Solitary wave solution, breather wave solution and rogue wave solution for a KP-equation

Zhenjie Niu[1] and Zenggui Wang[1]

[1]Liao Cheng University

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Abstract

In this paper, we investigate KP equation by the Hirota bilinear method and obtain its bilinear form successfully. On the basis of above bilinear form, a number of explicit solutions including one-solitary wave solution, two-solitary wave solution and their generalized form N-solitary wave solution are obtained successfully. Moreover, in view of the homoclinic breather limit method, we also express breather wave solutions and rogue wave solutions of KP-equation. Finally, with mathematical software Maple, we obtained overhead views, perspective views and wave propagation pattern of solutions in different parameter areas.

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Zhenjie Niu,* Zenggui Wang †

School of Mathematical Sciences, Liaocheng University, Liaocheng 252059, Shandong, PR China

Abstract: In this paper, we investigate KP equation by the Hirota bilinear method and obtain its bilinear form successfully. On the basis of above bilinear form, a number of explicit solutions including one-solitary wave solution, two-solitary wave solution and their generalized form N-solitary wave solution are obtained successfully. Moreover, in view of the homoclinic breather limit method, we also express breather wave solutions and rogue wave solutions of KP-equation. Finally, with mathematical software Maple, we obtained overhead views, perspective views and wave propagation pattern of solutions in different parameter areas.

keywords:KP equation, bilinear form, solitary solution, breather wave solution, rogue wave solution

1 Introduction

The existence of nonlinear physical phenomena leads to the derivation of many nonlinear evolution equations, therefore, seeking for exact solutions of these equations is always a significant and essential work. With longstanding and unremitting efforts, a number of powerful methods are put forward by researchers such as Hirota bilinear method [1-10], three-wave method [11], homoclinic breather limit method [12], tanh function method and extend tanh function method [13,14], Darboux and Bäcklund transformation method [15-20] and so on. As one kind of exact solutions, solitary wave solution is of great importance in solving nonlinear partial differential equation due to the high stability and particle property of solitary wave. And these solitary waves were discovered in fluid physics, solid state physics, plasma physics, and optical fibre communication. Therefore, an increasing number of scholars devote themselves to study the solitary wave theory. Besides, rogue wave, also referred to as extreme waves, abnormal waves, monster

*E-mail: nzj951001@163.com

†E-mail: wangzenggui@lcu.edu.cn, Corresponding author

waves, freak waves, was firstly found in deep ocean and it turns out that it also exists in biophysics, superfluids, Bose-Einstein condensates [21], financial market and other fields.

Kadomtsev-Petviashvili equation (KP-equation), firstly derived to simulate nonlinear fluctuations, attract many researchers to study and many useful results are obtained. Cheng [22] found group invariant solutions by Darboux transformation; Chen [23] found rogue wave solutions with employing homoclinic breather limit method; K.A.Gorshkov analyzed multisoliton solutions by using exact and approximate methods [24]; M.K. Elboree [12] studied an extended KP-equation by bilinear method, obtained its lump solutions and rogue wave solutions.

In this article, we would like to consider the following KP-equation

$$(u_t + uu_x + pu_{xxx})_x + \alpha b^2 u_{yy} = 0, \tag{1}$$

where p, α and b are three real numbers. We would like to investigate the KP-equation by Hirota bilinear method and obtain its N-solitary wave solution, breather wave solution and rogue wave solution. These solutions are useful to describe the evolution of long ion-acoustic waves of small amplitude propagating and the movement of water surface and internal waves.

This paper is organized as follows: in Section 2, we change Eq.(1) into bilinear form with a appropriate transformation and get its one-solitary wave solution, two-solitary wave solution and N-solitary wave solution. In Section 3, the breather wave solution and rogue wave solution are presented with the application of homoclinic breather limit method. In the end, some conclusions and discussions are given.

2 Bilinear form and solitary wave solution of Eq.(1)

With the transformation $u = [12p(\ln F)]_{xx}$ concluded from painlevè analysis, we can get

$$\begin{aligned} 12p(\ln F)_{xxxt} + 144p^2[(\ln F)_{xxx}]^2 + 144p^2(\ln F)_{xx}(\ln F)_{xxxx} \\ + 12p^2(\ln F)_{xxxxxx} + 12p\alpha b^2(\ln F)_{xxyy} = 0. \end{aligned} \tag{2}$$

