

VALIDATION OF A TAILORED GRAVITY FIELD MODEL FOR PRECISE QUASIGEOID MODELLING OVER LIMPOPO PROVINCE IN SOUTH AFRICA

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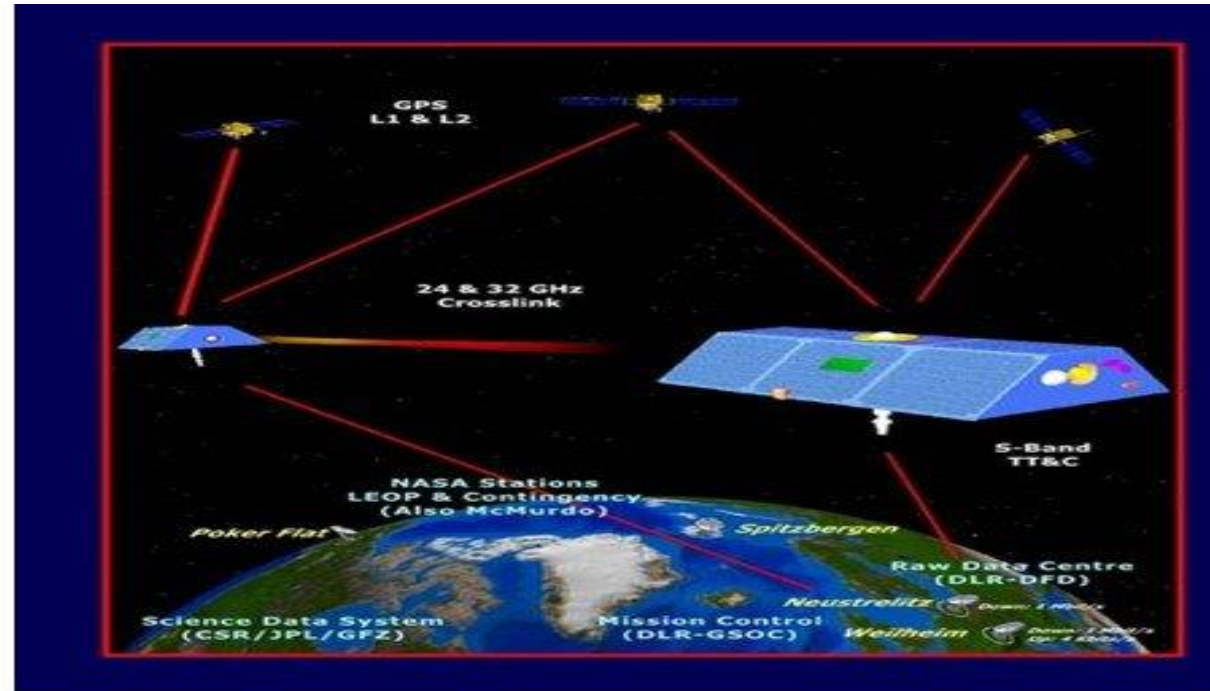
Abstract

Recently, a tailored gravity field model was developed to fit local terrestrial gravity data by integrating Global Gravitational Models (GGMs), terrestrial gravity data, and Digital Elevation Models (DEMs). The numerical analysis of the newly developed tailored gravity model showed a substantial improvement by means of its possible application for geophysical exploration by exhibiting known geological features over the Southern Benue Trough of Nigeria. In this study, we apply a similar technique to develop a tailored gravity field model at the Limpopo Province in South Africa using a total of 8,603 terrestrial gravity measurements. Validation of results indicates that our tailored gravity model could reproduce the observed gravity data with the accuracy specified by a standard deviation of 8.9 mGal and with a systematic bias less than 0.1 mGal within the study area. We then inspected a possibility of using our tailored gravity field model to improve the accuracy of existing geoid/quasi geoid models at the study area. For this purpose, we compute a new (quasi)geoid model by applying the Remove-compute-restore numerical technique that treats separately the detailed gravity pattern that is closely correlated spatially with the topographic relief, the higher-to-medium gravity signal that is mostly captured by local/regional gravity data, and the long-wavelength gravity signal that is modelled by using GGMs. The accuracy of the new (quasi)geoid model was assessed by using the most recent South African gravimetric quasi-geoid model CDSM09A and the latest hybrid quasi-geoid model of South Africa SAGEOID10. The comparison of our quasi-geoid model with the CDSM09A and SAGEOID10 quasi-geoid models was done at 7,225 quasi-geoid grid points. The comparison revealed that our new quasi-geoid model closely agrees with the CDSM09A and SAGEOID10 models. The differences between our and CDSM09A quasi-geoid models vary within -0.31 and 0.70 m, with a mean of 0.05 m and a standard deviation of 0.12 m. The corresponding differences between our and SAGEOID10 quasi-geoid models are between -0.35 and 0.70 m with a mean of 0.06 m and a standard deviation 0.12 m. The numerical analysis revealed that the new tailored gravity model could efficiently be used in various geophysical and geodetic applications.

