

# Optimal Transmission Expansion Planning considering Distributed Generations by using Non-dominated sorting genetic algorithm-II (NSGAI)

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## Abstract

Reconstructing power systems has changed the traditional planning of power systems and has raised new challenges in transmission expansion planning (TEP). In this paper, investment cost, cost of density and dependability have been considered three objectives of optimization. Also, multi-objective genetic algorithm NSGAI was used to solve this non-convex and mixed integer problem. A fuzzy decision method has been used to choose the final optimal answer from the Pareto solutions obtained from NSGAI. Moreover, to confirm the efficiency of NSGAI multi-objective genetic algorithm in solving TEP problem, the algorithm was implemented in an IEEE 24 bus system and the gained results were compared with previous works in this field.

**Keywords:** dynamic programming, TEP, NSGAI, fuzzy decision.

## 1 Introduction

Transmission Expansion Planning (TEP) is one of the main parts of planning development power systems aiming at identifying time, place and number of new transmission lines to optimize construction cost and efficiency of these lines. In order to achieve the adequacy of the power to the centers of load, TEP is usually classified into dynamic and static. In static planning the number and spots of needed power lines are determined for one year, while in dynamic planning, the needed construction time is also considered [1]. TEP is a nonlinear and complex problem one which is getting more complicated by increasing of the studied network scale. It was started by L. L. Grinver in 1970 to minimize efficiency cost and taking account generation constraints of power plants and power lines capacity using linear planning [2]. But in the recent year, most studies have been done on reconstructed power systems. The major difference between the TEPs in exclusive and competitive environments is that the main problem in

exclusive environments include generation, transmission, and distribution all together while in competitive environments these sections are considered separately [3]; [4]. Another important difference is that unlike exclusive environments which mostly include definite data, competitive environments include uncertainty data as a main parameter [5]; [6]. Objective function of exclusive environment is based on minimal cost while in competitive environment the objective function is maximum profit. Also, TEP solutions in competitive and traditional environments are classified into innovative optimization such as linear programming [6], dynamic programming [7], nonlinear programming [8] in mathematical optimization and mathematical methods such as genetic algorithm [9], objective-oriented models [10], metal plating [11], expert systems [12] and fuzzy theory [13].

The purpose of this paper is to study the effects of distributed generation on TEP in reconstructed environments. Since there is no contribution between generation companies and transmission companies in reconstructed environments, TEP needs to predict producers' behaviors. In this study, generation valuing method was used to predict producers' behaviors and planning was researched using dynamic approach in a five-year period. Moreover, the effects of distributed generation of windy and solar powers on TEP in reconstructed environments are considered. The rest of the paper is organized as follows: market exploitation model is introduced in section 2. Problem formulation and planning indexes is discussed in section 3. Simulation results and conclusion have been presented in section 4 and section 5.

## 2 Market exploitation model

In reconstructed power systems, independent system operator (ISO) manages generator productions to pro-

vide load with minimum cost while keeping dependability and quality of the power system. It is assumed that in a specified interval, all power producers inform to ISO their suggested price for one electric power and their minimum and maximum generation. Consumers send their minimum and maximum load demand and their suggestion of cutting off the load as well. Then, ISO executes optimum load distribution and determines quantity of Locational Marginal Price (LMP) and generations and loads. Market exploitation models have been modeled by applying the following equations [14]:

$$\text{Min} : J(P_G, P_D) = P_{Base}[P_G \cdot (a^T \cdot P_G + b) + C_D^T \cdot (P_D^{\max} - P_D)] \quad (1)$$

