# Bead on Hoop Problem

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### Abstract

We investigated near equilibrium behavior of a wire. A small perturbation from an equilibrium point either showed stable behavior with the size of oscillation correlated to the size of the perturbation or some sort of unstable behavior. The phase space and state space diagrams are the same shape except for units so for convinience the state space is plotted in each case.

## Introduction

Our system is a bead on a wire. The wire is in the form of spinning loop of radius R with constant angular velocity ($\Omega$). $\theta$ is defined as the CCW angle starting at the bottom of the loop. $\omega$ is investigated at two values 1 rad/sec and 5 rad/sec. In the first case we investigate $\theta = 0$ and $\theta = \pi$. In the second case we investigate the same equilibrium plus the additional equilibrium arccos(g/($\omega^2 * R$)).

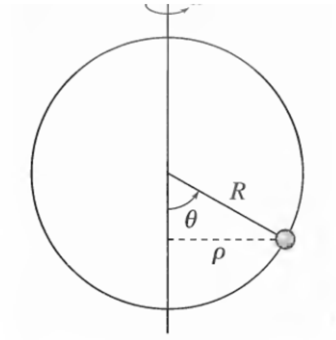

**Figure 7.9** **A bead is free to move around the frictionless wire hoop, which is spinning at a fixed rate $\omega$ about its vertical axis. The bead's position is specified by the angle $\theta$; its distance from the axis of rotation is $\rho = R \sin\theta$.**

Figure 1: Source: Taylor Mechanics Book

## Low angular speed for the hoop $\Omega = 1$

Fig. 2 shows oscillations about $\theta = 0$ a stable equilibrium point. Fig 3 shows oscillations about $\pi$ an unstable equilibrium that then oscillates about 0 or $2\pi$ radians.

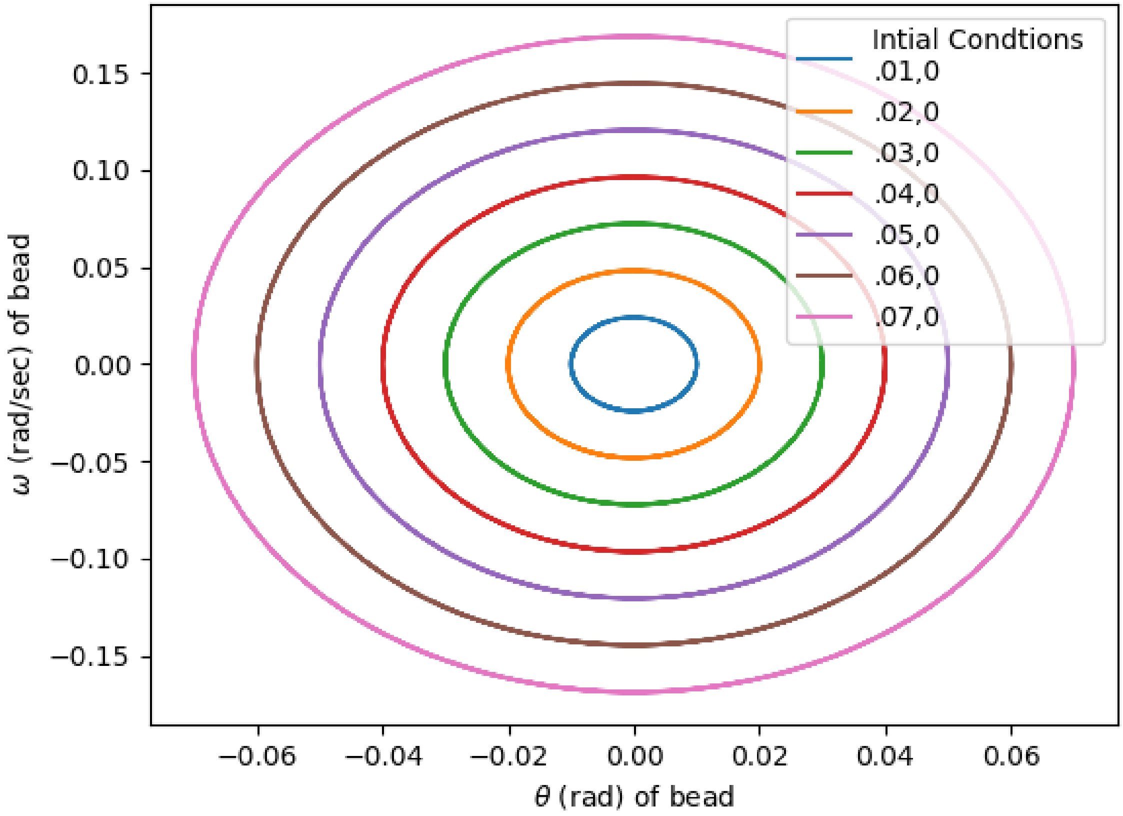

Figure 2: Near theta = 0

## High angular speed for the hoop $\Omega = 5$

Fig. 4 shows oscillations about $\theta = 0$ a stable equilibrium point. Fig 5 is a close up near shown in Fig. 4. It is a point of divergence. Fig 6 is shows the unstable equilibrium point behavior about $\theta = \pi$

