Novel solitons solutions of two different nonlinear PDEs appear in engineering and physics

Naeem Awan[1], MUHAMMAD IMRAN ASJAD[2], and Hamood Rehman[3]

[1]University of management and technology lahore
[2]University of Management and Technology Lahore
[3]University of the Punjab

April 05, 2024

Abstract

In this piece of research, our aim is to investigate the novel solitons solutions of nonlinear (4+1)-dimensional Fokas equation (FE) and (2+1)-dimensional Breaking soliton equation (BSE) via new extended direct algebraic method. New acquired solutions are bright, singular, dark, periodic singular, combined-dark bright and combined-dark singular solitons solutions along with hyperbolic and trigonometric functions solutions. The achieved distinct types of solitons solutions contain key applications in engineering and physics. By taking the appropriate values of involved parameters, numerous novel structures are also plotted. These solutions define the wave performance of the governing models, actually.

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Novel solitons solutions of two different nonlinear PDEs appear in engineering and physics

*Naeem Ullah[2], M. A. Imran[2], Hamood Ur Rehman[1].

[1]Department of Mathematics, University of Okara, Okara, Pakistan.

[2]Department of Mathematics, University of Management and Technology, Lahore, Pakistan.

Corresponding Author: *naeemullahjamali@gmail.com

Abstract

In this piece of research, our aim is to investigate the novel solitons solutions of nonlinear (4+1)-dimensional Fokas equation (FE) and (2+1)-dimensional Breaking soliton equation (BSE) via new extended direct algebraic method. New acquired solutions are bright, singular, dark, periodic singular, combined-dark bright and combined-dark singular solitons solutions along with hyperbolic and trigonometric functions solutions. The achieved distinct types of solitons solutions contain key applications in engineering and physics. By taking the appropriate values of involved parameters, numerous novel structures are also plotted. These solutions define the wave performance of the governing models, actually.

Keywords: (4+1)-dimensional Fokas equation (FE); (2+1)-dimensional Breaking soliton equation (BSE); New extended direct algebraic method.

1 Introduction

A large number of existing phenomena in science and engineering are connected to nonlinear partial differential equations (NLPDEs). These are much significant in explaining everyday problems growing in science and nature such as waves, geology, population ecology, solid-state physics, wave propagation, fluid dynamics, biology, computer science and birefringent fibers, etc. Many mathematical approaches have been efficiently applied to study the valuable results of NLPDEs [1-31]. Using these approaches, various novel properties of wave behavior of these NLPDEs have been observed. The observed results have much impact in many scientific phenomena such optics, optical fibers, long distance high speed transmission lines and in optics as temporal or spatial optical solitons. In this paper, we have constructed novel dark, singular, bright, periodic and mixed solitons solutions to (4+1)-dimensional FE and (2+1)-dimensional BSE equations by applying new extended direct algebraic method (EDAM) [32]. The (4+1)-dimensional FE is given by [33]

$$4\Phi_{tx} - \Phi_{xxxy} + \Phi_{xyyy} + 12\Phi_x\Phi_y + 12\Phi\Phi_{xy} - 6\Phi_{zw} = 0. \tag{1}$$

Here, Φ is a function of x, y, z, w *and* t. This model has been discovered from the Kadomtsev-Petviashvili (KP) and Davey-Stewartson (DS) equations, which have been frequently used to indicate internal and shallow waves in the canals [34]. One another model, deliberated in this paper, (2+1)-dimensional Breaking soliton equation (BSE) stated as which is used to explain the collaboration of propagating Riemann waves, which was first introduced by Degasperis and Calogero in 1977. This model was explored by considering HDM and so on [35-37].

$$\Phi_{xxxy} - 2\Phi_y\Phi_{xx} - 4\Phi_x\Phi_{xy} + \Phi_{xt} = 0. \tag{2}$$

2 Description of the new EDAM

The steps of new EDAM [32] are, as follows

Step 1. Let the PDE

$$G(\Phi, \Phi_x, \Phi_t, \Phi_{tx}, \Phi_{xx}, \Phi_{xt}, \Phi_{tt}, ...) = 0, \tag{3}$$

where $\Phi = \Phi(x, t)$ is a function.
Put on the wave transformation

$$\Phi(x, t) = P(\xi), \quad \xi = \mu(x - ct), \tag{4}$$

(3) converted into ODE as follows

$$H(P, P', P'', ...) = 0. \tag{5}$$

Step 2. Consider (5) has a solution as follows

$$P(\xi) = \sum_{j=0}^{N} F_j Q^j(\xi), \;\; F_N \neq 0, \tag{6}$$

where F_j $(0 \leq j \leq N)$ are constants and $Q(\xi)$ admits the ODE, as follows

$$Q'(\xi) = Ln(A)(\delta + \Theta Q(\xi) + \varsigma Q^2(\xi)), \quad A \neq 1, \; 0. \tag{7}$$

(7) gives the solution as

Family 1:When $\Theta^2 - 4\delta\varsigma < 0$ *and* $\varsigma \neq 0$,

$$Q_1(\xi) = -\frac{\Theta}{2\varsigma} + \frac{\sqrt{-(\Theta^2 - 4\delta\varsigma)}}{2\varsigma} tan_A(\frac{\sqrt{-(\Theta^2 - 4\delta\varsigma)}}{2}\xi),$$

$$Q_2(\xi) = -\frac{\Theta}{2\varsigma} - \frac{\sqrt{-(\Theta^2 - 4\delta\varsigma)}}{2\varsigma} cot_A(\frac{\sqrt{-(\Theta^2 - 4\delta\varsigma)}}{2}\xi),$$

$$Q_3(\xi) = -\frac{\Theta}{2\varsigma} + \frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{2\varsigma}tan_A(\sqrt{-(\Theta^2-4\delta\varsigma)}\xi) \pm \frac{\sqrt{-pq(\Theta^2-4\delta\varsigma)}}{2\varsigma}sec_A(\sqrt{-(\Theta^2-4\delta\varsigma)}\xi),$$

$$Q_4(\xi) = -\frac{\Theta}{2\varsigma} - \frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{2\varsigma}cot_A(\sqrt{-(\Theta^2-4\delta\varsigma)}\xi) \pm \frac{\sqrt{-pq(\Theta^2-4\delta\varsigma)}}{2\varsigma}csc_A(\sqrt{-(\Theta^2-4\delta\varsigma)}\xi),$$

$$Q_5(\xi) = -\frac{\Theta}{2\varsigma} + \frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{4\varsigma}tan_A(\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{4}\xi) - \frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{4\varsigma}cot_A(\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{4}\xi).$$

Family 2:When $\Theta^2 - 4\delta\varsigma > 0$ and $\varsigma \neq 0$,

$$Q_6(\xi) = -\frac{\Theta}{2\varsigma} - \frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2\varsigma}tanh_A(\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2}\xi),$$

$$Q_7(\xi) = -\frac{\Theta}{2\varsigma} - \frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2\varsigma}coht_A(\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2}\xi),$$

$$Q_8(\xi) = -\frac{\Theta}{2\varsigma} - \frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2\varsigma}tanh_A(\sqrt{(\Theta^2-4\delta\varsigma)}\xi) \pm \iota\frac{\sqrt{pq(\Theta^2-4\delta\varsigma)}}{2\varsigma}sech_A(\sqrt{(\Theta^2-4\delta\varsigma)}\xi),$$

$$Q_9(\xi) = -\frac{\Theta}{2\varsigma} - \frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2\varsigma}coth_A(\sqrt{(\Theta^2-4\delta\varsigma)}\xi) \pm \frac{\sqrt{pq(\Theta^2-4\delta\varsigma)}}{2\varsigma}csch_A(\sqrt{(\Theta^2-4\delta\varsigma)}\xi),$$

$$Q_{10}(\xi) = -\frac{\Theta}{2\varsigma} - \frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{4\varsigma}tanh_A(\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{4}\xi) - \frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{4\varsigma}coth_A(\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{4}\xi).$$

Family 3:When $\delta\varsigma > 0$ and $\Theta = 0$,

$$Q_{11}(\xi) = \sqrt{\frac{\delta}{\varsigma}}tan_A(\sqrt{\delta\varsigma}\xi),$$

$$Q_{12}(\xi) = -\sqrt{\frac{\delta}{\varsigma}}cot_A(\sqrt{\delta\varsigma}\xi),$$

$$Q_{13}(\xi) = \sqrt{\frac{\delta}{\varsigma}}tan_A(2\sqrt{\delta\varsigma}\xi) \pm \sqrt{pq\frac{\delta}{\varsigma}}sec_A(2\sqrt{\delta\varsigma}\xi),$$

$$Q_{14}(\xi) = -\sqrt{\frac{\delta}{\varsigma}}cot_A(2\sqrt{\delta\varsigma}\xi) \pm \sqrt{pq\frac{\delta}{\varsigma}}csc_A(2\sqrt{\delta\varsigma}\xi),$$

$$Q_{15}(\xi) = \frac{1}{2}(\sqrt{\frac{\delta}{\varsigma}}tan_A(\frac{\sqrt{\delta\varsigma}}{2}\xi) - \sqrt{\frac{\delta}{\varsigma}}cot_A(\frac{\sqrt{\delta\varsigma}}{2})).$$

Family 4: When $\delta\varsigma < 0$ *and* $\Theta = 0$,

$$Q_{16}(\xi) = -\sqrt{-\frac{\delta}{\varsigma}}tanh_A(\sqrt{-\delta\varsigma}\xi),$$

$$Q_{17}(\xi) = -\sqrt{-\frac{\delta}{\varsigma}}coth_A(\sqrt{-\delta\varsigma}\xi),$$

$$Q_{18}(\xi) = -\sqrt{-\frac{\delta}{\varsigma}}tanh_A(2\sqrt{-\delta\varsigma}\xi) \pm \iota\sqrt{-pq\frac{\delta}{\varsigma}}sech_A(2\sqrt{-\delta\varsigma}\xi),$$

$$Q_{19}(\xi) = -\sqrt{-\frac{\delta}{\varsigma}}coth_A(2\sqrt{-\delta\varsigma}\xi) \pm \sqrt{-pq\frac{\delta}{\varsigma}}csch_A(2\sqrt{-\delta\varsigma}\xi),$$

$$Q_{20}(\xi) = -\frac{1}{2}(\sqrt{-\frac{\delta}{\varsigma}}tanh_A(\frac{\sqrt{-\delta\varsigma}}{2}\xi) + \sqrt{-\frac{\delta}{\varsigma}}coth_A(\frac{\sqrt{-\delta\varsigma}}{2}\xi)).$$

