Caplan Microeconomics: HW #2

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1. From the class notes, we know that the equilibrium relative prices and allocations will be a function of the summation of all endowments and preferences:

$$\frac{p_x}{p_y} = \frac{\sum a_i \bar{y}_i}{\sum b_i \bar{x}_i}$$

If we assume 100 individuals, then we can normalize the utility functions, $a + b = 1$:

$$\frac{p_x}{p_y} = \frac{50 \cdot .5 \cdot 1 + 50 \cdot \frac{2}{3} \cdot 1}{50 \cdot .5 \cdot 1 + 50 \cdot \frac{1}{3} \cdot 1} = 1.4$$

Here, each individual has 2.4 units of income, because $1.4 \cdot 1 + 1 \cdot 1 = 2.4$. We can tell from the formula that the first agent will spend 1/2of their income on both x and y, since they have even preferences. The second type of agent will spent 2/3 of their income on x and 1/3 on y.

This means that the first type spends 1.2 units on each good, consuming $\frac{1.2}{1.4} = .857$ of good x. Good y is $\frac{1.2}{1} = 1.2$ units. This means that each first type of agent will sell .143 of x to receive .2 extra units. The second type of agent spends $\frac{2.4}{2/3} = 1.6$ on good x and $\frac{2.4}{1/3} = 0.8$ units on good y. Thus consuming $\frac{1.6}{1.4} = 1.143$ of good x and $\frac{.8}{1} = .8$ of good y. This means that the second type of agent buys .143 units of x with .2 units of y.

2. Using the first question's formula with new endowments:

$$\frac{p_x}{p_y} = \frac{50 \cdot .5 \cdot 0 + 50 \cdot \frac{2}{3} \cdot 2}{50 \cdot .5 \cdot 2 + 50 \cdot \frac{1}{3} \cdot 0} = \frac{4}{3}$$

Now the first type of individual has 8/3 units of income and the second type has 2 units. Since the income fractions remain the same, the first type consumes $\frac{8/3 \cdot .5}{4/3} = 1$ unit of x and $\frac{8/3 \cdot .5}{1} = 4/3$ units of good y. They would sell 1 unit of x for 4/3 units of y.

The second type of individual consumes $\frac{2 \cdot 2/3}{4/3} = 1$ unit of good x and $\frac{2/3}{1} = 2/3$ units of good y. They sell 4/3 of y to get 1 of x.

3. Now the price ratio will become

$$\frac{p_x}{p_y} = 1$$

Therefore, if x is cheaper than unit y, then all agents will only consume x (and vice versa). Since all agents start with 1 unit of each, the only equilibrium is where each person consumes exactly 1 x and 1 y.

4. To first find how much endowment must be taken from each type 2 to type 1 for the type 1 utility = 1, it is helpful to plug $\bar{x}, (2-\bar{x})$ respectively into the price formula:

$$\frac{p_x}{p_y} = \frac{50 \cdot .5 \cdot 1 + 50 \cdot \frac{2}{3} \cdot 1}{50 \cdot .5 \cdot 1 + 50 \cdot \frac{1}{3} \cdot (2-\bar{x})} = \frac{7}{\bar{x}+4}$$

This means that agents of type 1 have a total income of $\frac{7}{\bar{x}+4}(\bar{x}+1)$. They spend half their income on x still and half on y, thus consuming $\frac{4\bar{x}+2}{7}$ on x and $\frac{4\bar{x}+2}{\bar{x}+4}$ on y. Therefore, the utility of the first type of individual, set equal to .5 is:

$$\ln\left(\frac{4\bar{x}+2}{7}\right) + \ln\left(\frac{4\bar{x}+2}{\bar{x}+4}\right) = .5$$
$$\left(\frac{4\bar{x}+2}{7}\right) \cdot \left(\frac{4\bar{x}+2}{\bar{x}+4}\right) = e^{.5}$$
$$16x^{-2} + 4.46\bar{x} - 42.16 = 0$$

With the quadratic formula, the solution is $\bar{x} = 1.49$. Therefore, we would have to distribute .49 units of x from each type of agent 2 to each type of agent 1.

Putting this into the formula yields:

$$\frac{p_x}{p_y} = \frac{7}{1.49+4} = 1.275$$

Since the income of type 1 is now 2.9, they consume 1.137 units of x and 1.45 units of y. This meant that after the redistribution, .49 units of x to type 1, they can sell .353 units of x to buy .45 extra of y. The type 2 individual has .863 units of x and .55 units of y, with a utility of -.893.

5. There are two ways to analyze this situation, from both the demand and supply sides. First, we'll take the demand side. One significant way that a demand curve might shift outward, thus increasing the real wages of barbers have risen regardless of the technological progress, is by a general increase in income, which has happened since 1900. Secondly, looking to the supply of barbers, because other industries now pay more than before (thus making the opportunity cost of barber-ship higher), their real wages increased to continue attracting the equilibrium amount of barbers to the field. So, though hair-cutting has used the same technology (more or less… we could talk about buzzers) as has been used in the past hundred years, the symbiotic effect of both demand and supply exerts upward pressure on wages and downward pressure of supply.

6. One practical controversy that I find fascinating is the debate around the utility of school choice. The position I favor posits that mechanisms of school choice, charter schools, private school vouches, and educational savings accounts (which are no means the same, though can be aggregated for this purpose), are efficient responses to the coproduction of education by way of utilizing tacit knowledge available only at local levels and amongst those for whom incentives align the strongest toward the successful production of education (i.e. parents and students). One possible way to test this though a betting market would be offering \$100 to individuals if school choice enrollment rises AND educational attainment rises AND educational disciplinary issues decline AND public school enrollment declines. I could offer exactly the same for the opposite results in the same circumstances, if educational attainment drops and the need for disciplinary measures rise. By comparing the way people bet in these two markets, the aggregate preferences are revealed. For example, we could imagine that the first offer will be purchased in shares worth \$1 and the second in shares worth \$0.50. Therefore, there is a conditional probability that my position has a 2/3 probability of being correct.