References

Myself

Fig 1 Taylor Mechanics Book

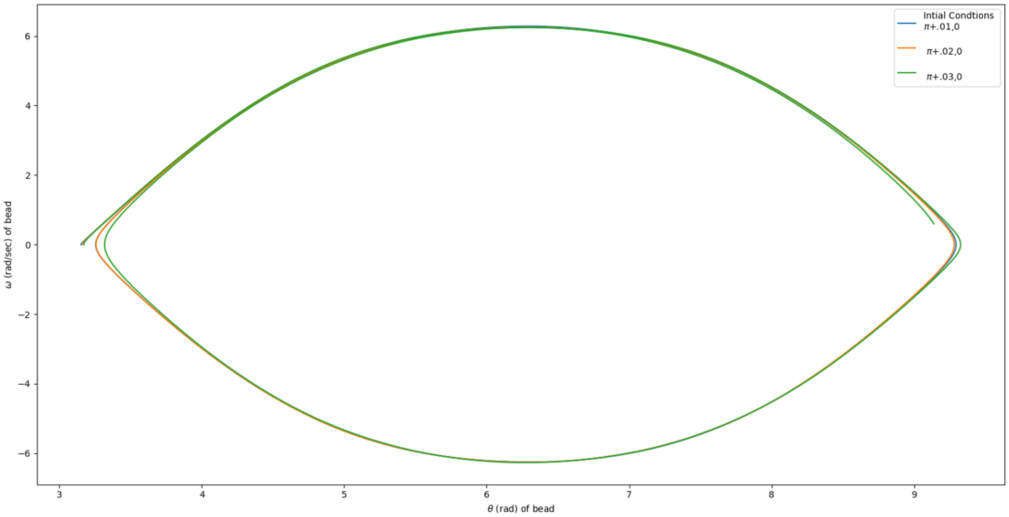

Figure 3: Near theta = pi

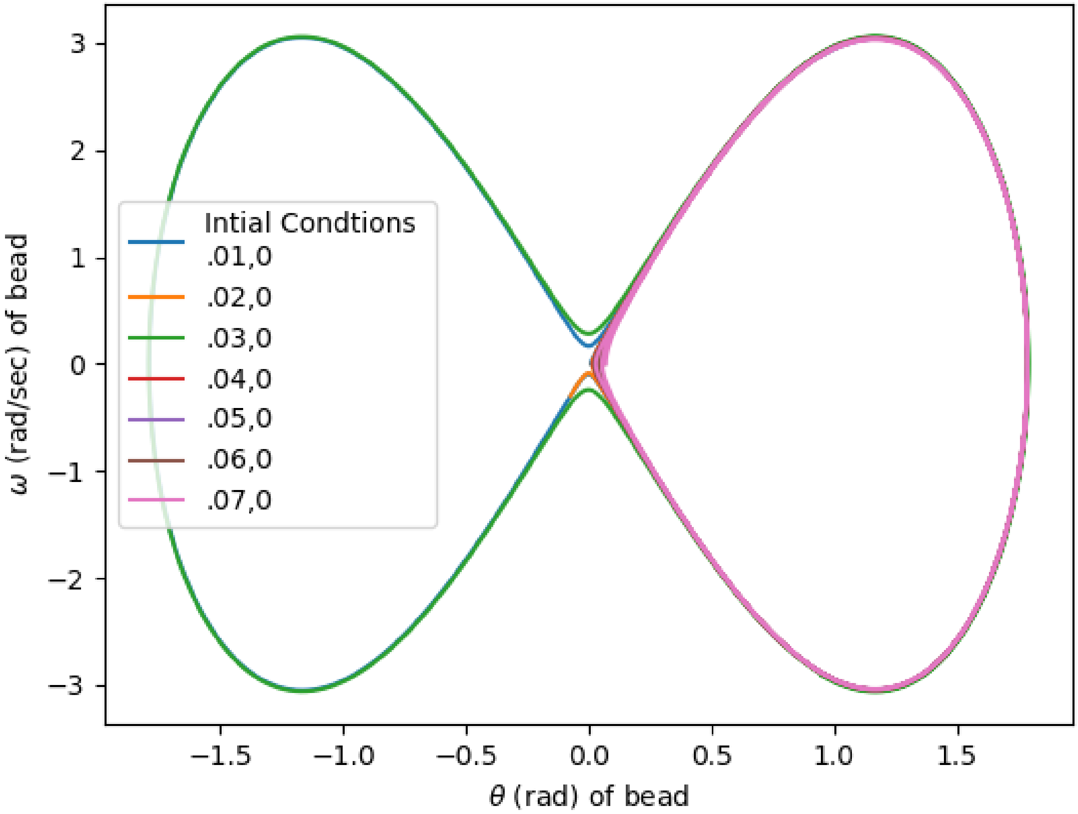

Figure 4: Behavior near theta = 0