Integrating Eq.(2) twice with respect to x, we have

$$(\ln F)_{xt} + 6p[(\ln F)_{xx}]^2 + p(\ln F)_{xxxx} + \alpha b^2(\ln F)_{yy} = 0, \tag{3}$$

which implies the following Hirota bilinear form of Eq. (1)

$$(D_x D_t + pD_x^4 + \alpha b^2 D_y^2)F \cdot F = 0, \tag{4}$$

where D_x, D_t, D_y are Hirota bilinear operators defined by

$$D_t^m D_x^n (f \cdot g) = (\partial_t - \partial_{t'})^m (\partial_x - \partial_{x'})^n f(t,x) g(t',x')|_{t'=t, x'=x}. \tag{5}$$

Suppose that F can be expanded as a series in term of ε,

$$F = 1 + \varepsilon F^{(1)} + \varepsilon^2 F^{(2)} + \varepsilon^3 F^{(3)} + \varepsilon^4 F^{(4)} + \cdots . \tag{6}$$

Substituting expansion (6) into Eq.(4) and collecting the coefficients of $\varepsilon^n (n = 1, 2, 3 \cdots)$, we can get the following formulas

$$\varepsilon : 2(F_{xt}^{(1)} + pF_{xxxx}^{(1)} + \alpha b^2 F_{yy}^{(1)}) = 0, \tag{7a}$$

$$\varepsilon^2 : 2(F_{xt}^{(2)} + pF_{xxxx}^{(2)} + \alpha b^2 F_{yy}^{(2)}) = -(D_x D_t + pD_x^4 + \alpha b^2 D_y^2)F^{(1)} \cdot F^{(1)}, \tag{7b}$$

$$\varepsilon^3 : 2(F_{xt}^{(3)} + pF_{xxxx}^{(3)} + \alpha b^2 F_{yy}^{(3)}) = -2(D_x D_t + pD_x^4 + \alpha b^2 D_y^2)F^{(1)} \cdot F^{(2)}, \tag{7c}$$

$$\varepsilon^4 : 2(F_{xt}^{(4)} + pF_{xxxx}^{(4)} + \alpha b^2 F_{yy}^{(4)}) = -(D_x D_t + pD_x^4 + \alpha b^2 D_y^2)(2F^{(1)} \cdot F^{(3)} + F^{(1)} \cdot F^{(2)}). \tag{7d}$$

2.1 One-solitary wave solution

By calculating Eq.(7a), we get the solution of $F^{(1)}$ with the form

$$F^{(1)} = 1 + e^{k_1 x + \mu_1 y + \omega_1 t + \xi_0^1}, \tag{8}$$

where $\omega_1 = -\frac{pk_1^4 + \alpha b^2 \mu_1^2}{k_1}$. Substituting $F^{(1)}$ into $Eq.(7b)$, we obtain

$$2(F_{xt}^{(2)} + pF_{xxxx}^{(2)} + \alpha b^2 F_{yy}^{(2)}) = 0. \tag{9}$$

If we choose $F^{(2)} = 0$, Eq.(7c) gives

$$2(F_{xt}^{(3)} + pF_{xxxx}^{(3)} + \alpha b^2 F_{yy}^{(3)}) = 0. \tag{10}$$

Similarly, we can choose $F^{(3)} = 0, F^{(4)} = 0, F^{(5)} = 0, \cdots$, when $\varepsilon = 1$, one-solitary wave solution can be expressed in the form

$$u = 12p[\ln(F^{(1)})]_{xx} = \frac{12pk_1^2 e^{k_1x+\mu_1y+\omega_1t+\xi_0^1}}{(1+e^{k_1x+\mu_1y+\omega_1t+\xi_0^1})^2}. \tag{11}$$

Based on solution (11), we can plot one-solitary waves of Eq.(1) with suitable parameters in Fig.1.

2.2 Two-solitary wave solution

Because Eq.(7a) is a linear equation, it has the solution in iterative form

$$F^{(1)} = e^{k_1x+\mu_1y+\omega_1t+\xi_0^1} + e^{k_2x+\mu_2y+\omega_2t+\xi_0^2}, \tag{12}$$

where $\omega_i = -\frac{pk_i^4+\alpha b^2\mu_i^2}{k_i}, i = 1, 2$. Substituting (12) into (7b), we have

$$F^{(2)} = A_{12}e^{\xi_1+\xi_2}, \tag{13}$$

where $\xi_i = k_ix + \mu_iy + \omega_it + \xi_0^i, i = 1, 2$, $A_{12} = \frac{\alpha b^2(k_1\mu_2-k_2\mu_1)^2-3pk_1^2k_2^2(k_1-k_2)^2}{\alpha b^2(k_1\mu_2-k_2\mu_1)^2-3pk_1^2k_2^2(k_1+k_2)^2}$, and the two-solitary wave solution is derived as follows

$$u = 12p[\ln(1+e^{\xi_1}+e^{\xi_2}+A_{12}e^{\xi_1+\xi_2})]_{xx}. \tag{14}$$

Similarly, the two-solitary wave solution (14) are plotted in Fig.(2) by choosing suitable parameters.

