ECON 812 Homework 2

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1. Suppose 50% of all agents in an economy have U=ln x + ln y, and the other 50% have U=2 ln x + ln y. All agents start with one unit of x and one unit of y. Find the general equilibrium relative prices and allocations.

On the second page of the notes, we see that $\frac{p_x}{p_y} = \frac{\Sigma a_i \overline{y_i}}{\Sigma \overline{x_i} - \Sigma a_i \overline{x_i}}$. Additionally, to normalize, $a + b = 1$ and there are 100 agents in total.

Using the formula, $\frac{p_x}{p_y} = \frac{(50)(.5)(1)+(50)(\frac{2}{3})(1)}{(50)(.5)(1)+(50)(\frac{1}{3})(1)} = 1.4$ This means that each agent has $(1.4)(1) + (1)(1) = 2.4$ units of income. The constant income fractions rule then demonstrates that the initial type of agent spends 50% of his or her income on each good while the latter type spends 2/3 on x and 1/3 on y.

The initial agent spends half of his or her income on good x and good y, therefore consuming $\frac{(.5)(2.4)}{1.4} =$ .857 units of good x and $\frac{(.5)(2.4)}{1} = 1.2$ units of good y. In essence, each initial agent sells .143 units of good x to attain .2 units of good y (in addition to the y he or she began with).

The latter agent type spends 2/3 of his or her income on good x which is $(2.4)(\frac{2}{3}) = 1.6$ units of good x and 1/3 of his or her income on good y which is $(2.4)(\frac{1}{3}) = 0.8$ units of good y. He or she consumes $\frac{1.6}{1.4} = 1.143$ units of good x and $\frac{0.8}{1} = 0.8$ units of good y. He or she sells 0.2 units of good y and buys 0.143 units of good x.

2. Re-do problem #1, assuming that the first type of agent starts with 2 units of x and 0 of y and the second type of agent starts with 2 units of y and 0 of x.

$\frac{p_x}{p_y} = \frac{(50)(.5)(0)+(50)(\frac{2}{3})(2)}{(50)(.5)(2)+(50)(\frac{1}{3})(0)} = \frac{4}{3}$ Now the initial agent has $\frac{8}{3}$ units of income and the latter agent has 2 units of income.

The initial agent consumes $\frac{(\frac{8}{3})(.5)}{(\frac{4}{3})} = 1$ unit of good x and $\frac{(\frac{8}{3})(.5)}{1} = \frac{4}{3}$ units of good y. He sells 1 unit of good x to attain $\frac{4}{3}$ units of good y.

The latter agent consumes $\frac{(2)(\frac{2}{3})}{\frac{4}{3}} = 1$ unit of good x and $\frac{\frac{2}{3}}{1} = \frac{2}{3}$ units of good y. She sells $\frac{4}{3}$ units of good y to attain 1 unit of good x.

3. Re-do problem #1, assuming that all agents have U=x+y. (Hint: At disequilibrium prices, agents want to consume only x or only y).

The price ration in this case would be $\frac{p_x}{p_y} = 1$. This infers that if a good is cheaper than the other, all agents would desire to consume only the cheaper good and none of the other. Assuming that agents start with one unit of good x and one unit of good y, each agent would only consume one unit of good x and one unit of good y.

4. Suppose you can redistribute x, but not y. Returning to problem #1, what exactly must you do to: (a) make the equilibrium utility of the first type of agents equal to .5 (b) give all agents of the second type the same utility, (c) and make type-2 agents' utility as high as possible conditional on (a)?

Letting the new initial agent type endowment be equal to $\overline{x}$, and the new latter agent type endowment $2-\overline{x}$.

$\frac{p_x}{p_y} = \frac{(50)(.5)(1)+(50)(\frac{2}{3})(1)}{(50)(.5)(\overline{x})+(50)(\frac{1}{3})(2-\overline{x})} = \frac{7}{\overline{x}+4}$ This means that the initial type of agents each have an income of $(\frac{7}{\overline{x}+4})(\overline{x}) + 1$ where they spend half of their income on good x and good y. This means that they consume $\frac{(\frac{7}{\overline{x}+4})(\overline{x})+1}{(2)\frac{7}{\overline{x}+4}} = \frac{4\overline{x}+2}{7}$ units of x and $\frac{\frac{7}{\overline{x}+4}(\overline{x})+1}{2} = \frac{4\overline{x}+2}{\overline{x}+4}$ units of y. Plugging this into their utility function gives: $U = \ln\frac{4\overline{x}+2}{7} + \ln\frac{4\overline{x}+2}{\overline{x}+4}$.

Setting this to 0.5 and solving gives: $0.5 = \ln\frac{4\overline{x}+2}{7} + \ln\frac{4\overline{x}+2}{\overline{x}+4}$ $(\frac{4\overline{x}+2}{7})(\frac{4\overline{x}+2}{\overline{x}+4}) = e^{0.5}$ $16\overline{x^2} + 4.46\overline{x} - 42.16 = 0$

$\overline{x} = 1.49$

$\frac{p_x}{p_y} = \frac{7}{1.49+4} = 1.275$ The income of the initial type of agents is equal to 2.9 which means that they consume $\frac{1.45}{1.275} = 1.137$ units of x and $\frac{1.45}{1} = 1.45$ units of y. In essence, after the redistrubtion of .49 units of x to the initial agents, they sell $1.49 - 1.137 = .353$ units of x to buy .45 additional units of y.

The latter type of agents have .863 units of x and .55 units of y with utitlity equal to -.893.