$$\text{s.t.} : B\delta = P_G - P_D - P_{tie} \quad (2)$$

$$-P_i^{\max} \leq H\delta \leq P_i^{\max} \quad (3)$$

$$P_G^{\min} \leq P_G \leq P_G^{\max} \quad (4)$$

$$P_D^{\min} \leq P_D \leq P_D^{\max} \quad (5)$$

In which  $J(P_G, P_D)$  represents total exploitation cost (\$/h),  $P_{Base}$  is base active power (MW),  $C_D^T$  is the transposed of suggested vector of cost load (\$/MWh),  $a$  and  $b$  are constant coefficients vector in the generator price suggestion function,  $P_G$  and  $P_D$  are vectors of generators output active power and active loads in perunit (P.U.) (these vectors are the output of Optimal Power Flow),  $P_{tie}$  is the output power vector from the studied area to other areas in P.U., and  $B$  is the linearized Jacobian matrix to P.U.. The term  $H$  is the linear matrix of the passing flow from lines to P.U. and  $\delta$  is the buses' voltage angle vector in radian. Objective function of Eq. (1) shows the total exploitation cost. The first part of this equation shows the exploitation cost of generators and the second part shows the deficiency cost of load. Equation (2) is for DC load distribution. The passing power limitation from lines is shown in equation (3). Equations (4) and (5) show generation limits and load limits, respectively. Losses are deleted in this model. Second-order optimization programming method can be used to solve this problem.

### 3 Formulation of problem

The main purpose of TEP in competitive environment is to provide a competitive, unprejudiced and sure environment for all the market actors and in minimum

cost. The prerequisite for providing such an environment is to consider some indices in designing and development of transmission network. Considered indexes in the paper are the level of competition, dependability, and investment cost which are presented in the following.

#### 3.1 Investment cost index

Economic justifying is important in competitive environment. Thus, development costs have to be considered in TEP to minimize investment budgets and transmission tariffs. Thus, the present value of total investment cost is formulated during the planning period as follows:

$$ic^k = \sum_{p=1}^{np} \frac{IC^P}{(1+r)^{p-1}} \quad p = 1, 2, \dots, np \quad (6)$$

While  $IC^P$  is the investment cost for the new lines installed in year  $p$  (\$) and  $ic^k$  is the present pure value of investment cost during programming horizon. Also,  $k$  is programming horizon.

#### 3.2 Lines density cost index

Density cost is a function of density level and its duration in a network. Here, density cost is calculated according to load peak over time and it is going to be minimized as a goal of TEP. Line density cost (as shown in Figure 1) is formulated as follows:

$$CC_i = (lmp_{i1} - lmp_{i2})P_{li1,i2} \quad i = 1, 2, \dots, N_i \quad (7)$$

In which  $CC_i$  is density cost in line in (\$/h),  $lmp_i$  is LMP in  $i_2$  base in (\$/MWh),  $P_{li1,i2}$  is the sent power from  $i_1$  base to  $i_2$  through line  $i$  and  $N_i$  is the number of lines of transmission network [14].

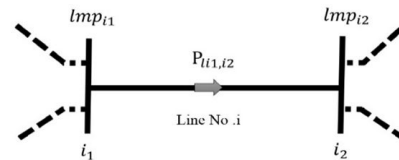


Figure 1:  $i^{th}$  line in sample power network [14]

$$TCC^P = \sum_{i=1}^{N_I^P} cc_I = \sum_{i=1}^{N_I^P} (lmp_{i1} - lmp_{i2}) P_{LI1,I2} \quad (8)$$

In which is network total density cost in  $P^{th}$  year from programming horizon and  $N_f^P$  is the number of installed lines until  $P^{th}$  year. In order to considering density cost in programming horizon, the present value of density cost should be calculated as follows:

$$CC^k = \sum_{p=1}^{np} \frac{TCC^P}{(1+r)^{p-1}} \quad (9)$$

While  $CC^k$  is the present value of pure density cost after adding planning  $k$  during programming period,  $np$  is the number of years of programming's horizon and  $r$  is the decline rate [14].

