Comparison between PID controller and a nonlinear control based on neural network modelling applied to a ball and plate system

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Abstract

In this work we present two proposals to control the position and the path of a sphere on a two degree of freedom flat surface. This pair is known as ball and plate and the first approach consists in a linear approximation for the original dynamical model in order to apply a PID controller to get some metrics and compare to the second approach which is a neural network evolutionary training that make possible include the non linearity of the ball and plate dynamics . Both of theses strategies are applied to a virtual simulation where we can predict the behavior of the sphere based on a set of constitutive equations derived taking into account all rigid body and non-slip bearing simplifications. As the controllers are adjusted, we enter as input the actual position of the ball with a pattern recognition algorithm to mimic a sensing done with cameras commonly seen in experimental benches for this system.

Introduction

The ball and plate system is a device found in experimental benches at some universities around the world[1], where researches can propose different approaches to define a control model that would be more effective and require less energy. Those strategies are slightly distinct one from another not because of the complexity of the mechanical system which is actually quite simple but because of the control variables involved.

The response of controlled variables behaves according to its nonlinear governing equations making it a bit harder to predict a trajectory without any simplification and it is also possible to increase the complexity if we include the effect of viscous damping, loss of contact, slip or other features besides external disturbances. Otherwise the unconsidered effects would act in a black box mode or simply would take the model away of a real situation.

Wherefore many controllers have been developed and investigated as options to optimize few parameters and more recently those that are based on evolutionary training algorithms for neural networks [2] have allowed us to reach even better solutions for the path. It means we are searching for an equilibrium among lowing the energetic cost and to execute a quick and stable movement.

Before we start the comparison between these two control strategies we should have a brief overview on dynamical models in order to understand the physics that governs our simulations.

Having the set of equations the next step is to linearize and rewrite it in a state-space representation then proportional–integral–derivative (PID) controller can be set up and provide us the first data to use in comparison with a smarter control.

Finally we leave the linear approximation and start to design the artificial neurons of the neural network (NN) to low energetic cost without giving up the stability and speed of the control getting functions for the actuation and more data to compare.

Model of ball and plate dynamics

This is a three-dimensional model so in this case it is preferable to get the movement equations by Lagrange's formulation of mechanics because it minimize the number of required coordinates to be used in the characterization of involved bodies kinematics. Firstly we get positions and velocities, secondly we write the conservative terms of energy and then we differentiate a set of Lagrange's equations. The variables of the ball and plate system are shown bellow but in the text some of them are being accompanied by subscript b for the ball and p for the plate.

Table 1: Variables used in the model of ball and plate mechanical system and its description.

Symbol	**Description**	**Unity**
m	mass	kg
J	moment of inertia of the ball	kg.m^2
I_{ii}	moment of inertia of the plate at ith direction	kg.m^2
$\vec{r}$	position vector in relation to rotating frame	m
$\vec{r'}$	position vector in relation to non-rotating frame	m
d	external diameter of the ball	m
h_i	center of mass position for the plate at ith direction	m
Θ	rotation matrix around x axis	Arc
Φ	rotation matrix around y axis	Arc
$\vec{\Omega}$	angular velocity vector	Arc/s
T	kinetic energy	J
U	gravitational potential energy	J
g	gravitational acceleration constant	m/s^2
c_i	viscous damping constant at ith coordinate	kg/s or m^{2}kg/s
τ_i	applied torque at ith coordinate	N·m

We have assumed for the plate that it is free to rotate θ and ϕ around two orthogonal cartesian coordinates x and y respectively which are fixed to a joint so this system can not translate.

The sphere displacement mechanism only admits to roll over the plate without slip or loss of contact so with such simplifications we can relate all the angular cinematic of the ball to a correspondent linear quantity. The actuators action is included in Lagrange equations as two non conservative torques in each direction which we discuss better in the next section and then we are able to merge both dynamical subsystems.

In such coordinate system the position of the center of mass of the sphere relative to the vertical axis z does not vary regardless of whether the ball is massive or hollow since it has a radial symmetric distribution of mass. We can write the center of mass position as:

$$\vec{r_b} = x\hat{i} + y\hat{j} + \frac{d}{2}\hat{k} \tag{1}$$

And its linear velocity and the ball angular velocity [3] can be written by:

$$\dot{\vec{r_b}} = \dot{x}\hat{i} + \dot{y}\hat{j} + (\dot{\theta}y - \dot{\phi}x)\hat{k} \tag{2}$$

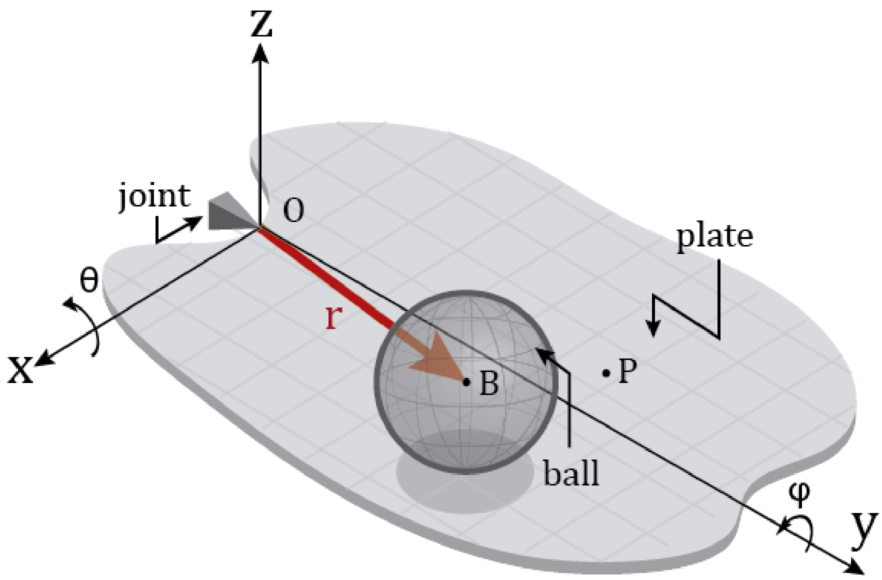

Figure 1: Group of coordinates considered to describe ball and plate dynamics. Point O represents the origin of rotating and non-rotating frame located near to the joint. Point P is the center of mass of the plate. Point B is the center of mass of the ball.