2.3 N-solitary wave solution

Similarly, we can figure out N-solitary wave solution of Eq.(1) with following form

$$u = 12p[\ln(F)]_{xx}, \quad F = \sum_{\lambda=0,1} e^{(\sum_{i=1}^{N}\lambda_i\xi_i + \sum_{1\le i<j\le N}\lambda_i\lambda_jA_{ij})},$$

$$\xi_i = k_ix + \mu_iy + \omega_it + \xi_0^i, \quad \omega_i = -\frac{pk_i^4+\alpha b^2\mu_i^2}{k_i},$$

$$e^{A_{ij}} = \frac{\alpha\, b^2\,(k_i\mu_j - k_j\mu_i)^2 - 3\,p{k_i}^2{k_j}^2\,(k_i-k_j)^2}{\alpha\, b^2\,(k_i\mu_j - k_j\mu_i)^2 - 3\,p{k_i}^2{k_j}^2\,(k_i+k_j)^2},$$

where $k_i, \mu_i, \omega_i (i = 1, 2, ..., N)$ are arbitrary constants, $\sum_{\lambda=0,1}$ represents the sum of all possible combinations of $\lambda_i, \lambda_j = 0, 1 (i, j = 1, 2, ..., N)$, $\sum_{1\le i<j\le N}$ is summation of all possible pairs taken from N elements with condition $(1 \le i < j \le N)$. In reference [25], the author obtained one-solitary wave solution, with the help of one-solitary wave solution, the author obtained two-solitary wave solution. Three-solitary wave solution and four-solitary wave solution are also obtained in that way. We just need to take N as the number we want in subsection 2.3, then,N-solitary wave solution (N=1,2,3,...) is obtained. If we choose appropriate parameters in subsection 2.3, we can get similarly results to [25].

3 Breather waves and Rogue waves

3.1 Breather waves

Considering that u_0 is a equilibrium solution of Eq.(1) which is an arbitrary constant, we use the transformation

$$u = u_0 + 12p[\ln(f)]_{xx}, \tag{15}$$

We can convert Eq.(1) into the following bilinear form by substituting (15) into Eq.(1)

$$(D_xD_t + u_0D_x^2 + pD_x^4 + \alpha b^2 D_y^2)F \cdot F = 0. \tag{16}$$

For finding out breather wave solution of Eq.(1), we suppose that f has the form

$$f = e^{-p_1(x+b_1y-at)} + \delta_1 \cos\left(p_2\left(x + b_2y + \beta t\right)\right) + \delta_2 e^{p_1(x+b_1y-at)}, \tag{17}$$

where $p_1, p_2, b_1, b_2, \delta_1, \delta_2, a, \beta$ are real constants to be determined. Taking (17) into (16), we can get a algebraic equation about $e^{p_1(x-b_1y+at)}$, collecting all coefficients of $e^{\pm p_1(x-b_1y+at)}\cos(p_2(x+b_2y+\beta t))$, $e^{\pm p_1(x-b_1y+at)}\sin(p_2(x+b_2y+\beta t))$ and equating them to zero, then we get a series of algebraic equations about $p_1, p_2, b_1, b_2, \delta_1, \delta_2, a, \beta$

$$\begin{aligned}&-2\,ap_1{}^2\delta_1\delta_2 - 2\,p_2{}^2\beta\,\delta_1\delta_2 + u_0\left(2\,p_1{}^2\delta_1\delta_2 - 2\,p_2{}^2\delta_1\delta_2\right) + p(2\,p_1{}^4\delta_1\delta_2\\ &- 12\,p_1{}^2p_2{}^2\delta_1\delta_2 + 2\,p_2{}^4\delta_1\delta_2) + \alpha\,b^2\left(2\,b_1{}^2p_1{}^2\delta_1\delta_2 - 2\,b_2{}^2p_2{}^2\delta_1\delta_2\right) = 0,\end{aligned}$$

$$\begin{aligned}&-2ap_1{}^2\delta_1 - 27p_2{}^2\beta\delta_1 + u_0(2p_1{}^2\delta_1 - 2\,p_2{}^2\delta_1) + p(2p_1{}^4\delta_1 - 12p_1{}^2p_2{}^2\delta_1\\ &+ 2p_2{}^4\delta_1) + \alpha b^2(2b_1{}^2p_1{}^2\delta_1 - 2b_2{}^2p_2{}^2\delta_1) = 0,\end{aligned}$$