### 3.3 Load deficiency cost

Load deficiency cost due to line failure in year is calculated as follows [14].

$$LC_i^P = (P_d^{p \max} - P_d^{i,p}) C_d \quad (10)$$

While  $P$  is maximum vector of demand in  $P^{th}$  year in (MW),  $P_d^{i,p}$  is provided load vector after failure of line  $i$  in  $P^{th}$  year of planning in (MW),  $C_d$  is the load deficiency vector due to failure of line  $i$  in  $P^{th}$  year in (MW),  $LC_i^P$  is load deficiency cost due to failure of line  $i$  in  $P^{th}$  year in (\$/h). Load deficiency cost due to failure of all lines in  $P^{th}$  year has been calculated as follows:

$$TLC^P = \sum_{i=1}^{N_i^P} LC_i^P = \sum_{i=1}^{N_i^P} (P_d^{p \max} - P_d^{i,p}) C_d \quad (11)$$

Whereas, the number of suggested lines is different in various answers, average of equation (11) has to calculate. Thus, average of load deficiency cost due to failure of all lines in  $P^{th}$  year is computed as below:

$$ALC^P = \frac{(TLC^P)}{N_i^P} \quad (12)$$

Average present value of load deficiency cost during the planning horizon can be calculated as follows:

$$ALC^k = \sum_{p=1}^{np} \frac{ALC^P}{(1+r)^{p-1}} \quad (13)$$

## 4 Modeling of distributed generation in TEP

In reconstructed environments, generation planning and transmission planning are separated and problem is faced a lot of uncertainty in generation and load. Type, capacity, and location of power plants may change during the operational phase which indeed may increase the uncertainty of input data [15]. DG units can be valued in two ways:

- 1- When the share and contribution of DG units be low in market, a unit of DG is usually modeled in the model of distribution network as a negative load and the distribution company constructs it when its cost is lower than buying electricity from market.
- 2- When DG influence level reaches to specific level, each DG can be considered as a standard power plant generation with technology  $j$  in bus  $i$  and its value is determined by Net Present Value (NPV) criterion which has been discussed in below section:

After calculating the knot price during the period of the lifetime of the power plant, financial circulation of the power plant  $j$  in year  $t$  is calculated as follows:

$$CF_i = (Z_i(t) - C_{VO \& M} - C_{fuel}) \times f_{cap} \times 8760 - C_{(FO \& M)} \quad (14)$$

In which  $C_{VO \& M}$ ,  $C_{FO \& M}$  and  $C_{fuel}$  are variable cost of repairs and maintenance, constant cost of repairs and maintenance and the fuel cost of technology  $j$ . The term  $f_{cap}$  shows the capacity coefficient of the power plant. Then, NPV is calculated thorough the below equation:

$$NPV_{i,j} = C_{Cap} + \sum_{t=1}^T (CF_t \times e^{-rt}) \quad (15)$$

While  $r$  is the interest rate without risk and  $C_{Cap}$  is the cost of constructing the power plant. However, it is possible to consider the government encouraging schemes in promoting of renewable resource in investments. Thus, according to equation (15), the current pure value of each technology in each base can be calculated. Then, candidate cases for generation are cases with higher NPV in electricity market.

## 5 Simulation Results

In the paper, multi-objective genetic algorithm was used to solve TEP problem in an IEEE standard 24-bus experimental system Fig. 2. The basis data of the network were provided from reference [16] and the data related to initial investment cost were taken from reference [15]. It is assumed that the system has to be developed for future condition in which load and generation demand is 2.2 times higher than the initial level (initial load was 3054, initial generation was 3404 MW, so 2.2 times higher equals a load of 6720 MW and generation level of 7490 MW. This is equal to increase rate of 8% per year in a five-year planning's horizon.

In addition, it is assumed that the candidate branches of network development can be done simultaneously in all current 34 lines, and 7 new lines to be added in future, the their data of which are presented in Table 1. It is noteworthy that the information of candidate lines which are in parallel with previous lines, be like them exactly. Due to environmental limitations, 3 lines can be installed in each route. It is also assumed that all generators can be upgraded to 1.3 times higher than their current capacity and if more capacity is needed, new plants have to be constructed.

