RLC circuits

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Introduction

The goal of this experiment was to investigate the main characteristics of three different types of electric circuits: resistor-capacitor (RC), resistor-inductor (RL), resistor-capacitor-inductor (RLC). The different types of circuit assembly will be dealt with through this report. The experiment also deals with the frequency dependency of these circuits. The experiment was useful to investigate how an RLC circuit behaves on a time dependent basis and also as a function of the frequency. Resonance is also an important part of this experiment. It is important to note that all the data acquired during the first week of the experiment was lost due to human error. The data used in this report was generously shared with us by our colleagues.

Part I

RL and RC circuit in AC

High pass filters

Experiment goal

The goal of this part of the experiment is to show the characteristic behavior of Rl and RC circuits.

Theoretical background

A high pass filter is a circuit that eliminates signals below its cut off frequency f_c, consider the circuit in fig 1.

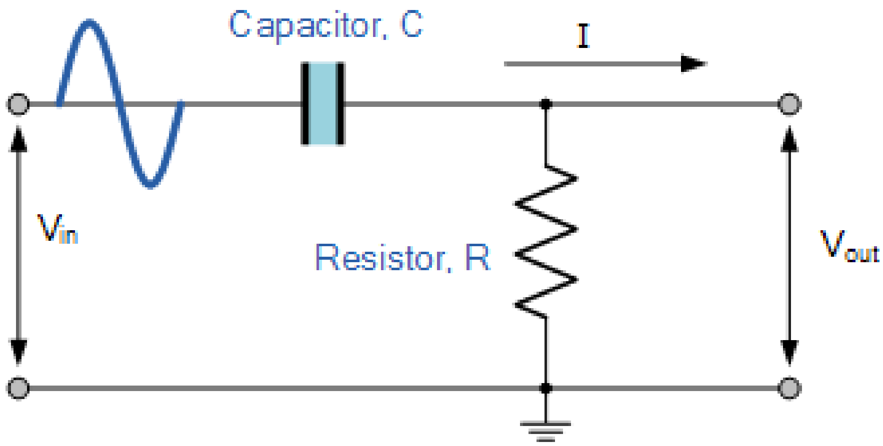

Figure 1: RC high pass circuit

The reactance of the capacitor at low frequencies is very high. It then acts like an open circuit and blocks the signal. The amplitude of the voltage across the resistor can be found using the circuits resistor gain:

$$G_R = \frac{|V_{out}|}{|V_{in}|} = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}} \tag{1}$$

where ω is the source frequency

Cut off frequency

The cut off frequency f_c is the frequency at which the energy flowing through the system is reduced. Its calculation will depend on the type of the circuit. In a RC circuit, the cutoff frequency will be given by:
$f_c \ = \ \frac{1}{2\pi\tau}$
$\tau \ = \ RC$
$f_c \ = \ \frac{1}{2\pi RC}$
Another way of identifying the cut off frequency is by a 45° phase shift of the output signal from the input signal.

Methods

In this part of the experiment, the goal was to study the characteristic of the capacitor in a high pass filter under AC. In order to achieve that, the following circuit was built:

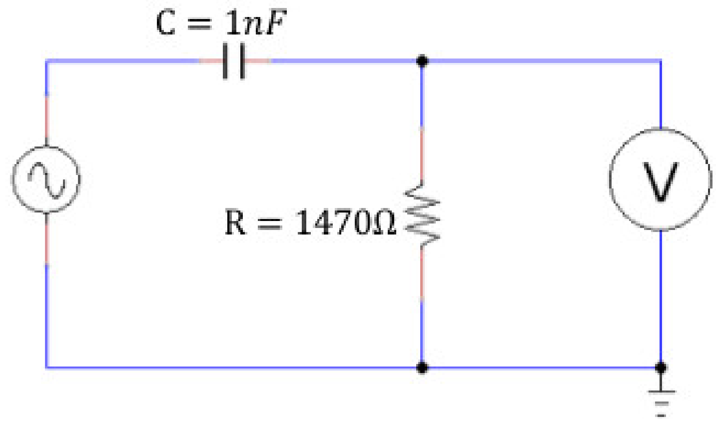

Figure 2: RC circuit - High pass filter

The cutoff frequency for this circuit is: $f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(1x10^{-9})(1470)} = 108268.67Hz$
$f_c = 108.3\ kHz$