Validation of a Tailored Gravity Field Model for Precise Quasigeoid Modelling over Limpopo Province in South Africa

AGU FALL MEETING

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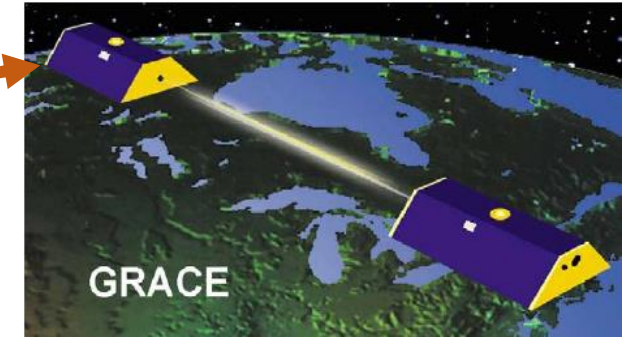
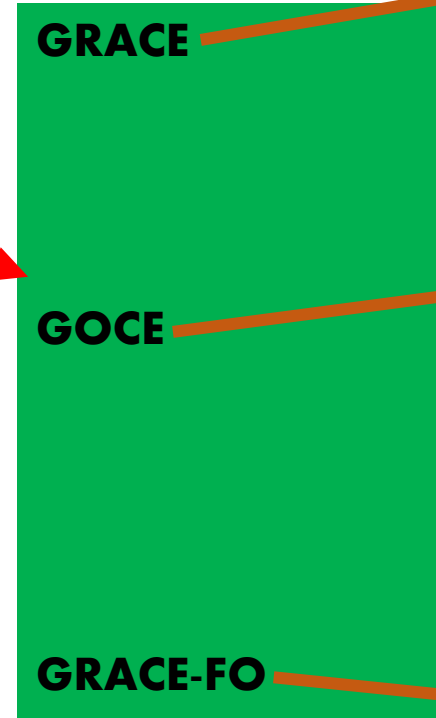
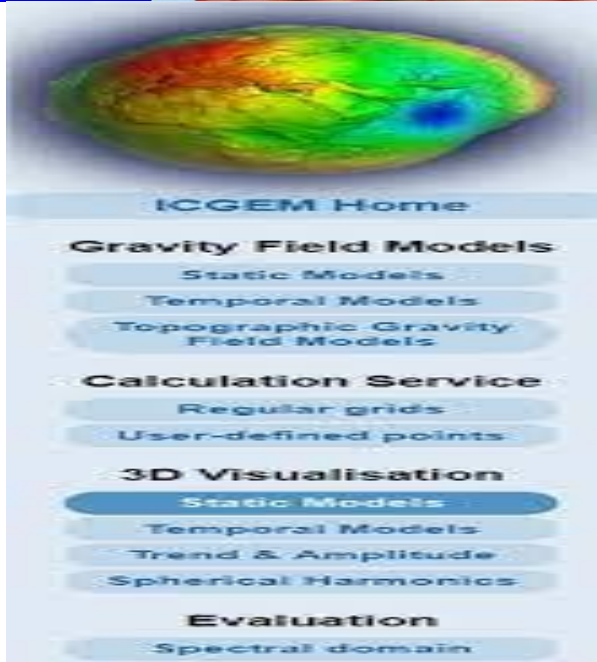
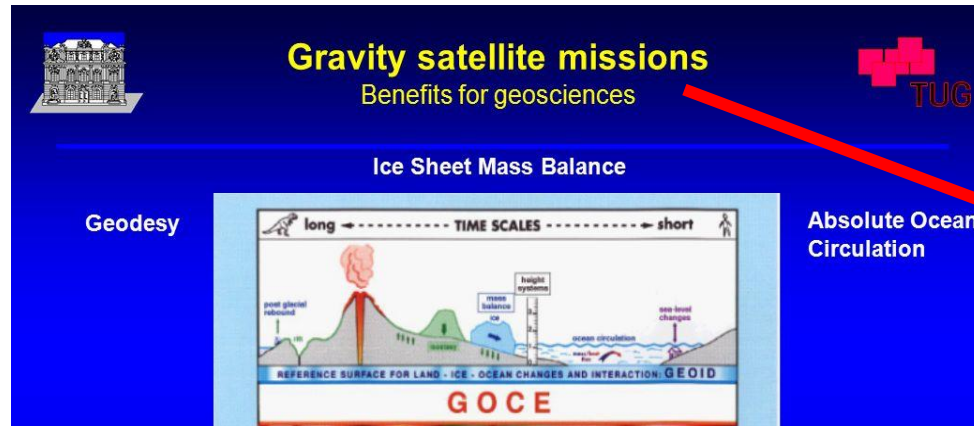
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SCIENCE LEADS THE FUTURE

