consists of a navigation message and a spreading code, the modulo-2 addition of these two elements will be modulated in BPSK by a sinusoidal carrier frequency. Certain GPS services have signals which do not contain a navigation message, these signals, called "pilot signals", allow the receiver to resolve problems of synchronization with the navigation message and offer better robustness in terms of signal processing [4,5,6]. In this work we interest to the SPS service, in the following all the methods are applied on the L1 C/A signal. After the generation of the C/A and P spreading codes, the navigation message will be modulated by these codes using a simple modulo 2 adder (exclusive OR gate). The result will be modulated by the carrier frequency L1 (1575.42 MHz) using BPSK modulation, the two P and C/A codes modulated with a phase shift of 90°, in phase and in quadrature. The mathematical formula of the GPS signal transmitted by each satellite in the L1 band is given by [4,5,7] :

$$s^k(t) = \sqrt{2\,P_c}(D^k(t) \oplus C^k(t))\cos(2\,\pi\, f_{L1}t) + \sqrt{2\,P_P}(D^k(t) \oplus P^k(t))\sin(2\,\pi\, f_{L1}t) \tag{1}$$

3.1 | C/A CODE

The C/A code belongs to the Pseudo-Random Noise (PRN) family known as the Gold code with a sequence length of 1023 bits. The C/A code is a two-phase modulated signal with a frequency of 1.023 MHz. The repetition period of each pseudo-random sequence is 1023/1.023 MHz or one ms. each chip has duration of approximately 977.5 ns (1/1.023 MHz). The codes are generated, as explained in FIGURE 1, by the product of two 1023-bit PRN sequences G1 and G2 these two sequences are generated by two 10-bit shift registers controlled by a 1.023 MHz clock [6,7].
The C/A code is a unique code for each satellite, for this reason each PRN code is associated with two different positions (TABLE 1). On G2 for example for the first PRN the two positions 2 and 6 of the register are combined with each other by a circuit or exclusive and returned to step 1. The polynomial which describes this architecture of shift register is:

$$G1 = 1 + X_2 + X_6 \tag{2}$$

On the other hand the sequence G2 is a fixed sequence for all the satellites and it is defined by the polynomial:

$$G2 = 1 + X_2 + X_3 + X_6 + X_8 + X_9 + X_10 \tag{3}$$

The first 32 elements of these PRN numbers are reserved for the space segment, five additional PRN numbers, PRN 33 to 37, are reserved for other uses such as terrestrial transmitters, and called pseudo satellites [7].

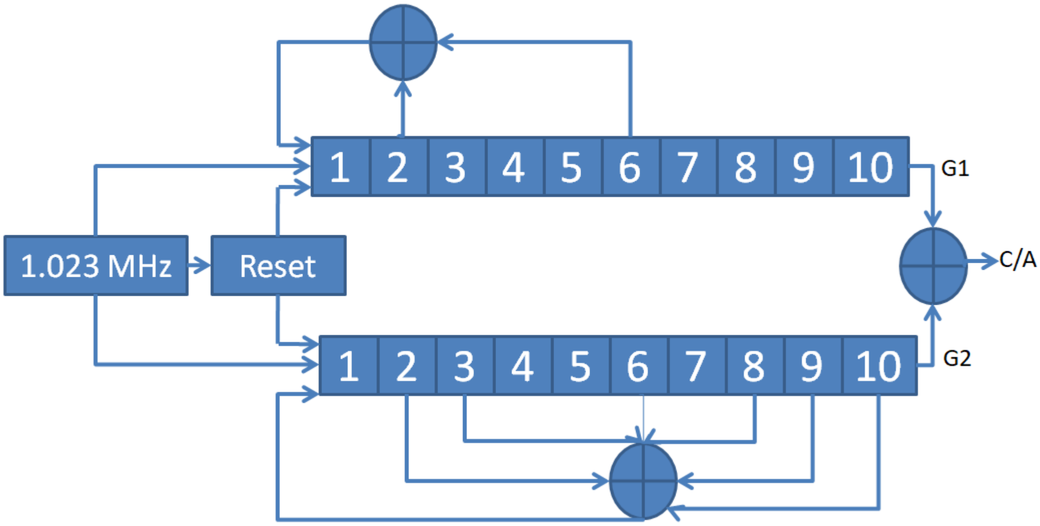


FIGURE 1 C/A code generator[6].

TABLE 1 G1 assignement for C/A code genetator.

SV ID	C/A G1	SV ID	C/A G1	SV ID	C/A G1	SV ID	C/A G1
01	02⊕06	09	03⊕10	17	01⊕04	25	05⊕07
02	03⊕07	10	02⊕03	18	02⊕05	26	06⊕08
03	04⊕08	11	03⊕04	19	03⊕06	27	07⊕09
04	05⊕09	12	05⊕06	20	04⊕07	28	08⊕10
05	01⊕09	13	06⊕07	21	05⊕08	29	01⊕06
06	02⊕10	14	07⊕08	22	06⊕09	30	02⊕07
07	01⊕08	15	08⊕09	23	01⊕03	31	03⊕08
08	02⊕09	16	09⊕10	24	04⊕06	32	04⊕09

3.2 | THE NAVIGATION MESSAGE

The GPS satellites continuously transmit a navigation message which contains the information using by the GPS receivers to calculate their positions (FIGURE 2). The navigation message which has 37,500-bit consists of 25 frames, each one containing 1,500 bits. Each frame is divided into 5 sub-frames of 300 bit; each one consists of 10 words of 30-bit. With a bit rate of 50 bps, each bit lasts 20 ms, the transmission of a sub-frame is done in 6s, a frame in 30s, and the entire navigation message takes 12.5 minutes. Sub-frames 1, 2, and 3 are repeated in each frame, TABLE 2 summarize information os sub-frame 1,2 and 3. The last sub-frames 4 and 5 have 25 different versions with the same structure, but different data. Each sub-frame begins with a telemetry word (TLM) containing an 8-bit synchronization pattern (10,001,011) and the word hand-over (HOW) contains the time of the week; the rest of the data in each one includes information necessary to determine the following: Transmission time, Satellite position, Satellite status, satellite clock correction, Ionospheric delay effects [7]. The information included in each sub-frame is [7]:

• subframe 1 contains health data indicating whether or not the data should be trusted, also contains the week number GPS, the accuracy of the spacecraft and its state of health, the clock correction terms satellite.

• subframe 2 and 3 contain the ephemeris data of the satellite, the ephemeris data are information on the orbit of the satellite and are necessary to calculate the position of the satellite.

• sub-frame 4 and 5, these last two sub-frames repeated every 12.5 minutes, which gives a total of 50 sub-frames. They contain almanac data which are the ephemeris and clock data with reduced precision. In addition, each satellite transmits almanac data for all GPS satellites while it transmits ephemeris data only for itself, the rest of subframes 4 and 5 contain various data, for example, the UTC parameters, health indicators, and ionospheric parameters.

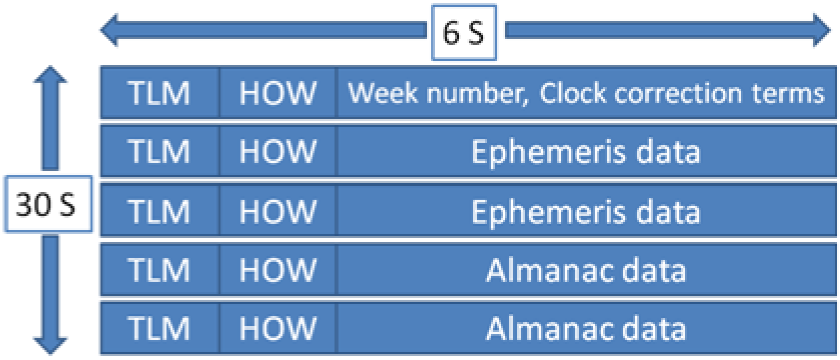


FIGURE 2 GPS navigation messge structure[5].

TABLE 2 GPS Ephemeris Data.

Data	Explanation	Unite
WN	Week Number	
Toc	Satellite clock correction	
af_2	Satellite clock correction	sec/sec^2
af_1	Satellite clock correction	sec/sec
af_0	Satellite clock correction	sec
$IODE_{sf3}$	Issue of data	
$\sqrt{A}$	Square Root of the Semi-Major Axis	$\sqrt{m}$
toe	Reference Time Ephemeris	sec
Δn	Mean Motion Difference From Computed Value	$radians/sec$
M_0	Mean Anomaly at Reference Time	$radians$
Ω	Argument of Perigee	$radians$
C_{us}	Amplitude of the Sine Harmonic Correction Term to the Argument of Latitude	$radians$
C_{uc}	Amplitude of the Sine Harmonic Correction Term to the Argument of Latitude	$radians$
C_{rs}	Amplitude of the Cosine Harmonic Correction Term to the Orbit Radius	$meters$
C_{rc}	Amplitude of the Cosine Harmonic Correction Term to the Orbit Radius	$meters$
C_{is}	Amplitude of the Sine Harmonic Correction Term to the Angle of Inclination	$radians$
C_{ic}	Amplitude of the Cosine Harmonic Correction Term to the Angle of Inclination	$radians$
i_0	The inclination angle at reference time	$radians$
$\dot{I}$	Rate of Inclination Angle	$radians/sec$
Ω_0	The longitude of the ascending node of orbit plane	$radians$
$\dot{\Omega}$	Rate of Right Ascension	$radians/sec$
e	Eccentricity	

4 | GPS RECEIVER

A GPS receiver generally consists of three blocks as shown in FIGURE 3. An RF part; represented by an antenna and a super-heterodyne receiver called Font-end. Its role is to capture the signals transmitted by visible satellites and convert these high frequency signals into low frequency signals so that they are easily processed by the following blocks. The second block is the signal processing block. In this block two methods are used. The first is the acquisition algorithm, the role of which is to identify the source of each signal received, and therefore identify the visible satellites. This process consists in correcting the Doppler Effect and the temporal phase shift of the code for each satellite. The second method in the signal processing block is the tracking; its role is to follow the variations due to the Doppler Effect and the phase shift of the code due to the propagation time. The third block is the data processing block, reserved firstly to reading the navigation message and secondly the position of the receiver will be calculated after correction of errors and increase its precision [5,7,8].

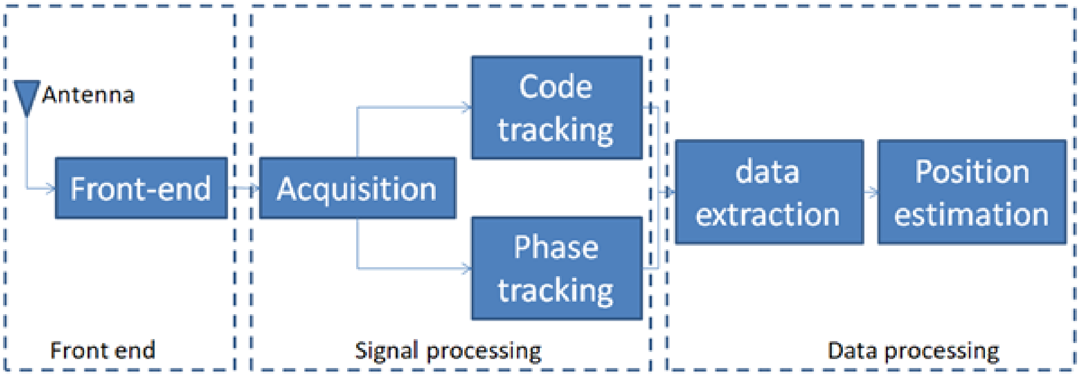


FIGURE 3 GPS receiver structure.