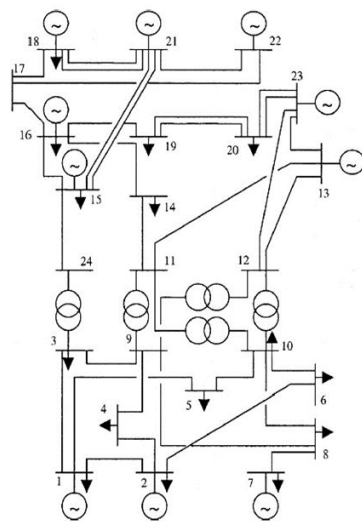


Figure 2: IEEE 24-bus system

### 5.1 The first scenario: two-objective optimizations

In the first scenario, two generators are installed in 11 and 24 buses from the second year onwards and each of them can produce electricity until maximum 1500 MW. Also, all generators can have produce 1.3 higher

Table 1: Configurations and investment costs of new lines

from	To	Construction cost (\$10000)
1	8	35
2	8	33
6	7	50
19	23	84
13	14	62
14	23	86
16	23	114

Table 2: Maximum and minimum of gained quantity for objective function in Pareto's diagram for objective functions, reducing the cost of construction and congestion of lines

	Min amount	Max. amount
Congestion cost	49404	54251
Construction cost	1.2396609	1.4473605

than their initial capacity. The increasing of generation can answer load growth until five next year, because we have to increase the generation as much as 4000 MW until end of the fifth year of development of transmission network with 3000 MW of which will be provided by two new installed power plants 1500 MW in the second year and 1000 MW will be supplied by increasing the generation level of existing power plants. Moreover, only two objective such as investment cost and distributed cost of lines have been considered and problem is solved as two-objective as seen in Table 2. Gained Pareto's diagram from simulation is shown in Fig. 3.

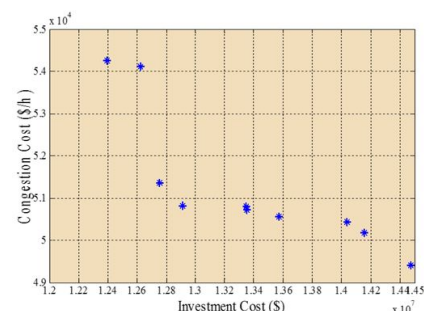


Figure 3: Pareto's diagram for solving considered problem with two objective functions, reducing the cost of construction and congestion of lines

Also, weighted values of 0.5 have been assigned to both objective functions. In Table 3, values of objective function of Pareto's spots and corresponding membership function are displayed. From compar-

Table 3: Pareto's spots and calculating their membership function and detecting the best spot for objective functions, reducing the cost of construction and congestion of lines

	Value of membership	Congestion cost
1	0.0513	54251
2	0.0513	49404
3	0.0731	51354
4	0.0472	54106
5	0.0509	50181
6	0.0512	50423
7	0.075	50818
8	0.0613	50568
9	0.0644	50797
10	0.065	50723
11	0.0613	50568
Construction cost		
1	1.2396609	
2	1.4473605	
3	1.2755295	
4	1.2625938	
5	1.4155655	
6	1.4036662	
7	1.2910195	
8	1.3570774	
9	1.334906	
10	1.3352972	
11	1.3570774	

Table 4: Final planning for construction of added lines in the network during 5 year

	first year	second year	third year
Line 1	24-3	24-3	4-1
Line 2	10-6	8-2	9-4
Line 3	10-5	8-7	6-2
Line 4	11-9	8-9	12-10
Line 5	18-15	13-12	23-16
Line 6	23-20	7-1	20-13
Line 7	14-13	4-8	2-16
Line 8	21-18	-	21-15
Line 9	-	-	2-1
	forth year		Fifth year
Line 1		8-23	24-3
Line 2		23-14	23-16
Line 3		-	21-15
Line 4		-	9-8
Line 5		-	2-1
Line 6		-	23-19
Line 7		-	23-14
Line 8		-	-
Line 9		-	-

Table 5: Comparing the gained results in the paper with reference [14]

	Proposed Method	Ref. [14]
Congestion Cost	50818	68916
Investment Cost	14.03	7.91

ing the membership function of Pareto's answers with Fig. 3 can be said that spot 7 is the best of problem's answer. Corresponding planning with this answer has been brought in Table 4. In Table 5, the obtained results for this scenario have been compared with reference [14]. The results states that however investment cost in the planning period in reference [14] is almost half of the gained investment cost in this paper, but the cost of congestion lines is very high in this reference.