After building the circuit shown in fig. 2, measurements were taken of the input voltage and the voltage on the resistor. An oscilloscope was used for this. The trigger function of the oscilloscope was set only on the channel measuring the input voltage, i.e. the voltage from the AC source.
The wave shape used was sine and the amplitude was set to 1.2 Volts. For each frequency, the amplitude-phase difference ratio was sampled and its graph appears at the end. This ratio was measured between the input and output voltage.
The frequency was scanned through different magnitude orders. The input and output voltages were measured for this frequency.

Results

When changing the frequency of the input there is a change in phase and amplitude of the output, as can be seen in figure 3 below.
Note: the yellow subplot is about 45° phase shift)

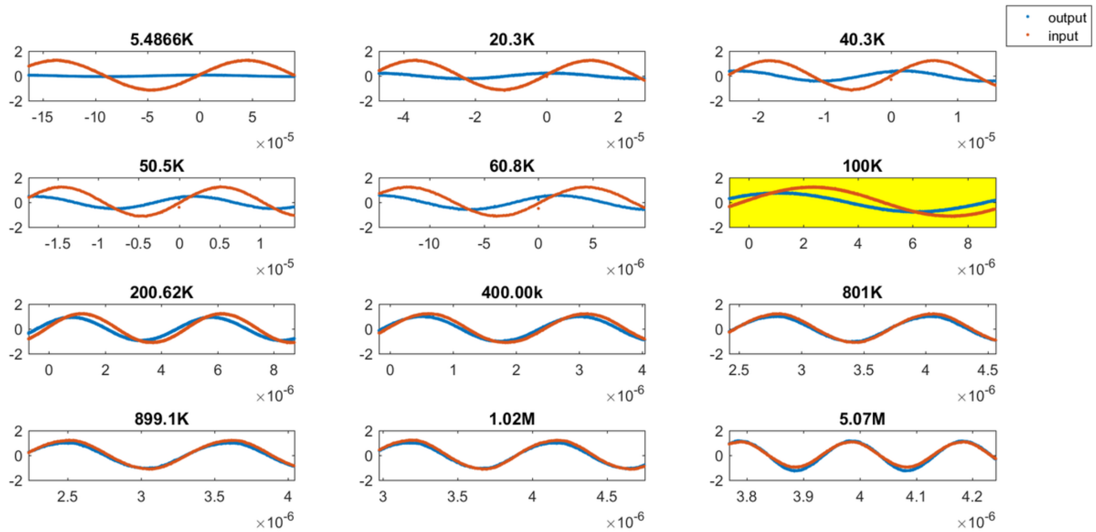

Figure 3: Input and output signals at different frequencies

The cut off frequency is expected to be at 45° angle, see fig 4 below.

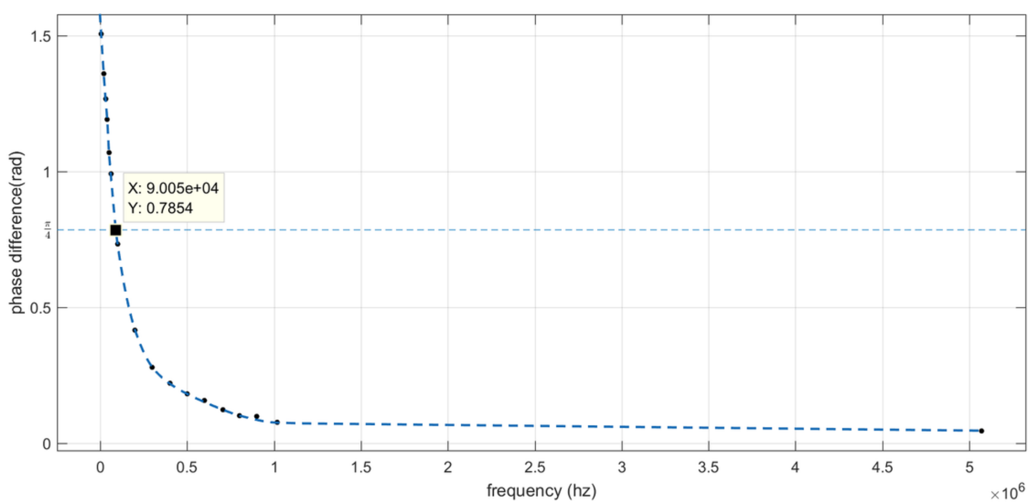

Figure 4: Frequency in relation to phase difference

When represented in a logarithmic scale the cut off frequency is expected at -3 decibels, see fig 5 below.

