Adaptive finite-time command filtered control for switched nonlinear systems with input quantization and output constraints

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Abstract

This article considers the problem of finite-time command filtered control for switched nonlinear systems with input quantization and output constraints. The unmeasurable state is estimated by designing a switched state observer. During the design process, to overcome the chattering problem effectively, the hysteresis quantization is designed as two bounded nonlinear functions. Furthermore, in order to restrict the output to an expected range, the barrier Lyapunov function (BLF) approach is introduced. The "explosion of complexity" (EOC) and the error compensation problems in the backstepping design are solved by using a finite-time command filtered (FTCF) approach. A first-order Levant differentiator (FLD) is used instead of the general command filter in this paper, which cannot only filter the intermediate signals accurately to get the differential signals, but also ensure the finite-time stability of the filter. Stability of the closed-loop system in the sense of semi-global practical finite-time stability (SGPFS) is proved by exploring a multiple Lyapunov functions approach. Finally, a simulation example is provided to verify the validity of the presented control method.

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RESEARCH ARTICLE

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Summary

This article considers the problem of finite-time command filtered control for switched nonlinear systems with input quantization and output constraints. The unmeasurable state is estimated by designing a switched state observer. During the design process, to overcome the chattering problem effectively, the hysteresis quantization is designed as two bounded nonlinear functions. Furthermore, in order to restrict the output to an expected range, the barrier Lyapunov function (BLF) approach is introduced. The “explosion of complexity" (EOC) and the error compensation problems in the backstepping design are solved by using a finite-time command filtered (FTCF) approach. A first-order Levant differentiator (FLD) is used instead of the general command filter in this paper, which cannot only filter the intermediate signals accurately to get the differential signals, but also ensure the finite-time stability of the filter. Stability of the closed-loop system in the sense of semi-global practical finite-time stability (SGPFS) is proved by exploring a multiple Lyapunov functions approach. Finally, a simulation example is provided to verify the validity of the presented control method.

KEYWORDS:
Switched system, state observer, finite-time command filter, output constrained, input quantization.

1 | INTRODUCTION

Switched systems, as a class of hybrid systems, are composed of a series of continuous or discrete-time subsystems and a specific switching law, which have attracted a lot of scholars' interest in the past few decades. Since many systems in engineering can be modeled as switched systems, they have been widely studied and used [1,2,3,4]. For a given switched system in [5], the problem of how to select the linear vector field to make the system exponentially stable was discussed. Stability analysis is still one of the most important problems in control issues of switched systems. For switched linear systems with time-varying delay, the stability problem in the sense of Hurwitz convex combination was studied in [6]. Besides, in [7], a new average dwell time method was proposed to solve the stability of switched nonlinear systems by T-S fuzzy modeling. With the rapid development of nonlinear control technology, adaptive backstepping method has become an effective tool for designing nonlinear system controllers. However, in the traditional backstepping method, the EOC and over parameterization problems caused by the repeated differentiation of virtual control signals

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hinder its further applications. In order to further develop the backstepping design method, the dynamic surface control (DSC) method was proposed to solve the EOC problem and over parametric problem in the backstepping process[8,9,10]. The DSC mainly introduces a first-order low-pass filter, which greatly simplifies the design steps and reduces the computational burden. Unfortunately, the DSC method does not consider the error compensation of the filter, which promotes further researches and improvements of the backstepping method. A practical extension of the backstepping nonlinear control approach was presented in[11], where the errors between filters and virtual controllers were compensated by designed compensating signals, which is a further modification of the backstepping method based on the DSC. On this basis, for the strict-feedback systems with parameter uncertainty and unmeasured states, an adaptive fuzzy output feedback control method based on observer and command filter was proposed in[12]. The command filtered control method was extended to multiple-input multiple-output (MIMO) nonlinear systems in[13], where the authors had considered MIMO nonlinear systems with input saturation and unknown direction control gains. Nevertheless, none of the above studies discussed non-smooth nonlinear factors like input quantization that would affect stability of systems.

As one of the most important non-smooth nonlinearities, quantization input is widely used in network control systems in recent years. The main reason for considering the quantization problem in the systems is its characteristics and practical applications. It possesses double advantages of high precision and there is no need for a high communication rate[14,15,16]. Meanwhile, in the actual quantized control systems, the transmission of control signal to plant is a piece-wise constant function of time. They make the quantized input get more attentions and related research results[17,18]. Stabilization of discrete-time linear systems with quantization of the input and output spaces has been studied in[19], where it assumed that the benchmark of the problem was how to quantify the input and output, rather than the degree of freedom of the design. To overcome the chattering problem caused by a quantizer in nonlinear systems, the authors in[20] first proposed a hysteretic quantizer that realizes the hysteresis mechanism by using two logarithmic input quantizers with dead zones. In[21], the hysteresis quantization input was divided into two bounded nonlinear functions, presented an adaptive fuzzy backstepping control method based on command filtered for switched nonlinear systems with input quantization. In this paper, the control problem for switched uncertain nonlinear systems with quantization inputs is considered by using the hysteretic quantizer.

On the other hand, it should be noted that due to the inherent physical limitations in the actual systems, the output constraint of the systems is a common phenomenon[22,23,24]. However, violation of output constraints may have harmful effects for the control systems, such as physical stoppage, performance degradation even system instability. In order to handle the problem of output constraint, the authors in[25] used BLF to constraint the output to a bounded range, since in BLF, as the parameter was close to some limit, the value of the function approaches infinity. In[26], an adaptive neural network control method for uncertain MIMO nonlinear systems with unmodeled dynamics and output constraints was proposed, by introducing the symmetric BLF, the constraint requirement of each element in the output was guaranteed. For the switched system to be studied in this paper, we not only quantize the input, but also constrain the output to a range to achieve the desired control effect. And in order to be closer to the practical application, the stability problem in finite-time is also considered.

In view of the fact that the actual system is complex and requires high precision performance, the time required for system stability is no longer infinite. The finite-time control can improve the control accuracy in a given settling time and make the system convergent to an arbitrary given accuracy range. Therefore, finite-time control has practical control significance, and many achievements have been researched in recent years[27,28,29,30]. In[31], a new FTCF backstepping method was proposed by the established new virtual controller and modified error compensation. The novel design technology not only solves the problem of EOC, but also eliminates the filtering error in finite-time. However, there are not any results on adaptive FTCF control for the switched nonlinear systems with input quantization and output constraints.

Inspired by the aforementioned results, an adaptive fuzzy FTCF control method is proposed for switched nonlinear systems with quantization input and output constraints. Different from the existing results, the contributions of this paper are as follows.

1)For the switched unknown nonlinear systems, the quantization input and output constraints are considered at the same time to make it closer to the practical application of complex and changeable. Due to the fact that many physical quantities cannot be measured or difficult to get the measured value, a switched state observer is designed to observe the unmeasurable states.

2)To solve the problems of over parameterization and EOC in the backstepping design process, a FTCF is used to improve our design method. The designed method not only has the advantages of the traditional command filter control method, but also ensures that the control result is stable in finite-time.

The explanations of the notations used in this paper are given in the table 1 as follows:

TABLE 1 Notation interpretation.

Notation	Interpretation
R	The set of real numbers
R^n	The n-dimensional Euclidean space
A^T	The transpose of matrix A
$\lambda_{\min}(Q_{\bar{m}})$	The minimum eigenvalue of matrix $Q_{\bar{m}}$
$\lambda_{\max}(Q_{\bar{m}})$	The maximum eigenvalue of matrix $Q_{\bar{m}}$
$\|\| \cdot \|\|$	The Euclidean norm of vectors or matrices

2 | PROBLEM FORMULATION AND PRELIMINARIES

Consider the uncertain switched nonlinear systems with input quantization and output constraints form

$$\begin{cases} \dot{x}_i = x_{i+1} + f_{i,\bar{\sigma}(t)}(\bar{x}_i) + d_{i,\bar{\sigma}(t)}(t) \\ \dot{x}_n = q_{\bar{\sigma}(t)}(u_{\bar{\sigma}(t)}) + f_{n,\bar{\sigma}(t)}(\bar{x}_n) + d_{n,\bar{\sigma}(t)}(t) \\ y = x_1, \quad i = 1, 2, ...n-1 \end{cases} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, ..., x_i]^T \in R^i (i = 1, 2, ..., n)$ are the state vectors, $y \in R$ is the output of the system. $\bar{\sigma}(t) : [0, \infty) \to \Xi \overset{def}{=} \{1, 2, ..., m\}$ is the switching signal. By utilizing the concept of output constraints, we mean that the output $y \in [-k_{c1}, k_{c1}]$. $f_{i,\bar{m}}(\bar{x}_i)(i = 1, 2, ..., n, \bar{m} \in \Xi)$ are unknown smooth nonlinear functions, $d_{i,\bar{m}}(t)$ are unknown but bounded external disturbances, we assumed that only the output y is measurable.

Furthermore, it is assumed that the solution of system (1) is continuous everywhere[24]. $q_{\bar{\sigma}(t)}(u_{\bar{\sigma}(t)}) \in R$ is the hysteresis quantiser, which will be defined in formula (2) where $u_{\bar{\sigma}(t)} = \bar{\rho}^{(1-i)} u_{\min}$ with integer $i = 1, 2, ..., n$, the parameters $u_{\min} > 0$ and $0 < \bar{\rho} < 1$, $\delta_{\bar{\sigma}(t)} = (1 - \bar{\rho}_{\bar{\sigma}(t)})/(1 + \bar{\rho}_{\bar{\sigma}(t)})$, $q_{\bar{\sigma}(t)}(u_{\bar{\sigma}(t)})$ is the set $\bar{U} = \{0, \pm u_{i,\bar{\sigma}(t)}, \pm u_{i,\bar{\sigma}(t)}(1 + \delta_{i,\bar{\sigma}(t)})\}$. The dead zone part of $q_{\bar{\sigma}(t)}(u_{\bar{\sigma}(t)})$ is expressed as $u_{\min}$. The quantizer $q_{\bar{\sigma}(t)}(u_{\bar{\sigma}(t)})$ represents the hysteretic quantizer as follows:

$$q_{\bar{\sigma}(t)}(u_{\bar{\sigma}(t)}) \overset{\Delta}{=} \begin{cases} u_{i,\bar{\sigma}(t)} \mathrm{sgn}(u_{\bar{\sigma}(t)}) & if \frac{u_{i,\bar{\sigma}(t)}}{1+\delta_{\bar{\sigma}(t)}} < |u_{\bar{\sigma}(t)}| \le u_{i,\bar{\sigma}(t)}, \dot{u}_{\bar{\sigma}(t)} < 0, or \\ & u_{i,\bar{\sigma}(t)} < |u_{\bar{\sigma}(t)}| \le \frac{u_{i,\bar{\sigma}(t)}}{1-\delta_{\bar{\sigma}(t)}}, \dot{u}_{\bar{\sigma}(t)} > 0 \\ u_{i,\bar{\sigma}(t)}(1 + \delta_{\bar{\sigma}(t)}) \mathrm{sgn}(u_{\bar{\sigma}(t)}) & if u_{i,\bar{\sigma}(t)} < |u_{\bar{\sigma}(t)}| \le \frac{u_{i,\bar{\sigma}(t)}}{1-\delta_{\bar{\sigma}(t)}}, \dot{u}_{\bar{\sigma}(t)} < 0, or \\ & \frac{u_{i,\bar{\sigma}(t)}}{1-\delta_{\bar{\sigma}(t)}} \le |u_{\bar{\sigma}(t)}| \le \frac{u_{i,\bar{\sigma}(t)}(1+\delta_{\bar{\sigma}(t)})}{1-\delta_{\bar{\sigma}(t)}}, \dot{u}_{\bar{\sigma}(t)} > 0 \\ 0 & if 0 \le |u_{\bar{\sigma}(t)}| < \frac{u_{\min}}{1+\delta_{\bar{\sigma}(t)}}, \dot{u}_{\bar{\sigma}(t)} < 0, or \\ & \frac{u_{\min}}{1+\delta_{\bar{\sigma}(t)}} \le |u_{\bar{\sigma}(t)}| \le u_{\min}, \dot{u}_{\bar{\sigma}(t)} > 0 \\ q_{\bar{\sigma}(t)}(u_{\bar{\sigma}(t)}(t^-)) & \dot{u}_{\bar{\sigma}(t)} = 0 \end{cases} \tag{2}$$

Remark 1. $\bar{\rho}_{\bar{\sigma}}$ is utilized to measure the quantization density. Smaller value of $\bar{\rho}_{\bar{\sigma}}$ will help to get a smaller width of quantizer. As $\bar{\rho}_{\bar{\sigma}(t)}$ decreases to 0, $\delta_{\bar{\sigma}(t)}$ is close to 1, $q_{\bar{\sigma}(t)}(u_{\bar{\sigma}(t)})$ will have a smaller quantization level.