Family 5:When $\Theta = 0$ *and* $\varsigma = \delta$,

$$Q_{21}(\xi) = tan_A(\delta\xi),$$

$$Q_{22}(\xi) = -cot_A(\delta\xi),$$

$$Q_{23}(\xi) = tan_A(2\delta\xi) \pm \sqrt{pq}sec_A(2\delta\xi),$$

$$Q_{24}(\xi) = -cot_A(2\delta\xi) \pm \sqrt{pq}csc_A(2\delta\xi).$$

$$Q_{25}(\xi) = -\frac{1}{2}(tan_A(\frac{\delta}{2}\xi) + cot_A(\frac{\delta}{2}\xi)).$$

Family 6:When $\Theta = 0$ *and* $\varsigma = -\delta$,

$$Q_{26}(\xi) = -tanh_A(\delta\xi),$$

$$Q_{27}(\xi) = -coth_A(\delta\xi),$$

$$Q_{28}(\xi) = -tanh_A(2\delta\xi) \pm \iota\sqrt{pq}sech_A(2\delta\xi),$$

$$Q_{29}(\xi) = -coth_A(2\delta\xi) \pm \sqrt{pq}csch_A(2\delta\xi),$$

$$Q_{30}(\xi) = -\frac{1}{2}(tanh_A(\frac{\delta}{2}\xi) + coth_A(\frac{\delta}{2}\xi)).$$

Family 7:When $\Theta^2 = 4\delta\varsigma$,

$$Q_{31}(\xi) = \frac{-2\delta(\Theta\xi Ln(A) + 2)}{\Theta^2\xi Ln(A)}.$$

Family 8:When $\Theta = k,\ \delta = mk\ (m \neq 0)\ and\ \varsigma = 0$,

$$Q_{32}(\xi) = A^{k\xi} - m.$$

Family 9:When $\Theta = \varsigma = 0$,

$$Q_{33}(\xi) = \delta\xi Ln(A).$$

Family 10:When $\Theta = \delta = 0$,

$$Q_{34}(\xi) = \frac{-1}{\varsigma\xi Ln(A)}.$$

Family 11:When $\delta = 0\ and\ \Theta \neq 0$,

$$Q_{35}(\xi) = -\frac{p\Theta}{\varsigma(cosh_A(\Theta\xi) - sinh_A(\Theta\xi + p))},$$

$$Q_{36}(\xi) = -\frac{\Theta(sinh_A(\Theta\xi) + cosh_A(\Theta\xi))}{\varsigma(sinh_A(\Theta\xi) + cosh_A(\Theta\xi + q))}.$$

Family 12:When $\Theta = k,\ \varsigma = mk,\ (m \neq 0)\ and\ \delta = 0$,

$$Q_{37}(\xi) = \frac{pA^{k\xi}}{q - mpA^{k\xi}}.$$

Note: The generalized hyperbolic and triangular functions are given as [39]

$$sinh_A(\xi) = \frac{pA^{\xi} - qA^{-\xi}}{2}, \quad cosh_A(\xi) = \frac{pA^{\xi} + qA^{-\xi}}{2},$$

$$tanh_A(\xi) = \frac{pA^{\xi} - qA^{-\xi}}{pA^{\xi} + qA^{-\xi}}, \quad coth_A(\xi) = \frac{pA^{\xi} + qA^{-}\xi}{pA^{\xi} - qA^{-\xi}},$$

$$sech_A(\xi) = \frac{2}{pA^{\xi} + qA^{-\xi}}, \quad csch_A(\xi) = \frac{2}{pA^{\xi} - qA^{-\xi}},$$

$$sin_A(\xi) = \frac{pA^{\iota\xi} - qA^{-\iota\xi}}{2\iota}, \quad cos_A(\xi) = \frac{pA^{\iota\xi} + qA^{-\iota\xi}}{2},$$

$$tan_A(\xi) = -\iota\frac{pA^{\iota\xi} - qA^{-\iota\xi}}{pA^{\iota\xi} + qA^{-\iota\xi}}, \quad cot_A(\xi) = \iota\frac{pA^{\iota\xi} + qA^{-\iota\xi}}{pA^{\iota\xi} - qA^{-\iota\xi}},$$

$$sec_A(\xi) = \frac{2}{pA^{\iota\xi} + qA^{-\iota\xi}}, \quad csc_A(\xi) = \frac{2\iota}{pA^{\iota\xi} - qA^{-\iota\xi}}.$$

where $p, q > 0$.
Step 3: The integer $N > 0$ can be determined by balancing rule in (3). Inserting (4) in (3), gets algebraic equations having powers of $Q^j(\xi)$ $(j = 0, 1, 2, ...)$ and equating the coefficients of powers of $Q(\xi)$ to zero, provides system of equations.
Step 4: Solve system of equations and putting results in (4) to retrieve the exact solutions of (1).

3 Applications of the new EDAM

3.1 New EDAM for (4+1)-dimensional Fokas equation

Here, new EDAM is applied to discover some new solutions of (1).

$$\Phi(x, y, z, t, w) = P(\xi) \quad \xi = \alpha x + \gamma y + \chi z + \tau w + \epsilon t, \tag{8}$$

into Eq. (1) the following ODE is obtained

$$(4\alpha\epsilon - 6\chi\tau)P'' + (\alpha\gamma^3 - \alpha^3\gamma)P^{(4)} + 12\alpha\gamma(PP')' = 0. \tag{9}$$

Integration (9) and gets the following equation

$$(4\alpha\epsilon - 6\chi\tau)P + (\alpha\gamma^3 - \alpha^3\gamma)P'' + 6\alpha\gamma P^2 = 0. \tag{10}$$

Using balancing rule on (10), it gives $N = 2$, which converts (6) as

$$P(\xi) = F_0 + F_1 Q(\xi) + F_2(Q(\xi))^2, \tag{11}$$

where F_0 , F_1 and F_2 are constants. Substituting (11) into (10) and equating the coefficients of polynomials of $Q(\xi)$ to zero, we yield a set of equations in $F_0,\ F_1,\ F_2\ and\ \epsilon$. On solving the set of equations, we get

$$F_0 = \frac{1}{6}((\alpha^2 - \gamma^2)(2\theta\lambda + \mu^2)Ln(A)^2,$$

$$F_1 = (\alpha^2 - \gamma^2)\theta\mu Ln(A)^2,$$

$$F_2 = (\alpha^2 - \gamma^2)\theta^2 Ln(A)^2,$$

$$\epsilon = \frac{6\tau\chi + \alpha\gamma(\alpha^2 - \gamma^2)(4\theta\lambda - \mu^2)Ln(A)^2}{4\alpha}. \tag{12}$$

From Eqns. (4), (11) and (12), we build new solutions of the model as follows.

Family 1: When $\Theta^2 - 4\delta\varsigma < 0$ and $\varsigma \neq 0$, then

$$\Phi_1(x,y,z,w,t) = \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-\frac{\Theta}{2\varsigma}-\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{2\varsigma} tan_A(\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{2}\xi)\Big)+6\theta^2\Big(-\frac{\Theta}{2\varsigma}-\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{2\varsigma}tan_A(\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{2}\xi)\Big)^2\Bigg), \quad (13)$$

$$\Phi_2(x,y,z,w,t) = \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-\frac{\Theta}{2\varsigma}-\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{2\varsigma} cot_A(\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{2}\xi)\Big)+6\theta^2\Big(-\frac{\Theta}{2\varsigma}-\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{2\varsigma}cot_A(\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{2}\xi)\Big)^2\Bigg), \quad (14)$$

$$\begin{aligned}\Phi_3(x,y,z,w,t) &= \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-\frac{\Theta}{2\varsigma}+\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{2\varsigma}\\ &\times tan_A(\sqrt{-(\Theta^2-4\delta\varsigma)}\xi)\pm\frac{\sqrt{-pq(\Theta^2-4\delta\varsigma)}}{2\varsigma}sec_A(\sqrt{-(\Theta^2-4\delta\varsigma)}\xi)\Big)\\ &+6\theta^2\Big(-\frac{\Theta}{2\varsigma}+\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{2\varsigma}tan_A(\sqrt{-(\Theta^2-4\delta\varsigma)}\xi)\\ &\pm\frac{\sqrt{-pq(\Theta^2-4\delta\varsigma)}}{2\varsigma}sec_A(\sqrt{-(\Theta^2-4\delta\varsigma)}\xi)\Big)^2\Bigg),\end{aligned} \quad (15)$$

$$\begin{aligned}\Phi_4(x,y,z,w,t) &= \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-\frac{\Theta}{2\varsigma}-\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{2\varsigma}\\ &\times cot_A(\sqrt{-(\Theta^2-4\delta\varsigma)}\xi)\pm\frac{\sqrt{-pq(\Theta^2-4\delta\varsigma)}}{2\varsigma}csc_A(\sqrt{-(\Theta^2-4\delta\varsigma)}\xi)\Big)\\ &+6\theta^2\Big(-\frac{\Theta}{2\varsigma}-\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{2\varsigma}cot_A(\sqrt{-(\Theta^2-4\delta\varsigma)}\xi)\\ &\pm\frac{\sqrt{-pq(\Theta^2-4\delta\varsigma)}}{2\varsigma}csc_A(\sqrt{-(\Theta^2-4\delta\varsigma)}\xi)\Big)^2\Bigg),\end{aligned} \quad (16)$$

$$\begin{aligned}
\Phi_5(x,y,z,w,t) &= \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-\frac{\Theta}{2\varsigma}+\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{4\varsigma}\\
&\times tan_A\frac{\sqrt{-(\Theta^2-4\delta\varsigma}}{4}\xi)-\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{4\varsigma}cot_A\frac{(\sqrt{-(\Theta^2-4\delta\varsigma)}}{4}\xi)\Big)\\
&+6\theta^2\Big(-\frac{\Theta}{2\varsigma}+\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{4\varsigma}tan_A(\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{4}\xi)\\
&-\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{4\varsigma}cot_A(\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{4}\xi)\Big)^2\Bigg). \qquad (17)
\end{aligned}$$