The simulation was performed under MATLAB using real data collected by the SE4110L front-end, this ASIC used as super-heterodyne receiver and analog-digital converter, to convert the GPS signal initially analog into a digital signal and to transform the frequency of this signal into another very low frequency easy to process.
In our case, the signal collected (FIGURE 4) has the following parameters [9]:

- IF intermediate frequency = 4.1304 MHz
- Sampling frequency = 16.3676 MHz

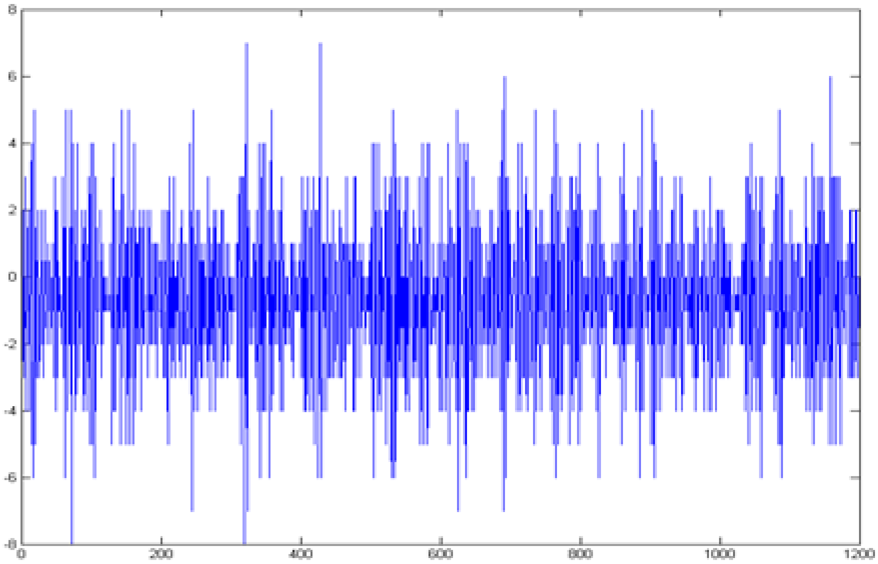


FIGURE 4 GPS signal.

5 | ACQUISITION ALGORITHM BASED ON THE FFT

The biggest problem with earth-level receivers when communicating with satellites that are not geostationary is materialized by changing the communication frequency to a new value from its original value. This phenomenon called Doppler Effect is due to the rapid movement of the satellites, exactly the important speed of the satellite and the relative speed of the rotation of the earth [10]. To solve this problem, the goal of our acquisition algorithm is to correlate the incoming signal with a copy of a locally generated GPS signal.

$$y(t) = x(t) * h(t) = \int x(t-\tau)h(\tau)d\tau \tag{4}$$

$$y(n) = \sum_{m=0}^{N-1} x(m)h(n-m) \tag{5}$$

Our algorithm [9]generates a signal that contains both the carrier and the PNR code. The signal generated also contains all possibilities for frequencies. The correlation performed is a circular correlation performed using the fast Fourier transform.

$$FFT\,[y(n)] = FFT\left[\sum_{m=0}^{N-1} x(m)h(n-m)\right] \tag{6}$$

This transform is necessary to convert the convolution operation in the time domain, a complicated operation, into a simple multiplication operation in the frequency domain.

$$Y(k) = \sum_{n=0}^{N-1}\sum_{m=0}^{N-1} x(m-n)h(m)\exp^{(-j2\pi km)/N} \tag{7}$$

$$Y(k) = \sum_{m=0}^{N-1} x(m)\left[\sum_{n=0}^{N-1} h(n-m)\exp^{(-j2\pi k(n-m))/N}\right]\exp^{(-j2\pi km)/N} \tag{8}$$

$$Y(k) = H(K)\sum_{m=0}^{N-1} x(m)\exp^{(-j2\pi km)/N} \tag{9}$$

$$Y(k) = H(K)X(K) \tag{10}$$

Once the correlation has been made in the frequency domain, the representation in the time domain can be accomplished by inverse Fourier transform.

$$y(t) = x(t) * h(t) = FFT^{-1}\,[H(K)X(K)] \tag{11}$$

If a peak is present in the correlation then we conclude that a satellite is visible. The index of this peak marks the phase of the PRN code and the frequency of the incoming signal.

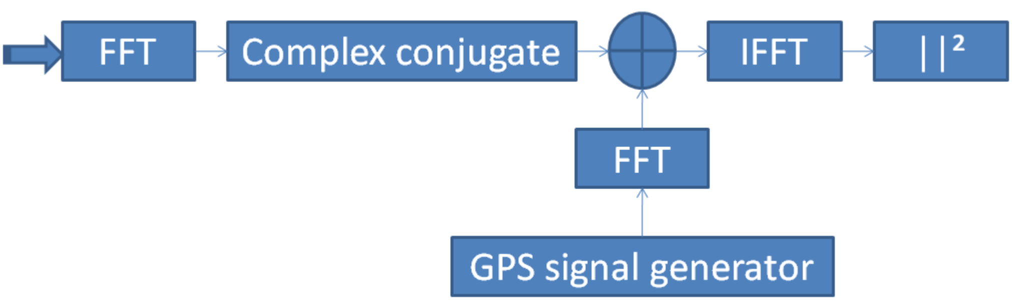


FIGURE 5 Acquisition Algorithm.

In our algorithm (FIGURE 5), the signal generated has a frequency which varies between -10 KHz and +10 KHz around the intermediate frequency of the receiver with a step of 500Hz (41 possibilities). The generated signal is correlated with the received signal. If the two have similar frequencies, we notice a significant value in the output of the algorithm.

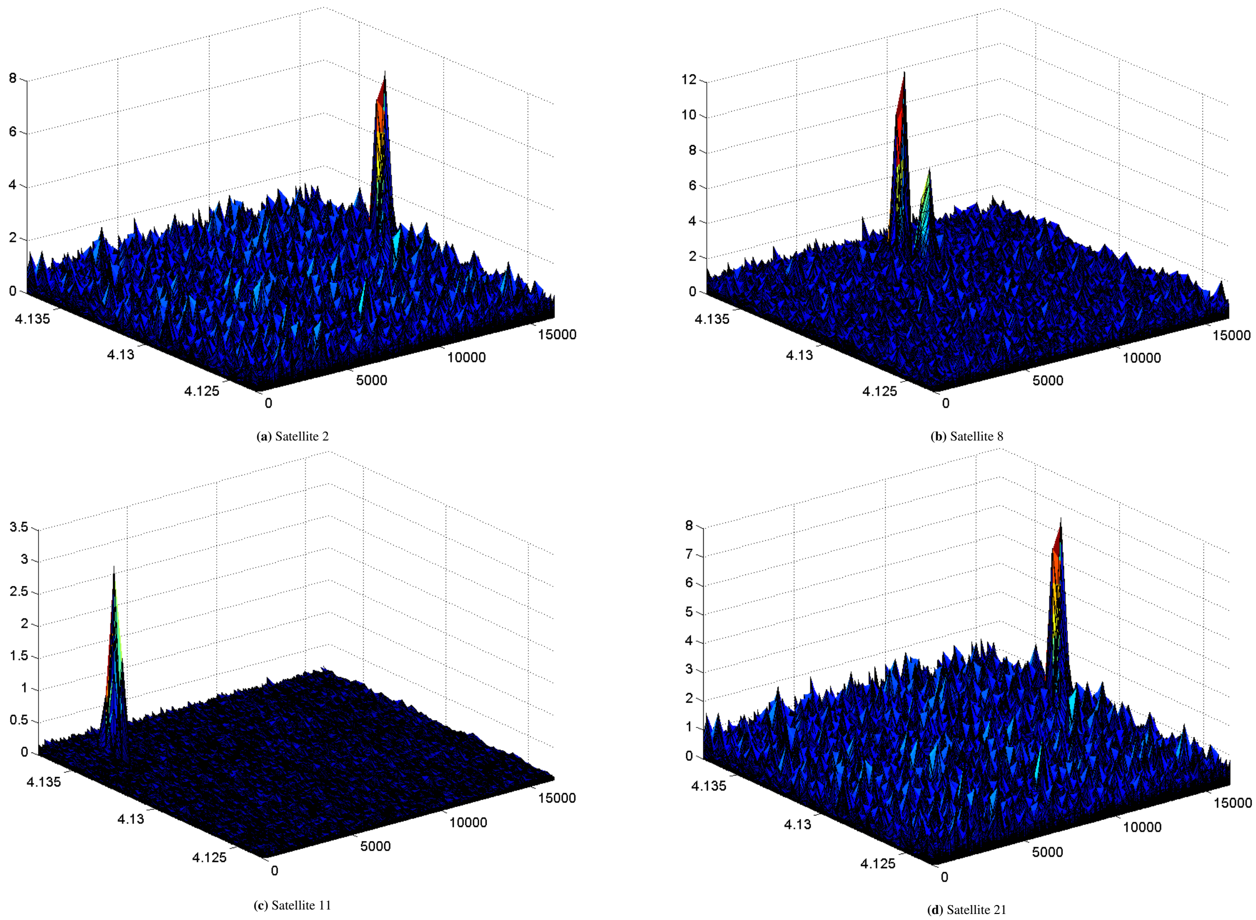


FIGURE 6 Algorithm output in case of visible satellite

FIGURE 6, illustrates the output of the algorithm in the case of a visible satellite, for example, satellite number 2,8,11 and 21.

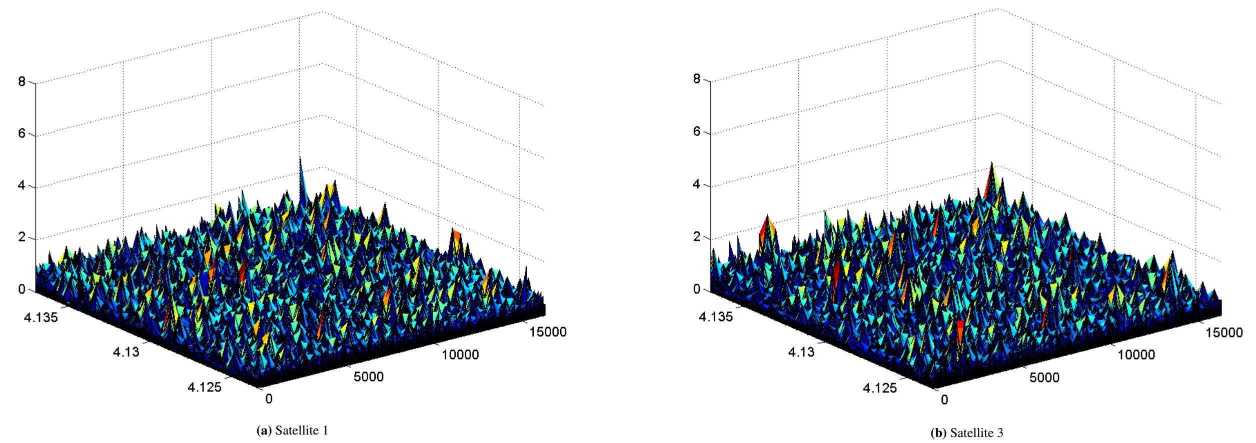


FIGURE 7 Algorithm output in case of invisible satellite

FIGURE 7, illustrates the output of the algorithm in the case of a no visible satellite, for example, satellite number 1 and 27.