Next, the hysteresis quantization $q_{\bar{\sigma}(t)}(u_{\bar{\sigma}(t)})$ is divided into two nonlinear parts to improve the effectiveness of the control:

$$q_{\bar{\sigma}}(u_{\bar{\sigma}}) = K_{\bar{\sigma}}(u_{\bar{\sigma}}) u_{\bar{\sigma}} + D_{\bar{\sigma}} \tag{3}$$

For the nonlinearities $K_{\bar{\sigma}}(u_{\bar{\sigma}})$ and $D_{\bar{\sigma}}$, we have the following lemmas.

Lemma 1[20]. The nonlinearities $K_{\bar{\sigma}}(u_{\bar{\sigma}})$ and $D_{\bar{\sigma}}$ satisfy the following inequality:

$$1-\delta_{\bar{\sigma}} \leq K_{\bar{\sigma}}(u_{\bar{\sigma}}) \leq 1+\delta_{\bar{\sigma}}, |D_{\bar{\sigma}}| \leq u_{\min} \tag{4}$$

Proof: By using sector bounded property, for $|u_{\bar{\sigma}(t)}| \geq u_{\min}$, we have:

$$1-\delta_{\bar{\sigma}} \leq \frac{q_{\bar{\sigma}}(u_{\bar{\sigma}})}{u_{\bar{\sigma}}} \leq 1+\delta_{\bar{\sigma}} \tag{5}$$

For $|u_{\bar{\sigma}(t)}| \leq u_{\min}$, $q_{\bar{\sigma}(t)}(u_{\bar{\sigma}(t)}) = 0$ and from the definition (3), we have:

$$0 = K_{\bar{\sigma}}(u_{\bar{\sigma}})u_{\bar{\sigma}} + D_{\bar{\sigma}} \tag{6}$$

Let

$$K_{\bar{\sigma}}(u_{\bar{\sigma}}) = \begin{cases} \frac{q_{\bar{\sigma}}(u_{\bar{\sigma}})}{u_{\bar{\sigma}}}, |u_{\bar{\sigma}}| > u_{\min} \\ 1, \qquad |u_{\bar{\sigma}}| \leq u_{\min} \end{cases} \tag{7}$$

and

$$D_{\bar{\sigma}} = \begin{cases} 0, \qquad |u_{\bar{\sigma}}| > u_{\min} \\ -|u_{\bar{\sigma}}|, |u_{\bar{\sigma}}| \leq u_{\min} \end{cases} \tag{8}$$

where $K_{\bar{\sigma}(t)}(u_{\bar{\sigma}(t)})$ and $D_{\bar{\sigma}(t)}(t)$ satisfy the inequality (4).

The following assumptions need to be made in this paper.

Assumption 1. $d_{i,\bar{m}}(t)(i = 1, 2, ..., n, \bar{m} \in \Xi)$ is a bounded disturbance, and $|d_{i,\bar{m}}(t)| \leq \breve{d}_{i,\bar{m}}$, $\breve{d}_{i,\bar{m}} > 0$.

Assumption 2. There is a known constant $L_{i,\bar{m}}$, which satisfies:

$$|f_{i,\bar{m}}(\bar{x}_i) - f_{i,\bar{m}}(\hat{\bar{x}}_i)| \leq L_{i,\bar{m}}||\bar{x}_i - \hat{\bar{x}}||$$

where $\hat{\bar{x}}_i$ is the estimation of $\bar{x}_i$.

Assumption 3. The positive constants $\mathcal{Y}_0, \breve{\mathcal{Y}}_0, \bar{\mathcal{Y}}_0, \mathcal{Y}_1, \mathcal{Y}_2, ..., \mathcal{Y}_n$ satisfying max $\{\breve{\mathcal{Y}}_0, \bar{\mathcal{Y}}_0\} \leq \mathcal{Y}_0$ make the reference signal y_d and its nth order derivatives are known and bounded, which satisfy $\breve{\mathcal{Y}}_0 \leq y_d(t) \leq \bar{\mathcal{Y}}_0$, $|\dot{y}_d(t)| \leq \mathcal{Y}_1$, $|\ddot{y}_d(t)| \leq \mathcal{Y}_2$, ...,$|y_d^{(n)}(t)| \leq \mathcal{Y}_n$, $\forall t \geq 0$.

The control objective is to propose an adaptive fuzzy quantized control scheme for the considered system (1) based on FTCF control, such that the system output y can track the desired signal y_d well in finite-time and all the signals in the closed-loop system are bounded.

In this part, we give the lemmas and definition needed in the article. Definition 1[27]. If we have all $\dot{x}(t_0) = x_0$, the equilibrium $x = 0$ of the nonlinear system $\dot{x} = f(x, q(u))$ is SGPFS, and have a constant $\bar{\imath} > 0$, a settling time $T_x(\bar{\imath}, x_0) < +\infty$ such that $|x(t)| < \bar{\imath}$, $\forall t > t_0 + T_x$.

Lemma 2[28]. For $x_i \in R$, $i = 1, 2..., n$, $0 < \chi \leq 1$, we have:

$$(\sum_{i=1}^{n} |x_i|)^{\chi} \leq n^{1-\chi}(\sum_{i=1}^{n} |x_i|)^{\chi} \tag{9}$$

Lemma 3[30]. The positive constants x, y and $\bar{\gamma}$ make any real variable $\mathcal{H}$, $\mathcal{Z}$ satisfied:

$$|\mathcal{H}|^x|\mathcal{Z}|^y \leq \frac{x}{x+y}\bar{\gamma}|\mathcal{H}|^{x+y} + \frac{y}{x+y}\bar{\gamma}^{-\frac{x}{y}}|\mathcal{Z}|^{x+y} \tag{10}$$

Lemma 4. For the system $\dot{x} = f(x, q(u))$, the function $V(x)$ is smooth and positive, if the positive scalars $\bar{c}$, $\bar{d}$ and $0 < \beta < 1$ exist, we have:

$$\dot{V}(x) \leq -\bar{c}V^{\beta}(x) + \bar{d}, t \geq 0 \tag{11}$$

It means that the system $\dot{x} = f(x, q(u))$ is SGPFS[30].

According to the literature[32], the FLD is designed as follows:

$$\dot{\psi}_{i,\bar{m},1} = \iota_{i,\bar{m},1} \tag{12}$$

$$\iota_{i,\bar{m},1} = -R_1|\psi_{i,\bar{m},1} - \alpha_{i-1}|^{\frac{1}{2}} sign(\psi_{i,\bar{m},1} - \alpha_{i-1}) + \psi_{i,\bar{m},2} \tag{13}$$

$$\dot{\psi}_{i,\bar{m},2} = -R_2 sign(\psi_{i,\bar{m},2} - \iota_{i,\bar{m},1}), i = 1, 2, ..., n-1 \tag{14}$$

where α_{i-1} is the input signal. The R_1, R_2 are design parameters.
Lemma 5[33]. The parameters R_1 and R_2 being properly chosen, when there is no noise at the input, after the finite time of the transient process, we have

$$\psi_{i,\bar{m},1} = \alpha_{i-1(0)}, \iota_{i,\bar{m},1} = \dot{\alpha}_{i-1(0)} \tag{15}$$

Lemma 6[34]. Continuous function $\Pi(x)$ and any constant $\mathfrak{I} > 0$, we have an FLS: $y(x) = \hat{\theta}^T \phi(x)$ satisfying:

$$\sup_{x \in R} |\Pi(x) - \hat{\theta}^T \phi(x)| \leq \mathfrak{I} \tag{16}$$

where $x = [x_1, ..., x_n]^T$ and y are the input and output of the FLS, respectively. $\hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, ..., \hat{\theta}_N]^T$ with N being the number of the fuzzy rules, $\phi(x) = [\phi_1(x), ..., \phi_N(x)]^T$, and $\varphi_j = \prod_{i=1}^{n} \mu_{F_i^j}(x_i) / \sum_{j=1}^{N} (\prod_{i=1}^{n} \mu_{F_i^j}(x_i))$ with $\mu_{F_i^j}(x_i)$ usually choosen Gaussian function.
Definition 2[35]. A scalar-valued function $V(x)$ on the open region M for $\forall x \in R^n$, which respects to the system $\dot{x} = f(x)$. For $\forall x \in M$, the positive definite function $V(x)$ is first-order partial differentiable. Meanwhile, for $\forall t \geq 0$, $V(x) \leq w$ with w is a positive constant, and when x closes to M, $V(x) \to \infty$. Then function $V(x)$ is BLF.

Choose the following BLF in[25] is used:

$$\bar{V}_{1,\bar{m}} = \frac{1}{2} \log \frac{k_{w1}^2}{k_{w1}^2 - v_1^2} \tag{17}$$

where $|v_1| < k_{w1}$ and k_{w1} is the constraint on v_1.
Lemma 7[36]. For constant $k_{w1} > 0$, we have:

$$\log \frac{k_{w1}^2}{k_{w1}^2 - v_1^2} < \frac{v_1^2}{k_{w1}^2 - v_1^2} \tag{18}$$

where $|v_1| < k_{w1}$.

3 | STATE OBSERVER DESIGN

In this section, a switched fuzzy state observer (SFSO) is designed to estimate the unmeasurable states.

System (1) can be rewritten as:

$$\begin{cases} \dot{x}_1 = x_2 + f_{1,\bar{m}}(x_1) + d_{1,\bar{m}} \\ \dot{x}_2 = x_3 + f_{2,\bar{m}}(\hat{\bar{x}}_2) + \Delta f_{2,\bar{m}} + d_{2,\bar{m}} \\ \vdots \\ \dot{x}_{n-1} = x_n + f_{n-1,\bar{m}}(\hat{\bar{x}}_{n-1}) + \Delta f_{n-1,\bar{m}} + d_{n-1,\bar{m}} \\ \dot{x}_n = f_{n,\bar{m}}(\hat{\bar{x}}_n) + q_{\bar{m}}(u_{\bar{m}}) + \Delta f_{n,\bar{m}} + d_{n,\bar{m}} \\ y = x_1 \end{cases} \tag{19}$$

where $\Delta f_{i,\bar{m}} = f_{i,\bar{m}}(\bar{x}_i) - f_{i,\bar{m}}(\hat{\bar{x}}_i)$, $i = 2, ..., n$.

Using Lemma 6, we have:

$$\begin{aligned} f_{i,\bar{m}}(\bar{x}_i | \hat{\theta}_{i,\bar{m}}) &= \hat{\theta}_{i,\bar{m}}^T \phi_{i,\bar{m}}(\bar{x}_i) \\ \hat{f}_{i,\bar{m}}(\hat{\bar{x}}_i | \hat{\theta}_{i,\bar{m}}) &= \hat{\theta}_{i,\bar{m}}^T \phi_{i,\bar{m}}(\hat{\bar{x}}_i) \end{aligned} \tag{20}$$

The optimal parameter vector $\theta_{i,\bar{m}}^*$ is defined as:

$$\theta_{i,\bar{m}}^* = \arg \min_{\hat{\theta}_{i,\bar{m}} \in \Omega_{i,\bar{m}}} [\sup_{\hat{\bar{x}}_i \in U_i} |\hat{f}_{i,\bar{m}}(\hat{\bar{x}}_i | \hat{\theta}_{i,\bar{m}}) - f_{i,\bar{m}}(\hat{\bar{x}}_i)]$$

Define the minimum approximation error $\mathfrak{I}_{i,\bar{m}}(\hat{\bar{x}}_i)$ as:

$$\mathfrak{I}_{i,\bar{m}}(\hat{\bar{x}}_i) = f_{i,\bar{m}}(\hat{\bar{x}}_i) - \hat{f}_{i,\bar{m}}(\hat{\bar{x}}_i | \theta_{i,\bar{m}}^*), \tag{21}$$

where $\mathfrak{I}_{i,\bar{m}}$ satisfies $|\mathfrak{I}_{i,\bar{m}}| \leq \breve{\mathfrak{I}}_{i,\bar{m}}$, $\breve{\mathfrak{I}}_{i,\bar{m}}$ is an unknown constant.