Family 2: When $\Theta^2-4\delta\varsigma>0.$ and $\varsigma\neq 0$, then

$$\Phi_6(x,y,z,w,t)=\frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-\frac{\Theta}{2\varsigma}-\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2\varsigma}$$
$$tanh_A(\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2}\xi)\Big)+6\theta^2\Big(-\frac{\Theta}{2\varsigma}-\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2\varsigma}tanh_A(\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2}\xi)\Big)^2\Bigg), \quad (18)$$

$$\Phi_7(x,y,z,w,t)=\frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-\frac{\Theta}{2\varsigma}-\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2\varsigma}$$
$$coth_A(\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2}\xi)\Big)+6\theta^2\Big(-\frac{\Theta}{2\varsigma}-\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2\varsigma}coth_A(\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2}\xi)\Big)^2\Bigg), \quad (19)$$

$$\begin{aligned}
\Phi_8(x,y,z,w,t) &= \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-\frac{\Theta}{2\varsigma}-\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2\varsigma}\\
&\times tanh_A(\sqrt{(\Theta^2-4\delta\varsigma)}\xi)\pm\iota\frac{\sqrt{pq(\Theta^2-4\delta\varsigma)}}{2\varsigma}sech_A(\sqrt{(\Theta^2-4\delta\varsigma)}\xi)\Big)\\
&+6\theta^2\Big(-\frac{\Theta}{2\varsigma}-\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2\varsigma}tanh_A(\sqrt{-(\Theta^2-4\delta\varsigma)}\xi)\\
&\pm\iota\frac{\sqrt{pq(\Theta^2-4\delta\varsigma)}}{2\varsigma}sech_A(\sqrt{(\Theta^2-4\delta\varsigma)}\xi)\Big)^2\Bigg), \qquad (20)
\end{aligned}$$

$$\begin{aligned}
\Phi_9(x,y,z,w,t) &= \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-\frac{\Theta}{2\varsigma}-\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2\varsigma}\\
&\times coth_A(\sqrt{(\Theta^2-4\delta\varsigma)}\xi)\pm\frac{\sqrt{pq(\Theta^2-4\delta\varsigma)}}{2\varsigma}csch_A(\sqrt{(\Theta^2-4\delta\varsigma)}\xi)\Big)\\
&+6\theta^2\Big(-\frac{\Theta}{2\varsigma}-\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2\varsigma}coth_A(\sqrt{(\Theta^2-4\delta\varsigma)}\xi)\\
&\pm\frac{\sqrt{pq(\Theta^2-4\delta\varsigma)}}{2\varsigma}csch_A(\sqrt{(\Theta^2-4\delta\varsigma)}\xi)\Big)^2\Bigg), \qquad (21)
\end{aligned}$$

$$\begin{aligned}\Phi_{10}(x,y,z,w,t) &= \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-\frac{\Theta}{2\varsigma}-\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{4\varsigma}\\ &\times tanh_A(\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{4}\xi)-\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{4\varsigma}\coth_A(\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{4}\xi)\Big)\\ &+6\theta^2\Big(-\frac{\Theta}{2\varsigma}-\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{4\varsigma}tanh_A(\frac{\sqrt{(\Theta^2-4\delta\varsigma}}{4}\xi)\\ &-\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{4\varsigma}coth_A(\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{4}\xi)\Big)^2\Bigg).\end{aligned} \tag{22}$$

Family 3:When $\delta\varsigma > 0$ *and* $\Theta = 0$, then

$$\begin{aligned}\Phi_{11}(x,y,z,w,t) = \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(\sqrt{\frac{\delta}{\varsigma}}tan_A(\sqrt{\delta\varsigma}\xi)\Big)\\ +6\theta^2\Big(\sqrt{\frac{\delta}{\varsigma}}tan_A(\sqrt{\delta\varsigma}\xi)\Big)^2\Bigg),\end{aligned} \tag{23}$$

$$\begin{aligned}\Phi12(x,y,z,w,t) = \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-\sqrt{\frac{\delta}{\varsigma}}cot_A(\sqrt{\delta\varsigma}\xi)\Big)\\ +6\theta^2\Big(-\sqrt{\frac{\delta}{\varsigma}}cot_A(\sqrt{\delta\varsigma}\xi)\Big)^2\Bigg),\end{aligned} \tag{24}$$

$$\begin{aligned}\Phi_{13}(x,y,z,w,t) = \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(\sqrt{\frac{\delta}{\varsigma}}tan_A(2\sqrt{\delta\varsigma}\xi)\\ \pm\sqrt{pq\frac{\delta}{\varsigma}}sec_A(2\sqrt{\delta\varsigma}\xi)\Big)+6\theta^2\Big(\sqrt{\frac{\delta}{\varsigma}}tan_A(2\sqrt{\delta\varsigma}\xi)\pm\sqrt{pq\frac{\delta}{\varsigma}}sec_A(2\sqrt{\delta\varsigma}\xi)\Big)^2\Bigg),\end{aligned} \tag{25}$$

$$\begin{aligned}\Phi_{14}(x,y,z,w,t) = \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-\sqrt{\frac{\delta}{\varsigma}}cot_A(2\sqrt{\delta\varsigma}\xi)\\ \pm\sqrt{pq\frac{\delta}{\varsigma}}csc_A(2\sqrt{\delta\varsigma}\xi)\Big)+6\theta^2\Big(-\sqrt{\frac{\delta}{\varsigma}}cot_A(2\sqrt{\delta\varsigma}\xi)\pm\sqrt{pq\frac{\delta}{\varsigma}}csc_A(2\sqrt{\delta\varsigma}\xi)\Big)^2\Bigg),\end{aligned} \tag{26}$$

$$\begin{aligned}\Phi_{15}(x,y,z,w,t) = \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(\frac{1}{2}(\sqrt{\frac{\delta}{\varsigma}}\\ tan_A\frac{(\sqrt{\delta\varsigma}}{2}\xi-\sqrt{\frac{\delta}{\varsigma}}cot_A(\frac{\sqrt{\delta\varsigma}}{2})\xi)\Big)+6\theta^2\Big(\frac{1}{2}(\sqrt{\frac{\delta}{\varsigma}}tan_A(\frac{\sqrt{\delta\varsigma}}{2}\xi)-\sqrt{\frac{\delta}{\varsigma}}cot_A(\frac{\sqrt{\delta\varsigma}}{2})\xi)\Big)^2\Bigg).\end{aligned} \tag{27}$$

Family 4:When $\delta\varsigma < 0$ *and* $\Theta = 0$, then

$$\Phi_{16}(x,y,z,w,t) = \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-\sqrt{-\frac{\delta}{\varsigma}}tanh_A(\sqrt{-\delta\varsigma}\xi)\Big) +6\theta^2\Big(-\sqrt{-\frac{\delta}{\varsigma}}tanh_A(\sqrt{-\delta\varsigma}\xi)\Big)^2\Bigg), \quad (28)$$

$$\Phi_{17}(x,y,z,w,t) = \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-\sqrt{-\frac{\delta}{\varsigma}}coth_A(\sqrt{-\delta\varsigma}\xi)\Big) +6\theta^2\Big(-\sqrt{-\frac{\delta}{\varsigma}}coth_A(\sqrt{-\delta\varsigma}\xi)\Big)^2\Bigg), \quad (29)$$

$$\Phi_{18}(x,y,z,w,t) = \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-\sqrt{\frac{\delta}{\varsigma}}cot_A(2\sqrt{\delta\varsigma}\xi)\Big) \pm\iota\sqrt{-pq\frac{\delta}{\varsigma}}csc_A(2\sqrt{\delta\varsigma}\xi)) + 6\theta^2\Big(-\sqrt{\frac{\delta}{\varsigma}}cot_A(2\sqrt{\delta\varsigma}\xi)\pm\iota\sqrt{-pq\frac{\delta}{\varsigma}}csc_A(2\sqrt{\delta\varsigma}\xi)\Big)^2\Bigg), \quad (30)$$

$$\Phi_{19}(x,y,z,w,t) = \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-\sqrt{\frac{\delta}{\varsigma}}cot_A(2\sqrt{\delta\varsigma}\xi \pm\sqrt{-pq\frac{\delta}{\varsigma}}csc_A(2\sqrt{\delta\varsigma}\xi)\Big) + 6\theta^2\Big(-\sqrt{\frac{\delta}{\varsigma}}cot_A(2\sqrt{\delta\varsigma}\xi\pm\sqrt{-pq\frac{\delta}{\varsigma}}csc_A(2\sqrt{\delta\varsigma}\xi)\Big)^2\Bigg), \quad (31)$$

$$\Phi_{20}(x,y,z,w,t) = \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-\frac{1}{2}(\sqrt{-\frac{\delta}{\varsigma}}tanh_A(\frac{\sqrt{-\delta\varsigma}}{2}\xi) +\sqrt{-\frac{\delta}{\varsigma}}coth_A(\frac{\sqrt{-\delta\varsigma}}{2}\xi)\Big) + 6\theta^2\Big(-\frac{1}{2}(\sqrt{-\frac{\delta}{\varsigma}}tanh_A(\frac{\sqrt{-\delta\varsigma}}{2}\xi)+\sqrt{-\frac{\delta}{\varsigma}}coth_A(\frac{\sqrt{-\delta\varsigma}}{2}\xi)\Big)^2\Bigg). \quad (32)$$

Family 5:When $\Theta = 0$ *and* $\varsigma = \delta$, then

$$\Phi_{21}(x,y,z,w,t) = \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(tan_A(\delta\xi)\Big)+6\theta^2\Big(tan_A(\delta\xi)\Big)^2\Bigg), \quad (33)$$

$$\Phi_{22}(x,y,z,w,t) = \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-cot_A(\delta\xi)\Big)+6\theta^2\Big(-cot_A(\delta\xi)\Big)^2\Bigg), \quad (34)$$

$$\Phi_{23}(x,y,z,w,t)=\frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(tan_A(2\delta\xi)\pm\sqrt{pq}\ sec_A(2\delta\xi)\Big)$$
$$+6\theta^2\Big(tan_A(2\delta\xi)\pm\sqrt{pq}\ sec_A(2\delta\xi)\Big)^2\Bigg),(35)$$

$$\Phi_{24}(x,y,z,w,t)=\frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-cot_A(2\delta\xi)\pm\sqrt{pq}\ csc_A(2\delta\xi)\Big)$$
$$+6\theta^2\Big(-cot_A(2\delta\xi)\pm\sqrt{pq}\ csc_A(2\delta\xi)\Big)^2\Bigg),(36)$$

$$\Phi_{25}(x,y,z,w,t)=\frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(\frac{1}{2}(tan_A(\frac{\delta}{2}\xi)-cot_A(\frac{\delta}{2}\xi)\Big)$$
$$+6\theta^2\Big(\frac{1}{2}(tan_A(\frac{\delta}{2}\xi)-cot_A(\frac{\delta}{2}\xi)\Big)^2\Bigg).\ (37)$$

