MATH207 - Ordinary Differential Equations

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Abstract

Pellentesque tincidunt lobortis orci non venenatis. Cras in justo luctus, pulvinar augue id, suscipit diam. Morbi aliquet fringilla nibh, vel pellentesque dui venenatis eget. Orci varius natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Donec ultricies ultrices magna gravida porta.

Question 1

$$\begin{array}{l} \frac{dS}{dt} = -rSI + \mu \\ \frac{dI}{dt} = rSI - kI \end{array}$$

a) Do a qualitative analysis of the model given there and show that the solutions are not periodic.

b) Modify the equations to incorporate the changes as prescribed. Do a qualitative analysis to show that solutions are cyclic.

Answer:

$$\begin{array}{l} \frac{dS}{dt} = -rSI + S\mu \\ \frac{dI}{dt} = rSI - kI \end{array}$$

Question 2

Let $x = x\left(t\right)$ be the number of thousands of animal species *A* at time *t*.

Let $y = y\left(t\right)$ be the number of thousands of animal species *B* at time *t*.

$$Suppose \begin{cases} \frac{dx}{dt} = x - 0.5xy \\ \frac{dy}{dt} = y - 0.5xy \end{cases} \tag{1}$$

(a) Is the interaction between species *A* and *B* symbiotic, competitive, or a predator-prey relationship?
Answer: Competitive

(b) What are the equilibrium populations?
Answer:

$$\frac{dx}{dt} = 0 \; when \; x = 0 \; and \; y = 2$$
$$\frac{dy}{dt} = 0 \; when \; x = 2 \; and \; y = 0$$

Coordinates:

$$\frac{dx}{dt} = \frac{dy}{dt} = 0$$
$$when: \quad x = 0 \;\rightarrow\; (0,0) \qquad y = 2 \;\rightarrow\; (2,2)$$

$$\therefore \; equilibrium \; populations \begin{cases} x = 0 \; and \; y = 0 \\ x = 2 \; and \; y = 2 \end{cases}$$

(e) If $x = 0$, what happens to $y(t)$? How is this indicated in the phase-plane? If $y = 0$, what happens to $x(t)$? How is this indicated in the phase-plane?
Answer:
As $x = 0$, $\frac{dy}{dt}$ increases, which means that y increases.
As $y = 0$, $\frac{dx}{dt}$ increases, which means that x increases.

(g) For each of the initial conditions given below, describe how the number of species of *A* and *B* change with time and what the situations will look like in the long run.

i) $x(0) = 2 \; and \; y(0) = 1.8$
Answer: Population of *A* will increase over time and the population of *B* will decrease since it is a competitive environment. In the long run, the population of *B* would tend to zero.

ii) $x(0) = 2 \; and \; y(0) = 2.3$
Answer: Population of *A* will decrease over time and the population of *B* will increase since it is a competitive environment. In the long run, the population of *A* would tend to zero.

iii) $x(0) = 2.2 \; and \; y(0) = 2$
Answer: Population of *B* will decrease over time and the population of *A* will increase since it is a competitive environment. In the long run, the population of *B* would tend to zero.

Question 3

$$Suppose \begin{cases} \frac{dx}{dt} = 0.03x - 0.001x^2 - 0.01xy \\ \frac{dy}{dt} = 0.05y - 0.001y^2 - 0.01xy \end{cases} \tag{2}$$

Answer:

$$\frac{dx}{dt} = 0 \; when \; x = 0 \; and \; y = 3 - x$$
$$\frac{dy}{dt} = 0 \; when \; x = 5 - y \; and \; y = 0$$

The population will tend to the equilibrium point $(0, 5)$, this means that in the long run the population of A will tend to zero and the population of B will tend to 5000.