Next, we can obtain that:

$$\begin{cases} \dot{x}_1 = x_2 + \theta_{1,\bar{m}}^{*T}\phi_{1,\bar{m}}(x_1) + \mathfrak{I}_{1,\bar{m}}(x_1) + d_{1,\bar{m}} \\ \dot{x}_2 = x_3 + \theta_{2,\bar{m}}^{*T}\phi_{2,\bar{m}}(\hat{\bar{x}}_2) + \mathfrak{I}_{2,\bar{m}}(\hat{\bar{x}}_2) + \Delta f_{2,\bar{m}} + d_{2,\bar{m}} \\ \vdots \\ \dot{x}_{n-1} = x_n + \theta_{n-1,\bar{m}}^{*T}\phi_{n-1,\bar{m}}(\hat{\bar{x}}_{n-1}) + \mathfrak{I}_{n-1,\bar{m}}(\hat{\bar{x}}_{n-1}) + \Delta f_{n-1,\bar{m}} + d_{n-1,\bar{m}} \\ \dot{x}_n = \theta_{n,\bar{m}}^{*T}\phi_{n,\bar{m}}(\hat{\bar{x}}_n) + \mathfrak{I}_{n,\bar{m}}(\hat{\bar{x}}_n) + q_{\bar{m}}(u_{\bar{m}}) + \Delta f_{n,\bar{m}} + d_{n,\bar{m}} \\ y = x_1 \end{cases} \tag{22}$$

By (22), a SFSO is designed as:

$$\begin{cases} \dot{\hat{x}} = A_{\bar{m}}\hat{x} + L_{\bar{m}}y + B_i\bar{F} + Bq_{\bar{m}}(u_{\bar{m}}) \\ \hat{y} = C\hat{x} \end{cases} \tag{23}$$

where $\hat{x} = [\hat{x}_1, ..., \hat{x}_n]^T$, $A_{\bar{m}} = \begin{bmatrix} -l_{1,\bar{m}} & & \\ \vdots & & I_{n-1} \\ -l_{n,\bar{m}} & \cdots & 0 \end{bmatrix}$, $\bar{F} = [\hat{f}_{1,\bar{m}}(x_1|\hat{\theta}_{1,\bar{m}}), \hat{f}_{2,\bar{m}}(\hat{\bar{x}}_2|\hat{\theta}_{2,\bar{m}}), ..., \hat{f}_{n,\bar{m}}(\hat{\bar{x}}_n|\hat{\theta}_{n,\bar{m}})]^T$.

Choose the appropriate $L_{\bar{m}}$ makes the matrix $A_{\bar{m}}$ is a Hurwitz matrix. And for a given $Q_{\bar{m}}^T = Q_{\bar{m}} > 0$, it has a positive definite matrix $P_{\bar{m}}^T = P_{\bar{m}} > 0$ as follow:

$$A_{\bar{m}}^T P_{\bar{m}} + P_{\bar{m}} A_{\bar{m}} = -Q_{\bar{m}} \tag{24}$$

Define $\diamond = x - \hat{x} = [\diamond_1, ..., \diamond_n]^T$, from (22) - (23), it has:

$$\dot{\diamond} = A_{\bar{m}}\diamond + \sum_{i=1}^{n} B_i\tilde{\theta}_{i,\bar{m}}^T\phi_{i,\bar{m}}(\hat{\bar{x}}_i) + \mathfrak{I}_{\bar{m}} + d_{\bar{m}} \tag{25}$$

where $\mathfrak{I}_{\bar{m}} = [\mathfrak{I}_{1,\bar{m}}, ..., \mathfrak{I}_{n,\bar{m}}]^T$, $d_{\bar{m}} = [d_{1,\bar{m}}, ..., d_{n,\bar{m}}]^T$ and $\tilde{\theta}_{i,\bar{m}} = \theta_{i,\bar{m}}^* - \hat{\theta}_{i,\bar{m}}$.

Consider the Lyapunov candidate as $V_{o,\bar{m}} = \diamond^T P_{\bar{m}}\diamond$, and the derivative of $V_{o,\bar{m}}$ with respect to t is:

$$\begin{aligned} \dot{V}_{o,\bar{m}} &= \dot{\diamond}^T P_{\bar{m}}\diamond + \diamond^T P_{\bar{m}}\dot{\diamond} \\ &= -\diamond^T Q_{\bar{m}} e + 2\diamond^T P_{\bar{m}}(\sum_{i=1}^{n} B_i\tilde{\theta}_{i,\bar{m}}^T\phi_{i,\bar{m}}(\hat{\bar{x}}_i) + \mathfrak{I}_{\bar{m}} + d_{\bar{m}}) \end{aligned} \tag{26}$$

Using Young's inequality, one has:

$$\begin{aligned} &2\diamond^T P_{\bar{m}}(\sum_{i=1}^{n} B_i\tilde{\theta}_{i,\bar{m}}^T\phi_{i,\bar{m}}(\hat{\bar{x}}_i) + \mathfrak{I}_{\bar{m}} + d_{\bar{m}}) \\ &\leq (n+2)\diamond^T\diamond + \| P_{\bar{m}}\|^2 \sum_{i=1}^{n} \tilde{\theta}_{i,\bar{m}}^T\tilde{\theta}_{i,\bar{m}} \\ &+ \| P_{\bar{m}}\|^2(\| \breve{\mathfrak{I}}_{\bar{m}}\|^2 + \| \breve{d}_{\bar{m}}\|^2) \end{aligned} \tag{27}$$

By substituting Eq. (27) into (26), we obtain:

$$\begin{aligned} \dot{V}_{o,\bar{m}} \leq &- \lambda_{o,\bar{m}} \| \diamond\|^2 + \| P_{\bar{m}}\|^2 \sum_{i=1}^{n} \tilde{\theta}_{i,\bar{m}}^T\tilde{\theta}_{i,\bar{m}} \\ &+ \| P_{\bar{m}}\|^2(\| \breve{\mathfrak{I}}_{\bar{m}}\|^2 + \| \breve{d}_{\bar{m}}\|^2) \end{aligned} \tag{28}$$

where $\lambda_{o,\bar{m}} = \lambda_{\min}(Q_{\bar{m}}) - n - 2$.

4 | CONTROLLER DESIGN AND STABILITY ANALYSIS

Define the tracking errors as follows:

$$\aleph_1 = y - y_d, \aleph_i = \hat{x}_i - x_{i,\bar{m},c}, \tag{29}$$

where $\aleph_i (i = 2, 3, ..., n)$ is the tracking error, $x_{i,\bar{m},c}$ is the output of command filter.

Remark 1. Different from the conventional command filters in [11], the FLD can precisely filter the intermediate signals and obtain differential signals, meanwhile, the filter is guaranteed to be stable in finite time. In [37], the FLD is used to replace the command filter.

Remark 2. The selection method of R_1 and R_2 is given in [38], while ensuring that R_1 and R_2 are large enough, choose R_2 first.

Let

$$v_i = \aleph_i - \xi_{i,\bar{m}}, \tag{30}$$

where $\xi_{i,\bar{m}}(i = 1, 2, ..., n)$ are compensating signals, v_i are the compensating tracking error signals.

Step 1: Since $x_2 = \hat{x}_2 + \diamond_2$ from (29) and (30), we can obtain the derivative of v_1 with respect to t as follows:

$$\begin{aligned}\dot{v}_1 =& x_2 + \theta^*_{1,\bar{m}}\phi_{1,\bar{m}}(\hat{x}_1) + \mathfrak{I}_{1,\bar{m}} + d_{1,\bar{m}} - \dot{y}_r - \dot{\xi}_{1,\bar{m}}\\ =& \aleph_2 + x_{2,\bar{m},c} + \diamond_2 + \hat{\theta}^T_{1,\bar{m}}\phi_{1,\bar{m}}(\hat{x}_1) + \tilde{\theta}^T_{1,\bar{m}}\phi_{1,\bar{m}}(\hat{x}_1)\\ &+ \mathfrak{I}_{\bar{m}} + d_{1,\bar{m}} - \dot{y}_r - \dot{\xi}_{1,\bar{m}}\end{aligned} \tag{31}$$

The Lyapunov function is selected as:

$$V_{1,\bar{m}} = V_{o,\bar{m}} + \bar{V}_{1,\bar{m}} + \frac{1}{2\gamma_{1,\bar{m}}}\tilde{\theta}^T_{1,\bar{m}}\tilde{\theta}_{1,\bar{m}} \tag{32}$$

where $\gamma_{1,\bar{m}}$ is a design positive parameter.

From (31) - (32), it has:

$$\begin{aligned}\dot{V}_{1,\bar{m}} =& \dot{V}_{o,\bar{m}} + \frac{v_1}{k^2_{w1} - v^2_1}[\aleph_2 + x_{2,\bar{m},c} + \diamond_2\\ &+ \hat{\theta}^T_{1,\bar{m}}\phi_{1,\bar{m}}(\hat{x}_1) + \mathfrak{I}_{1,\bar{m}} + d_{1,\bar{m}} - \dot{y}_r - \dot{\xi}_{1,\bar{m}}]\\ &+ \frac{1}{\gamma_{1,\bar{m}}}\tilde{\theta}^T_{1,\bar{m}}[\gamma_{1,\bar{m}}\phi_{1,\bar{m}}(\hat{x}_1)\frac{v_1}{k^2_{w1} - v^2_1} - \dot{\hat{\theta}}_{1,\bar{m}}]\end{aligned} \tag{33}$$

Using Young's inequality, it can be obtain:

$$\diamond_2\frac{v_1}{k^2_{w1} - v^2_1} + d_{1,\bar{m}}\frac{v_1}{k^2_{w1} - v^2_1} \le \frac{1}{2}\parallel \diamond\parallel^2 + \frac{v^2_1}{(k^2_{w1} - v^2_1)^2} + \frac{1}{2}\breve{d}^2_{1,\bar{m}} \tag{34}$$

$$\mathfrak{I}_{1,\bar{m}}\frac{v_1}{k^2_{w1} - v^2_1} \le \frac{1}{2}\breve{\mathfrak{I}}^2_{1,\bar{m}} + \frac{v^2_1}{2(k^2_{w1} - v^2_1)^2} \tag{35}$$

Substituting (34) - (35) into (33), one has:

$$\begin{aligned}\dot{V}_{1,\bar{m}} \le& \dot{V}_{o,\bar{m}} + \frac{v_1}{k^2_{w1} - v^2_1}[\frac{3v_1}{2(k^2_{w1} - v^2_1)} + \aleph_2 + x_{2,\bar{m},c}\\ &- \alpha_{1,\bar{m}} + \alpha_{1,\bar{m}} + \hat{\theta}^T_{1,\bar{m}}\phi_{1,\bar{m}}(\hat{x}_1) - \dot{y}_r - \dot{\xi}_{1,\bar{m}}]\\ &+ \frac{1}{\gamma_{1,\bar{m}}}\tilde{\theta}^T_{1,\bar{m}}[\gamma_{1,\bar{m}}\phi_{1,\bar{m}}(\hat{x}_1)\frac{v_1}{k^2_{w1} - v^2_1} - \dot{\hat{\theta}}_{1,\bar{m}}]\\ &+ \frac{1}{2}\parallel \diamond\parallel^2 + \frac{1}{2}\breve{d}^2_{1,\bar{m}} + \frac{1}{2}\breve{\mathfrak{I}}^2_{1,\bar{m}}\end{aligned} \tag{36}$$

Choose the virtual control signal $\alpha_{1,\bar{m}}$, adaptive law and compensating signal $\dot{\xi}_{1,\bar{m}}$ as:

$$\alpha_{1,\bar{m}} = -\hat{\mu}_{1,\bar{m}}\aleph_1 - \frac{3v_1}{4(k^2_{w1} - v^2_1)} - \hat{\theta}^T_{1,\bar{m}}\phi_{1,\bar{m}}(\hat{x}_1) + \dot{y}_r - \mu_{1,\bar{m}}v^{2\beta-1}_1 \tag{37}$$

$$\dot{\hat{\theta}}_{1,\bar{m}} = \gamma_{1,\bar{m}}\phi_{1,\bar{m}}(\hat{x}_1)\frac{v_1}{k^2_{w1} - v^2_1} - \sigma_{1,\bar{m}}\hat{\theta}_{1,\bar{m}} \tag{38}$$

$$\dot{\xi}_{1,\bar{m}} = -\hat{\mu}_{1,\bar{m}}\xi_{1,\bar{m}} + \xi_{2,\bar{m}} + (x_{2,\bar{m},c} - \alpha_{1,\bar{m}}) - \hbar_{1,\bar{m}}sign(\xi_{1,\bar{m}}) \tag{39}$$

where $\beta = 2p^* - 1/2p^* + 1$, and p^*, $\hbar_{1,\bar{m}}$, $\sigma_{1,\bar{m}}$, $\mu_{1,\bar{m}}$ and $\hat{\mu}_{1,\bar{m}}$ are design positive parameters.