Family 6:When $\Theta=0$ *and* $\varsigma=-\delta$, then

$$\Phi_{26}(x,y,z,w,t))=\frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-tanh_A(\delta\xi)\Big)+6\theta^2\Big(-tanh_A(\delta\xi)\Big)^2\Bigg),(38)$$

$$\Phi_{27}(x,y,z,w,t)=\frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-coth_A(\delta\xi)\Big)+6\theta^2\Big(-coth_A(\delta\xi)\Big)^2\Bigg),(39)$$

$$\Phi_{28}(x,y,z,w,t)=\frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-tanh_A(2\delta\xi)\pm\iota\sqrt{pq}sech_A(2\delta\xi)\Big)$$
$$+6\theta^2\Big(-tanh_A(2\delta\xi)\pm\iota\sqrt{pq}sech_A(2\delta\xi)\Big)^2\Bigg),(40)$$

$$\Phi_{29}(x,y,z,w,t)=\frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-coth_A(2\delta\xi)\pm\sqrt{pq}csch_A(2\delta\xi)\Big)$$
$$+6\theta^2\Big(-coth_A(2\delta\xi)\pm\sqrt{pq}csch_A(2\delta\xi)\Big)^2\Big),(41)$$

$$\Phi_{30}(x,y,z,w,t)=\frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-\frac{1}{2}(tanh_A(\frac{\delta}{2}\xi)+coth_A(\frac{\delta}{2}\xi)\Big)$$
$$+6\theta^2\Big(-\frac{1}{2}(tanh_A(\frac{\delta}{2}\xi)+coth_A(\frac{\delta}{2}\xi)\Big)^2\Bigg).(42)$$

Family 7:When $\Theta^2 = 4\delta\varsigma$, then

$$\Phi_{31}(x,y,z,w,t) = \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(\frac{-2\delta(\Theta\xi Ln(A)+2)}{\Theta^2\xi Ln(A)}\Big) +6\theta^2\Big(\frac{-2\delta(\Theta\xi Ln(A)+2)}{\Theta^2\xi Ln(A)}\Big)^2\Bigg). \quad (43)$$

Family 8:When $\Theta = k,\ \delta = mk\ (m \neq= 0)\ and\ \varsigma = 0$, then

$$\Phi_{32}(x,y,z,w,t) = \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(A^{k\xi}-m\Big)+6\theta^2\Big(A^{k\xi}-m)\Big)^2\Bigg).(44)$$

Family 9:When $\Theta = \varsigma = 0$, then

$$\Phi_{33}(x,y,z,w,t) = \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(\delta\xi Ln(A)\Big)+6\theta^2\Big(\delta\xi Ln(A)\Big)^2\Bigg).(45)$$

Family 10:When $\Theta = \delta = 0$, then

$$\Phi_{34}(x,y,z,w,t) = \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(\frac{-1}{\varsigma\xi Ln(A)}\Big)+6\theta^2\Big(\frac{-1}{\varsigma\xi Ln(A)})\Big)^2\Bigg).(46)$$

Family 11::When $\delta = 0\ and\ \Theta \neq 0$, then

$$\Phi_{35}(x,y,z,w,t) = \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(-\frac{p\Theta}{\varsigma(cosh_A(\Theta\xi)-sinh_A(\Theta\xi+p))}\Big) +6\theta^2\Big(-\frac{p\Theta}{\varsigma(cosh_A(\Theta\xi)-sinh_A(\Theta\xi+p))}\Big)^2\Bigg),(47)$$

$$\Phi_{36}(x,y,z,w,t) = \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu \Big(-\frac{\Theta(sinh_A(\Theta\xi)+cosh_A(\Theta\xi))}{\varsigma(sinh_A(\Theta\xi)+cosh_A(\Theta\xi+q))}\Big)+6\theta^2\Big(-\frac{\Theta(sinh_A(\Theta\xi)+cosh_A(\Theta\xi))}{\varsigma(sinh_A(\Theta\xi)+cosh_A(\Theta\xi+q))})\Big)^2\Bigg).(48)$$

Family 12:When $\Theta = k,\ \varsigma = mk,\ (m \neq 0)\ and\ \delta = 0$, then

$$\Phi_{37}(x,y,z,w,t) = \frac{1}{6}(\alpha^2-\gamma^2)Ln(A)^2\Bigg(2\theta\lambda+\mu^2+6\theta\mu\Big(\frac{pA^{k\xi}}{q-mpA^{k\xi}}\Big)+6\theta^2\Big(\frac{pA^{k\xi}}{q-mpA^{k\xi}})\Big)^2\Bigg).(49)$$

where
$\xi = \alpha x + \gamma y + \chi z + \tau w + \epsilon t$

3.2 EDAM for (2+1)-dimensional Breaking soliton equation

Here, (2+1)-dimensional BSE is used to build new solutions of (2), using following TWT

$$\Phi(x, y, t) = P(\xi), \qquad \xi = kx + ly - ct, \tag{50}$$

(2) converts to ODE as follows

$$k^3 l P^{(4)} - 3k^2 l((P')^2)' - kcP'' = 0. \tag{51}$$

Integrating (51), we get

$$k^3 l P''' - 3k^2 l(P'^2) - kcP' = 0. \tag{52}$$

for simplicity, when we can consider

$$V = P', \;\; V' = P'', \;\; V'' = P''', \tag{53}$$

$$k^3 l V'' - 3k^2 l V^2 - kcV = 0. \tag{54}$$

Using balancing rule in (54), it yields $N = 2$, (6) converts to

$$P(\xi) = F_0 + F_1 Q(\xi) + F_2(Q(\xi))^2, \tag{55}$$

Where F_0, F_1 and F_2 are constants. Substituting (55) into (54) and equating coefficients of polynomials of $Q(\xi)$ to zero, get a set of equations in $F_0,\ F_1,\ F_2,\ and\ c.$
On solving the set of equations, we get

$$F_0 = 2k\theta\lambda Ln(A)^2,$$

$$F_1 = 2k\theta\mu Ln(A)^2,$$

$$F_2 = 2k\theta^2 Ln(A)^2,$$

$$c = -4k^2 l\theta\lambda Ln(A)^2 + kl\mu^2 Ln(A)^2. \tag{56}$$

Using Eqns. (50), (55) and (56) the new solutions of (2) are obtained as follows
Family 1: When $\Theta^2 - 4\delta\varsigma < 0$ and $\varsigma \neq 0$, then

$$\Phi_1(x, y, t) = 2k\theta Ln(A)^2 \Bigg(\lambda + \mu\Big(-\frac{\Theta}{2\varsigma} + \frac{\sqrt{-(\Theta^2 - 4\delta\varsigma)}}{2\varsigma} tan_A(\frac{\sqrt{-(\Theta^2 - 4\delta\varsigma)}}{2}\xi)\Big)$$
$$+\theta\Big(-\frac{\Theta}{2\varsigma} + \frac{\sqrt{-(\Theta^2 - 4\delta\varsigma)}}{2\varsigma} tan_A(\frac{\sqrt{-(\Theta^2 - 4\delta\varsigma)}}{2}\xi)\Big)^2 \Bigg), \tag{57}$$

$$\Phi_2(x,y,t)=2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(-\frac{\Theta}{2\varsigma}+\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{2\varsigma}cot_A(\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{2}\xi)\Big)$$
$$+\theta\Big(-\frac{\Theta}{2\varsigma}+\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{2\varsigma}cot_A(\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{2}\xi)\Big)^2\Bigg),\quad (58)$$

$$\Phi_3(x,y,t)=2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(-\frac{\Theta}{2\varsigma}+\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{2\varsigma}tan_A(\sqrt{-(\Theta^2-4\delta\varsigma)}\xi)$$
$$\pm\frac{\sqrt{-pq(\Theta^2-4\delta\varsigma)}}{2\varsigma}sec_A(\sqrt{-(\Theta^2-4\delta\varsigma)}\xi)\Big)+\theta\Big(-\frac{\Theta}{2\varsigma}+\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{2\varsigma}\times$$
$$tan_A(\sqrt{-(\Theta^2-4\delta\varsigma)}\xi)\pm\frac{\sqrt{-pq(\Theta^2-4\delta\varsigma)}}{2\varsigma}sec_A(\sqrt{-(\Theta^2-4\delta\varsigma)}\xi)\Big)^2\Bigg),\quad (59)$$

$$\Phi_4(x,y,t)=2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(-\frac{\Theta}{2\varsigma}+\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{2\varsigma}cot_A(\sqrt{-(\Theta^2-4\delta\varsigma)}\xi)$$
$$\pm\frac{\sqrt{-pq(\Theta^2-4\delta\varsigma)}}{2\varsigma}csc_A(\sqrt{-(\Theta^2-4\delta\varsigma)}\xi)\Big)+\theta\Big(-\frac{\Theta}{2\varsigma}+\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{2\varsigma}\times$$
$$cot_A(\sqrt{-(\Theta^2-4\delta\varsigma)}\xi)\pm\frac{\sqrt{-pq(\Theta^2-4\delta\varsigma)}}{2\varsigma}csc_A(\sqrt{-(\Theta^2-4\delta\varsigma)}\xi)\Big)^2\Bigg),\quad (60)$$

$$\Phi_5(x,y,t)=2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(-\frac{\Theta}{2\varsigma}+\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{4\varsigma}tan_A(\frac{\sqrt{-(\Theta^2-4\delta\varsigma}}{4}\xi)$$
$$-\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{4\varsigma}cot_A(\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{4}\xi)\Big)+\theta\Big(-\frac{\Theta}{2\varsigma}+\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{4\varsigma}\times$$
$$tan_A(\frac{\sqrt{-(\Theta^2-4\delta\varsigma}}{4}\xi)-\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{4\varsigma}cot_A(\frac{\sqrt{-(\Theta^2-4\delta\varsigma)}}{4}\xi)\Big)^2\Bigg).\quad (61)$$

