Vegetation influence on stochastic coastal dune dynamics substantiated by process-based model

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Abstract

Coastal dunes are the highest natural features on the beach. They protect the beach communities and low-energy environments from storms by virtue of their elevation. Their formation is a result of delicate coupling between accretional and erosional processes. Here we study the influence of vegetation on dune growth and recovery under water-driven erosion utilizing a processbased coastal model under a stochastic framework. An equivalence of this model is first established with a recently developed stochastic model of dune evolution under water-erosional stress. From the model vegetation parameters: the vegetation growth time and colonization time are quantified and their relation with characteristic dune growth times is established. Vegetation causes an initial lag in dune formation due to the colonization time. Also, the dune growth under the influence of vegetation on dune recovery is quantified by the colonization time, and its competition with water-driven erosion is analyzed. This leads to the development of a phase space relating to flooding frequency, intensity, dune growth, and dune establishment times. The dune state transitions from high to barren based on the competing dune recovery time controlled by vegetation and the flooding frequency. Finally, a vulnerability indicator is obtained from the transition threshold as a minimum base elevation after an overwash required by the beach for vegetation to recover and establish dunes that overcome frequent flooding.

ARTICLE TYPE

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Abstract

Coastal dunes are the highest natural features on the beach. They protect the beach communities and low-energy environments from storms by virtue of their elevation. Their formation is a result of delicate coupling between accretional and erosional processes. Here we study the influence of vegetation on dune growth and recovery under water-driven erosion utilizing a process-based coastal model under a stochastic framework. An equivalence of this model is first established with a recently developed stochastic model of dune evolution under water-erosional stress. From the model vegetation parameters: the vegetation growth time and colonization time are quantified and their relation with characteristic dune growth times is established. Vegetation causes an initial lag in dune formation due to the colonization time. Also, the dune growth under the influence of vegetation is found to be divided into two regimes, stable and mobile. Within the stable regime, the influence of vegetation on dune recovery is quantified by the colonization time, and its competition with water-driven erosion is analyzed. This leads to the development of a phase space relating to flooding frequency, intensity, dune growth, and dune establishment times. The dune state transitions from high to barren based on the competing dune recovery time controlled by vegetation and the flooding frequency. Finally, a vulnerability indicator is obtained from the transition threshold as a minimum base elevation after an overwash required by the beach for vegetation to recover and establish dunes that overcome frequent flooding.

KEYWORDS

coastal dune, vegetation colonization, stochastic dune evolution, transition threshold

1 L **INTRODUCTION**

Coastal dunes are protective structures on the beach that preserve the ecosystem and provide a natural solution for reducing flooding and destruction from waves, storms, etc. Coastal dunes are generally the first line of defense for sandy beaches from flooding¹²³ of the sea and also provide ecosystem services⁴⁵⁶. Also, the presence of coastal dunes in several sandy beaches around the world makes them an ideal solution for reducing nuisance flooding of beaches⁷. Coastal dune formation and its evolution is complex as it depends on several variables such as preexisting beach morphology, sediment supply, vegetation and the climatic drivers such as wind, sea level, waves, and storms⁸⁹¹⁰¹¹¹²¹³.

The effect of vegetation on coastal dunes has been studied extensively to develop several governing processes and establish its drivers and control parameters. Vegetation dictates dune morphology by modifying the sediment flux to build and stabilize the sand dune^{12,14,8,15,16,17}. Vegetation species differ in their ability to colonize a location and capture sediment by reducing wind stress, and growth in response to sand deposition and water erosion for dune establishment^{18,19}. The studies identify a signature of a two-way interaction between vegetation and the landscape wherein vegetation contributes to building the dune through sand trapping and thus engineers an environment where it can develop more abundantly 20 . Thus a joint evolution of vegetation and morphological processes is utilized to develop numerical models and analyze the dune state^{21,22,23,24}. These eco-morphodynamic analyses combine the dynamics of vegetation with the dune thus developing a biophysical feedback mechanism between sand

1

flux, and vegetation growth to determine the resulting dune geomorphology^{25 26 27}. Studies such as²⁸ have identified the roles of vegetation in different aspects of coastal morphological processes and suggest the species that influence specific functionalities.

Due to the complexity associated with experimental and field study of dunes, in general, the development of aeolian dune dynamics, modeling has been used as an alternative to the empirical approaches. There have been several attempts to numerically simulate aeolian dunes^{29 30 31} and especially the continuous minimal model developed by³² has been extended to include several features such as a three-dimensional profile of dunes³³, dune collisions³⁴, vegetation growth³⁵ and sand transport³⁶. This model was applied to the coastal environment in¹⁶ to study the foredune development by simplifying the complex coastal eco-morphodynamic process. This process-based model identified variations in dune fields in response to vegetation and wind properties mainly vegetation growth rate, aeolian sand flux, etc. With the competition between the water-driven erosional processes and vegetation growth the model was analyzed in²³ to find a bi-stability present in barrier islands with coastal dunes relating the overtopping frequency to the vegetation growth and the growth of dunes driven by the sand supply.

The vegetation dynamics in the model are controlled by two major factors, colonization and growth rate. Colonization is a process affected by physical factors such as erosion, precipitation, salt spray and also on the species type^{37 38 39}. Through colonization, a minimum cover fraction of plant is required that starts affecting sand flux to cause sediment deposit and building of proto-dunes. This begins the feedback loop where morphology affects sand flux and causes growth of dune and the vegetation acts as an anchor holding the dune in place and enabling the dune to mature and stabilize. Thus initial cover fraction controls how long it takes for a proto-dune to establish and vegetation growth helps develop and sustain the mature dune. The change in these two characteristics can affect dune growth by varying both its growth time and maximum height.