Substituting (37) - (39) into (36), we have:

$$\begin{aligned}\dot{V}_{1,\bar{m}} \leq & \dot{V}_{o,\bar{m}} - \frac{\mu_{1,\bar{m}} v_1^{2\beta}}{k_{w1}^2 - v_1^2} - \dot{\mathbf{\mu}}_{1,\bar{m}} \frac{v_1^2}{k_{w1}^2 - v_1^2} - \frac{3v_1^2}{4(k_{w1}^2 - v_1^2)^2} \\ & + \frac{v_1}{k_{w1}^2 - v_1^2} v_2 + \frac{\sigma_{1,\bar{m}}}{\gamma_{1,\bar{m}}} \tilde{\theta}_{1,\bar{m}}^T \hat{\theta}_{1,\bar{m}} + \frac{1}{2} \parallel \diamond \parallel^2 \\ & + \frac{1}{2} \breve{d}_{1,\bar{m}}^2 + \frac{1}{2} \breve{\mathfrak{J}}_{1,\bar{m}}^2 + \hbar_{1,\bar{m}} \frac{v_1}{k_{w1}^2 - v_1^2} sign(\xi_{1,\bar{m}})\end{aligned} \tag{40}$$

Using Young's inequality, we have:

$$\frac{v_1}{k_{w1}^2 - v_1^2} v_2 \leq \frac{v_1^2}{4(k_{w1}^2 - v_1^2)^2} + v_2^2 \tag{41}$$

$$\hbar_{1,\bar{m}} \frac{v_1}{k_{w1}^2 - v_1^2} sign(\xi_{1,\bar{m}}) \leq \frac{v_1^2}{2(k_{w1}^2 - v_1^2)^2} + \frac{1}{2} \hbar_{1,\bar{m}}^2 \tag{42}$$

According to (28) and (40) - (42), it has:

$$\begin{aligned}\dot{V}_{1,\bar{m}} \leq & -(\lambda_{\min}(Q_{\bar{m}}) - n - 2 - \frac{1}{2}) \parallel \diamond \parallel^2 - \frac{\mu_{1,\bar{m}} v_1^{2\beta}}{k_{w1}^2 - v_1^2} - \dot{\mathbf{\mu}}_{1,\bar{m}} \frac{v_1^2}{k_{w1}^2 - v_1^2} \\ & + v_2^2 + \frac{\sigma_{1,\bar{m}}}{\gamma_{1,\bar{m}}} \tilde{\theta}_{1,\bar{m}}^T \hat{\theta}_{1,\bar{m}} + \frac{1}{2} \breve{d}_{1,\bar{m}}^2 + \frac{1}{2} \breve{\mathfrak{J}}_{1,\bar{m}}^2 + \frac{1}{2} \hbar_{1,\bar{m}}^2 \\ & + \parallel P_{\bar{m}} \parallel^2 \sum_{i=1}^{n} \tilde{\theta}_{i,\bar{m}}^T \tilde{\theta}_{i,\bar{m}} + \parallel P_{\bar{m}} \parallel^2 (\parallel \breve{\mathfrak{J}}_{\bar{m}} \parallel^2 + \parallel \breve{d}_{\bar{m}} \parallel^2)\end{aligned} \tag{43}$$

Step 2: According to $\aleph_2 = \hat{x}_2 - x_{2,\bar{m},c}$ and from (23) and (30), we have:

$$\begin{aligned}\dot{v}_2 = & \aleph_3 + x_{3,\bar{m},c} + l_{2,\bar{m}} \diamond_1 - \dot{x}_{2,\bar{m},c} + \hat{\theta}_{2,\bar{m}}^T \phi_{2,\bar{m}}(\hat{\bar{x}}_2) \\ & - \dot{\xi}_{2,\bar{m}} + \tilde{\theta}_{2,\bar{m}}^T \phi_{2,\bar{m}}(\hat{\bar{x}}_2) - \tilde{\theta}_{2,\bar{m}}^T \phi_{2,\bar{m}}(\hat{\bar{x}}_2)\end{aligned} \tag{44}$$

The Lyapunov function is selected as:

$$V_{2,\bar{m}} = V_{1,\bar{m}} + \frac{1}{2} v_2^2 + \frac{1}{2\gamma_{2,\bar{m}}} \tilde{\theta}_{2,\bar{m}}^T \tilde{\theta}_{2,\bar{m}} \tag{45}$$

where $\gamma_{2,\bar{m}}$ is a design positive parameter.

From (44) and (45), we can get:

$$\begin{aligned}\dot{V}_{2,\bar{m}} = & \dot{V}_{1,\bar{m}} + v_2(\aleph_3 + x_{3,\bar{m},c} + l_{2,\bar{m}} \diamond_1 - \dot{x}_{2,\bar{m},c} \\ & + \hat{\theta}_{2,\bar{m}}^T \phi_{2,\bar{m}}(\hat{\bar{x}}_2) - \dot{\xi}_{2,\bar{m}} - \tilde{\theta}_{2,\bar{m}}^T \phi_{2,\bar{m}}(\hat{\bar{x}}_2)) \\ & + \frac{1}{\gamma_{2,\bar{m}}} \tilde{\theta}_{2,\bar{m}}^T [\gamma_{2,\bar{m}} v_2 \phi_{2,\bar{m}}(\hat{\bar{x}}_2) - \dot{\hat{\theta}}_{2,\bar{m}}]\end{aligned} \tag{46}$$

Since $\theta_{i,\bar{m}}^T(\hat{\bar{x}}_i)\theta_{i,\bar{m}}(\hat{\bar{x}}_i) \leq 1$, we can obtain:

$$-v_2 \tilde{\theta}_{2,\bar{m}}^T \phi_{2,\bar{m}}(\hat{\bar{x}}_2) \leq \frac{1}{2} v_2^2 + \frac{1}{2} \tilde{\theta}_{2,\bar{m}}^T \tilde{\theta}_{2,\bar{m}} \tag{47}$$

Substituting (40) and (47) into (46), it has:

$$\begin{aligned}\dot{V}_{2,\bar{m}} \leq & \dot{V}_{o,\bar{m}} - \frac{\mu_{1,\bar{m}} v_1^{2\beta}}{k_{w1}^2 - v_1^2} - \dot{\mathbf{\mu}}_{1,\bar{m}} \frac{v_1^2}{k_{w1}^2 - v_1^2} + \frac{\sigma_{1,\bar{m}}}{\gamma_{1,\bar{m}}} \tilde{\theta}_{1,\bar{m}}^T \hat{\theta}_{1,\bar{m}} \\ & + \frac{1}{2} \parallel \diamond \parallel^2 + \frac{1}{2} \breve{d}_{1,\bar{m}}^2 + \frac{1}{2} \breve{\mathfrak{J}}_{1,\bar{m}}^2 + \frac{1}{2} \hbar_{1,\bar{m}}^2 + v_2(\frac{3}{2} v_2 + \aleph_3 \\ & + x_{3,\bar{m},c} + l_{2,\bar{m}} \diamond_1 - \alpha_{2,\bar{m}} + \alpha_{2,\bar{m}} - \dot{x}_{2,\bar{m},c} + \hat{\theta}_{2,\bar{m}}^T \phi_{2,\bar{m}}(\hat{\bar{x}}_2) \\ & - \dot{\xi}_{2,\bar{m}}) + \frac{1}{\gamma_{2,\bar{m}}} \tilde{\theta}_{2,\bar{m}}^T [\gamma_{2,\bar{m}} v_2 \phi_{2,\bar{m}}(\hat{\bar{x}}_2) - \dot{\hat{\theta}}_{2,\bar{m}}] + \frac{1}{2} \tilde{\theta}_{2,\bar{m}}^T \tilde{\theta}_{2,\bar{m}}\end{aligned} \tag{48}$$

Choose the virtual control signal $\alpha_{2,\bar{m}}$, adaptive law and compensating signal $\dot{\xi}_{2,\bar{m}}$ as:

$$\alpha_{2,\bar{m}} = -\dot{\mathbf{\mu}}_{2,\bar{m}}\aleph_2 - 2\aleph_2 - l_{2,\bar{m}}\diamond_1 + \dot{x}_{2,\bar{m},c} - \hat{\theta}^T_{2,\bar{m}}\phi_{2,\bar{m}}(\hat{\bar{x}}_2) - \mu_{2,\bar{m}}v_2^{2\beta-1} \tag{49}$$

$$\dot{\hat{\theta}}_{2,\bar{m}} = \gamma_{2,\bar{m}}v_2\phi_{2,\bar{m}}(\hat{\bar{x}}_2) - \sigma_{2,\bar{m}}\hat{\theta}_{2,\bar{m}} \tag{50}$$

$$\dot{\xi}_{2,\bar{m}} = -\dot{\mathbf{\mu}}_{2,\bar{m}}\xi_{2,\bar{m}} - 2\xi_{2,\bar{m}} + \xi_{3,\bar{m}} + (x_{3,\bar{m},c} - \alpha_{2,\bar{m}}) - \hbar_{2,\bar{m}}sign(\xi_{2,\bar{m}}) \tag{51}$$

where $\dot{\mathbf{\mu}}_{2,\bar{m}}$, $\sigma_{2,\bar{m}}$, $\mu_{2,\bar{m}}$ and $\hbar_{2,\bar{m}}$ are design positive parameters.