Family 2: When $\Theta^2-4\delta\varsigma>0$. and $\varsigma\neq 0$, then

$$\Phi_6(x,y,t)=2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(\frac{\Theta}{2\varsigma}+\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2\varsigma}tanh_A(\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2}\xi)\Big)$$
$$+\theta\Big(-\frac{\Theta}{2\varsigma}+\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2\varsigma}tanh_A(\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2}\xi)\Big)^2\Bigg),\quad (62)$$

$$\Phi_7(x,y,t)=2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(-\frac{\Theta}{2\varsigma}+\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2\varsigma}coth_A(\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2}\xi)\Big)$$
$$+\theta\Big(-\frac{\Theta}{2\varsigma}+\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2\varsigma}coth_A(\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2}\xi)\Big)^2\Bigg),\quad (63)$$

$$\Phi_8(x,y,t)) = 2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(-\frac{\Theta}{2\varsigma}+\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2\varsigma}tanh_A(\sqrt{(\Theta^2-4\delta\varsigma)}\xi)$$
$$\pm\frac{\sqrt{pq(\Theta^2-4\delta\varsigma)}}{2\varsigma}sech_A(\sqrt{(\Theta^2-4\delta\varsigma)}\xi)\Big)+\theta\Big(-\frac{\Theta}{2\varsigma}+\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2\varsigma}\times$$
$$tanh_A(\sqrt{(\Theta^2-4\delta\varsigma)}\xi)\pm\frac{\sqrt{pq(\Theta^2-4\delta\varsigma)}}{2\varsigma}sech_A(\sqrt{(\Theta^2-4\delta\varsigma)}\xi)\Big)^2\Bigg), \quad (64)$$

$$\Phi_9(x,y,t) = 2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(-\frac{\Theta}{2\varsigma}+\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2\varsigma}coth_A(\sqrt{(\Theta^2-4\delta\varsigma)}\xi)$$
$$\pm\frac{\sqrt{pq(\Theta^2-4\delta\varsigma)}}{2\varsigma}csch_A(\sqrt{(\Theta^2-4\delta\varsigma)}\xi)\Big)+\theta\Big(-\frac{\Theta}{2\varsigma}+\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{2\varsigma}\times$$
$$coth_A(\sqrt{(\Theta^2-4\delta\varsigma)}\xi)\pm\frac{\sqrt{pq(\Theta^2-4\delta\varsigma)}}{2\varsigma}csch_A(\sqrt{(\Theta^2-4\delta\varsigma)}\xi)\Big)^2\Bigg), \quad (65)$$

$$\Phi_{10}(x,y,t) = 2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(-\frac{\Theta}{2\varsigma}+\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{4\varsigma}tanh_A(\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{4}\xi)$$
$$-\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{4\varsigma}coth_A(\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{4}\xi)\Big)+\theta\Big(\frac{\Theta}{2\varsigma}+\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{4\varsigma}\times$$
$$tanh_A(\frac{\sqrt{(\Theta^2-4\delta\varsigma}}{4}\xi)-\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{4\varsigma}coth_A(\frac{\sqrt{(\Theta^2-4\delta\varsigma)}}{4}\xi)\Big)^2\Bigg). \quad (66)$$

Family 3:When $\delta\varsigma>0$ *and* $\Theta=0$, then

$$\Phi_{11}(x,y,t) = 2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(\sqrt{\frac{\delta}{\varsigma}}tan_A(\sqrt{\delta\varsigma}\xi)\Big)+\theta\Big(\sqrt{\frac{\delta}{\varsigma}}tan_A(\sqrt{\delta\varsigma}\xi)\Big)^2\Bigg), \quad (67)$$

$$\Phi_{12}(x,y,t) = 2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(\sqrt{\frac{\delta}{\varsigma}}cot_A(\sqrt{\delta\varsigma}\xi)\Big)+\theta\Big(\sqrt{\frac{\delta}{\varsigma}}cot_A(\sqrt{\delta\varsigma}\xi)\Big)^2\Bigg), \quad (68)$$

$$\Phi_{13}(x,y,t) = 2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(\sqrt{\frac{\delta}{\varsigma}}tan_A(2\sqrt{\delta\varsigma}\xi)\pm\sqrt{pq\frac{\delta}{\varsigma}}sec_A(2\sqrt{\delta\varsigma}\xi)\Big)$$
$$+\theta\Big(\sqrt{\frac{\delta}{\varsigma}}tan_A(2\sqrt{\delta\varsigma}\xi)\pm\sqrt{pq\frac{\delta}{\varsigma}}sec_A(2\sqrt{\delta\varsigma}\xi)\Big)^2\Bigg), \quad (69)$$

$$\Phi_{14}(x,y,t) = 2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(\sqrt{\frac{\delta}{\varsigma}}cot_A(2\sqrt{\delta\varsigma}\xi)\pm\sqrt{pq\frac{\delta}{\varsigma}}csc_A(2\sqrt{\delta\varsigma}\xi)\Big)$$
$$+\theta\Big(\sqrt{\frac{\delta}{\varsigma}}cot_A(2\sqrt{\delta\varsigma}\xi)\pm\sqrt{pq\frac{\delta}{\varsigma}}csc_A(2\sqrt{\delta\varsigma}\xi)\Big)^2\Bigg), \quad (70)$$

$$\Phi_{15}(x,y,t)=2k\theta Ln(A)^2\Bigg(\lambda+\mu\Bigg(\frac{1}{2}(\sqrt{\frac{\delta}{\varsigma}}tan_A(\frac{\sqrt{\delta\varsigma}}{2}\xi)-\sqrt{\frac{\delta}{\varsigma}}cot_A(\frac{\sqrt{\delta\varsigma}}{2})\xi)\Bigg)$$
$$+\theta\Bigg(\frac{1}{2}(\sqrt{\frac{\delta}{\varsigma}}tan_A(\frac{\sqrt{\delta\varsigma}}{2}\xi)-\sqrt{\frac{\delta}{\varsigma}}cot_A(\frac{\sqrt{\delta\varsigma}}{2})\xi)\Bigg)^2\Bigg). \quad (71)$$

Family 4:When $\delta\varsigma<0$ *and* $\Theta=0$, then

$$\Phi_{16}(x,y,t)=2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(-\sqrt{-\frac{\delta}{\varsigma}}tanh_A(\sqrt{-\delta\varsigma}\xi)\Big)$$
$$+\theta\Big(-\sqrt{-\frac{\delta}{\varsigma}}tanh_A(\sqrt{-\delta\varsigma}\xi)\Big)^2\Bigg), \quad (72)$$

$$\Phi_{17}(x,y,t)=2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(-\sqrt{-\frac{\delta}{\varsigma}}coth_A(\sqrt{-\delta\varsigma}\xi)\Big)$$
$$+\theta\Big(-\sqrt{-\frac{\delta}{\varsigma}}coth_A(\sqrt{-\delta\varsigma}\xi)\Big)^2\Bigg), \quad (73)$$

$$\Phi_{18}(x,y,t)=2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(-\sqrt{\frac{\delta}{\varsigma}}cot_A(2\sqrt{\delta\varsigma}\xi)\pm\iota\sqrt{-pq\frac{\delta}{\varsigma}}\ csc_A(2\sqrt{\delta\varsigma}\xi)\Big)$$
$$+\theta\Big(-\sqrt{\frac{\delta}{\varsigma}}cot_A(2\sqrt{\delta\varsigma}\xi)\pm\iota\sqrt{-pq\frac{\delta}{\varsigma}}\ csc_A(2\sqrt{\delta\varsigma}\xi)\Big)^2\Bigg), \quad (74)$$

$$\Phi_{19}(x,y,t)=2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(-\sqrt{\frac{\delta}{\varsigma}}cot_A(2\sqrt{\delta\varsigma}\xi\pm\sqrt{-pq\frac{\delta}{\varsigma}}\ csc_A(2\sqrt{\delta\varsigma}\xi)\Big)$$
$$+\theta\Big(-\sqrt{\frac{\delta}{\varsigma}}cot_A(2\sqrt{\delta\varsigma}\xi\pm\sqrt{-pq\frac{\delta}{\varsigma}}\ csc_A(2\sqrt{\delta\varsigma}\xi)\Big)^2\Bigg), \quad (75)$$

$$\Phi_{20}(x,y,t)=2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(-\frac{1}{2}(\sqrt{-\frac{\delta}{\varsigma}}tanh_A(\frac{\sqrt{-\delta\varsigma}}{2}\xi)+\sqrt{-\frac{\delta}{\varsigma}}coth_A(\frac{\sqrt{-\delta\varsigma}}{2}\xi)\Big)$$
$$+\theta\Big(-\frac{1}{2}(\sqrt{-\frac{\delta}{\varsigma}}tanh_A(\frac{\sqrt{-\delta\varsigma}}{2}\xi)+\sqrt{-\frac{\delta}{\varsigma}}coth_A(\frac{\sqrt{-\delta\varsigma}}{2}\xi)\Big)^2\Bigg). \quad (76)$$

Family 5:When $\Theta=0$ *and* $\varsigma=\delta$, then

$$\Phi_{21}(x,y,t)=2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(tan_A(\delta\xi)\Big)+\theta\Big(tan_A(\delta\xi)\Big)^2\Bigg), \quad (77)$$

$$\Phi_{22}(x,y,t)=2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(-cot_A(\delta\xi)\Big)+\theta\Big(-cot_A(\delta\xi)\Big)^2\Bigg), \tag{78}$$

$$\begin{aligned}\Phi_{23}(x,y,t)=2k\theta Ln(A)^2\Bigg(&\lambda+\mu\Big(tan_A(2\delta\xi)\pm\sqrt{pq}sec_A(2\delta\xi)\Big)\\&+\theta\Big(tan_A(2\delta\xi)\pm\sqrt{pq}sec_A(2\delta\xi)\Big)^2\Bigg),\end{aligned} \tag{79}$$

$$\begin{aligned}\Phi_{24}(x,y,t)=2k\theta Ln(A)^2\Bigg(&\lambda+\mu\Big(-cot_A(2\delta\xi)\pm\sqrt{pq}csc_A(2\delta\xi)\Big)\\&+\theta\Big(-cot_A(2\delta\xi)\pm\sqrt{pq}csc_A(2\delta\xi)\Big)^2\Bigg),\end{aligned} \tag{80}$$

$$\begin{aligned}\Phi_{25}(x,y,t)=2k\theta Ln(A)^2\Bigg(&\lambda+\mu\Big(\frac{1}{2}(tan_A(\frac{\delta}{2}\xi)-cot_A(\frac{\delta}{2}\xi))\Big)\\&+\theta\Big(\frac{1}{2}(tan_A(\frac{\delta}{2}\xi)-cot_A(\frac{\delta}{2}\xi))\Big)^2\Bigg).\end{aligned} \tag{81}$$