## 5.2 The second scenario: three-objective optimization with regard to distributed generation

In this scenario, in addition two objective, initial investment cost and congestion cost of transmission lines, average cost of cutting the load has been considered as the third objective of the optimization. Moreover, the effects of windy and solar distributed generations in TEP has been taken attention. In the scenario 1 and 2, it is assumed that generators increase the power output as much as 1.3 higher than their initial value and new generators in 11 and 24 buses

with capacity 1500 MW have been installed during the planning period in the second year. But, in this scenario for construction of the new generators, we will use generation valuation method and the generators with the highest value in market are the better options for the investors in generation section. Also, it's assumed that solar and windy power plant are only constructed in load buses. In order to considering the government's politics for encouraging applying of the renewable resource, a tariff coefficient is considered for solar and windy powers plants. The value of the electricity from wind and solar power plants is equal to the price of the spot market in its tariff. Requirements parameters for calculating of generation value have been stated in Table 6.

Also, candidate generation options assuming different tariff have been shown in Table 7 and Table 8. In calculation of Table 7, the values of tariff for windy and solar power plants have been considered 2. Then, it can be observed that with this tariff, solar power plants are not competitive case in electricity market and they have not any place among candidate options, but windy power plant are more competitive.



Table 6: Specifications of new generators

Technology	Solar
Initial Investment (M\$/MW)	4.9
Fixed production cost (\$/MW/Year)	-
Fixed production cost (\$/MWh)	45.5
life span (Year)	25
capacity (MW)	200*5
Capacity coefficient (%)	56
Technology	combined
Initial Investment (M\$/MW)	1.314
Fixed production cost (\$/MW/Year)	1550000
Fixed production cost (\$/MWh)	38.21
life span (Year)	30
capacity (MW)	1200
Capacity coefficient (%)	60
Technology	Coal
Initial Investment (M\$/MW)	2.239
Fixed production cost (\$/MW/Year)	7200000
Fixed production cost (\$/MWh)	17.02
life span (Year)	40
capacity (MW)	1500
Capacity coefficient (%)	85
Technology	Wind
Initial Investment (M\$/MW)	2.8
Fixed production cost (\$/MW/Year)	600000
Fixed production cost (\$/MWh)	-
life span (Year)	25
capacity (MW)	200*5
Capacity coefficient (%)	40

As observed in Table 8, with increasing the tariff coefficient of solar power plant to 3, this power plant converts to one of the options with higher generation value which can be replaced instead of hybrid power plant among the selective options. The achieved results show that these two renewable technology are not competitive with fossil power plant technology and they need government support in order to development of their market.

Now with considering two coal power plants 1000 MW in buses 11, 17 and also, five power plants 200 MW in bus 9, TEP has been done. Pareto's diagrams

Table 7: Generation Valuation Results (solar FIT=2, Wind FIT=2)

Technology	Cap. (MW)	Bus No.	NPV(M\$)
Coal	1500	11	14570
Coal	1500	20	13255.46
Combined	1500	11	11522.45
Wind	1000	17	8232.05
Wind	1000	11	6307.15

Table 8: Generation Valuation Results (solar FIT=3, Wind FIT=2)

Technology	Bus No.	Capacity (MW)	NPV(M\$)
Coal	17	1000	12850.32
Coal	11	1000	9650.31
Solar	9	5*200	7450.42
Solar	11	5*200	3740.11
Wind	1000	11	6307.15