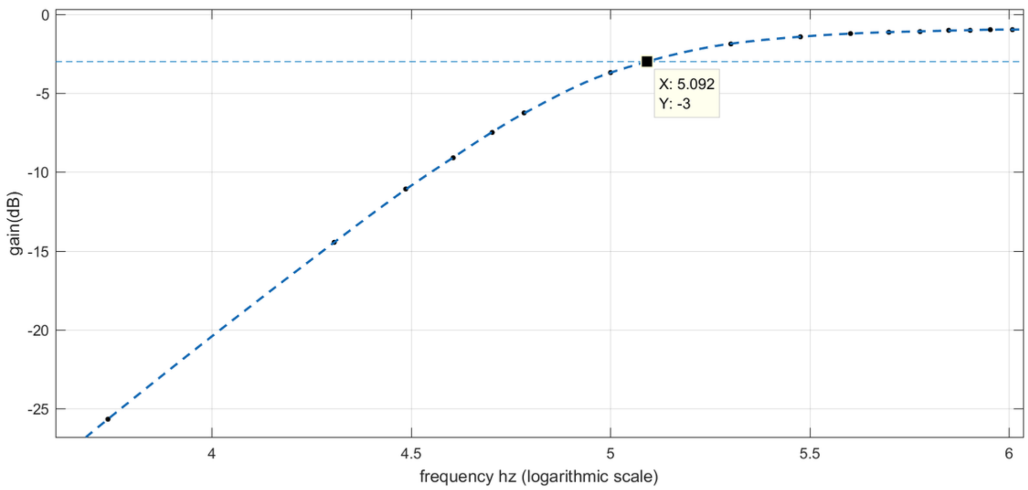

Figure 5: Frequency in relation to gain (logarithmic scale base 10)

Conclusion

The theory predicts a cut off frequency of 108 kHz.
Our measurements give values of 90 kHz when looked at the frequency in relation to phase (see fig 4) and 123 kHz when looked at frequency in relation to gain (see fig 5).
However, around that value our measurements fail to hit the mark. The reasons for it could be:

- The internal resistance of the power supply was not taken into account
- When fitting the data, Matlab failed to fit our theoretical model (see eq 1). Therefore, we interpolated the graph which may have skewed our results.
- Exact measurements of the capacitor and resistor were not taken, instead data supplied in the experiment instructions was used

Low pass filters

Theoretical background

A low pass filter is a circuit that eliminates signals above its cut off frequency f_c, consider the circuit in fig 6

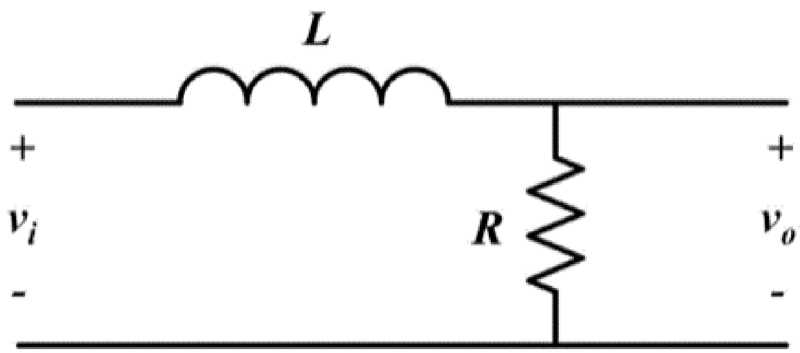

Figure 6: LR low pass filter

The impedance of the inductor at high frequencies is very high and so it resists the change in current effectively acting like a resistor and lowering the voltage. The amplitude of the voltage across the resistor can be found using the circuits resistor gain:

$$G_R = \frac{|V_{out}|}{|V_{in}|} = \frac{R}{\sqrt{R^2 + (\omega L)^2}} \tag{2}$$

where ω is the source frequency

Cut off frequency

The cut off frequency f_c is the frequency at witch the energy flowing through the system is reduced. Its calculation will depend on the type of the circuit. In a RL circuit, the cutoff frequency will be given by:
$f_c = \frac{1}{2\pi\tau}$
$\tau = \frac{L}{R}$
$f_c = \frac{R}{2\pi L}$
Another way of identifying the cut off frequency is by a -45° phase shift of the output signal from the input signal.

Methods

In this part of the experiment, the goal was to study the characteristic of the capacitor in a low pass filter under AC. In order to achieve that, the following circuit was built.

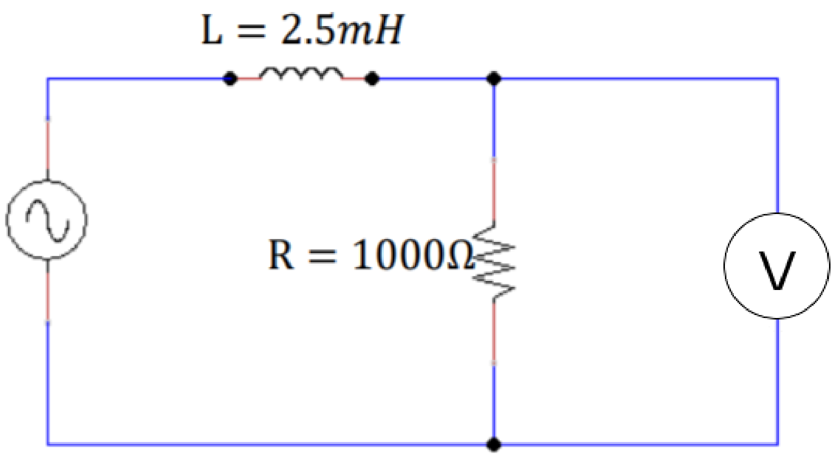

Figure 7: RL circuit - Low pass filter

The cutoff frequency for this circuit is: $f_c = \frac{R}{2\pi L} = \frac{1000}{2\pi(2.5x10^{-3})} = 63662Hz$
$f_c = 63.6\ kHz$

After building the circuit shown in fig 7, measurements were taken of the input voltage and the voltage on the resistor. An oscilloscope was used for this. Its trigger function was set only on the channel measuring the input voltage, i.e. the voltage from the AC source.
The wave shape used was sine and the amplitude was set to 1.2 Volts. For each frequency, the amplitude-phase difference ratio was taken and its graph appears at the end. This ratio was measured between the input and output voltage.
The frequency was scanned through different magnitude orders. The input and output voltages were measured for this frequency.

Results

When changing the frequency of the input there is a change in phase and amplitude of the output as can be seen in figure 8 below.