A recent study of stochastic properties of water erosional events on the beach described them as High water events (HWE), which corresponds to intermediate size erosional events with water levels above the beach disrupting the dunes and vegetation, in ⁴⁰ showed that they can be modeled as marked Poisson process with exponentially distributed marks. Thus the HWE can be defined by using only two parameters, mean frequency and mean intensity. This result enabled the development of a probabilistic point model describing the evolution of dune elevation at the crest, following the stochastic model of soil moisture dynamics⁴¹. This stochastic model predicts the dune state driven by the HWE characteristics and the existing morphology captured by two control parameters, rescaled intensity, and rescaled frequency⁴².

The stochastic model however includes a key assumption of vegetation being much faster than dune growth times thus disentangling the effects of vegetation dynamics. In order to study this effect the analytical model needs to be redesigned and solved with a vegetation dynamic equation. To simplify this the process-based model developed in²³ that involves vegetation dynamics for dune evolution is utilized. For this purpose, the validation of the stochastic model is performed by analyzing the evolution of dune along a transect of a barrier island with the process-based model following the assumptions undertaken in the stochastic model, such as fast vegetation, erosion only under dune overtopping and water-driven erosion following HWE properties. These simplifications are incorporated into the various modules to make the results comparable with the prediction of the stochastic model. This will enable the process-based model to be an equivalent tool to the stochastic model to study the vegetation influence. Thus through this analysis, an extension of the phase space of the dune state obtained in⁴² is carried out by relating the HWE properties and vegetation dynamics with respect to the growth and recovery of dunes to answer the question of how vegetation affects the dune dynamics.

2 | METHODS

2.1 | Process-based model

The coastal dune model describes the Eco-morphodynamics on the beach to study dune evolution driven by sand flux and vegetation. Mainly the temporal evolution of the sand surface elevation h(x, y, t), defined relative to the Mean High Water Level (MHWL), and the cover fraction $\rho_{veg}(x, y, t)$ describing the amount of vegetation in a numerical cell. It is governed by a set of differential equations for the physical and biological processes describing aeolian sand transport on a vegetated surface at the shore ^{43 44}. It resolves the co-evolution of the sand-surface elevation *h* relative to the water table, and the vegetation-cover fraction ρ_{veg} under a constant onshore wind ¹⁶. In the current analysis, the evolution of the dune on a barrier island profile is simulated to match the stochastic model conditions.

2.1.1 | Vegetation dynamics

Vegetation dynamics is represented in a simplified way, a single generic grass species is considered with a cover fraction ρ_{veg} with a logistic growth that is sensitive to erosion and accretion. It can increase up to the maximum cover $\rho_{veg} = 1$ during a characteristic time T_v^{16} . The growth is also sensitive to the distance from the shoreline such that plants can only grow land-ward of the vegetation limit L_{veg} :

$$\frac{d\rho_{veg}}{dt} = \rho_{veg} \frac{(1 - \rho_{veg})}{T_v} \theta(x - L_{veg}) - \gamma H_v^{-1} \left| \frac{\delta h}{\delta t} \right|$$
(1)

The plant mortality is driven by the rate of change in elevation at a location and is scaled by its sensitivity to accretion/erosion by γ while it can grow to an effective height of H_{ν} , the ratio of vegetation volume to cover area. In the model simulations, only pure vertical growth is considered and hence there is a requirement of a non-zero initial cover fraction ρ_0 for vegetation to start colonizing a location.

2.1.2 Dune formation process

The aeolian transport begins at the foreshore, at the first location where the wind shear stress is above the transport threshold τ_r , calculated based on sand wetness. Sand flux then steadily increases up to the maximum, saturated value that the wind can sustain. Under a constant onshore wind, sand blows continuously across the beach into the backshore where it is trapped by dunebuilding vegetation and a foredune begins to form. It is assumed that the spatial distribution of the vegetation is characterized by the minimum distance from the shoreline called vegetation limit L_{veg} needed by the plants to survive long enough to build a mature foredune. Closer to the shoreline, plant growth is hampered by wave runup, soil salinity, storms, etc., and any incipient foredune will be short-lived. Once a proto-dune emerges, its evolution is determined by its interaction with the wind flow, with the vegetation playing a secondary role as a passive roughness element anchoring the dune crest and thus preventing dune motion. Growth of the foredune produces a deceleration of the flow upwind thereby reducing the surface shear stress and leading towards a stable profile.

Dune formation is driven by the interaction of sand transported by the aeolian process with the morphology where there exists feedback between the change in morphology and the evolution of wind profile. The main components to modeling dune growth are - the calculation of the wind under the topographic influence, estimation of the sand flux carried by the perturbed wind, and finally the evolution of the sand surface due to sand erosion, deposition, and avalanches⁴³. The coupling of the aeolian sand transport and topographic evolution presents two different timescales, sand transport and wind flow changes that happen in the order of seconds and the erosion-deposition processes that change the surface elevation requiring hours or days to present significant changes. This enables the decoupling of the different processes thereby leading to stationary solutions for wind surface shear velocity u^* and for the sand flux \vec{q} that can be later used for the time evolution with the saturation length and saturation flux known based on the wind shear and sediment properties such as transport threshold etc. The spatial change of the sand flux defines the temporal change of the sand profile h(x, y) according to the mass conservation, $\delta h/\delta t = -\nabla .\vec{q}$. This topographic evolution thus creates a new disturbance on the wind field thereby completing a feedback loop of wind variation and topographic changes. The modelling of these processes is based on ³² and the improvements proposed by ³¹³³⁴⁵⁴⁴.

