

# Problem Statement<sup>1</sup>

Let a motion of the pursuer evolve in three-dimensional Euclidean space  $R^3$  and its dynamics be subject to the equation

$$\ddot{x} + \alpha \dot{x} = \rho u, \quad \|u\| \leq 1 \quad (1)$$

where  $x = (x_1, x_2, x_3)$  are geometric coordinates of the object. Here  $x_1, x_2$  denote coordinates in the horizontal plane and  $x_3$  a height. Vectors  $\dot{x} = \frac{dx}{dt}$  and  $\ddot{x} = \frac{d^2x}{dt^2}$  are velocity and acceleration, respectively;  $\alpha$  - friction coefficient;  $\rho > 0$  - resource coefficient;  $u$  - control, which is chosen in a unit ball centered at the origin of  $R^3$ ;  $\|x\| = \sqrt{(x, x)}$ , where by  $(\cdot, \cdot)$  is denoted a scalar product of vectors.

It is assumed that control  $u(t)$ ,  $t \geq 0$ , is Lebesgue measurable function of time. For simplicity's sake and in view of possible practical applications, it may be assumed that function  $u(t)$  is piecewise-continuous or even piecewise-constant.

The evader moves in the horizontal plane and his motion is described by the equation

$$\ddot{y} + \beta \dot{y} = \sigma \vartheta, \quad \|\vartheta\| \leq 1 \quad (2)$$

where  $y = (y_1, y_2)$  are coordinates of the object. Vectors  $\dot{y}$  and  $\ddot{y}$  denote velocity and acceleration of the evader at point  $y$ ,  $\beta > 0$  - coefficient of friction,  $\sigma > 0$  - coefficient of resources, and  $\vartheta$  - control of the evader, taking its values in a flat circle centered at the origin. In the sequel we shall sometimes write  $\tilde{y} = (y_1, y_2, 0)$  or even  $\tilde{y} = (y, 0)$  in order to treat  $y$  as vector in  $R^3$ .

The game  $(??)$ ,  $(??)$  will be analyzed from the pursuer's point of view. His goal is to achieve "soft meeting" with the evader at a finite instant of time:

$$\|x - \tilde{y}\| \leq \varepsilon_1, \quad \|\dot{x} - \dot{\tilde{y}}\| \leq \varepsilon_2 \quad (3)$$

where  $\varepsilon_1, \varepsilon_2$  are positive numbers, specifying the required proximity of the players. The hyperplane  $\{y_3 = 0\}$  stands for state constraint of the pursuer. The pursuer is allowed to move in this hyperplane, not intersecting it.

---

<sup>1</sup>Arkadii A. Chikrii, "Soft Landing of Moving Objects", Cybernetics Institute, Ukraine, Kyiv-1998

Without loss of generality, the initial state of the pursuer is assumed to lie in the upper halfspace, that is  $x_3^0 = x_3(0) > 0$ .

To simplify the treatment, we set  $\varepsilon_1 = \varepsilon_2 = 0$ , that is we shall study the precise “soft landing”. Note, that it is easy to pass from this problem to the problem (??)-(??) and the solution of problem (??)-(??) immediately follows from the solution of problem on precise “soft landing”.

For the sake of convenience, let us reduce the second order system (??), (??) to a system of first order but yet of larger dimension with the help of introduction of new variables

$$z_1 = x, \quad z_2 = \dot{x}, \quad z_3 = \tilde{y}, \quad z_4 = \dot{\tilde{y}}$$

Differentiating the above equalities in time and taking into account the equations (??), (??) we obtain an equivalent system

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= -\alpha z_2 + \rho u \\ \dot{z}_3 &= z_4 \\ \dot{z}_4 &= -\beta z_4 + \sigma \vartheta \end{aligned} \tag{4}$$

In the strict sense (??) is a system of  $12^{th}$  order, but, in fact, only of  $10^{th}$ , since although vectors  $z_1, z_2, z_3, z_4$  are three-dimensional,  $z_3$  and  $z_4$  have zero third components.

Thus, the pursuer strives to bring a trajectory of system (??) to a linear subspace

$$M_0 : z_1 = z_3, \quad z_2 = z_4 \tag{5}$$

or to a certain its neighbourhood for any admissible counteraction of the evader. In order to formulate the problem (??), (??) in a more general form and to develop a general approach for solution of the linear game problem we shall present the motion of a conflict-controlled process in the form

$$\dot{z} = Az + u - \vartheta, \quad u \in U, \quad \vartheta \in V, \quad z \in R^n \tag{6}$$

where  $A$  is a square matrix of order  $n$ ,  $U, V$  are nonempty compacts, and the terminal set is a cylindrical set

$$M^* = M_0 + M \tag{7}$$

Here  $M_0$  is a linear subspace of  $R^n$  and  $M$  is a compact from the orthogonal complement  $L$  to  $M_0$  in  $R^n$ . By  $\pi$  is usually denoted the operator of orthogonal projection from  $R^n$  to  $L$ .

One can easily see that the problem of “soft landing”, formulated in the form (??)-(??) or (??), (??), is a specific case of the differential game (??), (??). Pontryagin’s condition for the problem (??), (??) means the nonempty of set-valued mapping

$$W(t) = \pi e^{tA} U^* - \pi e^{tA} V \neq \emptyset \quad \forall t \geq 0 \quad (8)$$

The availability of information on a current state of the game to the pursuer will be specified separately for each particular method, presented in the paper. Denote states of the players (??), (??) by

$$\bar{x} = (x, \dot{x}) = (z_1, z_2), \quad \bar{y} = (\tilde{y}, \dot{\tilde{y}}) = (z_3, z_4)$$