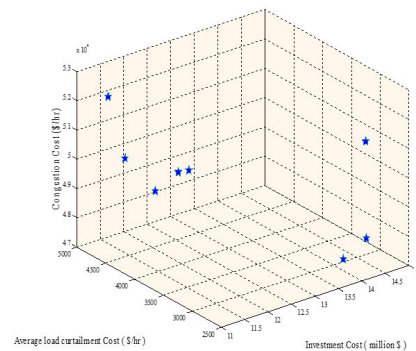


Figure 4: Pareto's diagram of three-objective algorithm NSGAII

of three-objective problem with the generation structure of the above mentioned has been displayed in Fig. 4. As seen in Fig. 4, the eight spot of Pareto have been recommended as final answer by Non dominated sorting genetic algorithm (NSGA-II) algorithm for TEP. Maximum and minimum value of each objective function based on Figure is according to the Table 9. In order to determining the membership function for each answers, equal weighting coefficients have been considered for three objective function. The obtained final answer (the biggest membership function) is according to Table 10. The gained optimization values of objective functions according to planning of Table 10 in comparison with results of reference [14] have been presented in Table 11.

As observed from results, the proposed method in the paper, with increasing of the investment cost as much as 44 percent, the cost of congestion lines as much as 32 percent and average cost of cutting off the load as much as 89 percent are decreased. Only problem of the optimization method is high calculating time, because of this reason for solving above problem in personal computer need one day. In order to decreasing calculating time can choose outlet of a number selected lines instead of outlet of every single line. As observed from results, the proposed method in the paper, with increasing of the investment cost as much as 44 percent, the cost of congestion lines as much as 32 percent and average cost of cutting off the load

Table 9: Minimum and maximum of gained quantity for objective function in three-objective Pareto's diagram

	Min amount	Max amount
Congestion cost	49041	52061
Construction cost	11.452	14.815
Av. Interr. Cost	2793.56	4983.06

Table 10: final planning for construction of added lines in network during 5 year for three-objective problem

	first year	second year	third year
Line 1	4-3	4-3	3-1
Line 2	10-6	8-10	9-4
Line 3	10-12	8-7	6-2
Line 4	16-7	8-9	12-10
Line 5	14-13	13-12	23-16
Line 6	23-20	9-10	20-13
Line 7	-	13-14	2-16
Line 8	-	-	21-15
Line 9	-	-	2-1
		forth year	fifth year
Line 1		8-2	24-3
Line 2		17-16	23-16
Line 3		-	21-15
Line 4		-	9-8
Line 5		-	2-1
Line 6		-	-
Line 7		-	-
Line 8		-	-
Line 9		-	-
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as much as 89 percent are decreased. Only problem of the optimization method is high calculating time, because of this reason for solving above problem in personal computer need one day. In order to decreasing calculating time can choose outlet of a number selected lines instead of outlet of every single line.

Table 11: Comparing the gained results in the paper with reference [14]

	Proposed Method	Ref. [14]
Congestion cost	47061	68916
Construction cost	14.21	7.91
Av. Interr. Cost	2938.52	2683.257

## 6 Conclusion

Condition and requirements of new reconstructed environment necessitates reviewing available classic methods in TEP problem. In the paper, a multi-objective model for TEP has been recommended for overcoming on challenges which are created in effect of reconstruction of electricity network. In this model, TEP is as a multi-objective nonlinear optimization with the minimizing of investment cost during planning period, distributed cost of transmission lines and average cost of load deficiency (dependability objective). Multi-objective genetic algorithm has been used for solving this problem. This method unlike the one-objective methods necessitates set of optimization answers which are lead to more flexibility in planning process. In order to obtaining a final optimization answer, Fuzzy membership function method has been applied for planning of network development from Pareto's answers. The gained results from this technique have been compared with other works in this field which is shown that distributed cost of line as much as 32 percent and average cost of cutting off the load as much as 89 percent can be decrease with more investing in planning period which are lead to better competitive and more dependability of system.

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