Substituting (49) - (51) into (48), it can be obtain that:

$$\begin{aligned}\dot{V}_{2,\bar{m}} \leq & \dot{V}_{o,\bar{m}} - \frac{\mu_{1,\bar{m}}v_1^{2\beta}}{k_{w1}^2 - v_1^2} - \dot{\mathbf{\mu}}_{1,\bar{m}}\frac{v_1^2}{k_{w1}^2 - v_1^2} - \mu_{2,\bar{m}}v_2^{2\beta} - \dot{\mathbf{\mu}}_{2,\bar{m}}{v_2}^2 \\ & + v_2v_3 + \sum_{j=1}^{2}\frac{\sigma_{j,\bar{m}}}{\gamma_{j,\bar{m}}}\tilde{\theta}^T_{j,\bar{m}}\hat{\theta}_{j,\bar{m}} + \frac{1}{2}\parallel\diamond\parallel^2 + \frac{1}{2}\breve{d}^2_{1,\bar{m}} + \frac{1}{2}\breve{\mathfrak{I}}^2_{1,\bar{m}} \\ & + \frac{1}{2}\hbar^2_{1,\bar{m}} + \frac{1}{2}\tilde{\theta}^T_{2,\bar{m}}\tilde{\theta}_{2,\bar{m}} + \hbar_{2,\bar{m}}v_2sign(\xi_{2,\bar{m}}) - \frac{1}{2}v_2^2\end{aligned} \tag{52}$$

From (28) and (52), we have:

$$\begin{aligned}\dot{V}_{2,\bar{m}} \leq & -(\lambda_{\min}(Q_{\bar{m}}) - n - 2 - \frac{1}{2})\parallel\diamond\parallel^2 - \frac{\mu_{1,\bar{m}}v_1^{2\beta}}{k_{w1}^2 - v_1^2} - \dot{\mathbf{\mu}}_{1,\bar{m}}\frac{v_1^2}{k_{w1}^2 - v_1^2} \\ & - \mu_{2,\bar{m}}v_2^{2\beta} - \dot{\mathbf{\mu}}_{2,\bar{m}}{v_2}^2 + v_2v_3 + \sum_{j=1}^{2}\frac{\sigma_{j,\bar{m}}}{\gamma_{j,\bar{m}}}\tilde{\theta}^T_{j,\bar{m}}\hat{\theta}_{j,\bar{m}} + \frac{1}{2}\breve{d}^2_{1,\bar{m}} \\ & + \frac{1}{2}\breve{\mathfrak{I}}^2_{1,\bar{m}} + \frac{1}{2}\hbar^2_{1,\bar{m}} + \frac{1}{2}\tilde{\theta}^T_{2,\bar{m}}\tilde{\theta}_{2,\bar{m}} + \parallel P_{\bar{m}}\parallel^2\sum_{i=1}^{n}\tilde{\theta}^T_{i,\bar{m}}\tilde{\theta}_{i,\bar{m}} \\ & + \parallel P_{\bar{m}}\parallel^2(\parallel\breve{\mathfrak{I}}_{\bar{m}}\parallel^2 + \parallel\breve{d}_{\bar{m}}\parallel^2) + \hbar_{2,\bar{m}}v_2sign(\xi_{2,\bar{m}}) - \frac{1}{2}v_2^2\end{aligned} \tag{53}$$

Step $i(3 \leq i \leq n-1)$: From (29) and (30), we have:

$$\begin{aligned}\dot{v}_i = & \aleph_{i+1} + x_{i+1,\bar{m},c} + l_{i,\bar{m}}\diamond_1 - \dot{x}_{i,\bar{m},c} \\ & + \hat{\theta}^T_{i,\bar{m}}\phi_{i,\bar{m}}(\hat{\bar{x}}_i) - \dot{\xi}_{i,\bar{m}} + \tilde{\theta}^T_{i,\bar{m}}\phi_{i,\bar{m}}(\hat{\bar{x}}_i) - \tilde{\theta}^T_{i,\bar{m}}\phi_{i,\bar{m}}(\hat{\bar{x}}_i)\end{aligned} \tag{54}$$

The Lyapunov function is selected as:

$$V_{i,\bar{m}} = V_{i-1,\bar{m}} + \frac{1}{2}v_i^2 + \frac{1}{2\gamma_{i,\bar{m}}}\tilde{\theta}^T_{i,\bar{m}}\tilde{\theta}_{i,\bar{m}} \tag{55}$$

where $\gamma_{i,\bar{m}}$ is a design positive parameter.

From (54) and (55), we can get that:

$$\begin{aligned}\dot{V}_{i,\bar{m}} = & \dot{V}_{i-1,\bar{m}} + v_i(\aleph_{i+1} + x_{i+1,\bar{m},c} + l_{i,\bar{m}}\diamond_1 - \dot{x}_{i,\bar{m},c} \\ & + \hat{\theta}^T_{i,\bar{m}}\phi_{i,\bar{m}}(\hat{\bar{x}}_i) - \dot{\xi}_{i,\bar{m}} + \tilde{\theta}^T_{i,\bar{m}}\phi_{i,\bar{m}}(\hat{\bar{x}}_i) - \tilde{\theta}^T_{i,\bar{m}}\phi_{i,\bar{m}}(\hat{\bar{x}}_i) \\ & + \frac{1}{\gamma_{i,\bar{m}}}\tilde{\theta}^T_{i,\bar{m}}[\gamma_{i,\bar{m}}v_i\phi_{i,\bar{m}}(\hat{\bar{x}}_i) - \dot{\hat{\theta}}_{i,\bar{m}}]\end{aligned} \tag{56}$$

Similar to (47), we have:

$$-v_i\tilde{\theta}^T_{i,\bar{m}}\phi_{i,\bar{m}}(\hat{\bar{x}}_i) \leq \frac{1}{2}v_i^2 + \frac{1}{2}\tilde{\theta}^T_{i,\bar{m}}\tilde{\theta}_{i,\bar{m}} \tag{57}$$

Substituting (52) and (57) into (56) results in:

$$\begin{aligned}\dot{V}_{i,\bar{m}} \leq & \dot{V}_{o,\bar{m}} - \frac{\mu_{1,\bar{m}} \upsilon_1^{2\beta}}{k_{w1}^2 - \upsilon_1^2} - \dot{\mathbf{\mu}}_{1,\bar{m}} \frac{\upsilon_1^2}{k_{w1}^2 - \upsilon_1^2} - \sum_{j=2}^{i-1} \dot{\mathbf{\mu}}_{j,\bar{m}} \upsilon_j^2 \\ & - \sum_{j=2}^{i-1} \mu_{j,\bar{m}} \upsilon_j^{2\beta} + \sum_{j=1}^{i-1} \frac{\sigma_{j,\bar{m}}}{\gamma_{j,\bar{m}}} \tilde{\theta}_{j,\bar{m}}^T \hat{\theta}_{j,\bar{m}} + \frac{1}{2} \sum_{j=2}^{i} \tilde{\theta}_{j,\bar{m}}^T \tilde{\theta}_{j,\bar{m}} \\ & + \frac{1}{2} \parallel \diamond \parallel^2 + \frac{1}{2} \breve{d}_{1,\bar{m}}^2 + \frac{1}{2} \breve{\mathfrak{I}}_{1,\bar{m}}^2 + \upsilon_i (\frac{1}{2} \upsilon_i + \upsilon_{i-1} \\ & + \aleph_{i+1} + x_{i+1,\bar{m},c} + l_{i,\bar{m}} \diamond_1 - \alpha_{i,\bar{m}} + \alpha_{i,\bar{m}} \\ & - \dot{x}_{i,\bar{m},c} + \hat{\theta}_{i,\bar{m}}^T \phi_{i,\bar{m}}(\hat{\bar{x}}_i) - \dot{\xi}_{i,\bar{m}}) \\ & + \frac{1}{\gamma_{i,\bar{m}}} \tilde{\theta}_{i,\bar{m}}^T [\gamma_{i,\bar{m}} \upsilon_i \phi_{i,\bar{m}}(\hat{\bar{x}}_i) - \dot{\hat{\theta}}_{i,\bar{m}}] \\ & + \sum_{j=2}^{i-1} \upsilon_j \hbar_{j,\bar{m}} sign(\xi_{j,\bar{m}}) - \frac{1}{2} \sum_{j=2}^{i-1} \upsilon_j^2 + \frac{1}{2} \hbar_{1,\bar{m}}^2 \end{aligned} \tag{58}$$

Choose the virtual control signal $\alpha_{i,\bar{m}}$, adaptive law and compensating signal $\dot{\xi}_{i,\bar{m}}$ as follows:

$$\alpha_{i,\bar{m}} = -\dot{\mathbf{\mu}}_{i,\bar{m}} \aleph_i - \aleph_i - \aleph_{i-1} + \dot{x}_{i,\bar{m},c} - l_{i,\bar{m}} \diamond_1 - \hat{\theta}_{i,\bar{m}}^T \phi_{i,\bar{m}}(\hat{\bar{x}}_i) - \mu_{i,\bar{m}} \upsilon_i^{2\beta-1} \tag{59}$$

$$\dot{\hat{\theta}}_{i,\bar{m}} = \gamma_{i,\bar{m}} \upsilon_i \phi_{i,\bar{m}}(\hat{\bar{x}}_i) - \sigma_{i,\bar{m}} \hat{\theta}_{i,\bar{m}} \tag{60}$$

$$\begin{aligned}\dot{\xi}_{i,\bar{m}} = & -\dot{\mathbf{\mu}}_{i,\bar{m}} \xi_{i,\bar{m}} - \xi_{i,\bar{m}} - \xi_{i-1,\bar{m}} + \xi_{i+1,\bar{m}} + (x_{i+1,\bar{m},c} - \alpha_{i,\bar{m}}) \\ & - \hbar_{i,\bar{m}} sign(\xi_{i,\bar{m}}) \end{aligned} \tag{61}$$

where $\dot{\mathbf{\mu}}_{i,\bar{m}}$, $\mu_{i,\bar{m}}$, $\sigma_{i,\bar{m}}$ and $\hbar_{i,\bar{m}}$ are design positive parameters.

Substituting(59) - (61) into (58), it can be obtain that:

$$\begin{aligned}\dot{V}_{i,\bar{m}} \leq & \dot{V}_{o,\bar{m}} - \frac{\mu_{1,\bar{m}} \upsilon_1^{2\beta}}{k_{w1}^2 - \upsilon_1^2} - \dot{\mathbf{\mu}}_{1,\bar{m}} \frac{\upsilon_1^2}{k_{w1}^2 - \upsilon_1^2} - \sum_{j=2}^{i} \dot{\mathbf{\mu}}_{j,\bar{m}} \upsilon_j^2 - \sum_{j=2}^{i} \mu_{j,\bar{m}} \upsilon_j^{2\beta} \\ & + \upsilon_i \upsilon_{i+1} + \sum_{j=1}^{i} \frac{\sigma_{j,\bar{m}}}{\gamma_{j,\bar{m}}} \tilde{\theta}_{j,\bar{m}}^T \hat{\theta}_{j,\bar{m}} + \frac{1}{2} \parallel \diamond \parallel^2 + \frac{1}{2} \breve{d}_{1,\bar{m}}^2 + \frac{1}{2} \breve{\mathfrak{I}}_{1,\bar{m}}^2 \\ & + \frac{1}{2} \sum_{j=2}^{i} \tilde{\theta}_{j,\bar{m}}^T \tilde{\theta}_{j,\bar{m}} + \sum_{j=2}^{i} \upsilon_j \hbar_{j,\bar{m}} sign(\xi_{j,\bar{m}}) - \frac{1}{2} \sum_{j=2}^{i} \upsilon_j^2 + \frac{1}{2} \hbar_{1,\bar{m}}^2 \end{aligned} \tag{62}$$

From (28) and (62), we have:

$$\begin{aligned}\dot{V}_{i,\bar{m}} \leq & -(\lambda_{\min}(Q_{\bar{m}}) - n - 2 - \frac{1}{2}) \parallel \diamond \parallel^2 - \frac{\mu_{1,\bar{m}} \upsilon_1^{2\beta}}{k_{w1}^2 - \upsilon_1^2} \\ & - \dot{\mathbf{\mu}}_{1,\bar{m}} \frac{\upsilon_1^2}{k_{w1}^2 - \upsilon_1^2} - \sum_{j=2}^{i} \mu_{j,\bar{m}} \upsilon_j^{2\beta} - \sum_{j=2}^{i} \dot{\mathbf{\mu}}_{j,\bar{m}} \upsilon_j^2 + \upsilon_i \upsilon_{i+1} \\ & + \sum_{j=1}^{i} \frac{\sigma_{j,\bar{m}}}{\gamma_{j,\bar{m}}} \tilde{\theta}_{j,\bar{m}}^T \hat{\theta}_{j,\bar{m}} + \frac{1}{2} \breve{d}_{1,\bar{m}}^2 + \frac{1}{2} \breve{\mathfrak{I}}_{1,\bar{m}}^2 + \frac{1}{2} \sum_{j=2}^{i} \tilde{\theta}_{j,\bar{m}}^T \tilde{\theta}_{j,\bar{m}} \\ & + \parallel P_{\bar{m}} \parallel^2 \sum_{i=1}^{n} \tilde{\theta}_{i,\bar{m}}^T \tilde{\theta}_{i,\bar{m}} + \parallel P_{\bar{m}} \parallel^2 (\parallel \breve{\mathfrak{I}}_{\bar{m}} \parallel^2 + \parallel \breve{d}_{\bar{m}} \parallel^2) \\ & + \sum_{j=2}^{i} \upsilon_j \hbar_{j,\bar{m}} sign(\xi_{j,\bar{m}}) - \frac{1}{2} \sum_{j=2}^{i} \upsilon_j^2 + \frac{1}{2} \hbar_{1,\bar{m}}^2 \end{aligned} \tag{63}$$

Step n: since $\aleph_n = \hat{x}_n - x_{n,\bar{m},c}$ and from (3) and (4), we have:

$$\begin{aligned}\dot{\upsilon}_n = & K_{\bar{m}}(u_{\bar{m}}) u_{\bar{m}} + D_{\bar{m}}(t) + l_{n,\bar{m}} \diamond_1 - \dot{x}_{n,\bar{m},c} + \hat{\theta}_{n,\bar{m}}^T \phi_{n,\bar{m}}(\hat{\bar{x}}_n) \\ & - \dot{\xi}_{n,\bar{m}} + \tilde{\theta}_{n,\bar{m}}^T \phi_{n,\bar{m}}(\hat{\bar{x}}_n) - \tilde{\theta}_{n,\bar{m}}^T \phi_{n,\bar{m}}(\hat{\bar{x}}_n) \end{aligned} \tag{64}$$

The Lyapunov function is selected as:

$$V_{n,\bar{m}} = V_{n-1,\bar{m}} + \frac{1}{2}v_n^2 + \frac{1}{2\gamma_{n,\bar{m}}}\tilde{\theta}_{n,\bar{m}}^T\tilde{\theta}_{n,\bar{m}} \tag{65}$$

where $\gamma_{n,\bar{m}}$ is a design positive parameter.