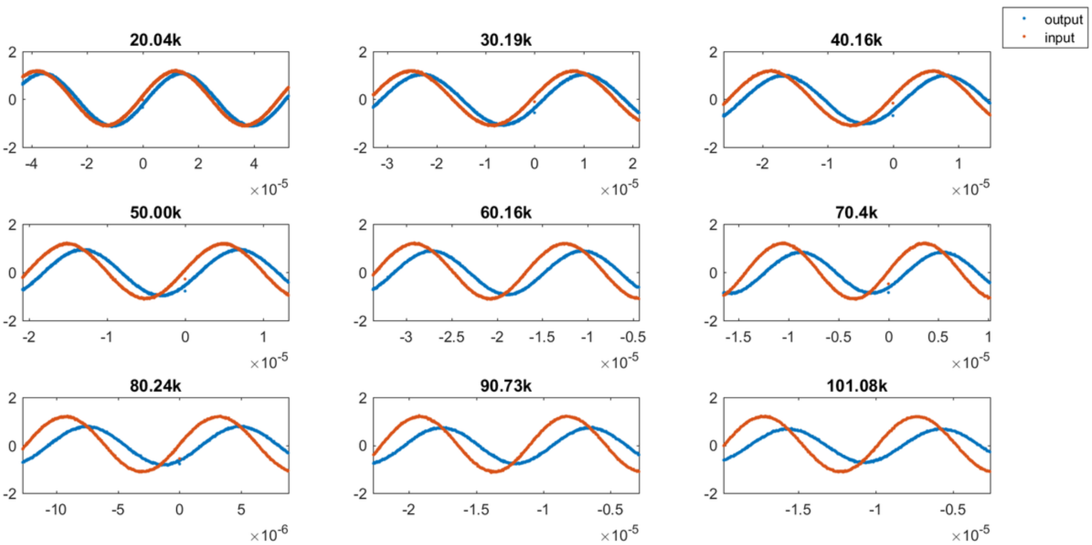

Figure 8: Input and output signals at different frequencies

The cut off frequency is expected to be at 45° angle, see fig 9 below.

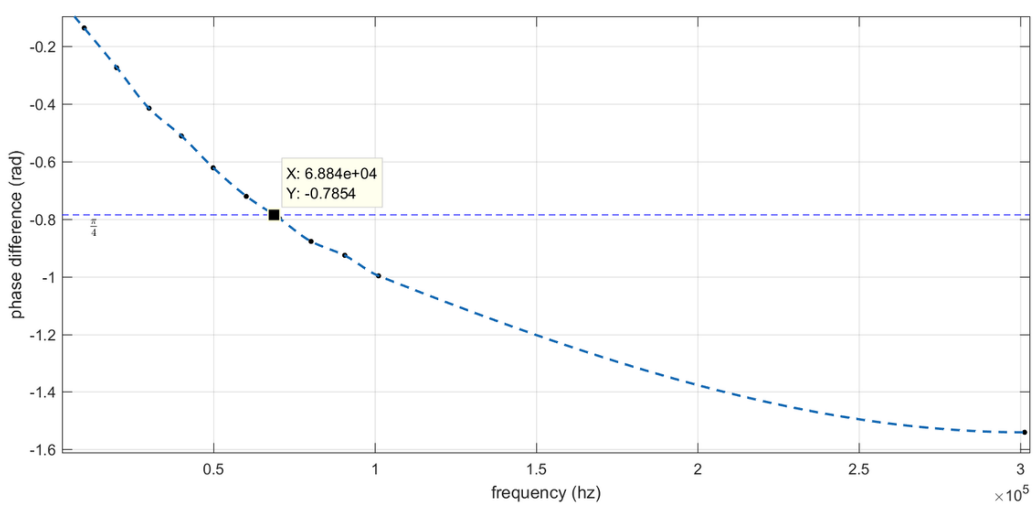

Figure 9: Frequency with relation to phase difference

When represented in a logarithmic scale the cut off frequency is expected at -3 decibels see fig 10 below.

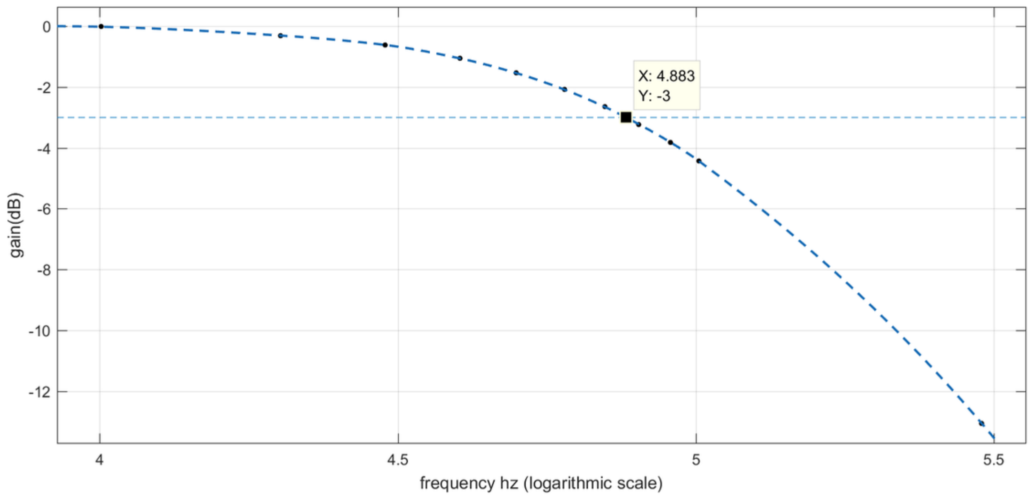

Figure 10: Frequency in relation to gain (logarithmic scale base 10)