Family 6:When $\Theta=0$ *and* $\varsigma=-\delta$, then

$$\Phi_{26}(x,y,t)=2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(-tanh_A(\delta\xi)\Big)+\theta\Big(-tanh_A(\delta\xi)\Big)^2\Bigg), \tag{82}$$

$$\Phi_{27}(x,y,t)=2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(-coth_A(\delta\xi)\Big)+\theta\Big(-coth_A(\delta\xi)\Big)^2\Bigg), \tag{83}$$

$$\begin{aligned}\Phi_{28}(x,y,t)=2k\theta Ln(A)^2\Bigg(&\lambda+\mu\Big(-tanh_A(2\delta\xi)\pm\iota\sqrt{pq}\ sech_A(2\delta\xi)\Big)\\&+\theta\Big(-tanh_A(2\delta\xi)\pm\iota\sqrt{pq}\ sech_A(2\delta\xi)\Big)^2\Bigg),\end{aligned} \tag{84}$$

$$\begin{aligned}\Phi_{29}(x,y,t)=2k\theta Ln(A)^2\Bigg(&\lambda+\mu\Big(-coth_A(2\delta\xi)\pm\sqrt{pq}\ csch_A(2\delta\xi)\Big)\\&+\theta\Big(-coth_A(2\delta\xi)\pm\sqrt{pq}\ csch_A(2\delta\xi)\Big)^2\Bigg),\end{aligned} \tag{85}$$

$$\Phi_{30}(x,y,t)=2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(-\frac{1}{2}(tanh_A(\frac{\delta}{2}\xi)+coth_A(\frac{\delta}{2}\xi)\Big)$$
$$+\theta\Big(-\frac{1}{2}(tanh_A(\frac{\delta}{2}\xi)+coth_A(\frac{\delta}{2}\xi)\Big)^2\Bigg). \tag{86}$$

Family 7:When $\Theta^2=4\delta\varsigma$, then

$$\Phi_{31}(x,y,t)=2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(\frac{-2\delta(\Theta\xi Ln(A)+2)}{\Theta^2\xi Ln(A)}\Big)+\theta\Big(\frac{-2\delta(\Theta\xi Ln(A)+2)}{\Theta^2\xi Ln(A)}\Big)^2\Bigg). \tag{87}$$

Family 8:When $\Theta=k,\ \delta=mk\ (m\neq=0)\ and\ \varsigma=0$, then

$$\Phi_{32}(x,y,t)=2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(A^{k\xi}-m\Big)+\theta\Big(A^{k\xi}-m\Big)^2\Bigg). \tag{88}$$

Family 9:When $\Theta=\varsigma=0$, then

$$\Phi_{33}(x,y,t)=2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(\delta\xi Ln(A)\Big)+\theta\Big(\delta\xi Ln(A)\Big)^2\Bigg). \tag{89}$$

Family 10:When $\Theta=\delta=0$, then

$$\Phi_{34}(x,y,t)=2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(\frac{-1}{\varsigma\xi Ln(A)}\Big)+\theta\Big(\frac{-1}{\varsigma\xi Ln(A)}\Big)^2\Bigg). \tag{90}$$

Family 11:When $\Theta=\delta=0$, then

$$\Phi_{35}(x,y,t)=2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(-\frac{p\Theta}{\varsigma(cosh_A(\Theta\xi)-sinh_A(\Theta\xi+p))}\Big)$$
$$+\theta\Big(-\frac{p\Theta}{\varsigma(cosh_A(\Theta\xi)-sinh_A(\Theta\xi+p))}\Big)^2\Bigg), \tag{91}$$

$$\Phi_{36}(x,y,t)=2k\theta Ln(A)^2\Bigg(\lambda+\mu\Big(-\frac{\Theta(sinh_A(\Theta\xi)+cosh_A(\Theta\xi))}{\varsigma(sinh_A(\Theta\xi)+cosh_A(\Theta\xi+q))}\Big)$$
$$+\theta\Big(-\frac{\Theta(sinh_A(\Theta\xi)+cosh_A(\Theta\xi))}{\varsigma(sinh_A(\Theta\xi)+cosh_A(\Theta\xi+q))}\Big)^2\Bigg). \tag{92}$$

Family 12:When $\Theta = k,\ \varsigma = mk,\ (m \neq 0)\ and\ \delta = 0$, then

$$\Phi_{37}(x,y,t) = 2k\theta Ln(A)^2\left(\lambda + \mu\Big(\frac{pA^{k\xi}}{q - mpA^{k\xi}}\Big) + \theta\Big(\frac{pA^{k\xi}}{q - mpA^{k\xi}}\Big)^2\right). \tag{93}$$

where
$\xi = kx + ly - ct.$

4 Results and Discussions

In this study, we effectively construct novel solitons solutions along with hyperbolic and trigonometric function solutions for the (4+1)-dimensional Fokas equation and (2+1)-dimensional Breaking soliton equation using new extended direct algebraic method. This technique is measured as most recent scheme in this field and that is not utilized to these equations earlier. For physical analysis, 3-D, 2-D and contour plots of some of these solutions are comprised with suitable parameters. The obtained solutions discover their application in communication to convey information because solitons have the capability to spread over long distances without reduction and without changing their forms. In this paper, we only included specific figures to avoid overloading the document. All the developed results are novel and distinct from that reported results. For graphical representation for (1), the physical behavior of (13) using the proper values of parameters $\alpha_2 = 2.3,\ \alpha = 1.3,\ \gamma_2 = 1.5,\ \Theta = 1.3,\ \lambda = 0.4,\ \mu = 0.7,\ w = 2,\ p = 0.98,\ q = 0.95,\ k = 2,\ A = 3,\ \delta = 2,\ \varsigma = 1.9,\ \tau = 0.85,\ c = 4,\ z = 3,\ \chi = 2.6,\ \epsilon = 2,\ \beta = 0.5\ \ y = 2.6$ and $t = 1$ are shown in Fig.1, the physical behavior of (18) using the appropriate values of parameters $\alpha_2 = 2.2,\ \alpha = 1.3,\ \gamma_2 = 1.6,\ \Theta = 2.3,\ \lambda = 3.4,\ \mu = 3.7,\ \ w = 2.2,\ p = 0.98,\ q = 0.95,\ k = 2,\ A = 2.4,\ \delta = -2,\ \varsigma = 1.9,\ \tau = 0.85,\ \ c = 2.9,\ z = 3,\ \chi = 1.5,\ \epsilon = 3,\ \beta = 1.6,\ y = 2.5$ and $t = 1$ are shown in Fig. 2, the physical behavior of (29) using the proper values of parameters $\alpha_2 = 2.3,\ \alpha = 1.1,\ \gamma_2 = 1.5,\ \Theta = 2.1,\ \lambda = 3.4,\ \mu = 1.7,\ \ w = 2,\ p = 0.98,\ q = 0.95,\ k = 2,\ A = 2.3,\ \delta = 2,\ \varsigma = 1.9,\ \tau = 0.85,\ \ c = 4,\ z = 3,\ \chi = 2.6,\ \epsilon = 3,\ \beta = 1.6,\ y = 2.6$ and $t = 1$ are shown in Fig. 3, the absolute behavior of (40) using the proper values of parameters $\alpha_2 = 2.2,\ \alpha = 1.4,\ \gamma_2 = 1.4,\ \theta = 2.3,\ \lambda = 3.3,\ \mu = 1.7,\ w = 2,\ p = 0.98,\ q = 0.95,\ k = 2,\ A = 2.6,\ b = 2,\ \delta = -2,\ \varsigma = 2,\ \tau = 0.82,\ c = 3.1,\ z = 3,\ \chi = 1.8,\ \epsilon = 1.9,\ \beta = 1.4,\ y = 2.7.$ and $t = 1$ are shown in Fig. 4. For graphical representation for (2), the absolute behavior of (58) using the suitable values of parameters $\theta = 2.3,\ \lambda = 3.3,\ \mu = 1.8,\ \Theta = 1.5,\ p = 0.98,\ q = 0.95,\ k = 2,\ A = 2.6,\ \delta = -2,\ \varsigma = 2,\ \ c = 2,\ y = 2.7,\ l = 1.5$ and $t = 1$ are shown in Fig. 5, the absolute behavior of (63) with the suitable values of parameters $\theta = 2.5,\ \lambda = 3.1,\ \mu = 1.5,\ \Theta = 1.4,\ p = 0.98,\ q = 0.95,\ k = 2.1,\ A = 2.8,\ \delta = -2,\ \varsigma = 2,\ c = 2.3,\ y = 1.7,\ l = 1.9$ and $t = 1$ are shown in Fig. 6, the absolute behavior of (71) using the proper values of parameters $\theta = 1.5,\ \lambda = 2.2,\ \mu = 2.9,\ \Theta = 1.7,\ p = 0.98,\ q = 0.95,\ k = 2.2,\ A = 2.9,\ \delta = 2,\ \varsigma = 2.1,\ \ c = 3.1,\ y = 2.7,\ l = 3.8$

and $t=1$ are shown in Fig. 7, the absolute behavior of (82) using the proper values of parameters $\theta = 2.4,\ \ \lambda = 2.2,\ \ \mu = 1.2,\ \ \Theta = 1.7,\ \ p = 0.98,\ \ q = 0.95,\ \ k = 2.4,\ \ A = 2.1,\ \ \delta = 2,\ \ \varsigma = 2,\ \ \ c = 3.3,\ \ y = 2.8,\ \ l = 1.1$ and $t = 1$ are shown in Fig. 8.