$$\begin{aligned}&-2ap_1p_2\delta_1\delta_2 + 2p_1p_2\beta\,\delta_1\delta_2 + 4\,u_0p_1p_2\delta_1\delta_2 + p(8p_1{}^3p_2\delta_1\delta_2 - 8p_1p_2{}^3\delta_1\delta_2)\\ &+ 4\alpha b^2b_1b_2p_1p_2\delta_1\delta_2 = 0,\end{aligned}$$

$$\begin{aligned}&2ap_1p_2\delta_1 - 2p_1p_2\beta\delta_1 - 4u_0p_1p_2\delta_1 + p(-8p_1{}^3p_2\delta_1 + 8p_1p_2{}^3\delta_1)\\ &- 4\alpha b^2b_1b_2p_1p_2\delta_1 = 0,\end{aligned}$$

$$\begin{aligned}&8pp_2{}^4\delta_1{}^2 - 2p_2{}^2u_0\delta_1{}^2 - 2\alpha b^2b_2{}^2p_2{}^2\delta_1{}^2 + 32pp_1{}^4\delta_2 + 8\,p_1{}^2u_0\delta_2\\ &- 2p_2{}^2\beta\,\delta_1{}^2 - 8ap_1{}^2\delta_2 + 8\alpha b^2b_1{}^2p_1{}^2\delta_2 = 0.\end{aligned}$$

Solving the above equation system, we get the following result

$$a = \frac{b^2b_1{}^2p_1{}^2\alpha + 2\,b^2b_1b_2p_2{}^2\alpha - b^2b_2{}^2p_2{}^2\alpha + pp_1{}^4 - 2\,pp_1{}^2p_2{}^2 - 3\,pp_2{}^4 + p_1{}^2u_0 + p_2{}^2u_0}{p_1{}^2 + p_2{}^2},$$

$$\beta = \frac{b^2b_1{}^2p_1{}^2\alpha - 2\,b^2b_1b_2p_1{}^2\alpha - b^2b_2{}^2p_2{}^2\alpha - 3\,pp_1{}^4 - 2\,pp_1{}^2p_2{}^2 + pp_2{}^4 - p_1{}^2u_0 - p_2{}^2u_0}{p_1{}^2 + p_2{}^2},$$

$$\delta_2 = \frac{1}{4}\,\frac{p_2{}^2\delta_1{}^2\left(b^2b_1{}^2p_1{}^2\alpha - 2\,b^2b_1b_2p_1{}^2\alpha + b^2b_2{}^2p_1{}^2\alpha - 3\,pp_1{}^4 - 6\,pp_1{}^2p_2{}^2 - 3\,pp_2{}^4\right)}{p_1{}^2\left(b^2b_1{}^2p_2{}^2\alpha - 2\,b^2b_1b_2p_2{}^2\alpha + b^2b_2{}^2p_2{}^2\alpha + 3\,pp_1{}^4 + 6\,pp_1{}^2p_2{}^2 + 3\,pp_2{}^4\right)},$$

where $b_1, b_2, p_1, p_2, \delta_1$ are real constants. Substituting the above results into Eq.(15), solution of Eq.(16) can be expressed as trigonometric form

$$u = u_0 + 12p\frac{ff_{xx} - f_x^2}{f^2}, \tag{18}$$

where f is given by (17), and f_x and f_{xx} are as follows

$$f_x = -p_1 e^{-p_1(x+b_1y-at)} - p_2\delta_1 \sin(p_2(x + b_2y + \beta t)) + p_1^2\delta_2 e^{p_1(x+b_1y-at)},$$

$$f_{xx} = p_1^2 e^{-p_1(x+b_1y-at)} - p_2^2\delta_1 \cos(p_2(x + b_2y + \beta t)) + p_1\delta_2 e^{p_1(x+b_1y-at)},$$

By choosing suitable parameters, we can plot solution (18) in Fig. 3 and Fig. 4.

