Problem Set # 1 Advanced Macroeconomics

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1.Derive an expression for the profit function π.

The profit function can be written as $\pi = p \cdot an^{\alpha} - n - c_f$.

2. Derive an expression for the labor policy function $n(a)$.

The labor policy function can be obtained by taking first order derivative w.r.t n, which is, $n(a) = (\alpha pa)^{\frac{1}{1-\alpha}}$

3. Computation

Given the parameters and productivity grids, the model can be solved computationally. The following results conclude the equilibrium features of interest:

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Results:
The price (p*) is                                          0.983329
The exit threshold (a*) is                                 4.00703
The measure of entrants (m*) is                            0.0799328
The entry/exit rate is                                     0.125499
The aggregate employment (or average firm size) is         65.5552
The aggregate output is                                    100
The aggregate profit is                                    20.0393
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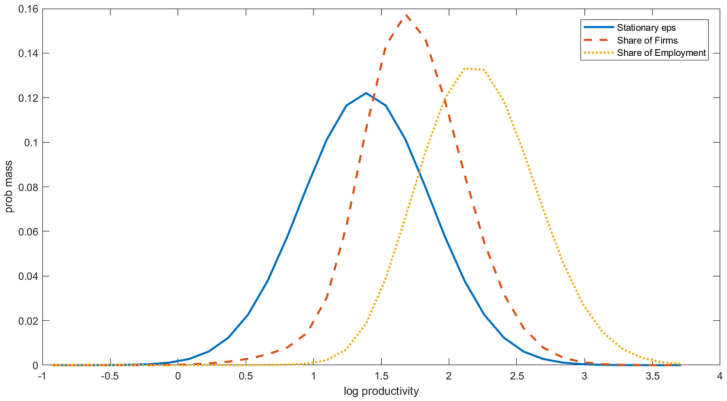

Figure 1: Stationary Distributions with D=100

The distributions are shown in **Figure 1**. It's obvious that the stationary distributions of firms and employment are shifted to the right side of the distribution of productivity levels. The reason that the distribution of firms deviates to the right of the productivity is that firms can always choose not to be in the industry if they find it not profitable given the fixed cost of staying and entering in the industry. That is especially true when the productivity is low and below the cutoff value. Share of employment is moving further right because labor demand is a strictly increasing and convex function of productivity meaning that higher productivity firms are more likely to employ more labor. This shifts the distribution of labor demand right to the distribution firms.

4. A Higher Demand

If the fixed demand is changed from 100 to 120, the results are shown below:

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Results:
The price (p*) is                                    0.983329
The exit threshold (a*) is                           4.00703
The measure of entrants (m*) is                      0.0959194
The entry/exit rate is                               0.125499
The aggregate employment (or average firm size) is   78.6663
The aggregate output is                              120
The aggregate profit is                              24.0471
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Clearly there's no change in the equilibrium price, exit threshold or entry/exist rate but the measure of entrants is increased from 0.08 to 0.096. Average firm size also increases to 78.67 with an elevated aggregate firm profit.

The reason that the demand increase has such an impact is that given the labor as a numeraire and taking price and wage rate as given, incumbents would employ more labor and increase output, hence increases the expected payoff of the entrants and attracts more entrants until free entry condition rebinds. It's also computationally clear as the process of determining equilibrium price has nothing related to the demand. This also explains why a larger measure of entrants is seen.

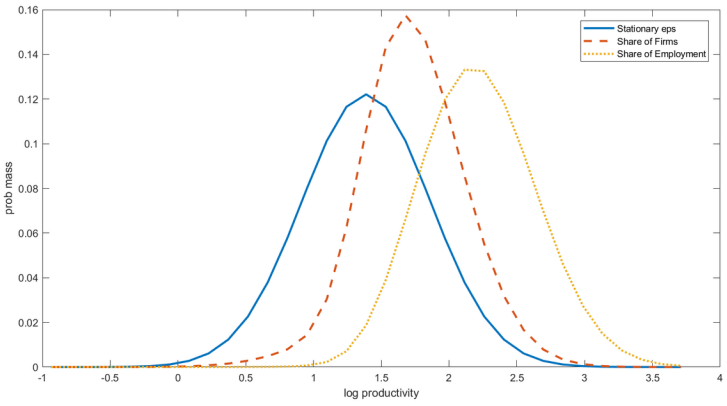

Figure 2: Stationary Distributions with D=120

The stationary distributions also remain the same as the previous case. To see the reason, note that the mass of firms is homogenous of m, or $\mu = m(1-\Psi)^{-1}g$. The overall measure $\int_A \mu$ is also homogenous of

m. Thus the share of firm remains the same if m is in both denominator and nominator. The implication is that the additional entrants due to a higher D only increase the measure of firms proportionally across the distribution of productivity. Same reason for the share of employment.

5. Research Question

In the Hopenhayn model, firm size distribution (or equivalently productivity) is stochastically increasing in the age of the cohort. And new entrants are set to start with a relatively lower productivity hence a smaller size compared with older firms. However, in Foster et al. (2008), they find that earlier literature (including the Hopenhayn's model) understates new producers' productivity advantages since previous work usually did no distinguish between the revenue productivity and physical productivity. They show that physical productivity is inversely correlated with price while revenue productivity is positively correlated with price. And furthermore, young producers charge lower prices than incumbents.

To capture this feature, a model of demand differentiation could be added into the Hopenhayn's model. Specifically, in addition to the firm-specific productivity, new entrants start with a demand of a smaller consumer base (*e.g.* local markets). Once they become incumbents, they can try to move to new markets (*e.g.* state market) and expand their consumer bases by invoking marketing costs and draw a new productivity from the same distribution as the initial one where they drew to become an entrant.

Equivalently, it can be thought as a combination of markets of different sizes with different entering costs that incumbents could sequentially choose in the Hopenhayn's model. Under this setting, higher entering cost markets imply higher market prices and higher revenues on average (if the result of Q4 applies), but does not imply a growing physical productivity over time or across the markets, which is what Foster et al. (2008) describes.

References

Lucia Foster, John Haltiwanger, and Chad Syverson. Reallocation Firm Turnover, and Efficiency: Selection on Productivity or Profitability? *American Economic Review*, 98(1):394–425, feb 2008. doi: 10.1257/aer.98.1.394. URL https://doi.org/10.1257%2Faer.98.1.394.