From (64) and (65), we have:

$$\begin{aligned}\dot{V}_{n,\bar{m}} =& \dot{V}_{n-1,\bar{m}} + v_n(K_{\bar{m}}(u_{\bar{m}})u_{\bar{m}} + D_{\bar{m}}(t) + l_{n,\bar{m}}\diamond_1 \\ &- \dot{x}_{n,\bar{m},c} + \hat{\theta}_{n,\bar{m}}^T\phi_{n,\bar{m}}(\hat{\bar{x}}_n) - \dot{\xi}_{n,\bar{m}} - \tilde{\theta}_{n,\bar{m}}^T\phi_{n,\bar{m}}(\hat{\bar{x}}_n)) \\ &+ \frac{1}{\gamma_{n,\bar{m}}}\tilde{\theta}_{n,\bar{m}}^T[\gamma_{n,\bar{m}}v_n\phi_{n,\bar{m}}(\hat{\bar{x}}_n) - \dot{\hat{\theta}}_{n,\bar{m}}]\end{aligned} \tag{66}$$

According to (57) and (4), it can be obtain that:

$$v_nD_{\bar{m}}(t) \leq \frac{1}{2}v_n^2 + \frac{1}{2}u_{\min}^2 \tag{67}$$

$$-v_n\tilde{\theta}_{n,\bar{m}}^T\phi_{n,\bar{m}}(\hat{\bar{x}}_n) \leq \frac{1}{2}v_n^2 + \frac{1}{2}\tilde{\theta}_{n,\bar{m}}^T\tilde{\theta}_{n,\bar{m}} \tag{68}$$

Substituting (67) and (68) into (66) results in:

$$\begin{aligned}\dot{V}_{n,\bar{m}} \leq& \dot{V}_{o,\bar{m}} - \frac{\mu_{1,\bar{m}}v_1^{2\beta}}{k_{w1}^2 - v_1^2} - \dot{\mathbf{\mu}}_{1,\bar{m}}\frac{v_1^2}{k_{w1}^2 - v_1^2} - \sum_{j=2}^{n-1}\dot{\mathbf{\mu}}_{j,\bar{m}}v_j^2 - \sum_{j=2}^{n-1}\mu_{j,\bar{m}}v_j^{2\beta} \\ &+ \sum_{j=1}^{n-1}\frac{\sigma_{j,\bar{m}}}{\gamma_{j,\bar{m}}}\tilde{\theta}_{j,\bar{m}}^T\hat{\theta}_{j,\bar{m}} + \frac{1}{2}\parallel\diamond\parallel^2 + \frac{1}{2}\breve{d}_{1,\bar{m}}^2 + \frac{1}{2}\breve{\mathfrak{I}}_{1,\bar{m}}^2 \\ &+ \frac{1}{2}u_{\min}^2 + \frac{1}{2}\sum_{j=2}^{n}\tilde{\theta}_{j,\bar{m}}^T\tilde{\theta}_{j,\bar{m}} + v_n(v_n + v_{n-1} + K_{\bar{m}}(u_{\bar{m}})u_{\bar{m}} \\ &+ l_{n,\bar{m}}\diamond_1 - \dot{x}_{n,\bar{m},c} + \hat{\theta}_{n,\bar{m}}^T\phi_{n,\bar{m}}(\hat{\bar{x}}_n) - \dot{\xi}_{n,\bar{m}}) \\ &+ \frac{1}{\gamma_{n,\bar{m}}}\tilde{\theta}_{n,\bar{m}}^T[\gamma_{n,\bar{m}}v_n\phi_{n,\bar{m}}(\hat{\bar{x}}_n) - \dot{\hat{\theta}}_{n,\bar{m}}] + \sum_{j=2}^{n-1}v_j\hbar_{j,\bar{m}}sign(\xi_{j,\bar{m}}) \\ &- \frac{1}{2}\sum_{j=2}^{n-1}v_j^2 + \frac{1}{2}\hbar_{1,\bar{m}}^2\end{aligned} \tag{69}$$

Choose the control signal $u_{\bar{m}}$, adaptive law and compensating signal $\dot{\xi}_{n,\bar{m}}$ as:

$$\begin{aligned}u_{\bar{m}} =& \frac{1}{1-\delta_{\bar{m}}}(-\dot{\mathbf{\mu}}_{n,\bar{m}}\aleph_n - \aleph_{n-1} - \frac{3}{2}\aleph_n - l_{n,\bar{m}}\diamond_1 + \dot{x}_{n,\bar{m},c} \\ &- \hat{\theta}_{n,\bar{m}}^T\phi_{n,\bar{m}}(\hat{\bar{x}}_n) - \mu_{n,\bar{m}}v_n^{2\beta-1})\end{aligned} \tag{70}$$

$$\dot{\hat{\theta}}_{n,\bar{m}} = \gamma_{n,\bar{m}}v_n\phi_{n,\bar{m}}(\hat{\bar{x}}_n) - \sigma_{n,\bar{m}}\hat{\theta}_{n,\bar{m}} \tag{71}$$

$$\dot{\xi}_{n,\bar{m}} = -\dot{\mathbf{\mu}}_{n,\bar{m}}\xi_{n,\bar{m}} - \xi_{n-1,\bar{m}} - \frac{3}{2}\xi_{n,\bar{m}} - \hbar_{n,\bar{m}}sign(\xi_{n,\bar{m}}) \tag{72}$$

where $\dot{\mathbf{\mu}}_{n,\bar{m}}$, $\sigma_{n,\bar{m}}$, $\mu_{n,\bar{m}}$ and $\hbar_{n,\bar{m}}$ are design positive parameters.

From (4) and (70), it has:

$$\begin{aligned}K_{\bar{m}}(u_{\bar{m}})u_{\bar{m}} \leq& -\dot{\mathbf{\mu}}_{n,\bar{m}}\aleph_n - \aleph_{n-1} - \frac{3}{2}\aleph_n - l_{n,\bar{m}}\diamond_1 \\ &+ \dot{x}_{n,\bar{m},c} - \hat{\theta}_{n,\bar{m}}^T\phi_{n,\bar{m}}(\hat{\bar{x}}_n) - \mu_{n,\bar{m}}v_n^{2\beta-1}\end{aligned} \tag{73}$$

Substituting (70) - (73) into (69), it has:

$$\begin{aligned}\dot{V}_{n,\bar{m}} \leq & \dot{V}_{o,\bar{m}} - \frac{\mu_{1,\bar{m}} v_1^{2\beta}}{k_{w1}^2 - v_1^2} - \dot{\mu}_{1,\bar{m}} \frac{v_1^2}{k_{w1}^2 - v_1^2} - \sum_{j=2}^{n} \dot{\mu}_{j,\bar{m}} v_j^2 - \sum_{j=2}^{n} \mu_{j,\bar{m}} v_j^{2\beta} \\ & + \sum_{j=1}^{n} \frac{\sigma_{j,\bar{m}}}{\gamma_{j,\bar{m}}} \tilde{\theta}_{j,\bar{m}}^T \hat{\theta}_{j,\bar{m}} + \frac{1}{2} \parallel \diamond \parallel^2 + \frac{1}{2} \breve{d}_{1,\bar{m}}^2 + \frac{1}{2} \breve{\mathfrak{I}}_{1,\bar{m}}^2 + \frac{1}{2} u_{\min}^2 \\ & + \frac{1}{2} \sum_{j=2}^{n} \tilde{\theta}_{j,\bar{m}}^T \tilde{\theta}_{j,\bar{m}} + \sum_{j=2}^{n} v_j \hbar_{j,\bar{m}} sign(\xi_{j,\bar{m}}) - \frac{1}{2} \sum_{j=2}^{n} v_j^2 + \frac{1}{2} \hbar_{1,\bar{m}}^2 \end{aligned} \tag{74}$$

Using Young's inequality, it has:

$$\frac{\sigma_{j,\bar{m}}}{\gamma_{j,\bar{m}}} \tilde{\theta}_{j,\bar{m}}^T \hat{\theta}_{j,\bar{m}} \leq -\frac{\sigma_{j,\bar{m}}}{2\gamma_{j,\bar{m}}} \tilde{\theta}_{j,\bar{m}}^T \tilde{\theta}_{j,\bar{m}} + \frac{\sigma_{j,\bar{m}}}{2\gamma_{j,\bar{m}}} \theta_{j,\bar{m}}^{*T} \theta_{j,\bar{m}}^* \tag{75}$$

According to (28), (74) and (75), we have:

$$\begin{aligned}\dot{V}_{n,\bar{m}} \leq & -(\lambda_{\min}(Q_{\bar{m}}) - n - 2 - \frac{1}{2}) \parallel \diamond \parallel^2 - \frac{\mu_{1,\bar{m}} v_1^{2\beta}}{k_{w1}^2 - v_1^2} \\ & - \dot{\mu}_{1,\bar{m}} \frac{v_1^2}{k_{w1}^2 - v_1^2} - \sum_{j=2}^{n} \dot{\mu}_{j,\bar{m}} v_j^2 - \sum_{j=2}^{n} \mu_{j,\bar{m}} v_j^{2\beta} \\ & - \sum_{j=2}^{n} (\frac{\sigma_{j,\bar{m}}}{2\gamma_{j,\bar{m}}} - \parallel P_{\bar{m}} \parallel^2 - \frac{1}{2}) \tilde{\theta}_{j,\bar{m}}^T \tilde{\theta}_{j,\bar{m}} + \frac{1}{2} u_{\min}^2 \\ & - (\frac{\sigma_{j,\bar{m}}}{2\gamma_{j,\bar{m}}} - \parallel P_{\bar{m}} \parallel^2) \tilde{\theta}_{1,\bar{m}}^T \tilde{\theta}_{1,\bar{m}} + \sum_{j=1}^{n} \frac{\sigma_{j,\bar{m}}}{2\gamma_{j,\bar{m}}} \theta_{j,\bar{m}}^{*T} \theta_{j,\bar{m}}^* \\ & + \frac{1}{2} \breve{d}_{1,\bar{m}}^2 + \frac{1}{2} \breve{\mathfrak{I}}_{1,\bar{m}}^2 + \parallel P_{\bar{m}} \parallel^2 (\parallel \breve{\mathfrak{I}}_{\bar{m}} \parallel^2 + \parallel \breve{d}_{\bar{m}} \parallel^2) \\ & + \sum_{j=2}^{n} v_j \hbar_{j,\bar{m}} sign(\xi_{j,\bar{m}}) - \frac{1}{2} \sum_{j=2}^{n} v_j^2 + \frac{1}{2} \hbar_{1,\bar{m}}^2 \end{aligned} \tag{76}$$