3.2 Rogue waves

In this part, for getting rogue wave solution of Eq.(1), let $p_1 = p_2 = s$ and we have

$$a = \frac{\alpha b^2(b_1^2 + 2b_1b_2 - b_2^2) + 2u_0 - 4ps^2}{2},$$

$$\beta = \frac{\alpha b^2(b_1^2 - 2b_1b_2 - b_2^2) - 2u_0 - 4ps^2}{2},$$

$$\delta_2 = \frac{1}{4}\delta_1^2\frac{\alpha b^2(b_1 - b_2)^2 - 12ps^2}{\alpha b^2(b_1 - b_2)^2 + 12ps^2},$$

and take $\delta_1 = 1$, (17) can be rewritten as following form

$$f = e^{-s(x+b_1y-at)} + \cos(s(x+b_2y+\beta t)) + \frac{1}{4}(1 - \frac{24ps^2}{\alpha b^2(b_1 - b_1)^2 + 12ps^2})e^{s(x+b_1y-at)},$$

where $\frac{p}{\alpha} < 0$. Substituting the above formula into (18), we get

$$u = u_0 + \frac{4s^2\delta_2 - s^2 - 4s^2\sqrt{\delta_2}\sinh(sX_1)\sin(sX_2)}{[2\sqrt{\delta_1}\cosh(sX_1) - 2\cos(sX_2)]^2},$$

where $X_1 = (x + b_1y - at + \frac{1}{2}\ln\delta_2)$, $X_2 = (x + b_2y + \beta t)$. With using of Taylor expansion at $s = 0$, solution (18) can be rewritten as

$$u = u_0 + 12p\frac{\frac{4\delta_2 - 1}{s^2} - 4\sqrt{\delta_2}X_1X_2}{(\sqrt{\delta_2}X_1^2 - \frac{1}{2}X_2^2 + \frac{2\sqrt{\delta_2}+1}{s^2})^2}, \tag{19}$$

thus, the rogue wave of Eq.(1) is obtained with above expression. And we can plot its perspective view, overhead view and propagation pattern. And we can plot its perspective view, overhead view and propagation pattern in Fig. 5 and Fig. 6 with suitable parameters.

4 Conclusion

In this paper, we first found a transformation from Pninlevé, then we substituted the transformation into Eq.(1) and obtained its bilinear form. Solving the bilinear equation, one-solitary wave solution, two-solitary wave solution and N-solitary wave solution heve been presented, meanwhile, their views were plotted. Next, we apply homoclinic breather limit method to solve Eq.(16) and calculate breather wave solutions of Eq.(1) with suitable parameters. And further, the rogue wave solutions were also found out.

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Compliance with ethical standards

Conflict of Interest: The authors declare that they have no conflict of interest.

References

[1] M.J. Dong, S.F. Tian, X.W. Yan, L. Zou, Solitary waves, homoclinic breather waves and rogue waves of the (3+1)-dimensional Hirota bilinear equation, Comput. Math. Appl. 2018.

[2] W. Tan, Z.D. Dai, Z.Y. Yin, Dynamics of multi-breathers, N-solitons and M-lump solutions in the (2+1)-dimensional KdV equation, Nonlinear. Dynam. 2019.

[3] A. Yusuf, T.A. Sulaiman, M. Inc, M. Bayram, Breather wave, lump-periodic solutions and some other interaction phenomena to the Caudrey-Dodd-Gibbon equation, Eur. Phys. J. Plus. 2020, 135(7).

[4] X.W. Yan, S.F. Tian, M.J. Dong, L. Zhou, T.T. Zhang, Characteristics of solitary wave, homoclinic breather wave and rogue wave solutions in a (2+1)-dimensional generalized breaking soliton equation, Comput. Math. Appl. 2018.

[5] C.C. Hu, B. Tian, H.M. Yin, C.R. Zhang, Z. Zhang, Dark breather waves, dark lump waves and lump wave-soliton interactions for a (3+1)-dimensional generalized Kadomtsev-Petviashvili equation in a fluid, Comput. Math. Appl. 78(2019):166-177.

[6] L.L. Feng, S.F. Tian, X.B. Wang, T.T. Zhang, Rogue waves, homoclinic breather waves and soliton waves for the (2+1)-dimensional B-type KadomtsevCPetviashvili equation, Appl. Math. Lett. 65(2017):90-97.

[7] Y.F. Guo, D.L. Li, J.X. Wang, The new exact solutions of the fifth-order

Sawada-Kotera equation using three wave method, Appl. Math. Lett. 94(2019):232-237.

[8] Y.L.Ma, B.Q.Li. Mixed lump and soliton solutions for a generalized (3+1)-dimensional Kadomtsev-Petviashvili equation[J], Aims Math. 2020, 5(2):1162-1176.

[9] B.Q.Li. Loop-like kink breather and its transition phenomena for the Vakhnenko equation arising from high-frequency wave propagation in electromagnetic physics[J], Appl. Math. Lett. 2020, 112.

[10] B.Q.Li, Y.L.Ma. Interaction dynamics of hybrid solitons and breathers for extended generalization of Vakhnenko equation[J], Nonlinear Dynam. 2020:1-13.