2.1.3 Water-driven erosion process

The dune model was developed to include the water-driven erosional process on barrier islands in 23 by using a phenomenological approach to water runup affecting the beach profile. The erosion disrupts both the elevation profile and the vegetation and its effect on dune growth and recovery can be now tested. The erosional process is modeled as water events producing a runup on the beach and only their contribution to beach erosion is considered and the accretion due to the water events is not analyzed. The result from the probabilistic analysis in 40 is utilized to model the erosional events (HWE) on the beach as marked Poisson process with exponential marks. This means the events can be described using only two parameters, the mean inter-arrival time and mean intensity, the total water elevation defined as 2% exceedance wave runup above the mean water level on the beach.

There have been studies that attempted to model the water erosion process or use phenomenological expressions to obtain the sand profile evolution. The phenomenological approach used in²³ involves multiple tuning parameters such as iterating to obtain a stable erosion profile and redistribution of eroded sand, the efficacy of wet avalanches causing dune toe erosion, etc. These complications make it difficult to analyze the dune state and hence it is simplified to the following condition, erosion of profile happens only for elevations below the HWE intensity and is reduced to a flat beach profile without vegetation post erosion. The elevations above the event intensity and beyond this location in the cross-shore line remain unchanged from the storm event. This is done without the presence of wet avalanches that occur during erosion but with dry avalanches post erosion to relax the steep dune profiles caused by the storm events. This method allows for the building of the dune profile due to the sand flux possible in front of the eroded dune shape and thus the dune recovers its shape before it can start growing back. Concerning the vegetation, the eroded regions undergo destruction reducing the cover fraction to zero. Since seeding is always required to enable vegetation to grow in these locations as propagation is not possible the initial cover fraction ρ_0 is established at these points post erosion following the vegetation constraints.

2.2 Model conditions and parameters

The model input parameters can be classified based on the dynamics they correspond to namely aeolian transport, vegetation, and water-driven erosion.

2.2.1 | Aeolian transport

Wind surface shear velocity u* is the key parameter that defines the aeolian transport. Its range is based on the transport threshold of sediment and the sand properties considered in the model are in the range of 0.3 - 0.5m/s, of which 0.35m/s is used for all simulations. Sediment properties such as size, weight, wetting threshold, etc are fixed. The fraction of time wind blows above the threshold is kept as a scaling factor of time r as no wind above the threshold means no transport and hence no change in morphology. This scaling factor is utilized to compare the simulated growth rates to field measurement of the dune growth to rescale the simulation time to real-time for interpretation of the results.

2.2.2 | Vegetation

The vegetation parameters such as the sensitivity of vegetation to erosion, $\gamma = 2$, and the effective plant height $H_v = 1m$, to re-scale the effect of erosion/accretion are kept as a constant corresponding to measurements of a typical coastal grass species⁴⁶. The initial cover fraction of vegetation is considered $\rho_0 = 0.1$, uniformly seeded to all locations capable of having vegetation for the case with vertical growth, and the characteristic growth time of vegetation of $T_v = 3days$ controls its production rate. These parameters were chosen to ensure fast vegetation growth compared to the dune, a constraint for consistency with the assumption of the stochastic model. The vegetation limit, distance from the shoreline at which vegetation can start establishing based on salt spray resistance and the presence of dry sand, etc. is considered $L_{veg} = 20m$ for this analysis to have a stable dune height around $H_{max} = 3m$, based on the result in ¹⁶. These parameters are kept constant for simulations and thus only a fast vegetation case is considered for validating the stochastic model so that vegetation grows and recovers much faster than the dune and does not influence the dune recovery process.

2.2.3 Water-driven erosion

The water-driven erosion is an event producing large enough run-up on the beach that modifies the dune elevation. For the water-driven erosion, the mean intensity and mean frequency of HWE are the only required parameters as the HWE is modeled as a marked Poisson process with exponential marks⁴⁰. The dune flooding is modeled as an overwash of all regions over-topped by the runup of the event. The dune height remains the same if it is not over-topped. Thus only the region ahead of it lower than the intensity gets eroded and its elevation reduces down to the beach level along with the vegetation cover fraction going to zero, thereby having a complete erosion of profile and destruction of vegetation. Consider the HWE has an intensity *S* and the beach profile elevation with the dune is given by h(x) where *x* is the cross-shore distance from the shoreline. The erosion is modelled as, if S = h(x) at $x = x_i$ then $h(x) = h_b \forall x < x_i$ where h_b is the initial flat beach elevation profile. Also $\forall x < x_i$, $\rho_{veg}(x) = 0$, and

then in the next time step the vegetation is seeded by the initial cover fraction ρ_0 . This assumption simplifies the water erosion model and does not accurately capture dune toe erosion. But it stays consistent with the assumption of the stochastic model of overtopping only erosion of the dune.