From the Young's inequality, we have:

$$v_j \hbar_{j,\bar{m}} sign(\xi_{j,\bar{m}}) \leq \frac{1}{2} v_j^2 + \frac{1}{2} \hbar_{j,\bar{m}}^2 [sign(\xi_{j,\bar{m}})] \leq \frac{1}{2} v_j^2 + \frac{1}{2} \hbar_{j,\bar{m}}^2 \tag{77}$$

Substituting (77) into (76), we have:

$$\begin{aligned}\dot{V}_{n,\bar{m}} \leq & -(\lambda_{\min}(Q_{\bar{m}}) - n - 2 - \frac{1}{2}) \parallel \diamond \parallel^2 - \frac{\mu_{1,\bar{m}} v_1^{2\beta}}{k_{w1}^2 - v_1^2} - \dot{\mu}_{1,\bar{m}} \frac{v_1^2}{k_{w1}^2 - v_1^2} \\ & - \sum_{j=2}^{n} \dot{\mu}_{j,\bar{m}} v_j^2 - \sum_{j=2}^{n} \mu_{j,\bar{m}} v_j^{2\beta} - \sum_{j=2}^{n} (\frac{\sigma_{j,\bar{m}}}{2\gamma_{j,\bar{m}}} - \parallel P_{\bar{m}} \parallel^2 - \frac{1}{2}) \tilde{\theta}_{j,\bar{m}}^T \tilde{\theta}_{j,\bar{m}} \\ & + \frac{1}{2} u_{\min}^2 - (\frac{\sigma_{j,\bar{m}}}{2\gamma_{j,\bar{m}}} - \parallel P_{\bar{m}} \parallel^2) \tilde{\theta}_{1,\bar{m}}^T \tilde{\theta}_{1,\bar{m}} + \sum_{j=1}^{n} \frac{\sigma_{j,\bar{m}}}{2\gamma_{j,\bar{m}}} \theta_{j,\bar{m}}^{*T} \theta_{j,\bar{m}}^* \\ & + \frac{1}{2} \breve{d}_{1,\bar{m}}^2 + \frac{1}{2} \breve{\mathfrak{I}}_{1,\bar{m}}^2 + \parallel P_{\bar{m}} \parallel^2 (\parallel \breve{\mathfrak{I}}_{\bar{m}} \parallel^2 + \parallel \breve{d}_{\bar{m}} \parallel^2) + \sum_{j=1}^{n} \frac{1}{2} \hbar_{j,\bar{m}}^2 \end{aligned} \tag{78}$$

Let $\lambda_{\min}(Q_{\bar{m}}) - n - 2 - (1/2) > 0$, $c_{j,\bar{m}} > 0$, $\mu_{j,\bar{m}} > 0 (j = 1, ..., n)$, $(\sigma_{j,\bar{m}}/2\gamma_{j,\bar{m}}) - \parallel P_{\bar{m}} \parallel^2 - (1/2) > 0 (j = 2, ..., n)$, $(\sigma_{1,\bar{m}}/2\gamma_{1,\bar{m}}) - \parallel P_{\bar{m}} \parallel^2 > 0$, and $\Theta = \min\{ \frac{\lambda_{\min}(Q_{\bar{m}}) - n - 2 - (1/2)}{\lambda_{\max}(P_{\bar{m}})}, 2\dot{\mu}_{1,\bar{m}}, ..., 2\dot{\mu}_{n,\bar{m}}, 2\mu_{1,\bar{m}}, ..., 2\mu_{n,\bar{m}}, 2\gamma_{1,\bar{m}}(\frac{\sigma_{1,\bar{m}}}{2\gamma_{1,\bar{m}}} - \parallel P_{\bar{m}} \parallel^2), 2\gamma_{2,\bar{m}}(\frac{\sigma_{2,\bar{m}}}{2\gamma_{2,\bar{m}}} - \parallel P_{\bar{m}} \parallel^2 - \frac{1}{2}), ..., 2\gamma_{n,\bar{m}}(\frac{\sigma_{n,\bar{m}}}{2\gamma_{n,\bar{m}}} - \parallel P_{\bar{m}} \parallel^2 - \frac{1}{2}) \}$, $d_n = +\frac{1}{2} u_{\min}^2 + \sum_{j=1}^{n} \frac{\sigma_{j,\bar{m}}}{2\gamma_{j,\bar{m}}} \theta_{j,\bar{m}}^{*T} \theta_{j,\bar{m}}^* + \frac{1}{2} \breve{d}_{1,\bar{m}}^2 + \frac{1}{2} \breve{\mathfrak{I}}_{1,\bar{m}}^2 + \sum_{j=1}^{n} \frac{1}{2} \hbar_{j,\bar{m}}^2 + \parallel P_{\bar{m}} \parallel^2 (\parallel \breve{\mathfrak{I}}_{\bar{m}} \parallel^2 + \parallel \breve{d}_{\bar{m}} \parallel^2)$.

From Lemma 2, we have:

$$\begin{aligned}\dot{V}_{n,\bar{m}} \leq & -\Theta(\diamond^T P_{\bar{m}}\diamond) - \Theta\frac{v_1^2}{k_{w1}^2 - v_1^2} - \Theta(\frac{v_1^2}{k_{w1}^2 - v_1^2})^{\beta} + \Theta(\frac{v_1^2}{k_{w1}^2 - v_1^2})^{\beta} \\ & - \Theta\sum_{j=2}^{n} v_j^2 - \Theta(\sum_{j=2}^{n} v_j^2)^{\beta} + \Theta(\sum_{j=2}^{n} v_j^2)^{\beta} - \Theta\sum_{j=1}^{n}\frac{\tilde{\theta}_{j,\bar{m}}^T\tilde{\theta}_{j,\bar{m}}}{2\gamma_{j,\bar{m}}} \\ & - \Theta(\sum_{j=1}^{n}\frac{\tilde{\theta}_{j,\bar{m}}^T\tilde{\theta}_{j,\bar{m}}}{2\gamma_{j,\bar{m}}})^{\beta} + \Theta(\sum_{j=1}^{n}\frac{\tilde{\theta}_{j,\bar{m}}^T\tilde{\theta}_{j,\bar{m}}}{2\gamma_{j,\bar{m}}})^{\beta} - \frac{\Theta v_1^{2\beta}}{k_{w1}^2 - v_1^2} - \Theta\sum_{j=2}^{n} v_j^{2\beta} + d_n \end{aligned} \tag{79}$$

Choose $\mathcal{H} = \diamond^T P_{\bar{m}}\diamond$, $\mathcal{Z} = 1$ and $x = \beta$, $y = 1-\beta$, $\bar{\gamma} = \beta^{-1}$, using Lemma 3, we can obtain:

$$(\diamond^T P_{\bar{m}}\diamond)^{\beta} \leq \diamond^T P_{\bar{m}}\diamond + (1-\beta)\beta^{\frac{\beta}{1-\beta}} \tag{80}$$

Next, let $\mathcal{H} = \Theta$, $\mathcal{Z} = \Theta\sum_{k=1}^{n}(\tilde{\theta}_{j,\bar{m}}^T\tilde{\theta}_{j,\bar{m}}/2\gamma_{j,\bar{m}})$ and $x = 1-\beta$, $y = \beta$, $\bar{\gamma} = \beta^{\frac{\beta}{1-\beta}}$. we have:

$$\begin{aligned}\Theta(\sum_{j=1}^{n}\frac{\tilde{\theta}_{j,\bar{m}}^T\tilde{\theta}_{j,\bar{m}}}{2\gamma_{j,\bar{m}}})^{\beta} &= \Theta^{1-\beta}(\Theta\sum_{j=1}^{n}\frac{\tilde{\theta}_{j,\bar{m}}^T\tilde{\theta}_{j,\bar{m}}}{2\gamma_{j,\bar{m}}})^{\beta} \\ &\leq (1-\beta)\bar{\gamma}\Theta + \Theta\sum_{j=1}^{n}\frac{\tilde{\theta}_{j,\bar{m}}^T\tilde{\theta}_{j,\bar{m}}}{2\gamma_{j,\bar{m}}}\end{aligned} \tag{81}$$

Similar to (81), it can be obtain that:

$$\Theta(\frac{v_1^2}{k_{w1}^2 - v_1^2})^{\beta} \leq (1-\beta)\bar{\gamma}\Theta + \Theta\frac{v_1^2}{k_{w1}^2 - v_1^2} \tag{82}$$

$$\Theta(\sum_{j=2}^{n} v_j^2)^{\beta} \leq (1-\beta)\bar{\gamma}\Theta + \Theta\sum_{j=2}^{n} v_j^2 \tag{83}$$

According to (18), we have:

$$\begin{aligned}\bar{V}_{1,\bar{m}}^{\beta} &= (\frac{1}{2}\log\frac{k_{w1}^2}{k_{w1}^2 - v_1^2})^{\beta} \\ &\leq (\frac{1}{2}\frac{v_1^2}{k_{w1}^2 - v_1^2})^{\beta} \leq \frac{1}{2}\frac{v_1^{2\beta}}{k_{w1}^2 - v_1^2}\end{aligned} \tag{84}$$

Then, substituting (80) - (84) into (79) results in:

$$\begin{aligned}\dot{V}_{n,\bar{m}} \leq & -\Theta(\diamond^T P_{\bar{m}}\diamond)^{\beta} - \Theta(\frac{1}{2}\log\frac{k_{w1}^2}{k_{w1}^2 - v_1^2})^{\beta} \\ & - 2^{\beta}\Theta(\sum_{j=2}^{n} v_j^2)^{\beta} - \Theta\sum_{j=2}^{n} v_j^{2\beta} - \Theta(\sum_{j=1}^{n}\frac{\tilde{\theta}_{j,\bar{m}}^T\tilde{\theta}_{j,\bar{m}}}{2\gamma_{j,\bar{m}}})^{\beta} + d \\ \leq & -\Theta V_{n,\bar{m}}^{\beta} + d\end{aligned} \tag{85}$$

where $d = 4\Theta(1-\beta)\beta^{(\beta/1-\beta)} + d_n$.

Define

$$T_r = \frac{1}{(1-\beta)\kappa\Theta}[V_{n,\bar{m}}^{1-\beta}(\varsigma(0)) - (\frac{d}{(1-\kappa)\Theta})^{\frac{1-\beta}{\beta}}] \tag{86}$$

with $V_{n,\bar{m}}(\varsigma(0))$ denotes the initial value of $V_{n,\bar{m}}(\varsigma)$. From Lemma 4, $\forall t \geq T_r$, we have $V_{n,\bar{m}}^{\beta}(\varsigma) \leq (d/(1-\kappa)\Theta)$, which shows that all the signals have SGPFS according to standard analysis[39].

In addition, for $\forall t \geq T_r$, it is easy to get that

$$|y - y_d| \leq 2(\frac{d}{(1-\kappa)\Theta})^{\frac{1}{2\beta}} \tag{87}$$

The (87) shows that after a finite time T_r, the tracking error will converge near the origin.

In addition, from[12], the boundedness of $\xi_{1,\bar{m}}$ is obtained. It is worth to be noticed that $k_{w1} - k_{\xi} > 0$ needs to be guaranteed with the aim to obtain better tracking performance. Since $y = \aleph_1 + y_d$, $v_1 = \aleph_1 - \xi_{1,k}$, $|y_d| \leq \mathcal{Y}_0$ and $|\xi_{1,\bar{m}}| \leq k_{\xi}$, thus we can infer that $|y| < \mathcal{Y}_0 + k_{\xi} + k_{w1} = k_{c1}$, where k_{c1} is the system output constraint bounds.

So far, the following theorem can be established.

Theorem 1. Under the Lemmas 1-7, the SFSO (23), the virtual control signals (37), (49) and (59), the adaptive parameters (38), (50), (60) and (71), and the control input u_k in (70), for the system (1) under consideration, we can conclude that all signals in the closed-loop system are SGPFS for every switching signal $\bar{\sigma}(t)$. Besides, a small neighborhood in which the constrained output y in the system reaches the desired trajectory in finite time.