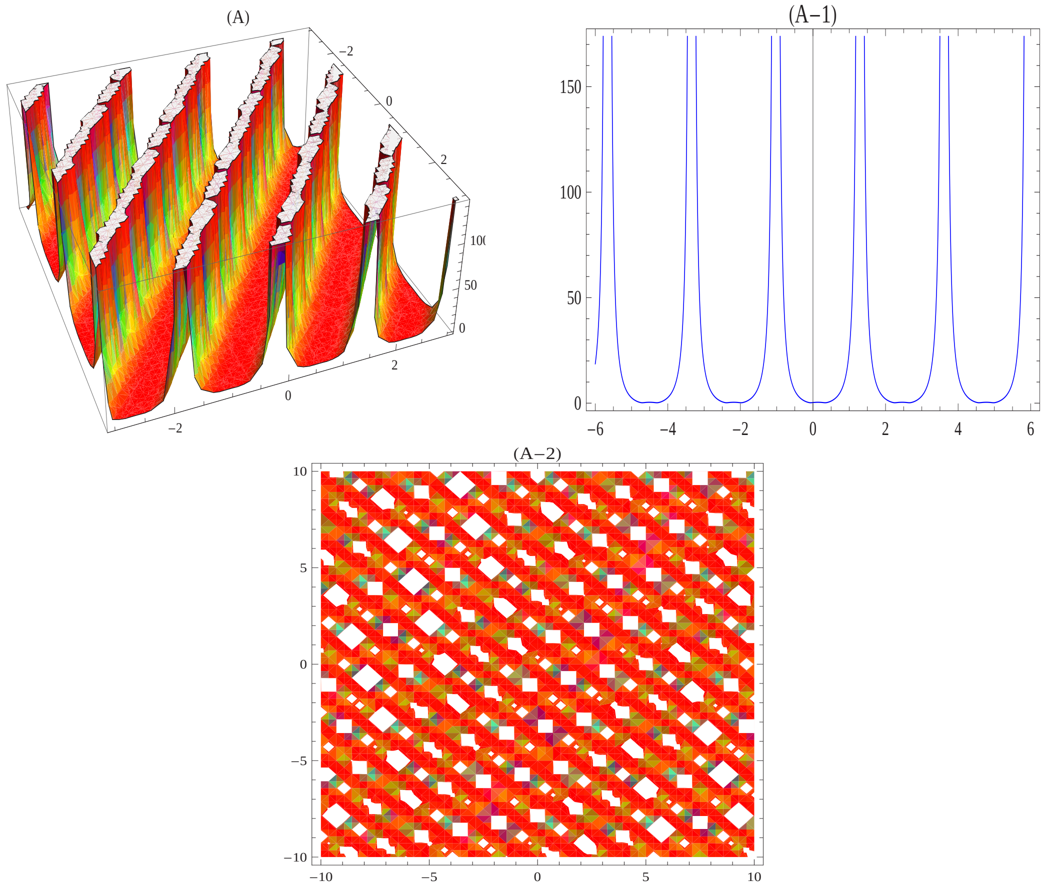

Figure 1: (A) 3D graph of (13) with $\alpha_2 = 2.3,\ \ \alpha = 1.3,\ \ \gamma_2 = 1.5,\ \ \Theta = 1.3,\ \ \lambda = 0.4,\ \ \mu = 0.7,\ \ \ w = 2,\ \ p = 0.98,\ \ q = 0.95,\ \ k = 2,\ \ A = 3,\ \ \delta = 2,\ \ \varsigma = 1.9,\ \ \tau = 0.85,\ \ \ c = 4,\ \ z = 3,\ \ \chi = 2.6,\ \ \epsilon = 2,\ \ \beta = 0.5\ \ \ y = 2.6$. (A-1) 2D plot of (13) with $t = 1$.(A-2) Contour graph of (13).

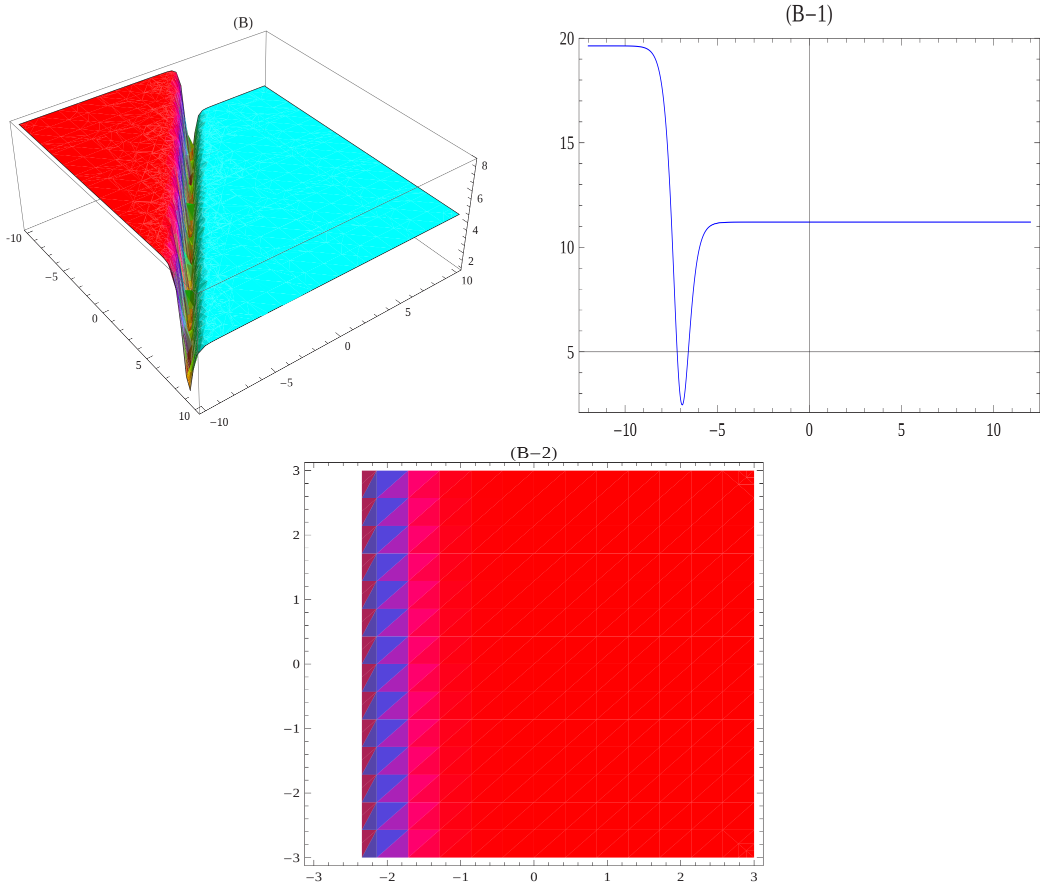

Figure 2: (B) 3D graph of (18) with $\alpha_2 = 2.2$, $\alpha = 1.3$, $\gamma_2 = 1.6$, $\Theta = 2.3$, $\lambda = 3.4$, $\mu = 3.7$, $w = 2.2$, $p = 0.98$, $q = 0.95$, $k = 2$, $A = 2.4$, $\delta = -2$, $\varsigma = 1.9$, $\tau = 0.85$, $c = 2.9$, $z = 3$, $\chi = 1.5$, $\epsilon = 3$, $\beta = 1.6$, $y = 2.5$. (B-1) 2D plot of (18) with $t = 1$.(B-2) Contour graph of (18).

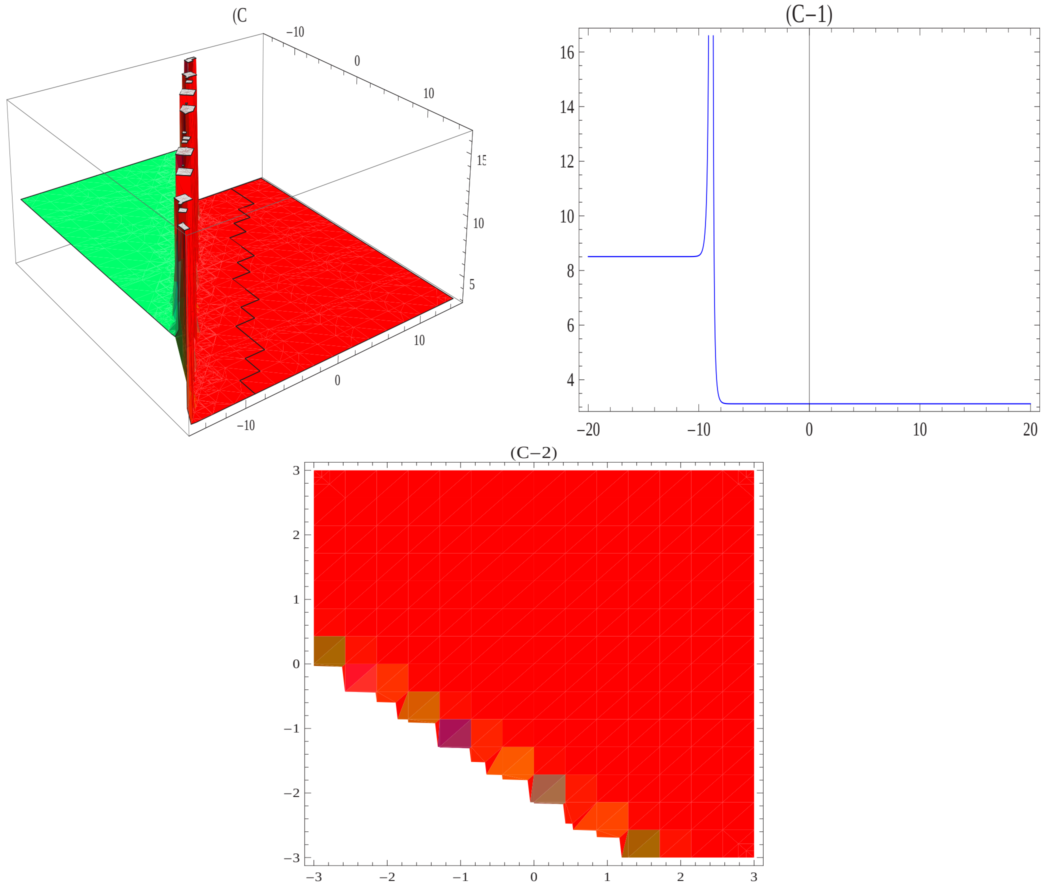

Figure 3: (C) 3D graph of (29) with $\alpha_2 = 2.3$, $\alpha = 1.1$, $\gamma_2 = 1.5$, $\Theta = 2.1$, $\lambda = 3.4$, $\mu = 1.7$, $w = 2$, $p = 0.98$, $q = 0.95$, $k = 2$, $A = 2.3$, $\delta = 2$, $\varsigma = 1.9$, $\tau = 0.85$, $c = 4$, $z = 3$, $\chi = 2.6$, $\epsilon = 3$, $\beta = 1.6$, $y = 2.6$. (C-1) 2D plot of (29) with $t = 1$.(C-2) Contour graph of (29).

