Exam 2

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**Q1 (a)**

By the Bayes’ rule,

$P[x]=\sum\_{i=1}^{J}P[x|y=i]P[y=i]=\sum\_{i=1}^{J}f\_{X}(x|y=i)π\_{i}$

$$\begin{matrix}P[y=i|x]&=\frac{P[x|y=i]P[y=i]}{P[x]}\\&=\frac{f\_{X}(x|y=i)P[y=i]}{P[x]}\\&=\frac{e^{−\frac{1}{2}\left(x−μ\_{i}\right)^{⊤}Σ\_{i}^{−1}\left(x−μ\_{i}\right)}π\_{i}}{(2π)^{p/2}det\left(\sum\_{i}^{}\right)^{1/2}}/P[x]\end{matrix}$$

**(b)** Since the denominator $P[x]$ and $(2π)^{p/2}$ do not depend on i, we can write them as a constant

$$P[y=i|x]=C\*f\_{X}(x|y=i)π\_{i}=\frac{Cπ\_{i}}{det\left(\sum\_{i}^{}\right)^{1/2}}e^{−\frac{1}{2}\left(x−μ\_{i}\right)^{⊤}Σ\_{i}^{−1}\left(x−μ\_{i}\right)}$$

Take the log of both sides, we have

$$logP[y=i|x]=logC+logπ\_{i}−\frac{1}{2}\left(x−μ\_{i}\right)^{T}Σ\_{i}^{−1}\left(x−μ\_{i}\right)−\frac{1}{2}log|Σ\_{i}|$$

Thus,

$$\begin{matrix}δ\_{i}(x)&=−\frac{1}{2}\left(x−μ\_{i}\right)^{T}Σ\_{i}^{−1}\left(x−μ\_{i}\right)−\frac{1}{2}log|Σ\_{i}|+logπ\_{i}\\&=−\frac{1}{2}x^{T}Σ\_{i}^{−1}x+x^{T}Σ\_{i}^{−1}^\_{i}−\frac{1}{2}^\_{i}^{T}Σ\_{i}^{−1}^\_{i}−\frac{1}{2}log\left|Σ\_{i}\right|+logπ\_{i}\end{matrix}$$

**(c)**



**(e)** Estimate sample mean and sample covariance for each class given the training data

$$^\_{j}=n\_{j}/n, ^\_{j}=\frac{1}{n\_{j}}\sum\_{x\_{i}\in C\_{j}}^{}x\_{i}, ^\_{j}=\frac{1}{n\_{j}−1}\sum\_{x\_{i}\in C\_{j}}^{}\left(x\_{i}−^\_{j}\right)\left(x\_{i}−^\_{j}\right)^{T}$$

where $j\in (1,2,3)$.

Thus, we have

$$^=0.57,^=0.37,^=0.06$$

$$^\_{1}=(−0.053,0.953)^{T},^\_{2}=(1.082,0.05067374)^{T},^\_{3}=(−0.825,1.241)^{T}$$

$$Σ\_{1}=\left[\begin{matrix}0.502&−0.219\\−0.219&0.460\end{matrix}\right], Σ\_{2}=\left[\begin{matrix}0.608&−0.159\\−0.159&0.421\end{matrix}\right], Σ\_{3}=\left[\begin{matrix}2.181&1.965\\1.965&1.816\end{matrix}\right]$$

The desicion boundary of QDA is given by $δ\_{1}(x)=δ\_{2}(x)=δ\_{3}(x)$.

**(f)**Scatter plot of the training data with labels

QDA: When n increased from 100 to 1000, the number of data points with predicted labels of 3 and 2 increased while the number of data points with predicted labels of 1 decreased.

$$^=0.606,^=0.284,^=0.11$$

$$^\_{1}=(−0.024,0.979)^{T},^\_{2}=(1.026,0.006)^{T},^\_{3}=(−1.157,0.923)^{T}$$

$$Σ\_{1}=\left[\begin{matrix}0.534&0.326\\0.326&0.500\end{matrix}\right], Σ\_{2}=\left[\begin{matrix}0.554&0.402\\0.402&0.535\end{matrix}\right], Σ\_{3}=\left[\begin{matrix}1.376&1.296\\1.296&1.323\end{matrix}\right]$$