Conclusion

The theory predicts a cut off frequency of 63.6 kHz.
Our measurements give values of 68.8 kHz when looked at the frequency in relation to phase (see fig 9) and 76.4 kHz when looked at frequency in relation to gain (see fig 10).
However,around that value our measurements fail to hit the mark. The reasons for could be:

- The internal resistance of the power supply was not taken into account
- When fitting the data Matlab failed to fit our theoretical model (see eq 2). Therefore, we interpolated the graph which may have skewed our results
- Exact measurements of the capacitor and resistor were not taken. Instead data supplied in the experiment instructions was used

Part II

RLC circuit

Responses of RLC circuits to quick changes in voltage

Experiment goal

The goal of this part of the experiment is to show the relation between the resistance of an RLC circuit and its oscillations.

Theoretical background

RLC circuit

An RLC circuit is an electrical circuit consisting of a resistor, an inductor and a capacitor.
When "disturbed", the voltage over its capacitor time behaves as a oscillator and is described by:

$$V_C = \frac{I(t)}{\omega C} = \frac{A}{\omega C} e^{-\alpha t} \cos(\omega_d t + \phi) \tag{3}$$

where: ω: is the resonant frequency of the circuit and equals to $\frac{1}{\sqrt{LC}}$
ω_d: is the frequency of the power supply

Damped harmonic oscillator

A harmonic oscillator is a system that when disturbed from its resting state experiences a restoring force. Damping influences the oscillating system and has the effect of reducing its oscillations and removing energy from the oscillator system. The oscillation of a damped harmonic oscillator are described by an expression of the form.

$$Ae^{-\alpha t} \cos(\omega t + \phi) \tag{4}$$

where
$Ae^{-\alpha t}$: is the amplitude of the oscillation
$\zeta = \frac{\alpha}{\sqrt{\alpha^2+\omega^2}}$: is the damping ratio which determines of the system is
Overdamped($\zeta > 1$ and the decay is exponential)
critically damped($\zeta = 1$ the system returns to steady state as quickly as possible)
Underdamped($\zeta < 1$ the system oscillates with the amplitude gradually decreasing)

Damping ratio of a series RLC circuit

The damping ratio of a series RLC circuit is

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L}} \tag{5}$$

Methods

We built the circuit depicted in fig 11

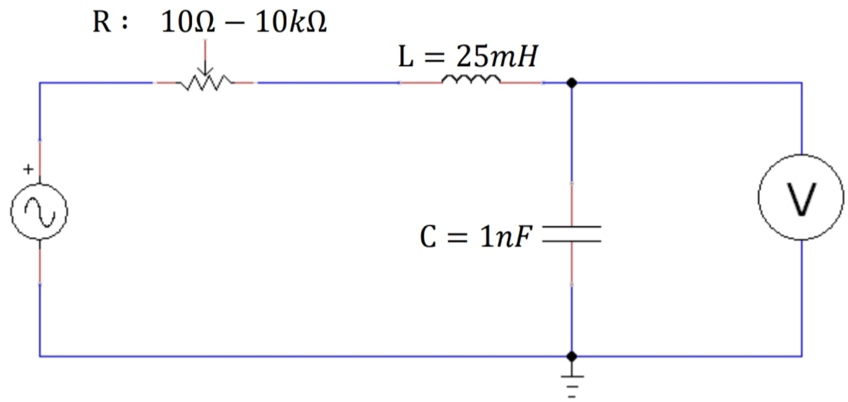

Figure 11: series rlc circuit

We set the power supply to a square wave of 25 volts and a frequency of 622 Hz. Then, we gradually changed the resistance of the variable resister while measuring the voltage over the capacitor.