6 | SIGNAL TRACKING

Once the acquisition process is complete, a decision on the visibility or not of a satellite is taken; in addition the two parameters necessary for synchronization are estimated. The frequency of the received signal and the phase of the code, for the first millisecond of the received signal. The main purpose of this algorithm is to refine the rough values provided by the acquisition to more precise values and to keep track of them.

6.1 | FREQUENCY TRACKING

In the acquisition process the Doppler shift is estimated using a step which increments by 500Hz for each calculation performed, this allows us to estimate the frequency of the received signal with an error of ± 250 Hz; this error can be minimized using a PLL loop that contains a filter with a discriminator [8]. The principle of this loop consists in continuously comparing the phase of the carrier frequency received from the satellite with that of the carrier frequency generated by the local oscillator of the receiver until their difference becomes zero [10,11]. So the PLL must compensate step by step the frequency offset between the carrier generated locally by the VCO and that of the received signal. The goal is that the locally generated frequency should approximate until it reaches the actual frequency of the received signal. This is done using an error detector, a loop filter which is used to control the oscillator and a comparator which makes a continuous comparison between the two frequencies [7,11].

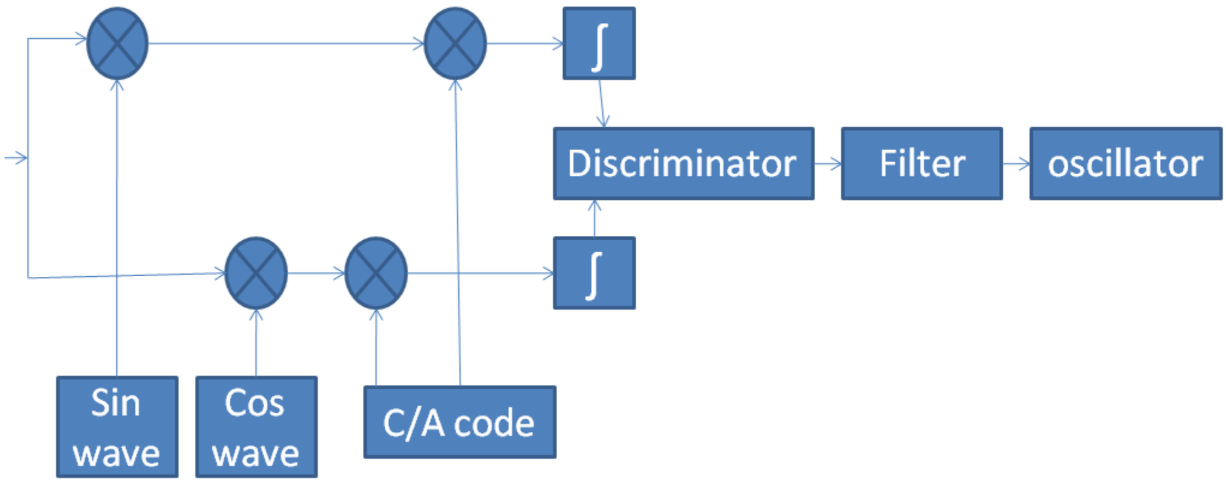


FIGURE 8 Frequency tracking algorithm[4].

This figure (FIGURE 8) illustrates the principle of a Costas loop used for the detection of the frequency offset. First a PRN code is multiplied by the incoming signal to extract the signal [8].

$$S(t).C(t) = C(t)\ D\ \cos(\omega_{IF}\ n)\ C(t) = D\ \cos(\omega_{IF}\ n) \tag{12}$$

Then the Costas loop uses two multiplications, a multiplication of the input signal and a local carrier wave and another multiplication between the input signal and a carrier wave 90 ° out of phase with the first [5,7,8].

$$D\ \cos(\omega_{IF}\ n)\ \ \cos(\omega_{IF}\ n + \Theta) = \frac{1}{2} D\ \cos(\Theta) + \frac{1}{2} D\ \cos(2\ \omega_{IF}\ n + \Theta) \tag{13}$$

$$D\ \cos(\omega_{IF}\ n)\ \ \sin(\omega_{IF}\ n + \Theta) = \frac{1}{2} D\ \sin(\Theta) + \frac{1}{2} D\ \sin(2\ \omega_{IF}\ n + \Theta) \tag{14}$$

The resulting signal will be passed through a low pass filter which gives

$$I = \frac{1}{2} D\ \cos(\Theta) \tag{15}$$

$$Q = \frac{1}{2} D\ \sin(\Theta) \tag{16}$$

At the end to calculate the frequency phase shift. The arctan discriminator is the most precise discriminator used by Costas loops. Except that it is much longer and requires a significant time of calculation [8,12].

$$\Theta = \tan^{-1}\left(\frac{Q}{I}\right) \tag{17}$$

The following figure (FIGURE 9) shows the output of the discriminator for the 4 visible satellites.

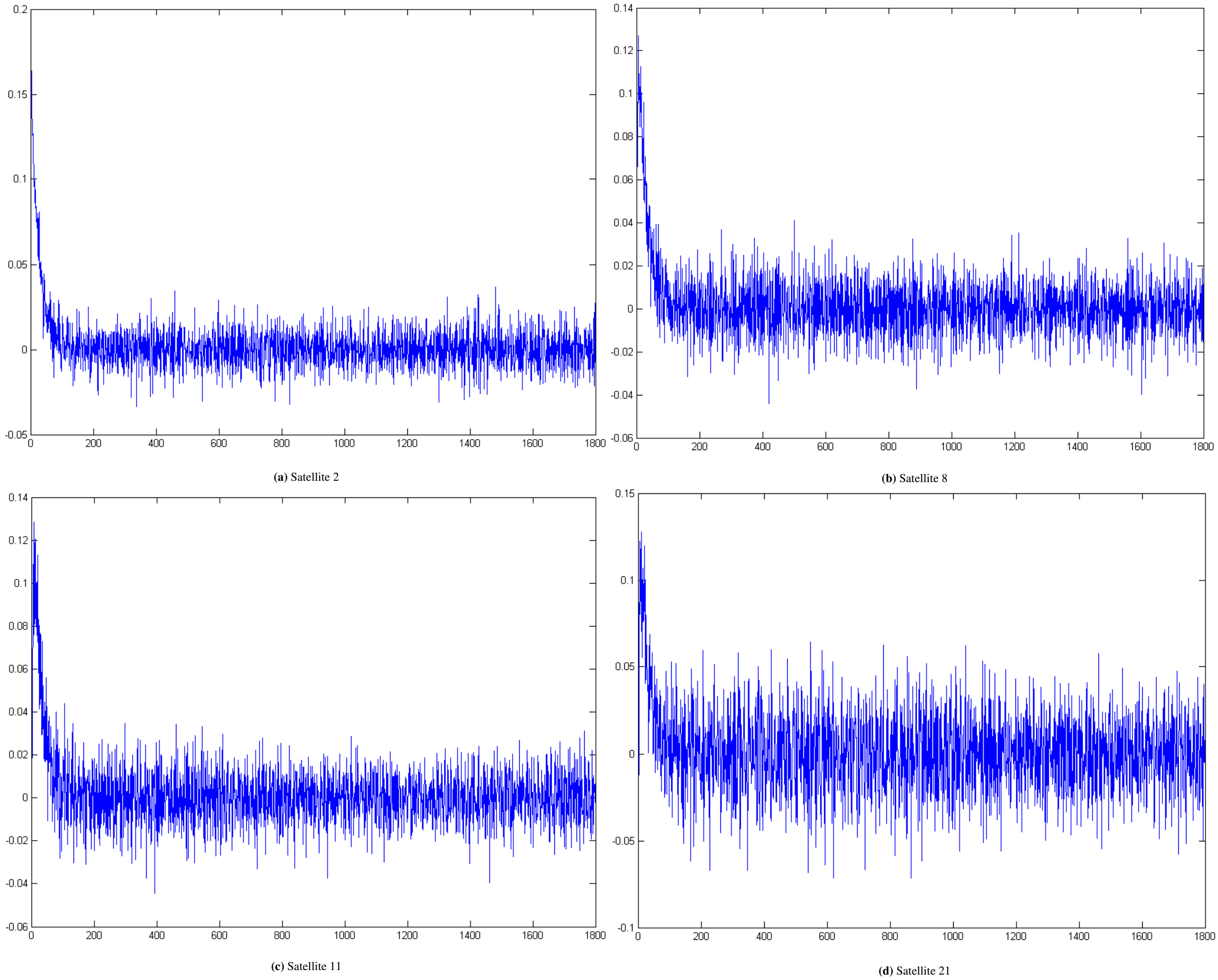


FIGURE 9 Code tracking algorithm output

6.2 | CODE TRACKING

The purpose of the code tracking loop (FIGURE 10) is to keep track of the phase shift of a specific code in the signal. To get a perfectly aligned replica of the C/A code of a targeted satellite that helps us to demodulate that signal. The code tracking loop used by GPS receivers is developed by a delay lock loop (DLL). Its principle consists in correlating the input signal with several replicas (prompt, Early and Late), with a spacing of 0, + ½ and -½ chip compared to the first code. Correlation outputs from these locally generated codes can be used to accurately determine the start of the C / A code in the input signal. This information is used to adjust the initial phase of the locally generated code to better match the phase of the code in the input signal [5,7,8,12]

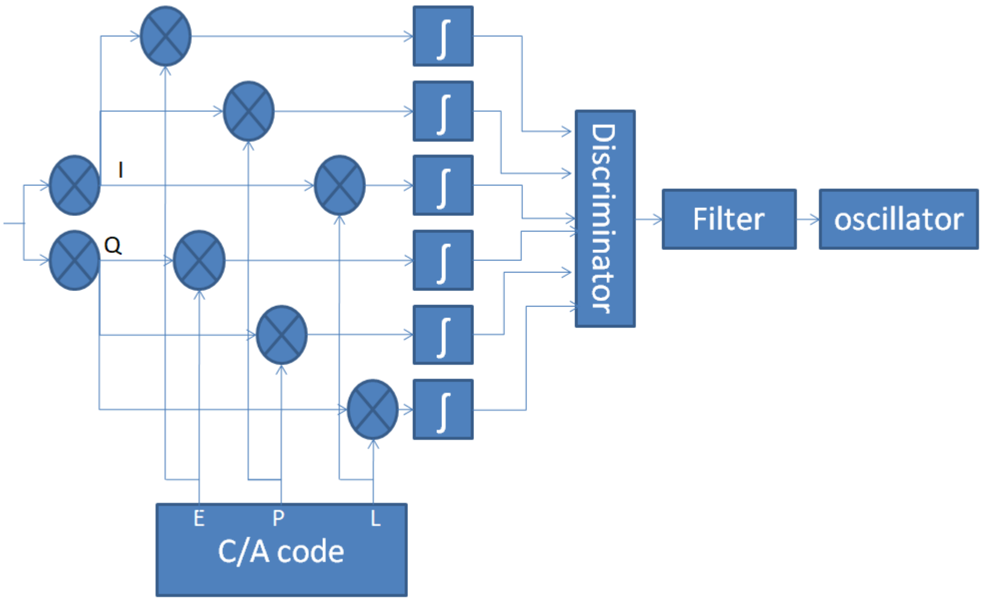


FIGURE 10 Code tracking algorithm[5].