Availability of Global Gravity field models (GGMs) make geodetic and geophysical modelling easier and faster but may not accurately fit local gravity field of a study area



- **GGMs are either satellite-only or combined, depending on the source(s) of data used for their compilation**

Tailoring of GGMs entails incorporating more locally suitable gravity and height data so as to enhance the accuracy of a GGM in a region and thereby develop locally synthetic gravity models that fit gravity field of that region.

A man wearing an undersized pair of trousers



A man wearing an oversized pair of trousers



A tailor fitting an oversized or undersized pair of trousers to the man's size

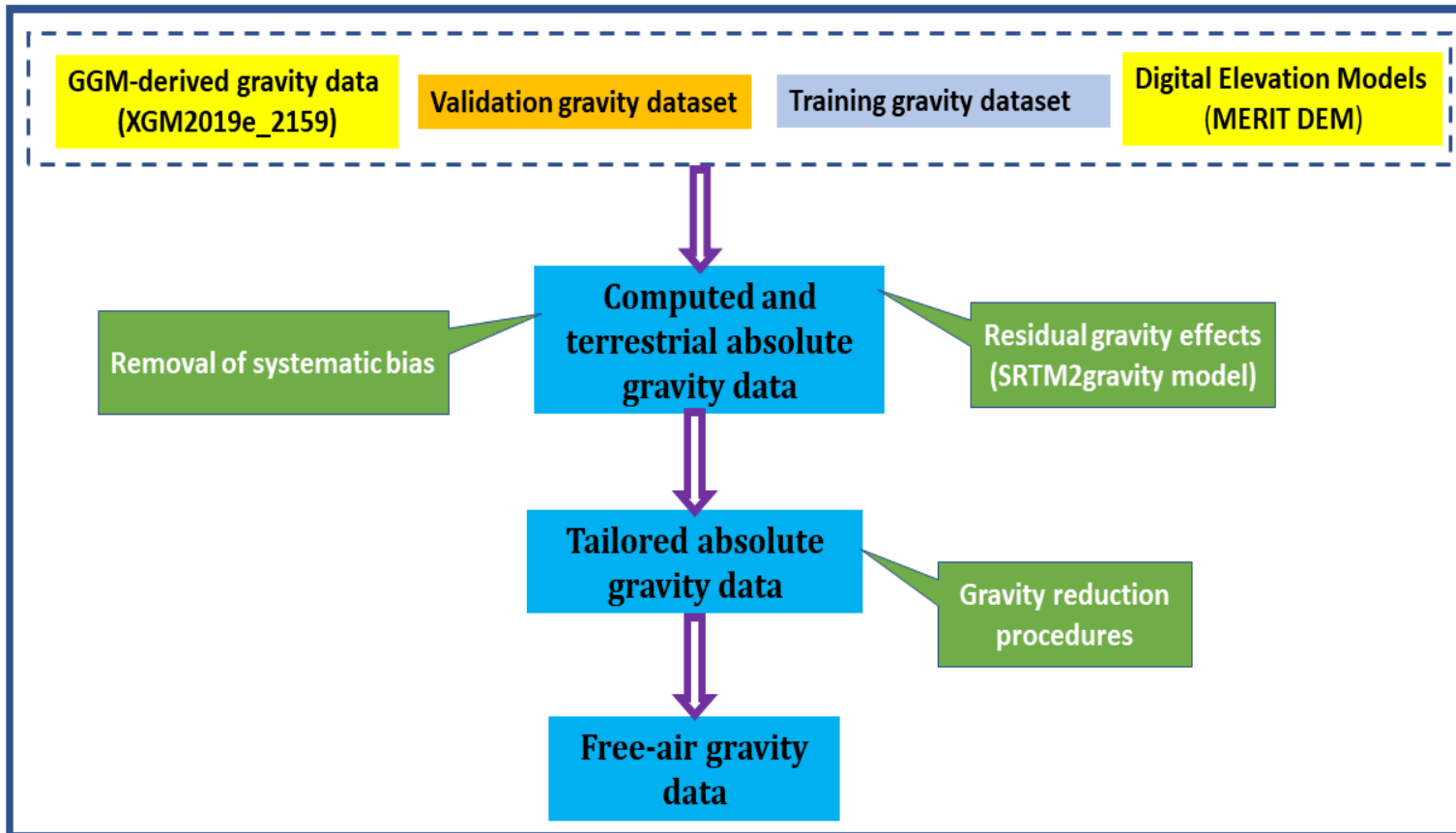
The man now wearing a fitted pair of trousers tailored to his size