Results

The theory predicts that the relation between the resistance and the damping coefficient is linear and is $\frac{1}{2}\sqrt{\frac{C}{L}} = 1 \cdot 10^{-4}$. The slope calculated by Matlab is $1.2 \cdot 10^{-4} \pm 5.8 \cdot 10^{-6}$.

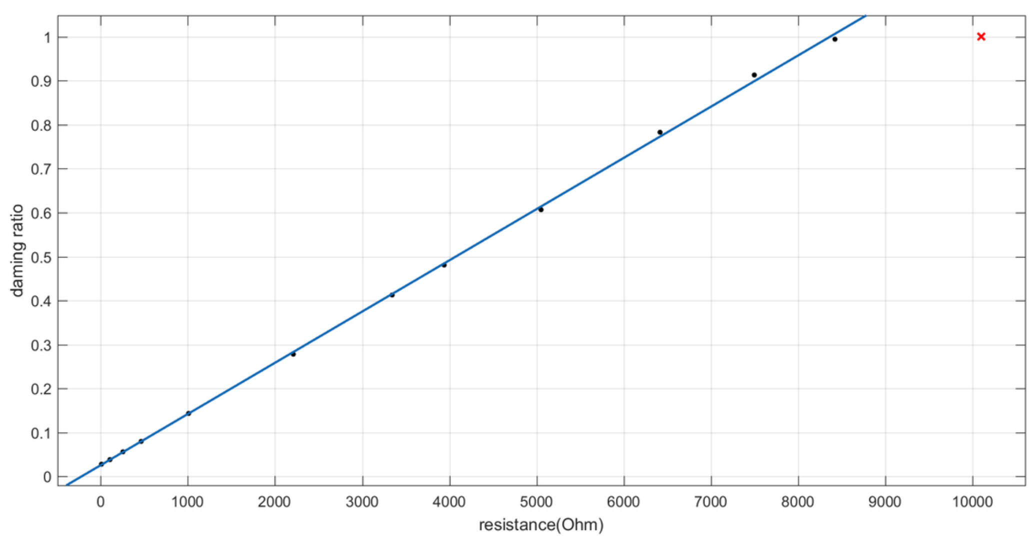

Figure 12: Damping in relation to resistance

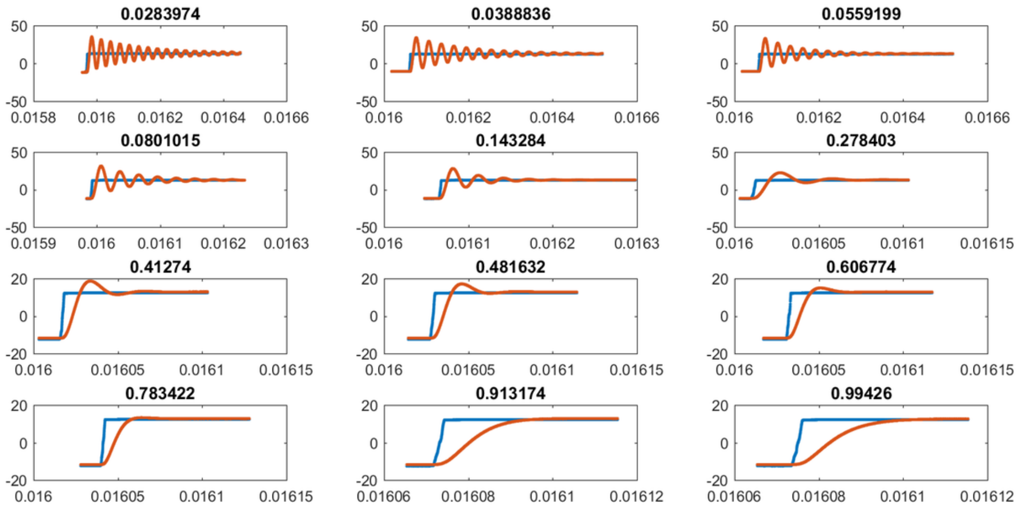

Figure 13: Oscillations and their respective damping ratio in the title

Conclusion

The data shows a small deviation from the theory. This deviation could be due to:

- Not taking the internal resistance of the power supply into account
- Error in the values taken for the inductor and capacitor