Problemas sobre centroides

JAIR [1]

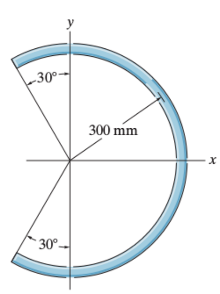
[1]Instituto Tecnológico Superior Zacatecas Occidente

2 de abril de 2019

Resumen

En el presente trabajo daremos solución a dos problemas sobre centroides de un volumen mediante formulas de integración, principalmente tendremos que determinar el tipo de centroide que se debe calcular, luego determinar el elemento diferencial y finalmente resolver las integrales y obtener el resultado.

9–1. Locate the center of mass of the homogeneous rod bent into the shape of a circular arc.



Prob. 9–1

Solución

1.- determinar el tipo de centroide

$$y = \frac{\int_{.} y\ dl}{\int_{.} dl} \qquad x = \frac{\int_{.} x\ dl}{dl}$$

$$x = R\cos\theta \qquad y = R\sin\theta$$

2.-elemento diferencial

$$dl = Rd\theta$$

3.- resolver integrales y obtener resultado

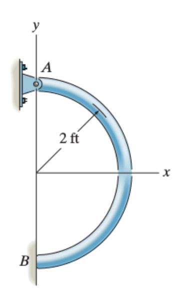
$$x = \frac{\int_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}} R^2 COS\theta d\theta}{\int_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}} Rd\theta} = \frac{R\int_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}} \cos\theta d\theta}{\int_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}} d\theta}$$

$$y = \frac{\int_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}} R^2 \sin\theta d\theta}{\int_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}} Rd\theta} = \frac{R\int_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}} \sin\theta d\theta}{d\theta}$$

$$x = \frac{R[\sin\theta]_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}}}{[\theta]_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}}} = \frac{R\sqrt{3}}{\left[\frac{2\pi}{3}+\frac{2\pi}{3}\right]} = \frac{3\sqrt{3}R}{4\pi} = 0.124m$$

$$y = \frac{R[-\cos\theta]_{\frac{2\pi}{3}}^{\frac{2\pi}{3}}}{[\theta]_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}}} = \frac{R[0.5+(-0.5)]}{\frac{4\pi}{3}} = 0$$

9–2. Locate the center of gravity $\bar{x}$ of the homogeneous rod bent in the form of a semicircular arc. The rod has a weight per unit length of 0.5 lb/ft. Also, determine the horizontal reaction at the smooth support *B* and the *x* and *y* components of reaction at the pin *A*.



Prob. 9–2

Solución

$x = 2\cos\theta$

$y = 2\sin\theta$

$dl = 2d\theta$

$x = \frac{\int_{.}^{\cdot} xdl}{\int_{.}^{\cdot} dl} = \frac{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\cos\theta 2d\theta}{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2d\theta} = \frac{4\sin\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\Big|}{2\theta\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\Big|} = \frac{4}{\pi}$

$y = \frac{\int_{.}^{\cdot} ydl}{\int_{,}^{\cdot} dl} = \frac{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\sin\theta 2d\theta}{\int_{-\frac{\pi}{2}}^{\frac{2}{\pi}} 2d\theta} = \frac{4[-\cos]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}}{2\theta\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\Big|} = \frac{3}{\pi}$

$x = \frac{\int_{.}^{\cdot} xdl}{\int_{.}^{\cdot} dl} = \frac{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\cos\theta 2d\theta}{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2d\theta} = \frac{4\sin\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\Big|}{2\theta\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\Big|} = \frac{4}{\pi}$