A code discriminator is used to detect the amount of error in the locally generated code. The simplest form of the discriminator used is that which calculates the difference between the two correlation amplitudes of the two copies of the EarlyLate signal[8]:

$$D = \frac{(I_E^2 + Q_E^2) - (I_L^2 + Q_L^2)}{(I_E^2 + Q_E^2) + (I_L^2 + Q_L^2)} \tag{18}$$

The response of the discriminant function is given by the FIGURE 11:

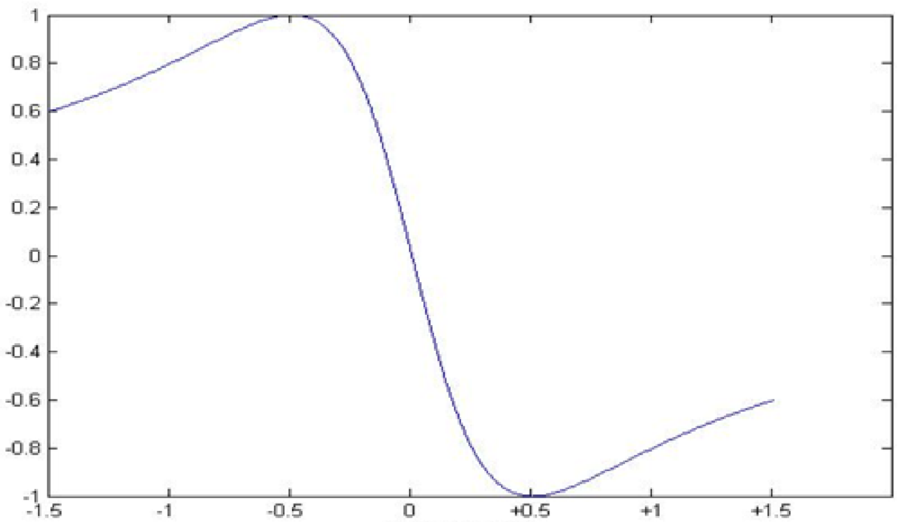
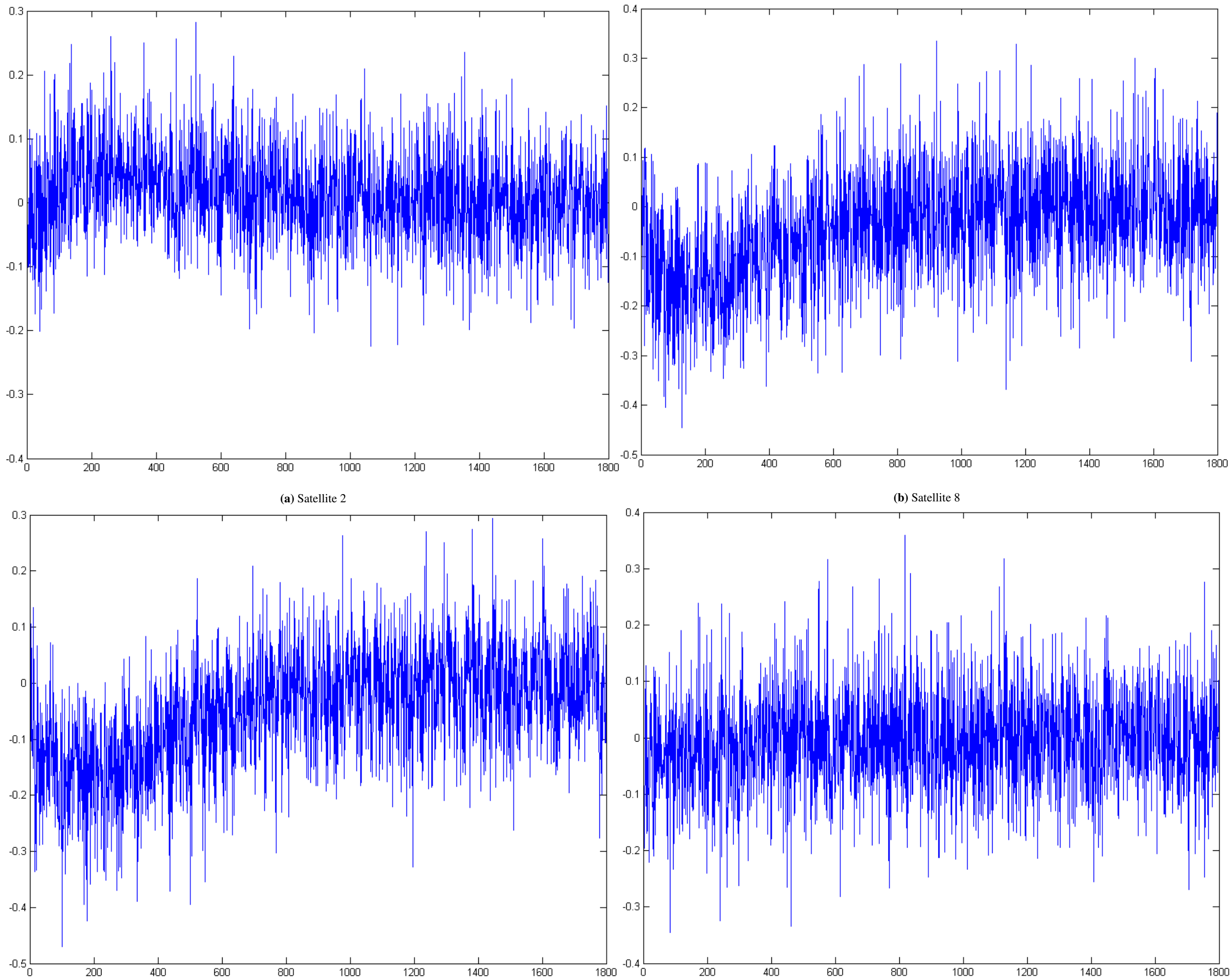


FIGURE 11 Discriminant response in function of real delay.

The output of this discriminant is given by the FIGURE 12 for the 4 visible satellites.



(a) Satellite 2 **(b)** Satellite 8

(c) Satellite 11 **(d)** Satellite 21

FIGURE 12 Frequency tracking algorithm output

7 | DEMODULATION OF THE GPS SIGNAL

The signal transmitted by each satellite in frequency L1 can be written as:

$$s^k(t) = \sqrt{2\,P_c}(D^k(t) \oplus C^k(t))\cos(2\,\pi\,f_{L1}t) + \sqrt{2\,P_P}(D^k(t) \oplus P^k(t))\sin(2\,\pi\,f_{L1}t) \tag{19}$$

k is the satellite Id with, P_C and P_P is the powers for the C/A and P codes respectively. The C/A code sequence is denoted by $C_k(t)$ and the P(Y) code sequence is denoted $P_k(t)$. $D_k(t)$ is the navigation data sequence.
The signal received by our front end is the component that contains only the C / A code:

$$s^k(n) = \sqrt{2\,P_c}(D^k(n) \oplus C^k(n))\cos(2\,\pi\,f_{L1}n) + N(n) \tag{20}$$

Where N represents noise.
First, the signal will be multiplied by a locally generated signal to remove the carrier:

$$s^k(n)\;\cos(\omega_{IF}\;n) = \sqrt{2\,P_c}(D^k(n) \oplus C^k(n))\cos(2\,\pi\,f_{L1}n)\cos(2\,\pi\,f_{L1}n) \tag{21}$$

$$s^k(n)\;\cos(\omega_{IF}\;n) = \frac{1}{2}\sqrt{2\,P_c}(D^k(n) \oplus C^k(n)) + \frac{1}{2}\sqrt{2\,P_c}(D^k(n) \oplus C^k(n))\cos(2\,\omega_{IF}\;n + \Theta) \tag{22}$$

After filtering by a low pass filter, the remaining component contains only the spreading code and the navigation message:

$$s'(n) = \frac{1}{2}\sqrt{2\,P_c}(D^k(n) \oplus C^k(n)) \tag{23}$$

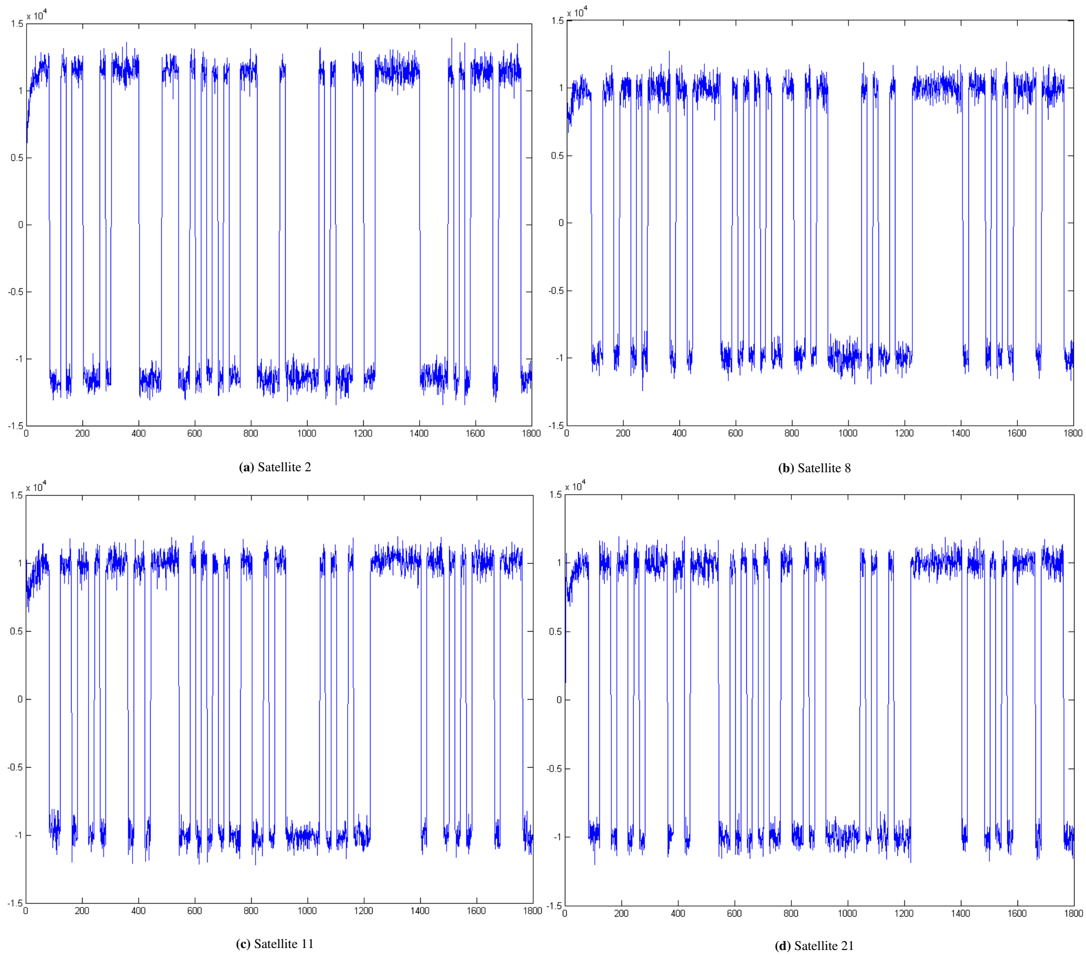
Then this signal will be multiplied by its spreading code to extract the navigation message:

$$s'(n) \oplus C^k(n)) = \frac{1}{2}\sqrt{2\,P_c}(D^k(n) \oplus C^k(n)) \oplus C^k(n)) \tag{24}$$

$$s'(n) \oplus C^k(n)) = \frac{1}{2}\sqrt{2\,P_c}(D^k(n)\sum_{n=0}^{N-1} C^k(n)) \oplus C^k(n)) \tag{25}$$

$$s'(n) \oplus C^k(n)) = \frac{1}{2}\sqrt{2\,P_c}N\,D^k(n) = \alpha D^k \tag{26}$$

The signal at the output (FIGURE 13) of the tracking algorithm is the navigation message multiplied by a coefficient α which represents the amplitude of the signal and the sum between the two spreading codes. TABLE 3 gives the necessary data contained in subframe 1,2 and 3.



(a) Satellite 2

(b) Satellite 8

(c) Satellite 11

(d) Satellite 21

FIGURE 13 Demodulated GPS message

This result which is a series of bits (-1 or +1) represents the navigation message included in the received signal in the form of a data stream with a bit rate of 50 bps, each bit having a duration of 20 ms [13]. The next step is to read this message to extract the data which contains the parameters necessary to calculate the positions and pseudos distances from the satellites such as the ephemeris data which contains precise orbital information and the correction values.

TABLE 3 Information transmitted from each satellite.

Data	Satellite a	Satellite b	Satellite c	Satellite d
Week Number	1321	1321	1321	1321
T_{GD}	-4.1910e-09	-2.7940e-09	-1.1642e-08	-6.0536e-09
IODC	503	250	487	499
t_{oc}	590384	590400	590400	590400
af_2	0	0	0	0
af_1	3.2969e-12	5.3433e-12	1.9327e-12	1.0232e-12
af_0	1.0680e-04	4.2776e-04	1.0180e-04	2.2032e-05
$IODE_{sf2}$	85	169	45	140
C_{rs}	42.8438	96.2500	97.1563	-61.5625
Δn	4.9302e-09	4.8066e-09	5.0834e-09	3.7627e-09
M_0	1.6539	-0.3383	-0.4479	1.1289
C_{uc}	2.2613e-06	4.8261e-06	5.0701e-06	-2.8890e-06
e	0.0068	0.0091	0.0098	0.0164
C_{us}	1.2351e-05	2.5816e-06	2.4214e-06	1.2413e-05
$\sqrt{A}$	5.1536e+03	5.1535e+03	5.1536e+03	5.1536e+03
t_{oe}	590384	590400	590400	590400
C_{ic}	-1.2293e-07	-3.5390e-08	1.7509e-07	-1.5832e-07
Ω0	-0.0923	1.1133	1.0811	-3.1334
C_{is}	5.9605e-08	-1.5832e-07	-5.9605e-08	1.6391e-07
i_0	0.9275	0.9618	0.9484	0.9866
C_{rc}	121.5938	329.1250	324.5938	158.2813
Ω	0.5959	2.3960	-3.1381	0.6846
$\dot{\Omega}$	-8.2071e-09	-8.5839e-09	-8.7064e-09	-7.7053e-09
$IODE_{sf3}$	85	169	45	140
$\dot{i}$	-3.0573e-10	4.0645e-10	3.7609e-10	-6.8931e-11
TOW	587454	587454	587454	587454

8 | CALCULATION OF THE SATELLITE POSITION

The information obtained from the ephemeris data is used to calculate the position of each satellite, this position will be calculated in the ECEF coordinate system in X, Y and Z using the Kepler equations (FIGURE 14).

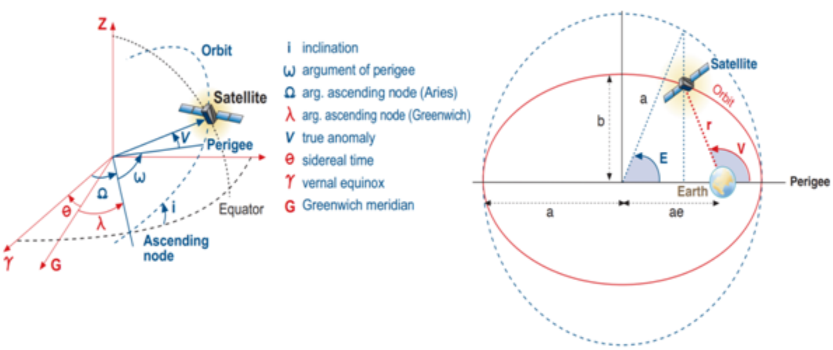


FIGURE 14 The Keplerian orbit element (ESA, Book).

In space three parameters are used to calculate the position of a satellite in X, Y and Z, these three parameters are [14,15]:
The angle between the accent node and the Greenwich meridian: Ω_{er}
The angle: φ
The distance between the satellite and the center of the earth r

By these parameters and using the following equation the position will be calculated:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} r\ \cos\Omega_{er}\ \cos\varphi + r\ \sin\Omega_{er}\ \cos i\ \sin\varphi \\ r\ \sin\Omega_{er}\ \cos\varphi + r\ \cos\Omega_{er}\ \cos i\ \sin\varphi \\ r\ \sin i\ \sin\varphi \end{pmatrix} \tag{27}$$

First the distance between the satellite and the center of the earth calculated by the following equation:

$$r = \frac{a\,(1 - e^2)}{1 + e\ \cos\upsilon} \tag{28}$$

The mean of the movement n, which is the average angular satellite speed, will be calculated:

$$n = \frac{T}{2\,\pi} = \sqrt{\frac{\mu}{a^3}} \tag{29}$$

T is the period of a revolution of the satellite.
μ is the universal gravitational parameter of the Earth = 9386005 $10^8 m^3/s^2$.
a is the semi-large axis of the orbit.
The value of n will be corrected using the value of correction obtained from ephemeris data Δn:

$$n = n + \Delta n = \sqrt{\frac{\mu}{a^3}} + \Delta n \tag{30}$$

The average anomaly M can be found as a function of the eccentric anomaly E

$$M = M_0 + n(t - t_{oe}) \tag{31}$$

Third, the eccentric anomaly E must be calculated from the average anomaly by:

$$M = E - e\sin E \tag{32}$$

In this equation the only unknown is E. This equation is not a linear equation; is it difficult to solve this equation analytically. The simplest way to solve it is to use the iteration method, a new value E can be obtained from a previous value. The above equation can be rewritten in an iterative format like [4]:

$$E_{i+1} = M + e\sin E_i \tag{33}$$

The solution of this equation gives as:

$$\cos\upsilon = \frac{\cos E - e}{1 - e\cos E} \tag{34}$$

The υ can be found from this equation:

$$\upsilon = \cos^{-1}\left(\frac{\cos E - e}{1 - e\cos E}\right) sign\left(\sin^{-1}\left(\frac{\sqrt{1 - e^2}\sin E}{1 - e\cos E}\right)\right) \tag{35}$$

And the angle φ can be found from the following equation:

$$\varphi = \upsilon + \omega \tag{36}$$

The argument ω can be found from the ephemeris data.
All the parameters calculated previously must be corrected using the correction terms transmitted by each satellite [13]:

$$\begin{aligned} \delta\varphi &= C_{us}\ sin(2\varphi) + C_{uc}\ cos(2\varphi) \\ \delta r &= C_{rs}\ sin(2\varphi) + C_{rc}\ cos(2\varphi) \\ \delta i &= C_{is}\ sin(2\varphi) + C_{ic}\ cos(2\varphi) \end{aligned} \tag{37}$$

These three terms are used to correct the following terms as shown in the following equations [13]:

$$\begin{aligned} \varphi &= \varphi + \delta\varphi \\ r &= r + \delta r \\ i &= i + \delta i \end{aligned} \tag{38}$$

The last element to calculate is the angle between the accent node and the Greenwich meridian [13]:

$$\Omega_{er} = \Omega - \dot{\Omega}_{ie}\, t_{er} = t \; + \; \Delta t \tag{39}$$

The following figure (FIGURE 15) gives the results for four visible satellites whose signals have been measured.