Remark 3. From (87) and the design parameters of $\aleph$ and d, the tracking error $|y - y_d|$ is depending on $\hat{\boldsymbol{\mu}}_{i,\bar{m}}$, $\mu_{i,\bar{m}}$ and $\gamma_{i,\bar{m}}$. We can increasing $\hat{\boldsymbol{\mu}}_{i,\bar{m}}$, $\mu_{i,\bar{m}}$, meanwhile decreasing $\gamma_{i,\bar{m}}$ to achieve ideal tracking performance.

5 | SIMULATION EXAMPLE

To verify the effectiveness of the proposed control strategy, we give an example in this part, and the unknown functions and unknown disturbance signals mentioned in this paper will be given in known form.

Consider the switched nonlinear system as follow:

$$\begin{cases} \dot{x}_1 = x_2 + f_{1,\bar{\sigma}(t)}(x_1) + d_{1,\bar{\sigma}(t)}(t) \\ \dot{x}_2 = q_{\bar{\sigma}(t)}(u_{\bar{\sigma}(t)}) + f_{2,\bar{\sigma}(t)}(\bar{x}_2) + d_{2,\bar{\sigma}(t)}(t) \\ y = x_1 \end{cases}$$

where $f_{1,1} = 0.1x_1 sin(x_1)/(1 + x_1^4)$, $d_{1,1} = 0.01sin(exp(-t^2))$, $f_{2,1} = -sin(x_1)\ /exp(1 + x_2^4)$, $d_{2,1} = 0.01exp(-t^2)sin(t)$, $f_{1,2} = 0.1x_1 cos(x_1)/(1 + x_1^2)$, $d_{1,2} = 0.1/(1 + t^3)$, $f_{2,2} = -0.5x_2 sin(x_1)/(1 + 0.5x_2^4)$, $d_{2,2} = 1/(1 + t^2)$. The switching signal is chosen as $\bar{\sigma}(t) = 1, 2$ and the reference tracking signal is $y_d = 0.5 * sin(t) + 0.5 * sin(0.5t)$. Constrained output is $|y| \leq k_{c1} = 0.91$. The parameters δ and $u_{\min}$ are given by $\delta = 0.4$, $u_{\min} = 0.2$.

From (23), we have:

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + \hat{f}_{1,\bar{m}}(\hat{x}_1|\hat{\theta}_{1,\bar{m}}) + l_{1,\bar{m}}(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_2 &= q_{\bar{m}}(u_{\bar{m}}) + \hat{f}_{2,\bar{m}}(\hat{\bar{x}}_2|\hat{\theta}_{2,\bar{m}}) + l_{2,\bar{m}}(x_1 - \hat{x}_1) \\ \hat{y}_1 &= \hat{x}_1 \end{aligned}$$

Choose $Q = diag\{6, 6\}$, the positive-definite symmetric matrix is:

$$P = \begin{bmatrix} 4.5000 & -3.0000 \\ -3.0000 & 1.5000 \end{bmatrix}$$

The compensating signals are defined as:

$$\begin{aligned} \dot{\xi}_{1,\bar{m}} =& -20\xi_{1,\bar{m}} + \xi_{2,\bar{m}} + (x_{2,\bar{m},c} - \alpha_1) - h_{1,\bar{m}} sign(\xi_{1,\bar{m}}) \\ \dot{\xi}_{2,\bar{m}} =& -75\xi_{2,\bar{m}} - \frac{5}{2}\xi_{2,\bar{m}} - h_{2,\bar{m}} sign(\xi_{2,\bar{m}}) \end{aligned}$$

Design the virtual control signal $\alpha_{1,\bar{m}}$, the control input $u_{\bar{m}}$, and adaptive laws as:

$$\begin{aligned} \alpha_{1,\bar{m}} =& -20\aleph_1 - \frac{3\aleph_1}{4(k_{w1}^2 - v_1^2)} - \hat{\theta}_{1,k}^T \phi_{1,k}(\hat{x}_1) + 0.5\cos(t) + 0.25\cos(0.5t) \\ u_{\bar{m}} =& \frac{1}{1 - 0.4}[-75\aleph_2 - \frac{5}{2}\aleph_2 - 13\diamond_1 + \dot{x}_{2,k,c} - \hat{\theta}_{2,1}^T \phi_{2,1}(\hat{\bar{x}}_2) - 0.1v_2^{2\times\frac{99}{101}-1}] \end{aligned}$$

$$\begin{aligned} \dot{\hat{\theta}}_{1,\bar{m}} =& 0.01\frac{v_1}{k_{w1}^2 - v_1^2}\phi_{1,\bar{m}}(\hat{x}_1) - 2\hat{\theta}_{1,\bar{m}} \\ \dot{\hat{\theta}}_{2,\bar{m}} =& 0.1v_2\phi_{2,\bar{m}}(\hat{\bar{x}}_2) - 0.3\hat{\theta}_{2,\bar{m}} \end{aligned}$$

The non-zero initial conditions are chosen as $x_1(0) = 0.07$, $x_2(0) = 0.08$, $\hat{x}_1(0) = 0.5$, $\hat{x}_2(0) = 0.5$, $\hat{\theta}_{1,\bar{m}}(0) = 0.5$, $\hat{\theta}_{2,\bar{m}}(0) = 0.5$. $k_{w1} = 0.1$.

Fig. 1 means signal y_d tracks the trajectory of output y; Fig. 2 expresses the tracking error; Fig. 3 shows the unknown state x_1 and its estimated value $\hat{x}_1$; Fig. 4 shows the unknown state x_2 and its estimated value $\hat{x}_2$; Fig. 5 expresses the adaptive laws θ_1 and θ_2; Fig. 6 exhibits the input u and its quantized input $q(u)$; and Fig. 7 shows the trajectory of switching signal.

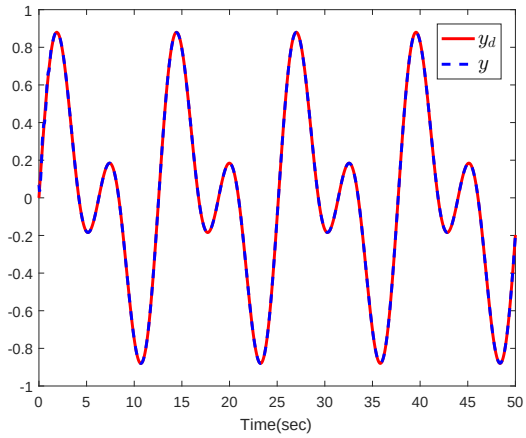

Fig.1 Trajectories of y and y_d.

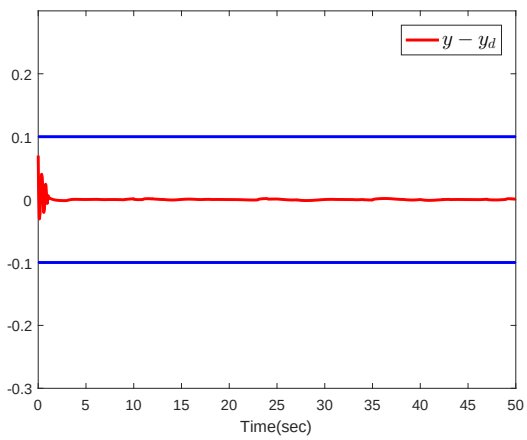

Fig.2 Trajectory of tracking error.

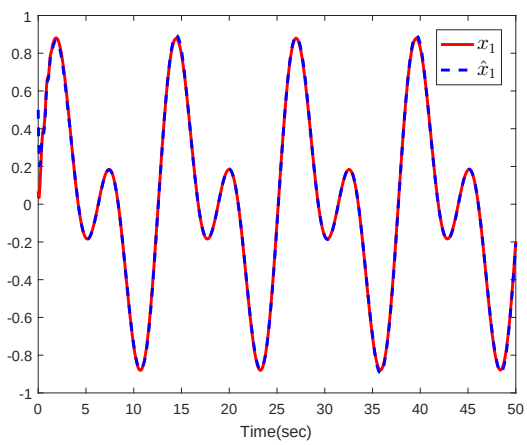

Fig.3 Trajectories of x_1 and $\hat{x}_1$.

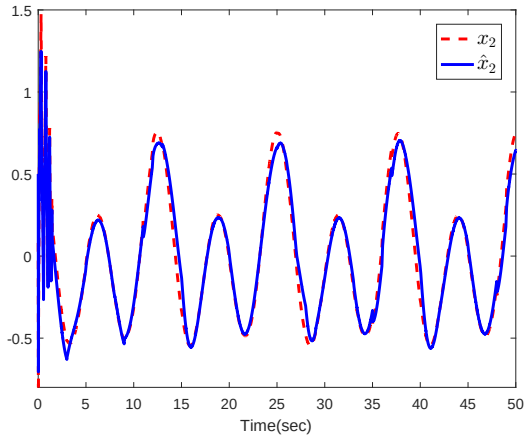

Fig.4 Trajectories of x_2 and $\hat{x}_2$.

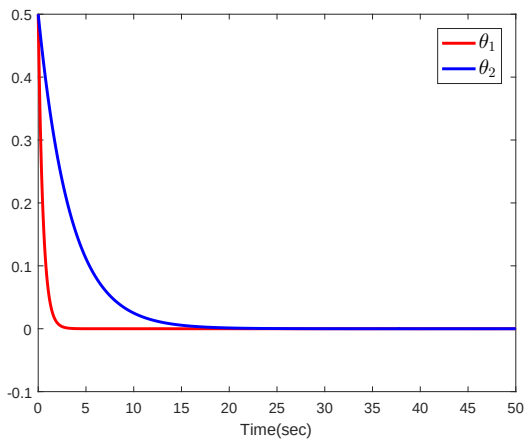

Fig.5 Trajectories of θ_1 and θ_2.

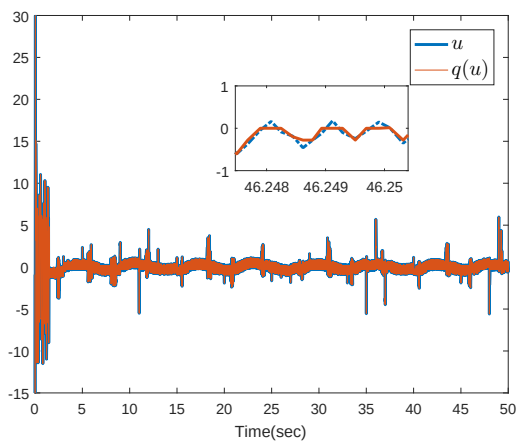

Fig.6 Trajectories of u and $q(u)$.

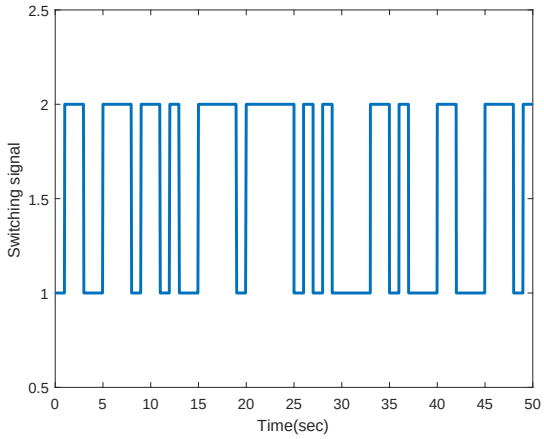

Fig.7 The switching signal.

6 | CONCLUSIONS

In this paper, a FTCF adaptive backstepping control scheme has been proposed for switched nonlinear systems with input quantization and output constraints. Using the FLS approximation abilities, a SFSO has been designed to estimate the unmeasured state. The FLD has been introduced to replace the traditional command filter, it ensured that the derivative of the virtual signal is approximated in finite-time. Satisfactory tracking performance can be obtained under input quantization and output constraints. The theoretical results are verified by the presented a simulation example.

7 | ACKNOWLEDGEMENT

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