[11] B. He, Q. Meng, Bilinear form and new interaction solutions for the sixth-order Ramani equation, Appl. Math. Lett. 98(2019):411-418.

[12] M.K. Elboree, Lump solitons, rogue wave solutions and lump-stripe interaction phenomena to an extended (3+1)-dimensional KP equation, Chinese. J. Phys. 63(2020):290-303.

[13] E.H.M. Zahran, M.M.A. Khater, Modified extended tanh-function method and its applications to the Bogoyavlenskii equation, Appl. Math. Model. 40(2016):1769-1775.

[14] Y.Y. Wang, Y.P. Zhang, C.Q. Dai, Re-study on localized structures based on variable separation solutions from the modified tanh-function method, Nonlinear. Dynam. 2016.

[15] Y.S. Zhang, L.J. Guo, J.S. He, Z.X. Zhou. Darboux Transformation of the Second-Type Derivative Nonlinear Schrödinger Equation, Lett. Math. Phys. 105(2015):853-891.

[16] L.M. Ling, L.C. Zhao, B.L. Guo, Darboux transformation and multi-dark soliton for N-component nonlinear Schrödinger equations, Nonlinearity, 28(2015):3243-3261.

[17] Z.J. Xiao, B. Tian, H.L. Zhen, J. Chai, X.Y. Wu, Multi-soliton solutions and Bäcklund transformation for a two-mode KdV equation in a fluid, Wave. Random. Complex. (2016):1-14.

[18] X.H. Zhao, B. Tian, X.Y. Xie, X.Y. Wu, Y.J. Guo, Solitons, bäcklund transformation and lax pair for a (2+1)-dimensional davey-stewartson system on surface waves of finite depth, Wave. Random. Complex, 28(2017), 1-11.

[19] B.Q.Li, Y.L.Ma. Extended generalized Darboux transformation to hybrid rogue wave and breather solutions for a nonlinear Schördinger equation[J], Appl. Math. Comput. 2020, 386:125469.

[20] B.Q.Li, Y.L.Ma. N-order rogue waves and their novel colliding dynamics for a transient stimulated Raman scattering system arising from nonlinear optics[J], Nonlinear. Dynam. 2020, 101(4):1-13.

[21] J.S. He, E.G. Charalampidis, P.G. Kevrekidis, D.J. Frantzeskakis, Rogue waves in nonlinear Schrödinger models with variable coefficients: application to Bose-Einstein condensates, Phys. Lett. A 378(2014):577-583.

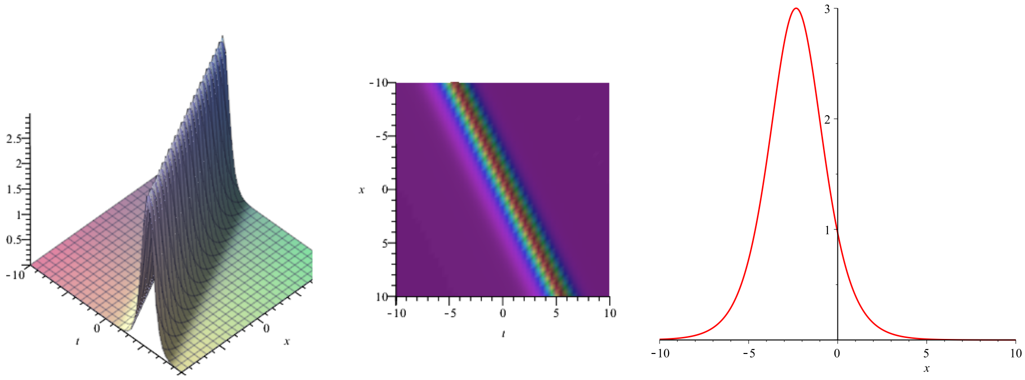
[22] X.P. Cheng, C.L. Chen, S.Y. Lou, Interactions among different types of nonlinear waves described by the Kadomtsev-Petviashvili equation, Wave Motion, 51(2014):1298-1308.

[23] H.L. Chen, Z.D. Dai, Z.H. Xu, Rogue wave for the (2+1)-dimensional Kadomtsev-Petviashvili equation, Appl. Math. Lett. 2014

[24] K.A.Gorshkov, D.E.Pelinovskii, Y.A.Stepanyants. Normal and anomalous scattering, formation and decay of bound states of two-dimensional solitons described by the Kadomtsev-Petviashvili equation[J], J Exp Theor Phys. 1993, 77:237-245.