The erosional events (HWE) occur with exponentially distributed inter-arrivals and intensity. A single parameter is needed to define the exponential distribution which is the mean of the distribution. The parameters are the mean intensity and mean inter-arrival time or mean frequency of HWE. From the mean inter-arrival frequency λ_0 , the events are generated using the formulation for Poisson distribution inter-arrival times.

$$T_i = -\ln(n)/\lambda_0 \tag{2}$$

where T_i is the inter-arrival time and *n* is randomly picked from a Uniform distribution $U_{[0,1]}$. Using the mean intensity \overline{S} and an exponential distribution generator the event intensity is associated with the corresponding event times of the HWE⁴⁰.

$$S_i = -\ln(n)/\overline{S} \tag{3}$$

2.2.4 Boundary and initial conditions

For the boundary conditions, an infinite reservoir of sediment influx is considered in the foreshore region of x < 0, and the foreshore is always replenished to the same beach slope. Sea level rise and shoreline migration are not included in the current analysis. No aeolian sand influx $\vec{q} = 0$ happens at the most seaward limit of the foreshore, where the surface elevation equals the mean low water level (MLWL), and thus the sand is effectively wet at all times and not available for aeolian transport.

The sand surface elevation *h* is initially defined as an inclined plane $h(x, y) = xtan(\alpha)$ at the foreshore (x < 0), and as a flat surface h(x, y) = 0 at the backshore (x > 0), where α is the beach slope, kept 1 in the simulations. All elevations in the result from the model are with reference to the mean high water level, MHWL. The initial vegetation cover fraction is uniform at all locations possible beyond the vegetation limit ($\rho_{veg} = \rho_0$). The simulations are performed on a domain with 100 m in the cross-shore *x* direction and 4 m in the alongshore *y* direction, with square cells of 1m * 1m. The number of cells in the alongshore direction must be a power of 2 to support the periodic boundary condition in Fourier space used in the solution of wind shear stress. This makes the simulation domain predominantly 2D and hence is analyzed using only a single cross-shore as shown in Fig.1.

2.2.5 | Control parameters

The stochastic model predicts the state of the dune by two control parameters - the rescaled frequency ξ_c^+ and rescaled intensity λ_d^+ .

$$\xi_c^+ = \frac{\overline{S}}{H - h_0} \tag{4}$$

$$\lambda_d^+ = \lambda_r T_d e^{-h_0/\overline{S}} \tag{5}$$

The dune morphology parameters are maximum dune elevation H, base elevation h_0 , and growth time T_d and the HWE parameters are the mean intensity \overline{S} and mean frequency at the beach elevation λ_r . These parameters are input to the process-based model and the simulation results are compared with the stochastic model predictions. The maximum dune elevation and the characteristic time for growth from the fit of the undisturbed dune growth are obtained using the assumed dune growth equation from ¹⁶, where the only factors affecting dune size are the vegetation limit and the sediment supply, which are fixed for all simulation runs. The base elevation is considered the same as the beach elevation for the cases considered. Using these the only control parameters for the fixed dune growth case are the HWE intensity and frequency. These results are used to validate the stochastic model using the process-based model.

2.3 Vegetation and dune characteristic times

These model simulations consider the influence of vegetation strongly affecting the dune growth by slowing the vegetation colonization and growth timescales to the order of dune growth times. These model runs are performed with undisturbed



FIGURE 1 A) Process-based model simulated dune beach profile displaying the various morphological features: base elevation h_0 , vegetation limit L_{veg} and maximum dune height *H*. All elevations are with respect to MSL B) Evolution of dune profile over time with undisturbed growth until saturation at the stable maximum dune height

conditions in the absence of water-driven erosion to purely relate the dune and vegetation growth characteristics. This is done mainly by modifying the two parameters of vegetation, growth time T_v and initial cover fraction ρ_0 . The values of vegetation sensitivity $\gamma = 2$ and maximum height of vegetation $H_v = 1m$ are fixed based on results from previous studies using the model³⁶. The vegetation limit and wind shear velocity above the sediment transport threshold are fixed to $L_{veg} = 20m$ and u = 0.35m/s. These values control the simulated dune's maximum height and growth time and are hence fixed to study only the influence of vegetation. The scaling factor r of wind above the transport threshold, representing the active sand flux, is used to convert the simulation time to an approximate time in real years. This is obtained as the ratio between the maximum dune growth rate of 0.5m/yr present in Oregon coastal dunes, from field data⁴⁷, with the simulation dune growth rate G, obtained as the initial slope of dune height time series. This value is approximately 50 for stable dune elevations and this factor is applied to the simulation years henceforth. In the model runs the evolution of the cross-shore profile is obtained from each time step and the dune crest, the maximum elevation, is extracted along with the maximum vegetation cover fraction including its location. Results from the model for parameters corresponding to fast vegetation and slow vegetation conditions display the initial lag and the slow growth of a mature dune under the influence of vegetation (Fig.2).

The simulation is performed for a sufficiently long time to ensure a steady maximum dune height is achieved. This resulting time series of dune growth is used to obtain the characteristic times that indicate the effect of initial vegetation and the dune growth rate. The parameters are identified and extracted from these times that can be used to quantify and relate the vegetation and dune growth and thus define the controls on the dune dynamics. The undisturbed dune growth equation can be approximated as,

$$\frac{dh}{dt} = \frac{H-h}{T_d} \tag{6}$$

, where H is the maximum dune height and T_d is characteristic dune growth time¹⁶. This means the dune follows an exponential saturation growth given by,

$$h(t) = H\left(1 - e^{-t/T_d}\right)$$
(7)

This result was however obtained considering that the vegetation growth timescale is much smaller than the dune growth times. However, in the current analysis, the vegetation growth is comparable to dune growth and the variation affects the dune

growth thus bringing another influence to the growth equation. This is captured using a modified equation of the form

$$h(t) = H\left(1 - e^{-\frac{t - T_{dy}}{T_d}}\right) \tag{8}$$

The dune initial growth time T_{dv} can be obtained by extending this fit to cross the zero elevation and the characteristic dune growth time T_d can be obtained from the exponential growth component (Fig.3). This fit is performed for elevations satisfying h/H > 0.1 as the initial dune growth in the process-based model is faster than the exponential following a power law due to volumetric growth.