$$\vec{\Omega_b} = -\frac{2\dot{y}}{d}\hat{i} + \frac{2\dot{x}}{d}\hat{j} \tag{3}$$

Then we can express the kinetic energy of the ball:

$$T_b = \frac{m_b}{2}\dot{\vec{r_b}} \cdot \dot{\vec{r_b}} + \frac{J_b}{2}\vec{\Omega_b} \cdot \vec{\Omega_b} \tag{4}$$

The kinetic energy for the plate can be also computed in relation to a frame whose orientation accompanies the rotation. Assuming that the frame is aligned to the directions of the inertia principal axes of the plate. The inertia tensor is generically written as following:

$$[I] = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \tag{5}$$

and the angular velocity is:

$$\vec{\Omega_p} = \dot{\theta}\hat{i} + \dot{\phi}\hat{j} \tag{6}$$

And finally we have the last term of kinetic energy:

$$T_p = \frac{1}{2}\vec{\Omega_p} \cdot \left([I]\vec{\Omega_p}\right) \tag{7}$$

We have to consider a second frame which is non-rotating in addition to be in a state of equilibrium and then we can calculate a vertical position of the center of mass for both the ball and the plate in order to compute gravitational potential energy. There is a relation between these two frames given by a transformation calculated with a product of rotating matrices $[\Phi]$ and $[\Theta]$.

$$[\Theta] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix} \tag{8}$$

$$[\Phi] = \begin{bmatrix} \cos\phi & 0 & -\sin\phi \\ 0 & 1 & 0 \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \tag{9}$$

If we consider the position vectors related to non-rotating frame $\vec{r'}$ are being calculated in this order:

$$\vec{r} = [\Phi][\Theta]\vec{r'(10)}$$

We can compute the gravitational potential energy of an object relative to the chosen frame by:

$$U = m.g(\vec{r' \cdot \hat{k})}(11)$$

Defining the expression $L = T_b + T_p - U_b - U_p$ [4] as an appropriate Lagrangian, we differentiate it with respect to the generalized coordinates x, y, θ and ϕ that are being represented in equation below by q_k. To include non conservative effects to the system we must replace Q_k terms for such non conservative forces and torque like damping or friction but mainly the actuators action.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q_k}}\right) - \frac{\partial L}{\partial q_k} = Q_k \tag{12}$$

And we got these two PDE's for the position x and y of the ball:

$$\left(m_b + \frac{4J}{d^2}\right)\ddot{x} = -c_x\dot{x} - m_b(\dot{\theta}y - \dot{\phi}x)\dot{\phi} - m_b g\sin\phi \tag{13}$$

$$\left(m_b + \frac{4J}{d^2}\right)\ddot{y} = -c_y\dot{y} - m_b(\dot{\theta}y - \dot{\phi}x)\dot{\theta} - m_b g\cos\phi\sin\theta \tag{14}$$

And also these required torques in each of the coordinates x and y:

$$\begin{aligned} \tau_x = (m_b y^2 + I_{xx})\ddot{\theta} - xym_b\ddot{\phi} + (2\dot{y}ym_b + c_\theta)\dot{\theta} - m_b(\dot{x}y + \dot{y}x)\dot{\phi} \\ - \left[(m_b y - m_p h_y)\cos\theta + \frac{m_b d}{2}\sin\theta\right] g\cos\phi \end{aligned} \tag{15}$$

$$\begin{aligned} \tau_y = (m_b x^2 + I_{yy})\ddot{\phi} - xym_b\ddot{\theta} + (2\dot{x}xm_b + c_\phi)\dot{\phi} - m_b(\dot{x}y + \dot{y}x)\dot{\theta} \\ - \left[(-m_b y + m_p h_y)\sin\theta\sin\phi + \frac{m_b d}{2}\cos\theta\sin\phi + (-m_b x + m_p h_x)\cos\phi\right] g\cos\phi \end{aligned} \tag{16}$$

Model of actuators dynamics and integration

Linearization to a state-space representation

Linearizing the equations (13), (14), (15) and (16) obtained from ball and plate model using an first order approximation by Taylor series we find respectively:

$$\left(m_b + \frac{4J}{d^2}\right)\ddot{x} = -c_x\dot{x} - m_b g\phi \tag{17}$$

$$\left(m_b + \frac{4J}{d^2}\right)\ddot{y} = -c_y\dot{y} + m_b g\theta \tag{18}$$

$$I_{xx}\ddot{\theta} = -c_\theta\dot{\theta} + m_b g y + m_b g\frac{d}{2}\theta + \tau_x \tag{19}$$

$$I_{yy}\ddot{\phi} = -c_\phi\dot{\phi} + m_b g x + m_b g\frac{d}{2}\phi + \tau_y \tag{20}$$

PID controller: implementation and analysis

Neural Network evolutionary training: implementation and analysis

Conclusions

Acknowledgements

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