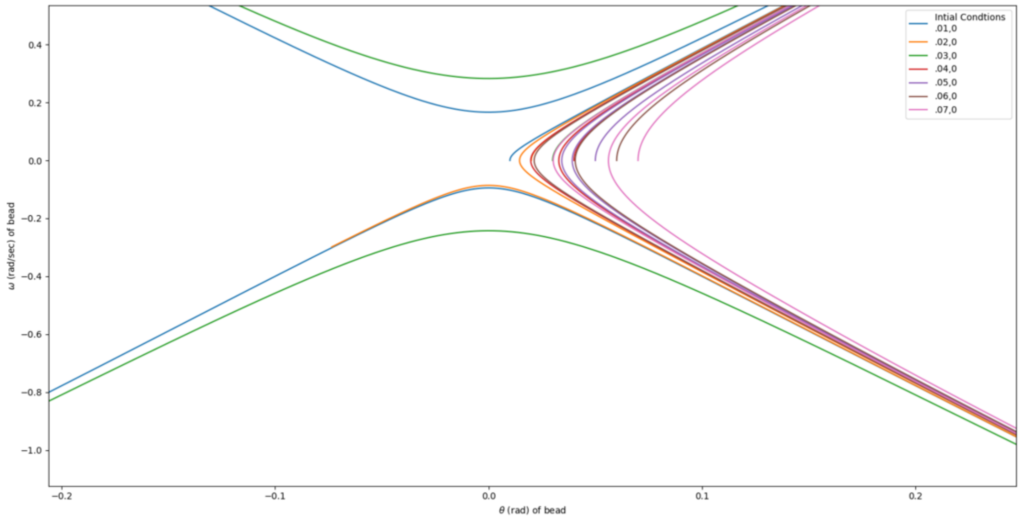

Figure 5: Close up of behavior near theta = 0

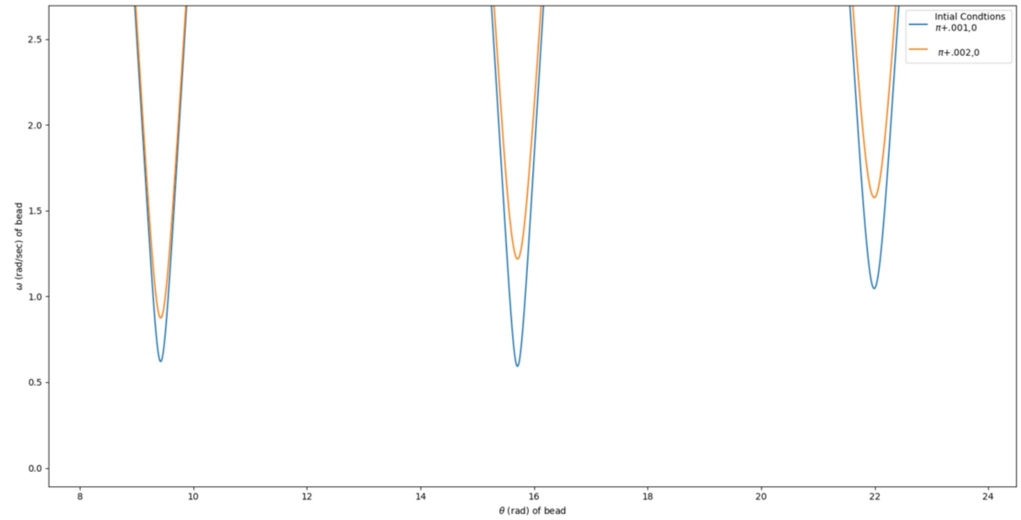

Figure 6: Behavior when initial conditions are near pi