Similar to the growth of dunes, vegetation growth characteristic times can be extracted. Vegetation growth is modeled as a logistic curve of cover fraction at a location. This analysis considers vertical growth only and does not include lateral propagation effects. Thus it follows an exponential asymptotic saturation after establishment. The result obtained from the model for the cover fraction at the dune crest is fit using the logistic equation to obtain the two required values, vegetation growth time T_v that depends on the growth rate factor and the vegetation colonization time T_{v0} which is controlled by the initial cover fraction ρ_0 at the location (Fig.3). The growth time T_v is an input value provided to the model and is in the order of simulation days. To obtain the colonization time the logistic growth equation is solved as follows,

$$\frac{d\rho(t)}{dt} = \frac{\rho(1-\rho)}{T_{v}}$$

$$\rho(t) = 1 - \frac{1-\rho_{0}}{1-\rho_{0}+\rho_{0}e^{t/T_{v}}}$$

$$\rho(t) = 1 - \frac{1-\rho_{0}}{\rho_{0}}e^{-t/T_{v}}$$

$$T_{v0} = T_{v}log\left(\frac{1-\rho_{0}}{\rho_{0}}\right)$$
(9)

For large times the solution of the growth simplifies to the exponential asymptotic curve that starts from the time $T_{\nu 0}$ with the condition that $\rho_0 < 0.5$ to ensure vegetation colonization time is always higher than 0. These provide all the required vegetation and dune characteristic times for analyzing the undisturbed dune growth simulations.

3 | RESULTS

3.1 Model validation

The Distribution functions (probability density, PDF and cumulative density, CDF) of rescaled dune elevation from the processbased model are compared with that predicted by the stochastic model for the same HWE parameters. The stability of the numerical simulations is verified by analyzing the moving average and standard deviation of the dune height time series. The comparison of the distributions for the three characteristic dune elevation states: high dune state, barren state, and mixed state with both modes coexisting, shown in Fig.4, 5, and 6 illustrates that the elevation modes are captured effectively.

The validation of the stochastic process is further substantiated by comparing its result with the process-based model for different output variables namely the mean transition time, mean excursion time below the steady state minimum rescaled elevation, and the average dune elevation under steady state. As shown in Fig.7 the analytical and simulation results agree well for the cases with sufficient statistics to reach the steady state desired. The discrepancies can be seen in the points corresponding to inadequate statistics and cannot be deemed as the equivalent steady state result. Also, the greatest variation among the steady state points occurs in the mid-range of the mean which corresponds to the mixed state case, thereby indicating the need for more statistics for these states. These results are also visualized in the phase space only for the points with long enough simulation time (Fig.8). Here we can see that the only factors affecting the state of the dune are the rescaled intensity ξ_c^+ and frequency λ_d^+ of the HWE. Increasing the frequency makes the state barren and intensity reduces the duration for which the dune can stay high as it starts to feel the effect of water erosion. The importance of the simulation time is evident from the comparison as certain states require longer series in order to represent the steady state solution accurately. Fig.8 shows this by the error percentage between the analytical and numerical rescaled mean dune height indicated by the size of points.

3.2 | Vegetation influence on dune dynamics

The dune growth time from the undisturbed model runs can be compared to the vegetation growth time and colonization time. From (Fig.9) it is evident that it is independent of the colonization time but follows a positive trend with growth time beyond a certain threshold. This leads to the two regimes caused by vegetation growth on dune stabilization where below this threshold the vegetation is fast and stabilizes the dune by reducing transport but in the case of slow vegetation, the vegetation is killed by sand burial and thus leads to mobile dunes that grow in size moving farther from the shoreline. In this case, the dune continues to grow in size over a longer growth time. Thus slow vegetation growth destabilizes the dune, and increases its size and effective vegetation limit, which is the distance from shore to dune crest or first line of vegetation establishment⁴⁸. The undisturbed dune growth under these vegetation characteristics evolves from a stable state first to a metastable state, where the dune crest shifts and the maximum elevation increases, finally leading to an unstable mobile dune as shown in Fig.10.

We can see this effect of dune stabilization in the (Fig.11). The maximum dune height is stable within a band of 2 to 3 m for fast vegetation whereas slow vegetation leads to dunes that keep growing till a maximum size much higher than 3 m. The effect is also visible from the movement of the dune crest with respect to the shoreline. The greater this distance the larger the dune, as mobile dunes can grow due to the presence of non-zero sand flux in the wider beach. This relation between maximum dune height and distance to the shoreline was studied in ¹⁶ and here as seen a similar trend is obtained. In the current analysis, the presence of mobile dunes is avoided thereby restricting the maximum dune height and its growth time. This enables us to fix the parameter of rescaled intensity ξ_C^+ associated with the HWE as the mean intensity is constant at the beach and the profile has a fixed maximum dune elevation.