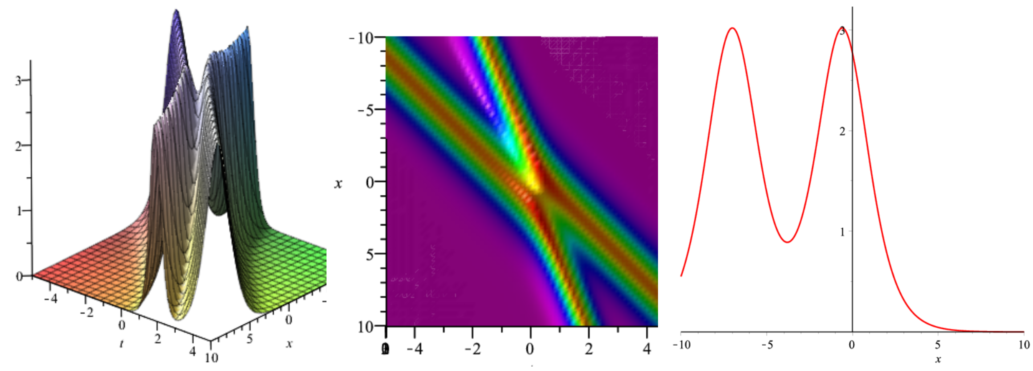
[25] A.M.Wazwaz. Multiple-soliton solutions for the KP equation by Hirota's bilinear method and by the tanh-coth method[J], Appl. Math. Comput. 2007, 190(1):633-640.

Figures of Solitary wave solution, breather wave solution and rogue wave solution for a KP-equation



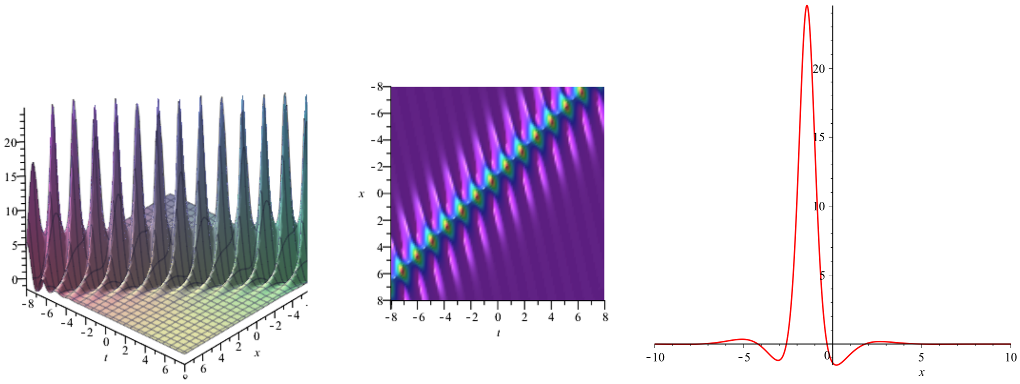
(a) (b) (c)

Figure 1: One-soliton solution (11) of Eq.(1) with parameters $k_1 = 1, \mu_1 = 1, \alpha = 1, b = 1, p = 1, y = 1$. (a) Perspective view of (11). (b) Overhead view of (11). (c) The mode of wave propagation along the x axis.



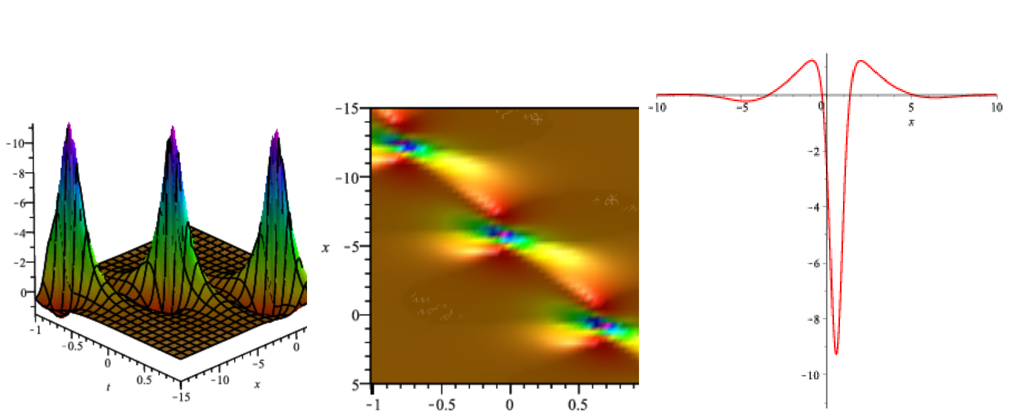
(a) (b) (c)

Figure 2: Two-soliton solution (14) of Eq.(1) with parameters $k_1 = 1, \mu_1 = 1, \alpha = 1, b = 1, p = 1, k_2 = -2, \mu_1 = 2, y = 1$. (a) Perspective view of (14). (b) Overhead view of (14). (c) The mode of wave propagation along the x axis.