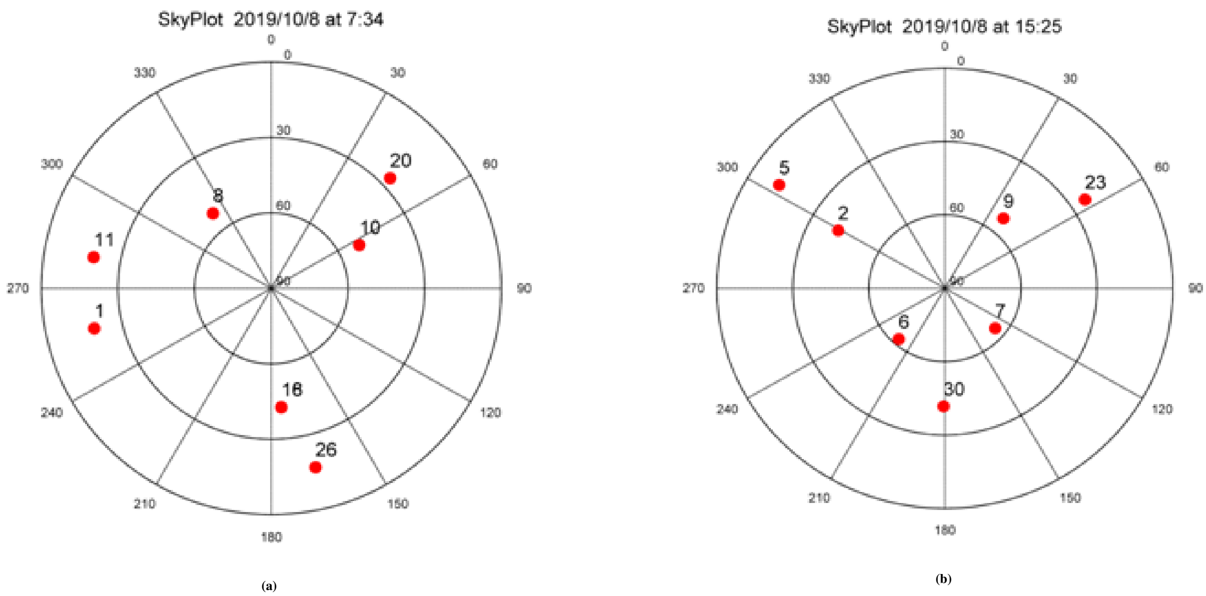


FIGURE 15 GPS satellite position

9 | CALCULATION OF THE PSEUDO-DISTANCE

This step consists in estimating the pseudo-distance between the satellites and our receiver. The method used in our work consists in giving an initial value which is a reference value, thereafter calculating the distances between each satellite and the receiver on the basis of this initial value and at the end correcting the errors of the measurement. We know that the value of the travel time of a signal between a GPS satellite and a receiver varies between 65 and 90 ms. The satellite closest to Earth is the satellite with the first subframe arriving first. The first value is therefore 65 ms being used as a reference to calculate the initial pseudo-distance for the first signal received, all the GPS satellites transmit their signals at the same time, so that the travel time of all the satellites is calculated on the base of the reference signal by estimating the offset of the reception time between the signals [7]. In FIGURE 16, the dots represent digital data of our signal and are separated by 61 ns because the sampling frequency is 16.3676 MHz. This sampling frequency leads to a pseudo-random precision of 4 m. The first point of subframe 1 is used as a reference point. All the first points of subframe 1 from different satellites are transmitted at the same time, and because the subframe from different satellites are received at different times, this time differs due to the dissimilarity of satellite propagation at receiver. This distance can be calculated by the following equation:

$$\begin{aligned} p_1 &= 65 * 3\ 10^8 \\ p_2 &= 65 * 3\ 10^8 (N * 61\ 10^{-9}) * 3\ 10^8 \end{aligned} \tag{40}$$

Where N represents the number of samples between the first point in the first subframe of the first signal and the first point in the first subframe of the second signal.

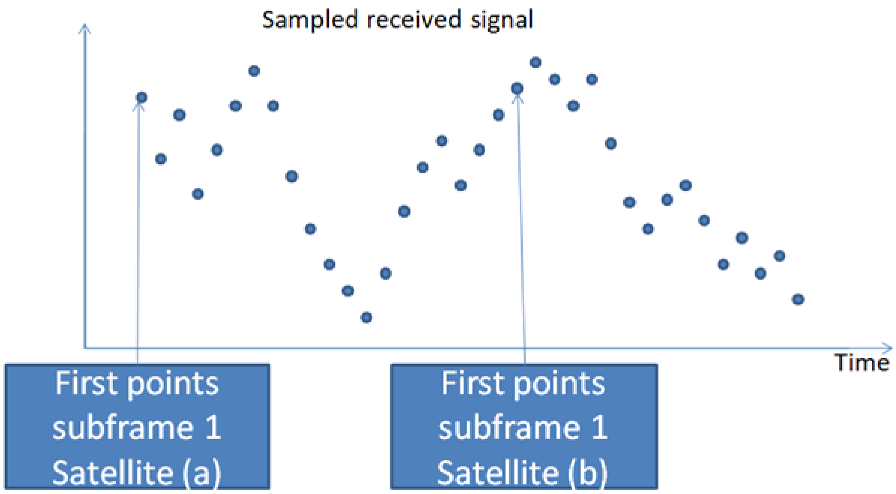


FIGURE 16 Delay between two successive signals.

10 | CORRECTION OF GPS ERRORS

Errors that influence the GPS signal can introduce a deviation of 50 to 100 meters. There are several sources of error, the most important of which are errors due to atmospheric conditions, particularly the two layers of the atmosphere, the ionosphere and the troposphere, which influence the signal transmitted from the satellite to the receiver. The speed of the GPS signal that propagates in the ionosphere and the troposphere are different from the speed of the latter signal in space. So the distance calculated previously will be different from the actual distance traveled. The second type of error is the error due to equipment such as the offset of the satellite clocks and also the measurement noise. The actual pseudo-distance is equal to the measured pseudo-distance plus various disturbance factors due to the different sources of error, as shown below [10,15]:

$$\rho = \rho_e + c * \delta_r + c * (\Delta T + \Delta I + \Delta t) \tag{41}$$

Or:
ρ_e: The estimated distance between the satellite and the receiver.
δ_r: The error in calculating travel time
ΔT: The tropospheric error
ΔI: The ionosphere error
Δt: The satellite polarization clock error

11 | TROPOSPHERIC INFLUENCE

This part deals with the tropospheric delay of the pseudo-distances. For frequencies lower than 30 GHz the troposphere behaves essentially like a non-dispersive medium, that is to say that the refraction is independent of the frequency of the signals which cross it [19]. The effective height of the troposphere is around 40 km, typically, the tropospheric refraction is treated in two parts. The first part is the hydrostatic component which contains ideal gases. It is responsible for an overhead delay of approximately 240 cm, which can be calculated with precision from the pressure measured at the receiver antenna, the second part is the humidity component, also called the non-hydrostatic component. It is responsible for a delay of 40 cm in the zenithal direction, therefore the ZTD is the sum of the hydrostatic delay (ZHD) due to the effects of dry gas, and the ZWD due to the presence of water vapor [17,18]. It can be defined as the integrated refractivity along the signal path between the antenna and the atmosphere [19]:

$$ZTD = 10^{-6} \int_{antenna}^{atmosphere} N(s)\, ds \tag{42}$$

Where N is the neutral atmospheric refractivity, which is the sum of the hydrostatic refractivity (Nh) and the non-hydrostatic refractivity (Nw). Therefore, the equation for ZTD can be rewritten as [19,20]:

$$ZTD = 10^{-6} \int_{antenna}^{atmosphere} \left(N_h(s) + N_w(s)\right)\, ds \tag{43}$$

$$N = N_h + N_w = k_1 \frac{p_d}{T} + k_2 \frac{e}{T} + k_3 \frac{e}{T^2} \tag{44}$$

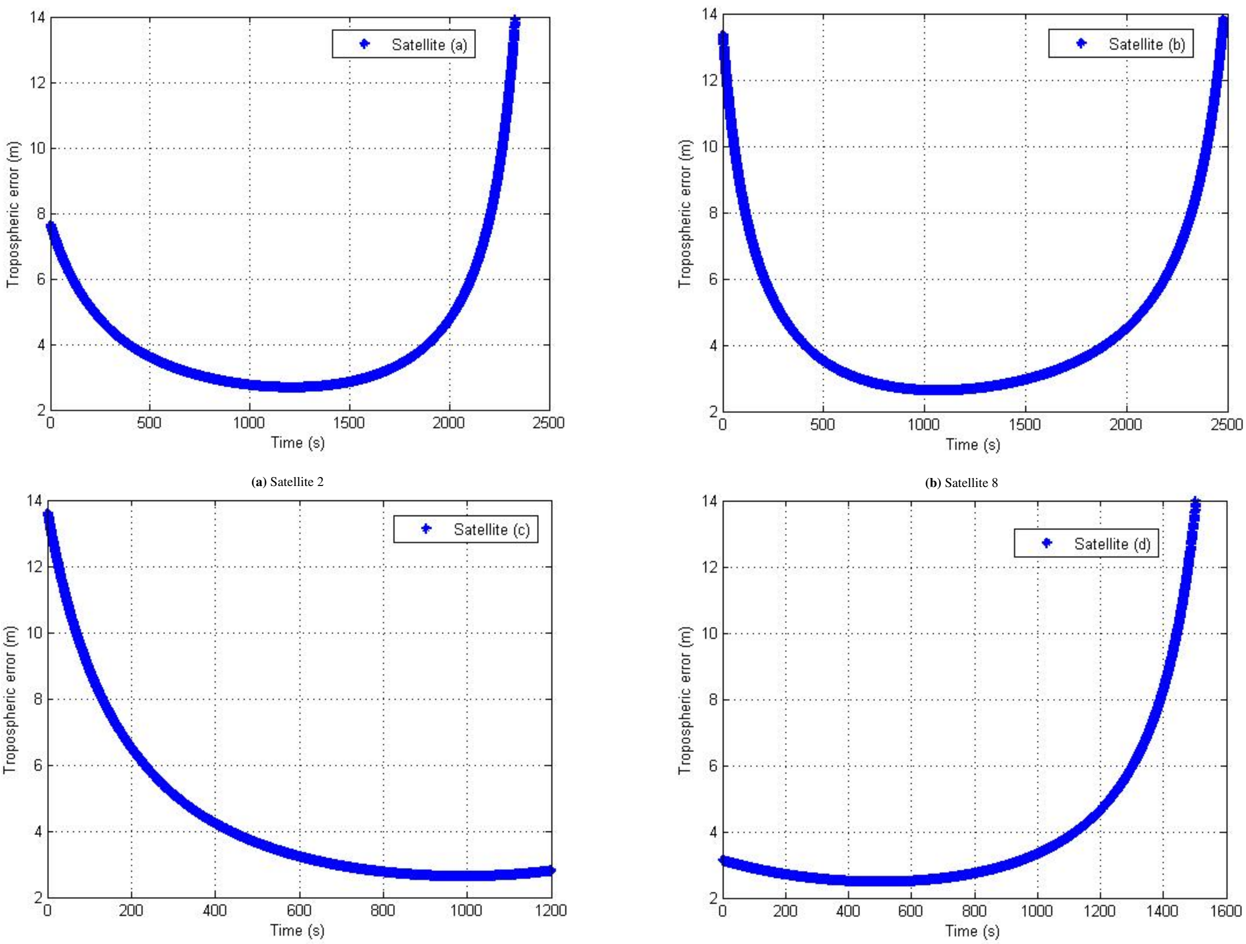
Where Pd is the pressure of the water vapor.

The ZTD can also be estimated using one of the many mathematical models developed. In our work, we used the Saastamoinen model, in which the ZTD can be expressed as [20]:

$$ZTD_{sam} = \frac{0.002277}{\cos z}\, f(\varphi, h) \left[P_s + \left(\frac{1.255}{T_s} + 0.05 \right) e_s \, - \, B \, tan^2 \, Z \right] + \Delta R \tag{45}$$

$$f(\varphi, h) = 1 \, + \, 0.0026 \, cos(2\,\varphi) + \, 0.00028 \, h \tag{46}$$

Where z is the angle of the zenith relative to the satellite, Ts is the surface temperature, Ps is the pressure on the surface, es the partial pressure of the water vapor in mbar, h the height of the station above sea level, φ the latitude of the station, B and ΔR are the correction terms. FIGURE 17 gives the troposphere delay for the 4 visible satellite.