Also, the initial growth time of dune T_{dv} obtained for the model using the fit for different cases of ρ_0 and T_v , is found to be dependent only on the vegetation colonization time T_{v0} and independent of its growth time T_v (Fig.12). This makes the establishment of dunes post overwash solely dependent on the ability of vegetation to colonize the location and to present enough cover fraction to affect the sand flux. For the phase space analysis, only the stable dune regime is considered where there is the fast growth of vegetation and maximum dune height *H* is around 3 m by the stabilization and with the HWE mean intensity \overline{S} fixed at 0.35 m, the rescaled intensity parameter ξ_c^+ is kept constant around 0.13 for the analysis. By varying colonization time the model runs are performed to study its effect combined with the HWE conditions.

3.3 Extended phase space of dune state with vegetation effects

With the established stable dune regime the impact of water-driven erosion is now analyzed. For different initial seeding conditions ρ_0 with dune growth in the stable regime multiple model runs are performed by varying the HWE mean inter-arrival time and for all model runs the mean intensity is fixed at $\overline{S} = 0.35m$. The idea of a rescaled frequency $\lambda_d^+ = \lambda_0 T_d$ to describe the competition between dune growth time and the inter-arrival time of HWE corresponding to the required elevation can be applied now to the vegetation effect. The vegetation does not modify the dune rescaled frequency λ_d^+ , as the growth rate of dune *G* is independent of the vegetation conditions in the stable regime considered in the analysis (Fig.12) and is controlled by sediment supply whereas the base elevation h_0 is local morphology depending on past geology of location. Thus vegetation condition is solely represented by the vegetation colonization time T_{v0} that influences the initial dune growth time T_{dv} . This new time for dune to establish depending on vegetation colonization shifts the competition between dune and HWE and introduces a new rescaled frequency $\lambda_{dv}^+ = \lambda_0 T_{dv}$ which captures the competition between HWEs and dune establishment driven by vegetation colonization. This helps to analyze an extended phase space that studies the impact of initial cover fraction ρ_0 as a parameter on dune state and recovery under the presence of HWEs.

This provides a way to describe the dune state as high, barren, or mixed described by rescaled frequency of plant colonization λ_{dv}^+ and dune formation times λ_d^+ . Indicators of the dune state similar to the transition time obtained in ⁴² are developed in this analysis such as the mean rescaled dune height, obtained as the mean of the dune crest elevation extracted in the model runs referred to the flat beach elevation, and the average vegetation cover which is the non-zero average of the cover fraction extracted at the maximum location of dune crest. The values of these can be used to obtain the dune state for each condition of vegetation (ρ_0, T_v) and HWE (λ_0, \overline{S}) from the model result time series. An important thing to consider regarding these indicators is the requirement of an adequate amount of HWEs that can lead to overwash of the dunes to study its recovery and to provide enough statistics that lead to a steady dune state. This generally requires more events to describe the mixed island type than that required to define a high or low state.

The transition from one state to another state as seen in Fig.13 can be driven by either the HWEs or by the vegetation colonization. This can be attributed to either vegetation slowing or dune growth slowing thus causing one of the two processes in dune growth and recovery to not be able to counter the HWE effects. To understand these effects the competition between the time between HWE flooding a certain elevation above the beach and the time required for the dune to grow to the same elevation is studied. Considering the elevation at the mean intensity \overline{S} above the base elevation h_0 , the frequency of HWE flooding this elevation is given by,

$$\lambda = \lambda_r e^{-(h_0 + \bar{S})/\bar{S}}$$

$$\lambda = \lambda_0 e^{-1}$$
(10)

Here λ_r is the rescaled frequency at the reference beach elevation obtained as constant 18 events/year from the stochastic analysis⁴⁰. The beach rescaled frequency is obtained as $\lambda_0 = \lambda_r e^{-h_0/\overline{S}}$, which is the value at the base elevation h_0 above the reference beach. In the case of rapid vegetation recovery with respect to dune growth, the time taken for the dune to reach an elevation \overline{S} is $T_s \approx \overline{S}/G$. The dune growth is controlled by its rate $G = (H - h_0)/T_d$. The dune outcompetes the flooding if this growth time is below the mean inter-arrival time of the HWE. This can be analytically represented by the constraint,

$$T_{s}\lambda < 1$$

$$\frac{\overline{S}}{\overline{G}}\lambda_{0}e^{-1} < 1$$

$$\frac{\overline{S}}{H-h_{0}}\frac{H-h_{0}}{G}\lambda_{0} < e$$

$$\xi_{c}^{+}\lambda_{d}^{+} < e$$
(11)

The product of the rescaled frequency and rescaled intensity of HWE can be used to separate regions where dunes can recover before flooding and where flooding dominates dune growth. So, when the product is greater or equal to e, the effect of flooding starts to dominate the dune recovery. However, in the general case, the vegetation is not very fast in recovery and colonization post erosion and grows at a rate comparable to dune growth. Then the time taken by the dune to reach an elevation is obtained as the sum of the dune's initial growth time T_{dv} and the time to grow to that elevation. This modifies the constraint as follows,

$$(T_{dv} + T_s)\lambda < 1$$

$$\left(\frac{\overline{S}}{G} + T_{dv}\right)\lambda_0 < e$$

$$\xi_c^+ \lambda_d^+ + \lambda_{dv}^+ < e$$
(12)

This gives the new transition threshold in the phase space, $\lambda_{dv}^+ + \lambda_d^+ \xi_c^+ = e$ represented on the Fig.13 as the red line. This is the constraint for which the dune growth is capable of overcoming the HWE and can remain in a high state. Below this threshold, the dune will be competing with the HWE and will transition to a barren state.