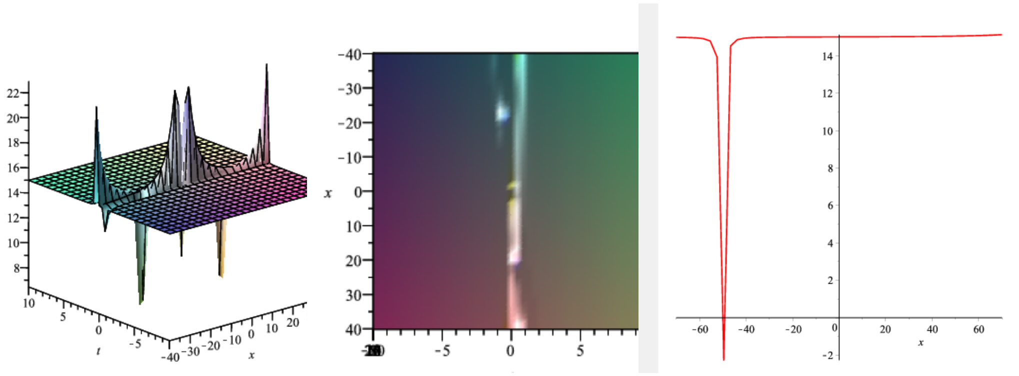
(a) (b) (c)

Figure 3: Breather wave solution (18) of Eq.(1) with parameters $p_1 = 1, p_2 = 1, b_1 = 1, b_2 = 2, p = 1, \alpha = -\frac{108}{7}, b = 1, \delta_1 = 1, u_0 = \frac{61}{7}, y = 1$. (a) Perspective view of (18). (b) Overhead view of (18). (c) The mode of wave propagation along the x axis.



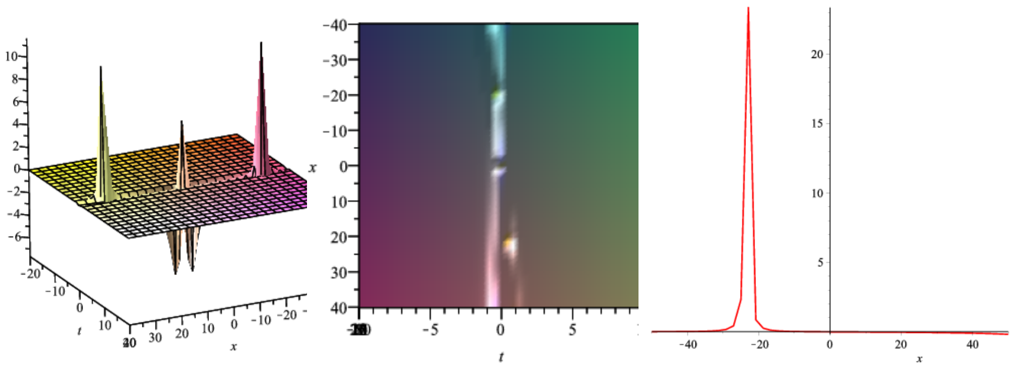
(a) (b) (c)

Figure 4: Breather wave solution (18) of Eq.(1) with parameters $p_1 = \frac{3}{5}, p_2 = \frac{4}{5}, b_1 = 3, b_2 = 1, p = -\frac{7}{25}, \alpha = \frac{3}{8}, b = 2, \delta_1 = 1, u_0 = -1, y = 2$. (a) Perspective view of (18). (b) Overhead view of (18). (c) The mode of wave propagation along the x axis.



(a) (b) (c)

Figure 5: Rogue wave solution (19) of Eq.(1) with parameters $s = -1, b_1 = 1, b_2 = 2, p = -1, \alpha = 20, b = 1, \delta_1 = 1, u_0 = 15, y = 0$. (a) Perspective view of (19). (b) Overhead view of (19). (c) The mode of wave propagation along the x axis.



(a) (b) (c)

Figure 6: Rogue wave solution (19) of Eq.(1) with parameters $s = 1, b_1 = 1, b_2 = 2, p = 1, \alpha = -20, b = 1, \delta_1 = 1, u_0 = 0, y = 0$. (a) Perspective view of (19). (b) Overhead view of (19). (c) The mode of wave propagation along the x axis.