Past tailoring efforts by geoscientists produced good results in the context of local gravity field modelling

Study Area	Method of analysis	Results	Authors
Iran	Harmonic expansion	± 19.7 to about ± 5.2 mGal	Weber and Zomorrodian (1988)
Scandinavia, Canada, and Australia	Harmonic expansion	8.85 to 6.39 mGal in Scandinavia, 5.39 to 2.97 mGal in Canada and 7.86 to 4.36 mGal in Australia	Kearsley and Forsberg (1990)
Europe	Harmonic expansion	15.77 to 5.96 mGal	Bašić et al. (1990)
Australia	Forward modelling technique through the Newtonian numerical integration	differences are less than ± 20 mGal	Baran et al. (2006)
Bavaria, Switzerland, Australia, and Slovakia	Combined spatial and spectral gravity forward modelling techniques	Computed residual gravity effects could detect small-scale or near-surface mass density anomalies	Hirt <i>et al.</i> (2019)
Southern Benue in Nigeria	Stochastic combination technique	16.7 to 6.4 mGal and differences are within ± 10 mGal	Apeh and Tenzer (2022)

We applied the tailoring procedure developed by Apeh and Tenzer (2022) to compile high-resolution gravity data over Limpopo Province in South Africa

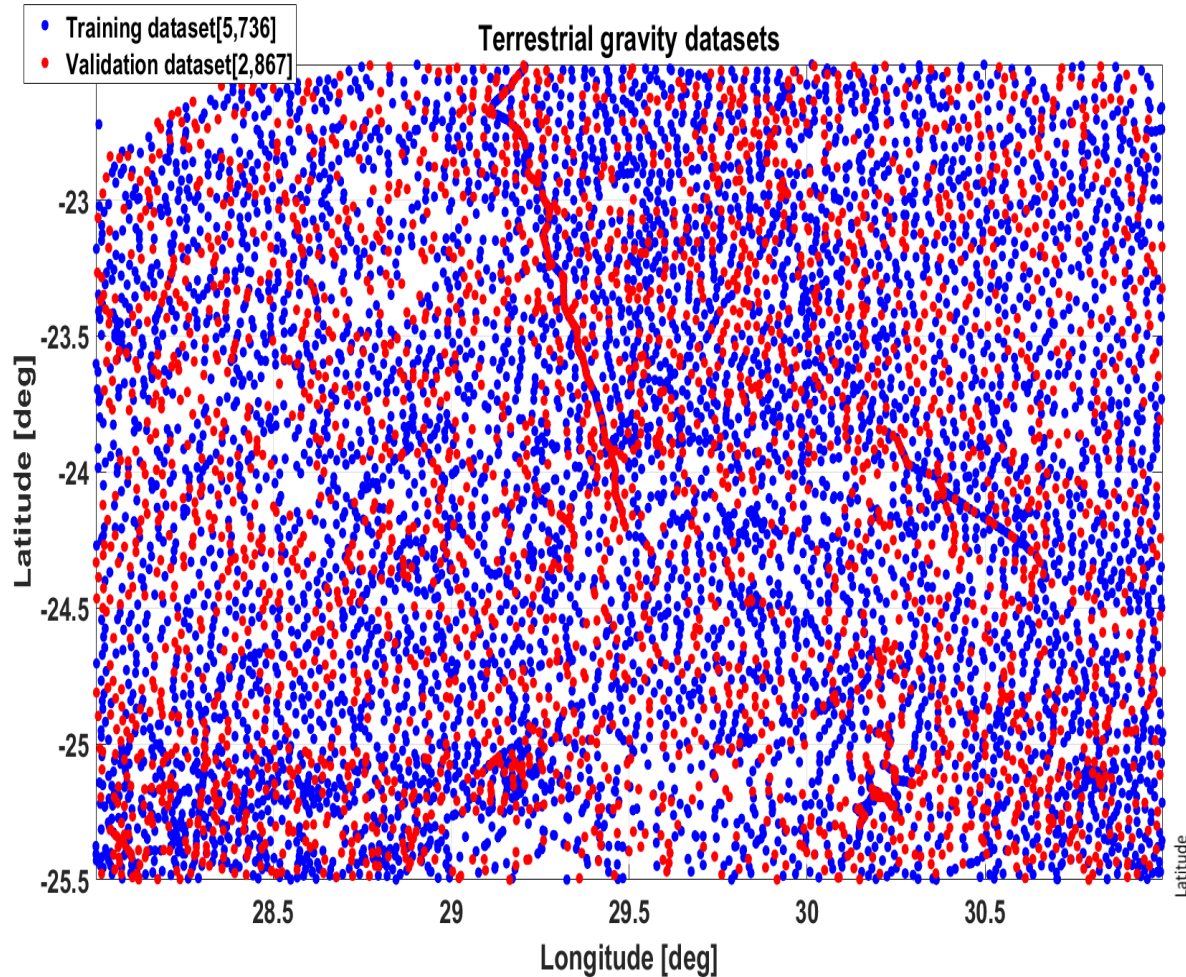


Tailored gravity model over Limpopo Province

$$\text{TGM} = \text{Abs}_{\text{ICGEM}} + \text{RTM}_{\text{Hirt}} + 1.47 \text{ [mGal]}$$

where **TGM** refers to tailored gravity model, $\text{Abs}_{\text{ICGEM}}$ is absolute gravity computed on the MERIT topographic surface from the XGM2019e_2159 via the ICGEM online platform while setting appropriate parameters during the computation, RTM_{Hirt} is the residual gravity effect extracted from the SRTM2gravity model using the coordinates of gravity sites, and **+1.47 mGal** is the detected systematic bias between the terrestrial and XGM2019e_2159 derived absolute gravity data after algebraically adding the residual gravity effects

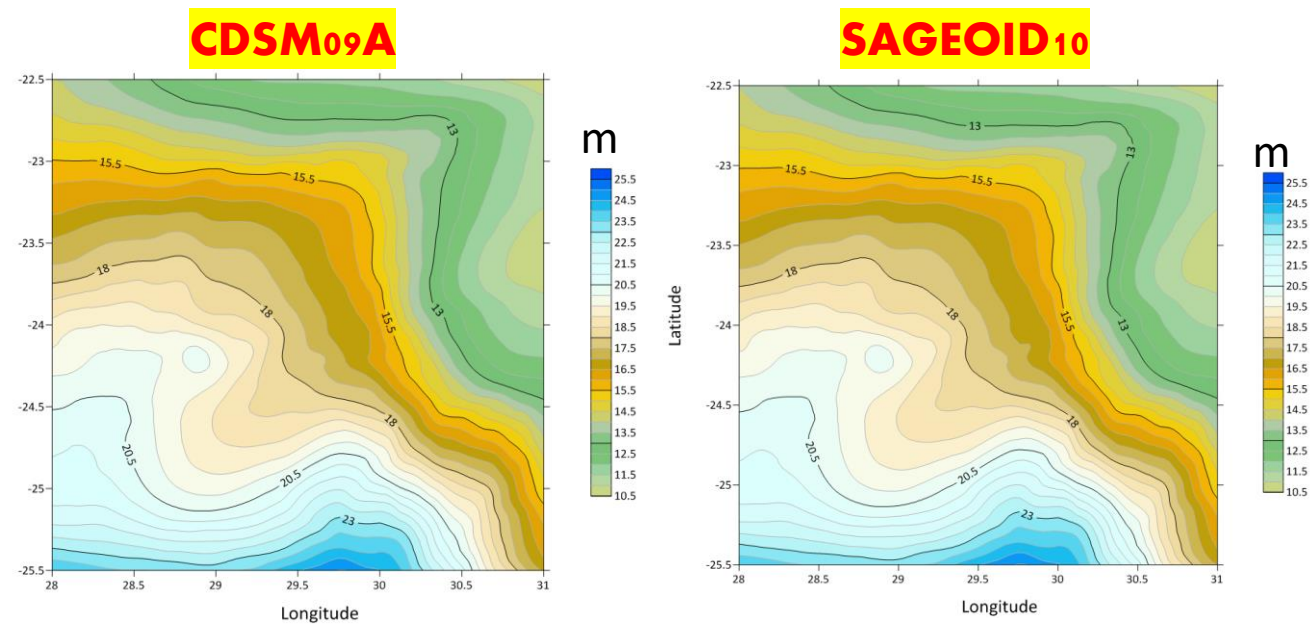
Available terrestrial gravity data over Limpopo Province and existing quasigeoid models in South Africa were used to validate the tailored gravity data



CDSM_{09A} = latest gravimetric quasigeoid model over South Africa 2010 (Chandler & Merry, 2010)

SAGEOID₁₀ = latest Hybrid quasigeoid model over South Africa 2010 (Chandler & Merry, 2010)