4 DISCUSSION

The transition threshold distinguishes the regions where the dune recovers at least to the mean intensity of HWE before flooding and where the flooding dominates. This parametric line aligns with the transition of the dune state from high to barren through the mixed state. Hence, this transition parameter can be used as a vulnerability indicator to assess the state of the dune under both vegetation and HWEs. Expanding the parameter in terms of the base elevation gives,

$$h_{0c} = \overline{S} \ln \frac{\lambda_r (T_{dv} + \overline{S}/G)}{e}$$

This gives a critical threshold for the base elevation h_{0c} depending on the vegetation colonization time T_{v0} , as it directly controls the dune initial growth time T_{dv} , and the dune growth rate G to indicate the dune state transition. This value can be

estimated for locations to verify its capability in assessing the vulnerability of barrier island dunes. For example, in Virginia barrier islands for a dune growth rate of G = 0.3m/yr, $\overline{S} = 0.3m$ and for a $T_{v0} = 0.5years$ the $h_{0c} = 0.7m$. This explains why in overwash fans such as in Metompkin Island, Virginia, USA, with elevations around 0.5 m vegetation cannot colonize due to the frequent flooding and the dune is unable to recover. Depending on the vegetation time this threshold controls the ability of dune and vegetation to recover from overwash as shown in Fig.14.

This critical elevation can also be applied to beaches fronting upland such as in Oregon. There this elevation can be interpreted as the minimum threshold above which the formation of dunes occurs. The erosion of these dunes however migrates the sand upland and aids in the formation of dune fields. Instead of controlling the state, it establishes the seaward limit to initiate dune formation. In Oregon, the dune growth rate is around G = 0.5m/yr and the mean HWE intensity is $\overline{S} = 0.3m^{47,40}$. Considering a seasonal vegetation with $T_{v0} = 0.5years$ for vegetation colonization the critical base elevation is $h_{0c} = 0.6m$. The beach elevation in Oregon is predicted to be 3.5 m⁴⁰ and so this indicates that the vegetation colonization and subsequent dune growth starts to occur around and above 4.1 m. This result matches with that presented in ⁴⁷ where dunes are found to grow from around 4 m.

The importance of the initial cover fraction part of the vegetation colonization process in delaying dune growth and recovery suggests a need for further investigation of its controls such as species type, precipitation, salt spray, etc. A future analysis on these lines will prove fruitful for a better understanding of this control parameter. Primarily a comparison of the model with real data is needed to validate the effect of vegetation colonization as a key factor in determining the dune state and in its recovery from HWE overwashes. This requires beach vegetation data and the development of empirical parameters to quantify various vegetation effects such as its species, growth rate, etc. to obtain the colonization time of vegetation post-overwash.

There were relaxations and assumptions used in this analysis to simplify the process-based model such as single species of vegetation with vertical growth, 2D model domain, relaxed storm erosion, homogeneous seeding, etc. Many of these can be improved by extending the current analysis for specific purposes. The vegetation impact on the dune state and the importance of certain species to govern growth and recovery can be addressed by a more complex simulation. This can be performed by including multiple vegetation species based on their primary function and also incorporating the lateral growth of the vegetation with random seeding in a 3D simulation of the beach with coastal dunes. These results can shed more light on the complexity of competition between species, and succession of vegetation thereby highlighting the importance of initial salt-tolerant species that enable proto-dune growth and later fast growth enabling vegetation that builds and stabilizes mature tall dunes. The maximum dune height was kept constant in the regime of analysis by excluding the slow-growing vegetation leading to mobile dunes. However, this regime of migrating dunes brings another effect to the dynamics due to the varying dune height and its interplay with beach width, dune migration, etc. which opens an interesting research question. However, the model structure provides room for such improvement and exploration of interesting physics associated with drivers and controllers of dune dynamics.

5 | CONCLUSION

The stochastic model of dune evolution is validated using a process-based model. This enabled an extension of the analysis tool to study the influence of vegetation on dune growth. When vegetation timescales are comparable to that of the dune coupled dynamics evolve under the stress of water erosion. The vegetation characteristics such as growth rate, and initial cover fraction causing delay in the dune growth process were quantified in terms of time scales that could be related to characteristic dune times. They were obtained by analyzing the dune height and vegetation cover fraction time series from the model for various cases ranging from slow to fast vegetation and using approximate equations of dune and vegetation growth. It provided the initial lag required for the dune to establish saturating growth which is found to be dependent on the vegetation colonization time. Using these estimates two distinct dune growth regimes were then identified, stable with dune height reaching saturation and unstable with mobile dunes increasing in dune height. These two regimes were separated by a threshold in the vegetation growth time which follows the ideology that the main controller for dune stabilization is the vegetation. The stable regime is selected to avoid the complex process of unstable mobile growing dune thereby having a constant HWE rescaled intensity parameter. The results from the model runs belonging to this regime presented the three different states of the coastal dune- high, barren, and mixed, that is not only driven by the HWE frequency but also by the lag in dune growth and recovery caused by vegetation colonization. With this new controller, the phase space of the dune state is extended with a rescaled frequency relating the storm frequency to the dune's initial growth time that depends on the vegetation colonization time. Analyses of the phase space with the competing timescales helped to obtain the dune state transition parameter as a combination of the rescaled frequencies and intensity λ_{dv} , λ_d and ξ_c that shows the relative contribution to dune vulnerability against flooding. This transition threshold can

be converted in terms of a critical base elevation after an overwash h_{0c} . This is the minimum elevation required for vegetation to colonize post an overwash and overcome the HWE flooding to establish dunes, thus providing a vulnerability indicator.

