Doctoral thesis proposal:
Meshless modelling of thermo-mechanics of low-frequency electromagnetic direct chill casting

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1 Introduction

1.1 Direct-chill casting

In direct-chill casting the degassed melt arriving from the furnace is lead by the trenches in the casting table to a DC casting machine (Eskin, 2008). The melt enters the mold from the top and starts solidifying on contact with the dummy block. When the solidified part is strong enough to support the metallostatic pressure caused by the liquid phase, the dummy block starts to retract from the mold, removing the solidified alloy. The solidified alloy takes on the role of the dummy block in the heat removal as well as in preventing the melt to escape the mold. The speed of retraction of the dummy block is set such that a stationary state is obtained with regards to thermal profile in the mold.

The direct-chill (DC) casting is a well established technology used to cast majority of aluminium intended for production of sheet ingots, extrusion billets and electrical conductors. Although the process has been widely used for almost a century, there are still many technological problems to be understood. Due to the complex interaction between various physical phenomena, the best way to understand the technological problems is by development of advanced numerical models.

One of the most serious casting defects is hot tearing, the irreversible formation of a crack in the semisolid casting. The importance of hot tearing is reflected in many studies trying to understand various aspects of the phenomenon (Eskin et al., 2004; Eskin, 2008; Li, 2010; Stangeland et al., 2004) as well as in the development of numerical models to predict it's occurrence (Hao et al., 2010; Sistaninia, 2013).

To better understand the physical reasons for hot tearing, we should consider the solidification process of an aluminium alloy. Based on the permeability of the solid network, the process can be divided into four stages (adapted from (Eskin, 2008)).

1. *Mass feeding*, in which both, the already solidified dendrites and liquid melt are free to move.

2. *Interdendritic feeding*, in which the fluid has to flow through the already coherent solid skeleton.

3. *Interdendritic separation*, in which the liquid network becomes fragmented. With increasing solid fraction, liquid is isolated in pockets or immobilized by surface tension.

4. *Interdendritic bridging* or *solid feeding*, in which the billet has developed considerable strength and solid state creep compensates for further contraction.

Hot tearing usually occurs in the last two stages in conditions, in which the liquid feeding is not sufficient to account for the shrinkage of the material. The shrinkage can be in general divided in two parts, the solidification shrinkage and thermal shrinkage. Thermal shrinkage is well known thermal isotropic contraction, while the reason for the solidification shrinkage is the difference in the densities of solid and liquid phases. The thermal shrinkage is the main factor for stress occurrence in already solidified material, while the solidification shrinkage is most relevant during the third stage of the solidification, when the liquid network becomes fragmented.

As a result of the shrinkage stresses develop in the ingot. The existence of areas with tensile stress is one of the most important conditions for hot tearing. The occurrence of hot tears is also facilitated by the fact that the yield stress decreases with increasing temperature. If the tensile stresses in an area increase over this value, the dendritic bridges that have formed collapse, initiating a hot tear.

The alloy composition is also an important factor determining the hot-tearing probability. Different thermo-mechanical properties, different solidification range and variances in micro structure resulting from different chemical composition can make hot-tearing even more difficult to predict.

1.2 Meshless methods

In conventional methods the mesh generation is an important step during the numerical solution. In case of the FEM and finite volume method (FVM) the domain has to be polygonized, which is an demanding and time consuming task. The finite difference method (FDM) is defined on a regular grid, which severely reduces the applicability of the method for irregular geometries.

The many variants of meshless methods try to alleviate both problems (Liu, 2010). In general the meshless methods either use the variational approach of the FEM or try to generalize the FDM method. The latter approach is the one we are interested in. The main reason is the simplicity of the implementation and applicability to many different physical phenomena.

Over the years various approaches to generalizing the FDM (Liszka and Orkisz, 1980; Shirobokov, 2006; Sadat and Prax, 1996) were proposed. These methods were able to obtain quite accurate results, but they were plagued by ill-conditioned interpolation problem for some node arrangements. To bypass this problem, the finite difference formulas were suggested to be calculated using a set of (conditionally) positive definite functions, meaning that the interpolation problem was always well defined. One such set are the radial basis functions, in particular the multiquadrics.

Multiquadrics (MQ) were first used for PDE solving in the two fundamental articles published by Kansa in 90's (Kansa, 1990b; Kansa, 1990a). In these articles, the MQs were used in spectral mode, meaning that the solution was interpolated on the whole computational domain, and not only on the small stencils as in the case of FDM. The global interpolation approach limits the number of discretization points that can be used. Since all the functions used are of the same shape, only their origin is shifted, the functions with origins close enough become linearly dependent, which causes the system of equations to become ill conditioned.

Improvements were proposed in the direction of abandoning the global interpolation and interpolating the solution only locally (Lee et al., 2003; Tolstykh and Shirobokov, 2003). This approach has turned out to be very robust, allowing for great flexibility of the method. The local radial basis function collocation method (LRBFCM) has since been successfully applied to many physical problems (heat transfer (Šarler and Vertnik, 2006), solidification (Vertnik et al., 2006), turbulent fluid flow (Vertnik and Šarler, 2009), macro segregation (Kosec et al., 2011), magnetohydrodynamics (Mramor et al., 2013)) and applied to modelling processes in industry (Vertnik, 2010; Mramor, 2015; Hanoglu, 2015).

2 Intended work and expected results

The main aim of the thesis is development of a meshless solver for thermomechanics model of the DC casting process. In the process of developing we expect to formulate and test the LRBFCM for solving linear thermoelasticity and further explore the capabilities of the method when applied to mechanical problems. Also, we expect to develop and test a multiphysics framework based on LRBFCM, extend it to solve viscoplastic problems and compare it with commercial programs. The developed solver for thermomechanics will be coupled to the adjacent model of heat and mass transfer during the DC process. The resulting model will be able to provide insight in development of hot tearing of aluminium billets and the impact of the process parameters on the quality of the billet.

3 Performed work

The LRBFCM method has been tested for solving time-independant linear thermoelasticity problems and the precision of the method has been studied with regards to the method parameters. This work was presented at THERMACOMP2014 at Lake Bled (Mavrič and Šarler, 2014a) and was published in International Journal of Numerical Methods in Heat and Fluid Flow as an invited contribution (Mavrič and Šarler, 2015b). The developed model was further used to model the stresses during DC casting. Improvements on the model were presented in conferences ICASP-4 in London (Mavrič and Šarler, 2014b), COUPLED2015 in Venice (Mavrič and Šarler, 2015d) and MCWASP XIV in Osaka (Mavrič and Šarler, 2015c). Futhermore, a multiphysics modelling framework was developed and a coupled thermoelasticity problem was used to benchmark the performance. This work was presented at NHT2015 (Mavrič and Šarler, 2015a) in Warsaw and has also been invited for publication in International Journal of Numerical Methods in Heat and Fluid Flow.

4 Originality

The method will be for the first time used to solve thermomechanical and viscoplastic problems, which will provide new insights on the method performance with respect to various free parameters of the method. Also, the LRBFCM will be for the first time extended to modelling of thermomechanics during the DC casting of aluminium billets.

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