GeomQG = GPS/levelling quasigeoid (height anomaly) over the Province



Statistical analyses of the tailored gravity data indicate that our model could reproduce the observed gravity data at similar accuracy meaning that it can be used to densify terrestrial gravity network over the Province

5736	Terrestrial absolute gravity data (A)	Absolute gravity data computed without residual gravity effects (B)	Absolute gravity data computed with residual gravity effects (C)	Differences (A-B)	Differences (A-C)
MIN	978382.01	978342.09	978342.28	-147.84	-146.90
MAX	978740.48	978723.00	978722.04	170.64	150.57
MEAN	978586.73	978587.19	978585.25	-0.46	1.47
STD	57.51	59.78	58.22	12.22	9.48
RMS				12.23	9.59

2867	Terrestrial absolute gravity data	Tailored Absolute gravity data	Differences (mGal)
MIN	978382.01	978358.87	-65.36
MAX	978740.48	978717.29	74.11
MEAN	978586.73	978583.40	0.10
STD	57.51	58.45	8.86
RMS			8.86

Geodetic techniques used to compute quasigeoid model over the Limpopo Province in South Africa based on the tailored gravity data

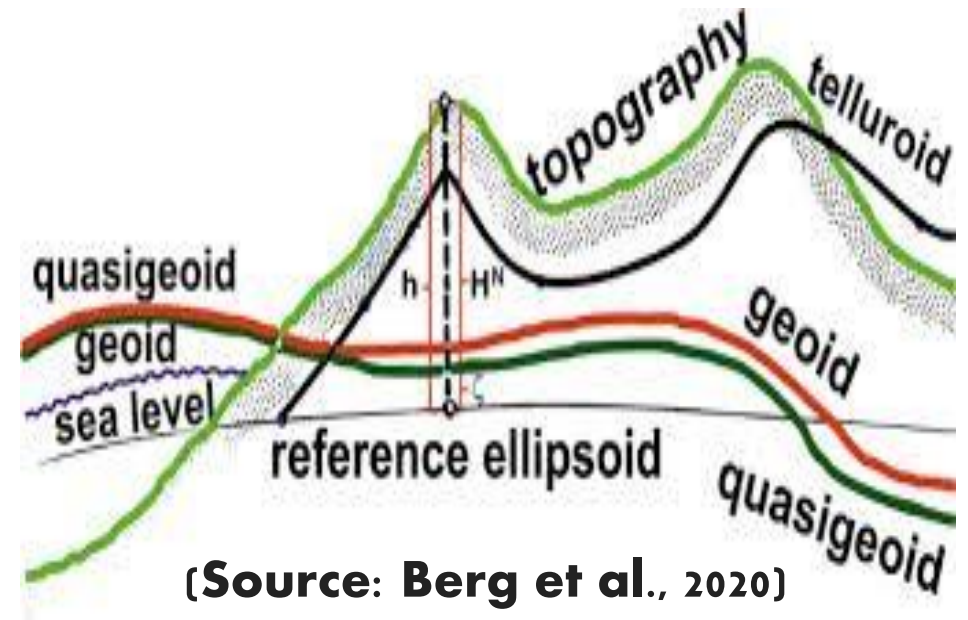
$$\zeta = \zeta_L + \zeta_S,$$

$$\zeta_L = \frac{GM_g}{r\gamma} \cdot \sum_{n=2}^{n_{max}} \left(\frac{a_g}{r}\right)^n \cdot \sum_{m=0}^n (\Delta\bar{C}_{n,m} \cos m\lambda + \Delta\bar{S}_{n,m} \sin m\lambda) \cdot \bar{P}_{n,m}(\sin \bar{\varphi})$$

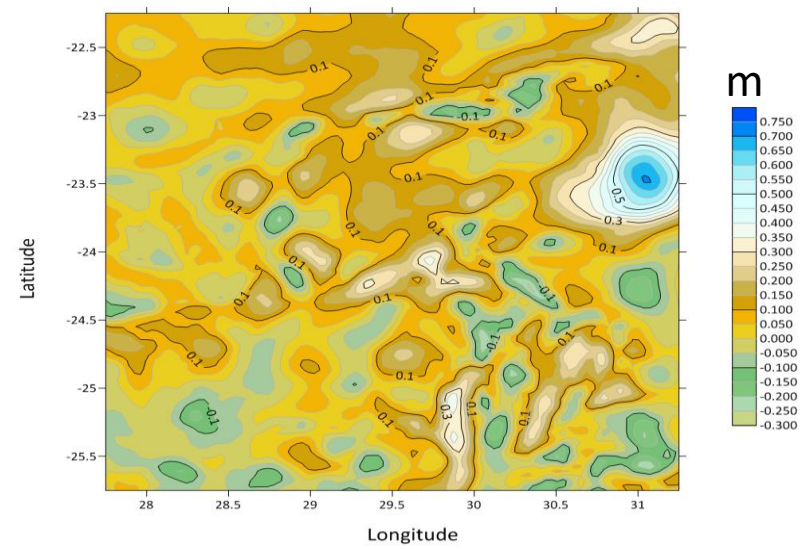
$$\zeta_S = \frac{R}{4\pi\gamma} \iint_{\sigma} \Delta g_r \cdot S(\psi) d\sigma$$