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12

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FIGURE 2 Process-based model results of A) dune height and B) Vegetation cover fraction time series under different vegetation conditions: Fast vegetation with both initial cover fraction ρ_0 and growth rate T_{ν} higher than the slow case. The vegetation not only affects the growth of the dune but also the initial time required for the dune to establish by vegetation affecting the sand flux



FIGURE 3 A) Exponential fit (red line) on the dune height time series (black line) to obtain the characteristic dune growth times. The blue point indicates the location of initial growth time T_{dv} , the minimum time to begin the exponentially saturating dune growth, and the green point is when the dune growth time T_d is reached. B) Asymptotic fit (red dashed line) on the vegetation cover fraction time series (blue line) to obtain the characteristic vegetation times. The blue point represents the location of initial growth time T_{v0} and the green point indicates when the vegetation growth time T_v is reached. The horizontal black dashed line indicates the vegetation cover fraction for the exponential dune growth to begin



FIGURE 4 High elevation state - A) Snippet of the dune elevation time series as the dune growth competes with the eroding HWE (red points). A fast dune recovery with respect to the frequency of events makes the dune recover to a high state. Comparison of the stochastic model (Analytical solution) and process-based model results using the B) PDF and C) CDF. The result holds for the different initial conditions: initial profile with barren flat beach h(t = 0) = 0 and with fully developed dune h(t = 0) = H. Stability is ensured by monitoring the simulation mean. D) The analytical mean predicted by the stochastic model agrees well with the simulation mean



FIGURE 5 Barren elevation state - A) Snippet of the dune elevation time series as the dune growth competes with the eroding HWE (red points). A higher frequency of events holds the dune in a barren state. Comparison of the stochastic model (Analytical solution) and process-based model results using the B) PDF and C) CDF. The result holds for the different initial conditions: initial profile with barren flat beach h(t = 0) = 0 and with fully developed dune h(t = 0) = H. Stability is ensured by monitoring the simulation mean. D) The analytical mean predicted by the stochastic model agrees well with the simulation mean



FIGURE 6 Mixed elevation state - A) Snippet of the dune elevation time series as the dune growth competes with the eroding HWE (red points). Comparison of the stochastic model (Analytical solution) and process-based model results using the B) PDF and C) CDF. The result holds for the different initial conditions: initial profile with barren flat beach h(t = 0) = 0 and with fully developed dune h(t = 0) = H. Stability is ensured by monitoring the simulation mean. D) The analytical mean predicted by the stochastic model agrees well with the simulation mean



FIGURE 7 Comparison of A) rescaled transition time and B) rescaled mean dune height of the analytical solution from the stochastic model with the numerical simulation for the same HWE parameters. Discrepancies are exhibited by mixed state points that present a lower simulation mean than expected due to the requirement of large statistics to converge due to dune recovery timescales. The simulation time of each run is used to highlight the importance of numerical convergence to reach a steady state



FIGURE 8 Phase space of rescaled intensity ξ_c^+ against rescaled frequency λ_d^+ showing the transition of the dune from high state to low state. The contour represents the rescaled mean dune height which acts as an indicator of the dune's steady state. The size of the point indicates the rescaled error percentage between the analytical and numerical rescaled mean dune height



FIGURE 9 A) Dune growth time T_d is independent of the vegetation growth time T_v till a threshold beyond which the impact of slow vegetation is established. This helps to identify regimes of stable dune growth and unstable mobile dunes where vegetation cannot recover and control transport. The mobile unstable points have dunes continuing to grow and are restricted by the simulation size and time. B) Also, the dune initial growth time T_{dv} is found to be independent of the vegetation growth T_v



FIGURE 10 Time evolution of elevation profile of undisturbed dune growth under different vegetation conditions showing the regimes of dune stabilization by vegetation: A) Stable, B) Metastable, and C) Unstable



FIGURE 11 A) Stabilization of dune depending on the vegetation growth time. B) Stable dunes are fixed close to the vegetation limit L_{veg} but in the unstable regime become mobile and continue growing with the onset of migration of dune crest



FIGURE 12 A) Dependency of dune initial growth time T_{dv} to the vegetation colonization time T_{v0} for simulations belonging to the stable dune region. B) The characteristic dune growth time T_d is independent of the colonization time T_{v0}



FIGURE 13 Phase space indicating the dune state using A) the average vegetation cover fraction and B) the rescaled mean dune height. The dune state is governed by both HWE frequency and the vegetation colonization time. The red line indicates the transition threshold for the dune to shift from a high state to a barren state



FIGURE 14 A) The critical base elevation estimate for two locations: Virginia and Oregon, is based on known parameters varying the vegetation colonization time. This presents the required elevation above the beach for establishing vegetation and building dunes to compete with the beach flooding events. The characteristic berm elevation of 0.4 m \pm 0.1 m, considered in the stochastic model, is included to present the minimum elevation above which the dune building process can start. B) DEM of Metompkin Barrier Island in Virginia with the critical elevation constraint of 0.5 m above the beach (black contours). The contours correspond to the region of mature and proto-dunes capable of recovery from frequent flooding