(a) Satellite 2

(b) Satellite 8

(c) Satellite 11

(d) Satellite 21

FIGURE 17 Tropospheric delay

12 | THE IONOSPHERIC EFFECT

The ionosphere is the region of the atmosphere that lies between 50 and 1500 km above the earth. This area is characterized by the presence of free electrons and ions, the number of these free elections varies in time and space. The ionospheric effect is a major source of error in GPS measurement; the amount of ionospheric delay associated with a GPS signal can vary from a few meters to more than twenty meters in a day. In general, it is difficult to model the ionospheric effects due to complicated physical interactions between the electromagnetic field and solar activities, the ionosphere is a dispersive medium, that is to say that the ionospheric effect depends on the frequency. In addition to that, this delay is related to the number of free electrons along the transmission line from the satellite to the receiver, this number of free electrons is measured per unit of TEC which is equal to the number of free electrons in a column $1m^2$, with $1TEC = 10^{16} el/m^2$. Total electronic content (TEC) in the overhead direction can be defined as [21,22]:

$$TEC = \int N_e ds \tag{47}$$

Where Ne is the electron density.

Klobuchar [23] has developed a simple analytical model for ionospheric delay. We have used this ionospheric correction model to correct this error in our work. The GPS satellites broadcast the parameters of the Klobuchar ionospheric model in the navigation message. Using the parameters of the model, the ionospheric effects can be calculated and corrected. The steps to follow to calculate the delay due to the ionospheric effect are [13]:

Calculate the angle relative to the center of the Earth Ψ

$$\Psi = \frac{0.0137}{E + 0.11} - 0.22 \tag{48}$$

Calculate the sub-spherical latitude, ϕ_I in function of the satellite azimuth A

$$\phi_I = \Phi + \Psi \, cosA \tag{49}$$

$$\begin{aligned} &If \;\; \phi_I \geqslant +0.416 \;\; so \;\; \phi_I = +0.416 \\ &If \;\; \phi_I \leqslant -0.416 \;\; so \;\; \phi_I = -0.416 \end{aligned} \tag{50}$$

Calculate sub-spherical longitude

$$\lambda_I = \lambda + \frac{\Psi \sin A}{\cos \varphi_I} \tag{51}$$

Find the geomagnetic latitude, ϕ_m,

$$\phi_m = \phi_I + 0.064 \cos(\lambda_I - 1.617) \tag{52}$$

Find local time

$$t = 4.32 \; 10^4 \; \lambda_I + t_{GPS} \tag{53}$$

$$\begin{aligned} &If \;\; t \geqslant 86400 \;\; so \;\; t = t - 86400 \\ &If \;\; t \leqslant 86400 \;\; so \;\; t = t + 86400 \end{aligned} \tag{54}$$

Calculate the slope factor, F

$$F = 1 + 16(0.53 - E)^3 \tag{55}$$

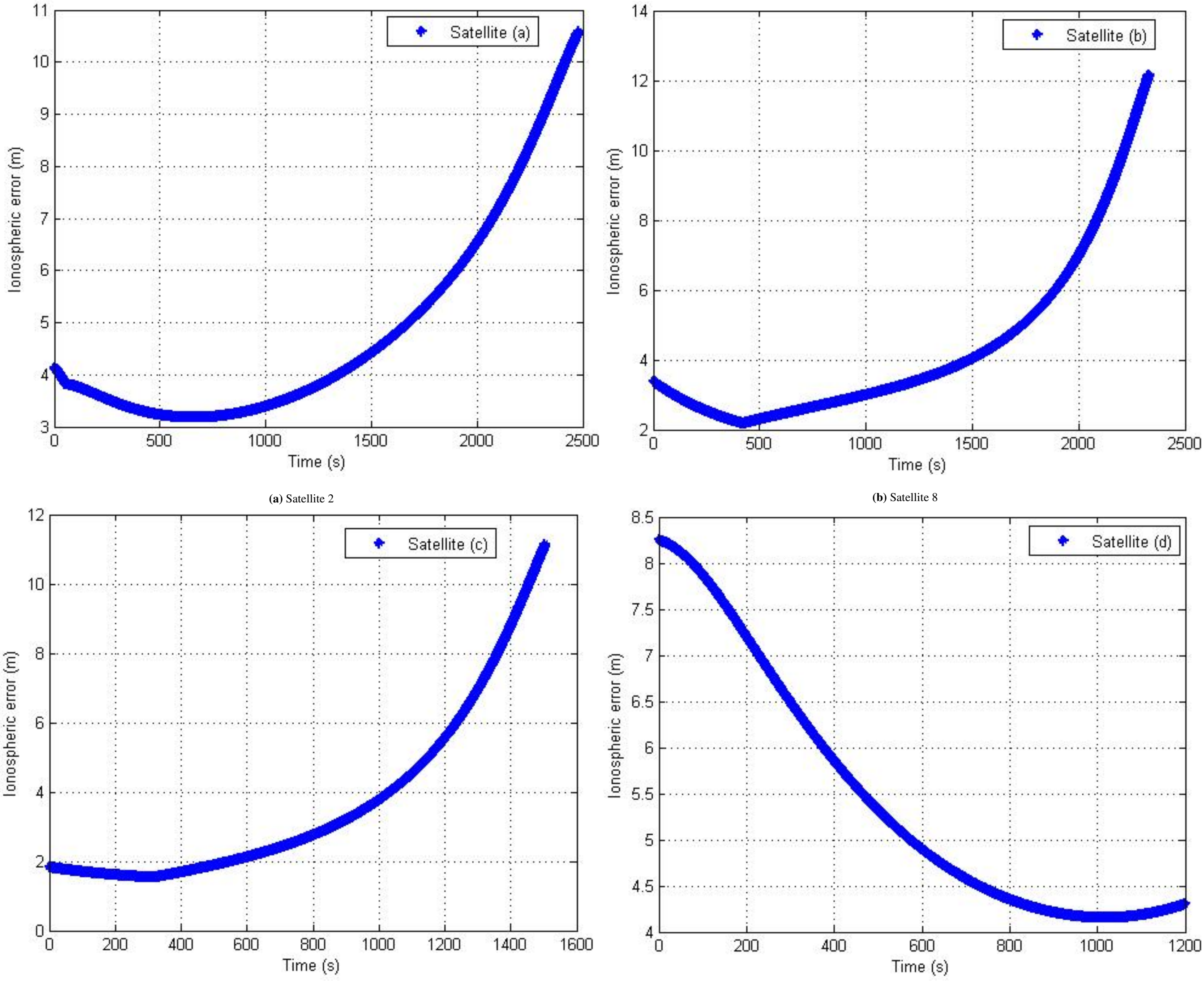
Calculate ionospheric time delay by calculating x

$$x = \frac{2\,\pi(t - 50400)}{\sum_{n=1}^{4} \beta_n \, \phi_n^m} \tag{56}$$

Calculate the ionospheric delay by:

$$\begin{aligned} &If \;\; |x| \geqslant 1.57 \;\; T_{iono} = F \; 5 \; 10^{-9} \\ &If \;\; |x| < 1.57 \;\; T_{iono} = F \; 5 \; 10^{-9} + \textstyle\sum_{n=1}^{4} \alpha_n \, \beta_n \left(1 - \frac{x^2}{2} \frac{x^4}{24}\right) \end{aligned} \tag{57}$$

The following figure (FIGURE 18) is the ionospheric error calculated by the Klobuchar model.



(a) Satellite 2

(b) Satellite 8

(c) Satellite 11

(d) Satellite 22

FIGURE 18 Ionospheric delay

13 | USER ALGORITHM FOR SV CLOCK CORRECTION

In this part we determine the offset of the satellite clocks (The polynomial correction coefficients transmitted in subframe 1 af0, af1 and af2 are used to determine this parameter. The approximate term for satellite clock correction is given by the following polynomial [13]:

$$\Delta_s = af_0 + af_1(t - t_{oe}) + af_2(t - t_{oe}) + \Delta t_R - T_{gd} \tag{58}$$

Tgd is the group delay. tr is the term of relativistic correction (seconds) which is given by:

$$\Delta t_R = Fe\sqrt{a}\sin E \tag{59}$$

The orbit parameters (e, a, e) used here are described in the discussions of the data contained in sub-frames 2 and 3, while F is a constant whose value is:

$$F = -2\frac{\sqrt{\mu}}{c^2} \tag{60}$$

FIGURE 19 is the clock error for each visible satellite.

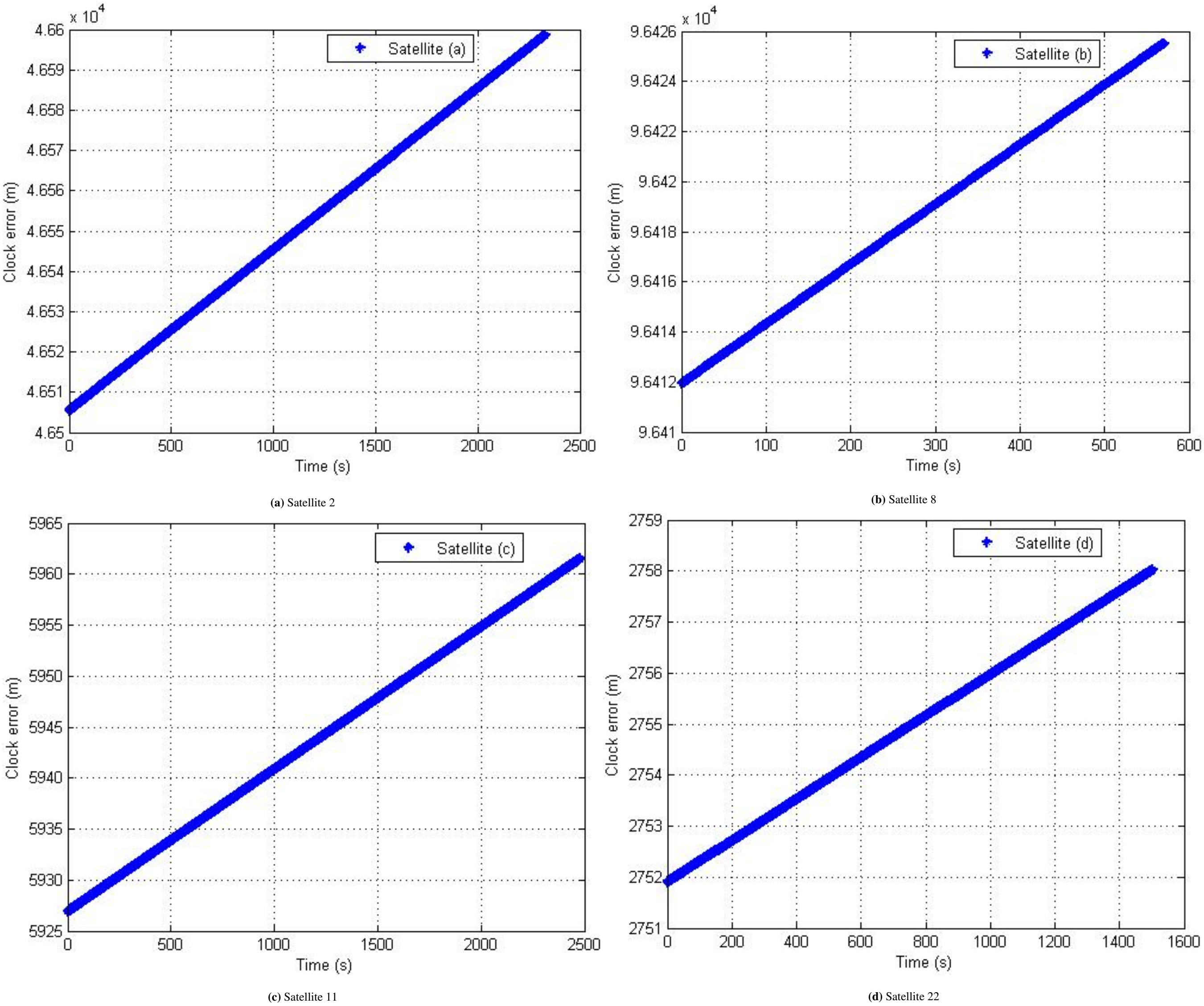


FIGURE 19 SV clock error

14 | POSITION CALCULATION

After correction of all errors, the navigation equation of each satellite contains four parameters which are the pseudo-distance and the three coordinates of the satellite X, Y and Z. The unknowns of this equation are the three coordinates of the receiver in addition to the error of estimation of the propagation time. We have four unknowns therefore we need four equations to solve this problem and calculate the position. So at least 4 observations from 4 different satellites are necessary [7]:

$$\rho_i = \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} + c\Delta t \tag{61}$$

The most used method to solve this equation is the one which exploits the linearization. The derivative of equation (61) gives:

$$\delta\rho_i = \frac{(x_i - x)\delta x + (y_i - y)\delta y + (z_i - z)\delta z}{\sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2}} + \delta t \tag{62}$$

$$\delta\rho_i = \frac{(x_i - x)\delta x + (y_i - y)\delta y + (z_i - z)\delta z}{\rho_i - c\Delta t} + \delta t \tag{63}$$

With $\delta x, \delta y, \delta z$ and δt unknowns. The above equation becomes a linear equation and can be written in matrix form:

$$\begin{bmatrix} \delta\rho_1 \\ \delta\rho_2 \\ \delta\rho_3 \\ \vdots \\ \delta\rho_n \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & 1 \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & 1 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_{n1} & \alpha_{n2} & \alpha_{n3} & 1 \end{bmatrix} + \begin{bmatrix} \delta x \\ \delta y \\ \delta z \\ \delta t \end{bmatrix}$$

with :
$\alpha_{i1} = (x_i - x)/(p_i - b)$
$\alpha_{i2} = (y_i - y)/(p_i - b)$
$\alpha_{i3} = (z_i - z)/(p_i - b)$
The solution of this equation is therefore:

$$\alpha x = [\alpha^T \alpha]^{-1} \alpha^T \delta p \tag{64}$$

The following figure (FIGURE 20) gives the position of a fixed point calcullated 1000 times.

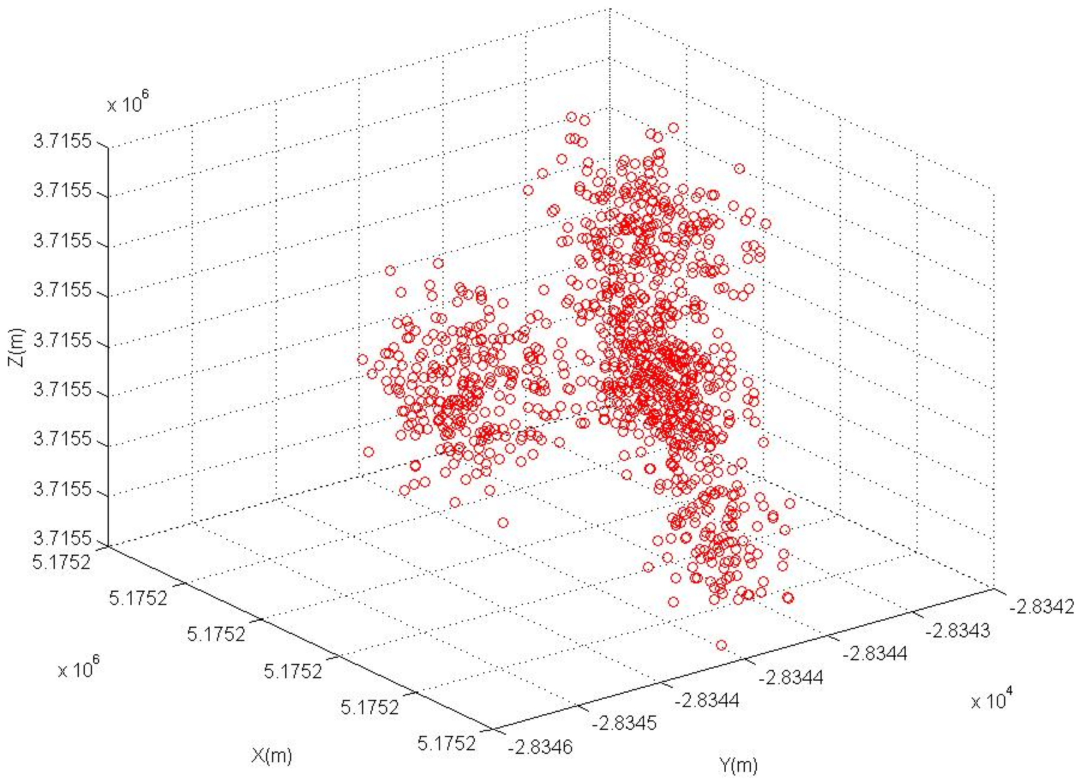


FIGURE 20 Position of a fixe point in ECEF frame.

The coordinates previously calculated are calculated in the ECEF coordinate system, therefore in X, Y and Z. To convert these coordinates into latitude longitude and altitude we use the following steps [4,15]:
The longitude λ of the receiver is defined as the angle between the receiver and the x-axis of the ECEF coordinate system measured in the x-y plane and can be calculated by:

$$\lambda = \begin{bmatrix} \arctan \frac{x}{y} & IF \ x \geqslant 0 \\ 180 + \arctan \frac{x}{y} & IF \ x < 0 \ \& \ y \geqslant 0 \\ 180 - \arctan \frac{x}{y} & IF \ x > 0 \ \& \ y < 0 \end{bmatrix} \tag{65}$$

The latitude φ of a point is defined as the angular distance which separates this point from the equator, expressed in degrees, this value can be calculated from Cartesian coordinates by the following equations:

$$\varphi = \arctan\left[\frac{z + e'^2 h \sin^3 \mu}{p - e^2 a \cos^3 \mu}\right] \tag{66}$$

With:

$$p = \sqrt{x^2 + y^2} \tag{67}$$

And

$$\mu = \arctan\left[\frac{z}{p}\frac{a}{b}\right] \tag{68}$$

The altitude h is the geodesic height, can be defined as the minimum distance between the receiver and the reference ellipsoid, the direction of this distance to the reference ellipsoid will be in the same direction of the vector p:

$$h = \sqrt{x^2 + y^2} cos\phi + z \sin\phi - a\sqrt{1 - e^2 \sin^2\phi} \tag{69}$$

The FIGURE 21 is the transformation of the calculated position to the geographic coordinate.

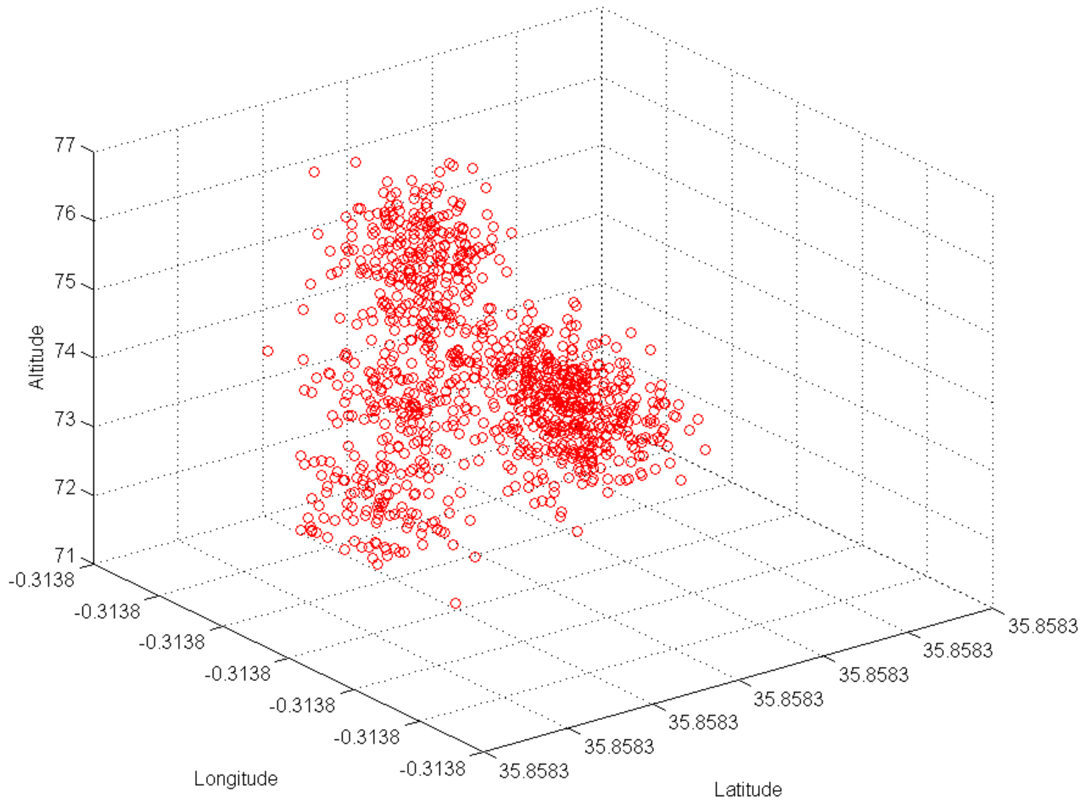


FIGURE 21 Position in Latitude, Longitude and Altitude.

15 | CONCLUSION

In this paper a GPS receiver is developed based on SDR technology and advanced signal processing methods. In this work we started with an overview on satellite location systems, an explanation on the location principle is given after the study done on the principle of satellite orbits. In the following we provide a detailed description on the structure of the signals of the American GPS navigation system, the navigation message, the spreading codes used and the frequencies used by this system for data transmission. All the all the techniques of signal processing (Correlation,PLL, DLL and FFT) used in positioning are presented.

Befor obtaining the users position, the errors introduce on the signal are corrected as well as the Ionospheric and Tropospheric errors.

CONFLICT OF INTEREST

The authors declare no conflict of interest. The funding agencies had no role in the design, writing or results analyzing of this article.

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