$$\Delta g_r = \Delta g_f - \Delta g_{GGM} + G_1$$

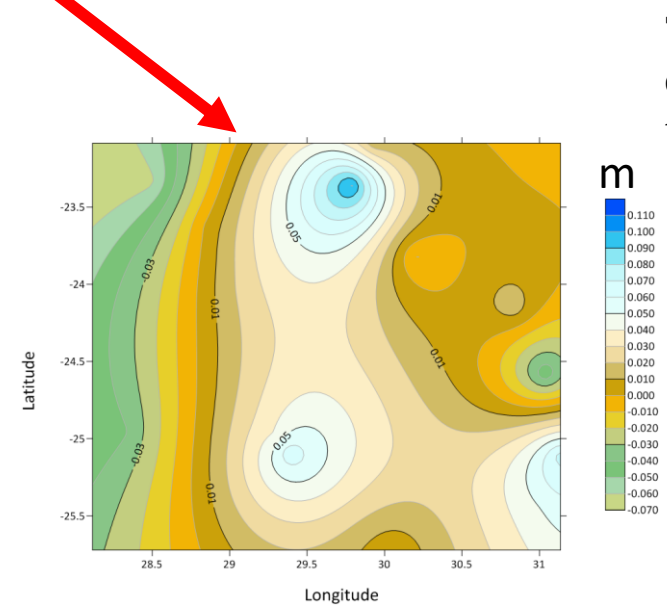
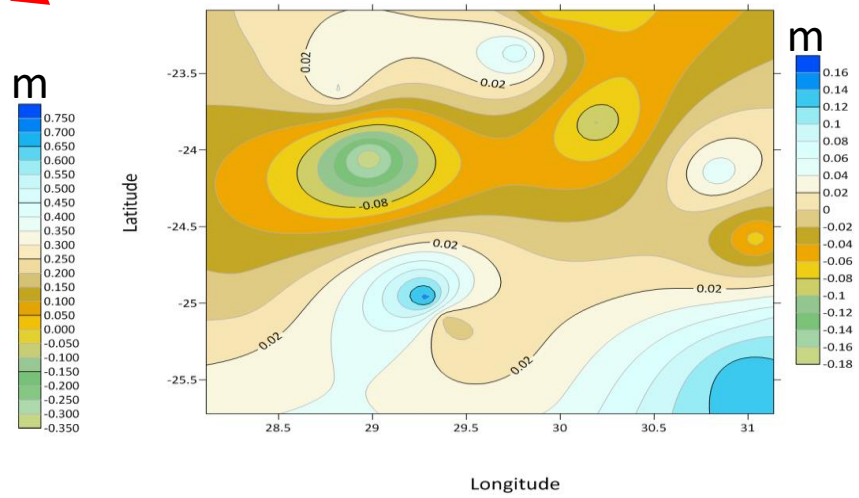
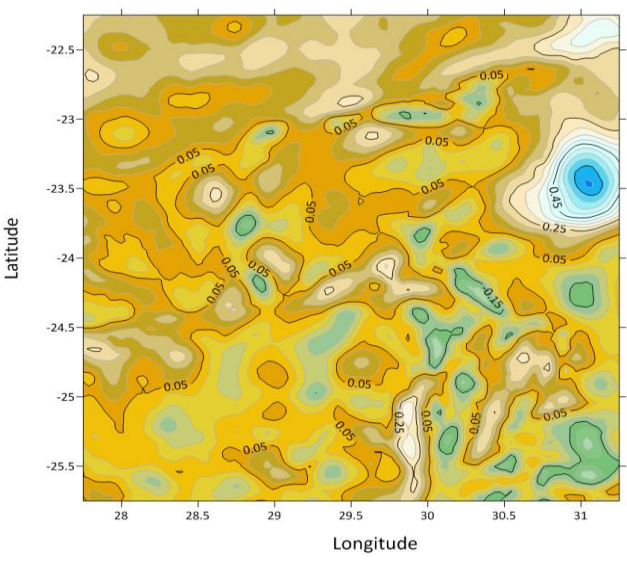
$$G_1 = \frac{\Delta\varphi\Delta\lambda}{2\pi} \left[(H \cdot \Delta g) * \frac{1}{\ell^3} - H_P \left(\Delta g * \frac{1}{\ell^3} \right) \right]$$



Geodetic validation of the tailored gravity data reveal that our technique can efficiently be used to model a high-resolution quasigeoid over Limpopo Province in South Africa

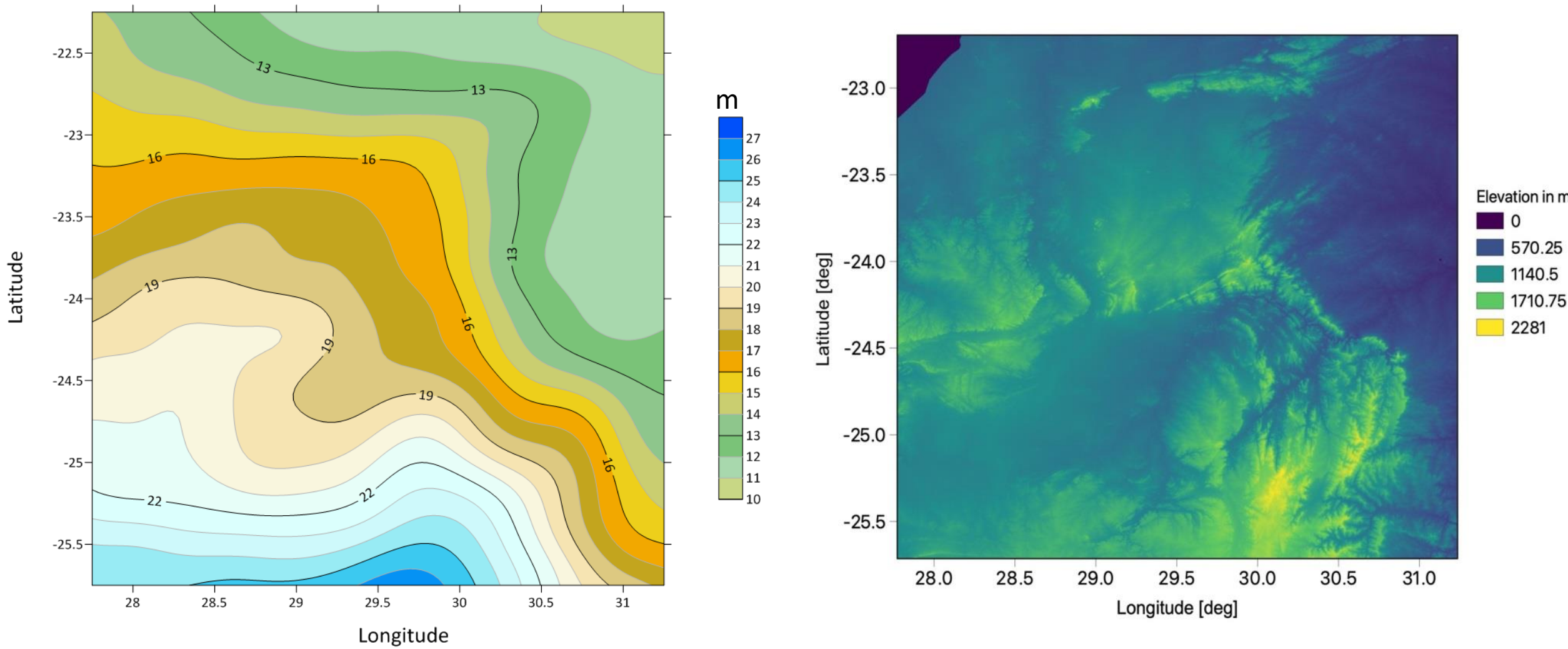


Differences	Points	Min (cm)	Max (cm)	Mean (cm)	SD (cm)
TGQG-CDSM09A	7225	-30.59	69.69	5.29	12.06
TGQG-SAGEOID10	7225	-34.87	69.80	6.14	12.67
GeomQG-TGQG	15	-18.40	15.40	1.25	5.22
GeomQG-CDSM09A	15	-6.30	10.30	1.19	4.80



TGQG = gravimetric quasigeoid based on tailored gravity data over the Province

Quasi-geoid model over the Limpopo province conform with geomorphological features of that Province.



In summary, the newly developed tailored gravity model fits local gravity field over the Limpopo Province for geodetic and geophysical applications



A cost-efficient and sustainable way of densifying terrestrial gravity data

This study implies that our approach could be beneficial for many developing countries and regions with insufficient coverage of terrestrial gravity data

Questions?

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