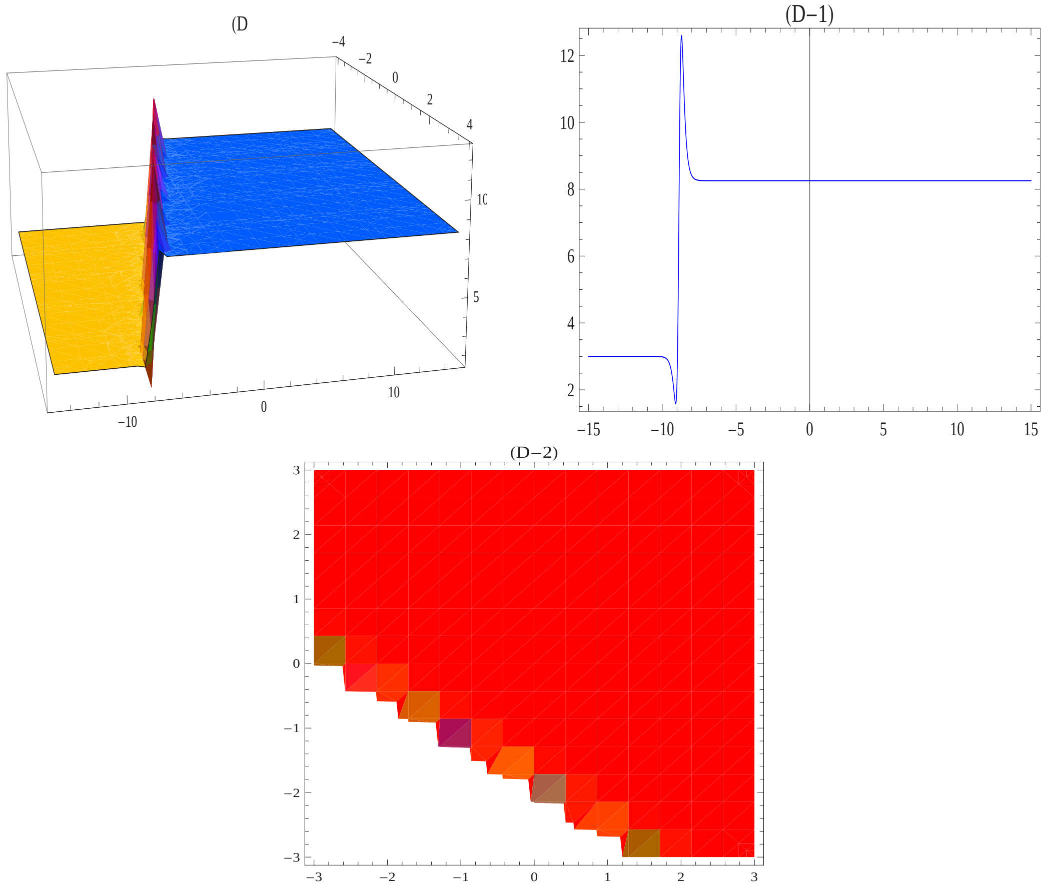

Figure 4: (D) 3D graph of (40) with $\alpha_2 = 2.2$, $\alpha = 1.4$, $\gamma_2 = 1.4$, $\theta = 2.3$, $\lambda = 3.3$, $\mu = 1.7$, $w = 2$, $p = 0.98$, $q = 0.95$, $k = 2$, $A = 2.6$, $b = 2$, $\delta = -2$, $\varsigma = 2$, $\tau = 0.82$, $c = 3.1$, $z = 3$, $\chi = 1.8$, $\epsilon = 1.9$, $\beta = 1.4$, $y = 2.7$. (D-1) 2D plot of (40) with $t = 1$.(D-2) Contour graph of (40).

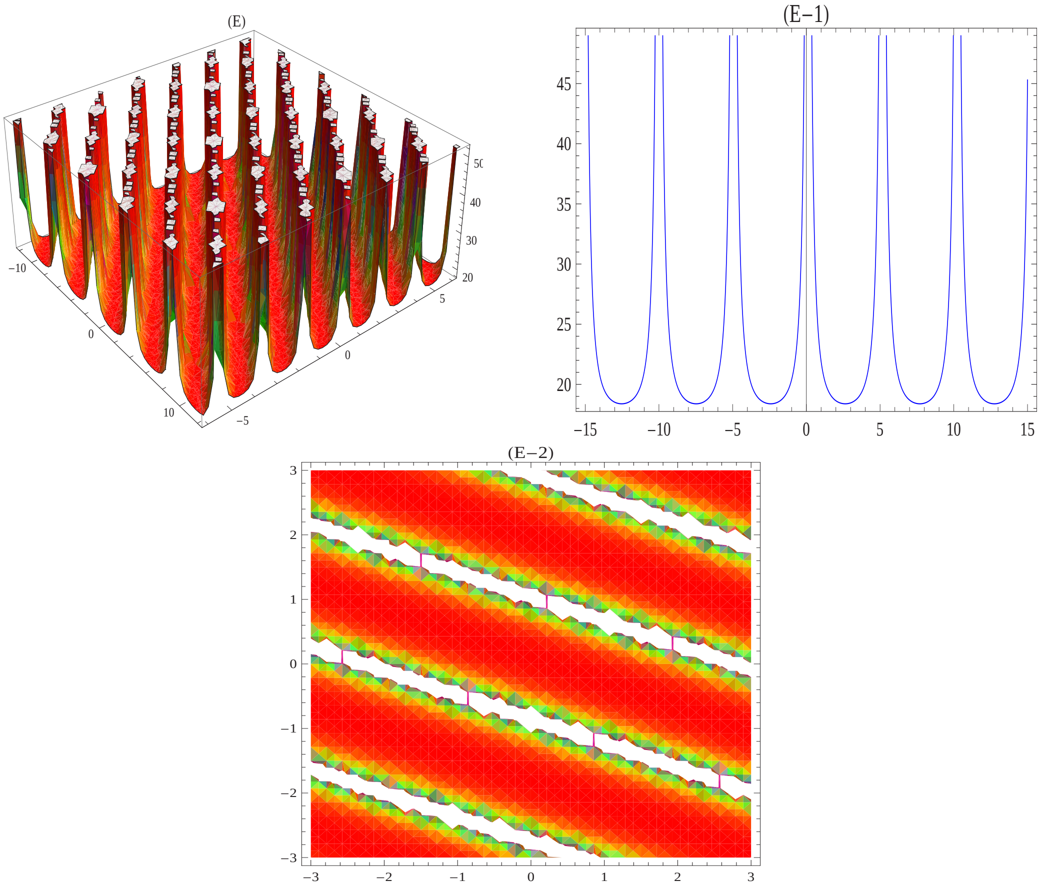

Figure 5: (E) 3D graph of (58) with $\theta = 2.3$, $\lambda = 3.3$, $\mu = 1.8$, $\Theta = 1.5$, $p = 0.98$, $q = 0.95$, $k = 2$, $A = 2.6$, $\delta = -2$, $\varsigma = 2$, $c = 2$, $y = 2.7$, $l = 1.5$. (E-1) 2D plot of (58) with $t = 1$.(E-2) Contour graph of (58).

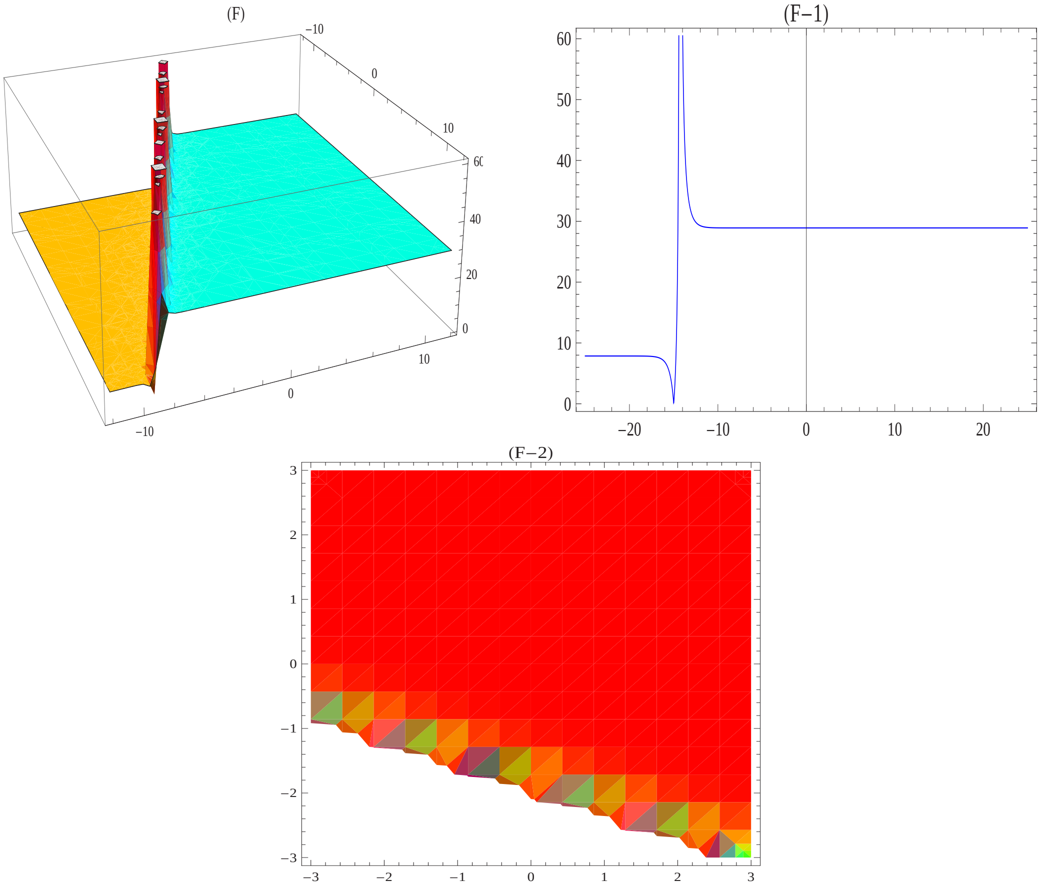

Figure 6: (F) 3D graph of (63) with $\theta = 2.5$, $\lambda = 3.1$, $\mu = 1.5$, $\Theta = 1.4$, $p = 0.98$, $q = 0.95$, $k = 2.1$, $A = 2.8$, $\delta = -2$, $\varsigma = 2$, $c = 2.3$, $y = 1.7$, $l = 1.9$. (F-1) 2D plot of (63) with $t = 1$.(F-2) Contour graph of (63).

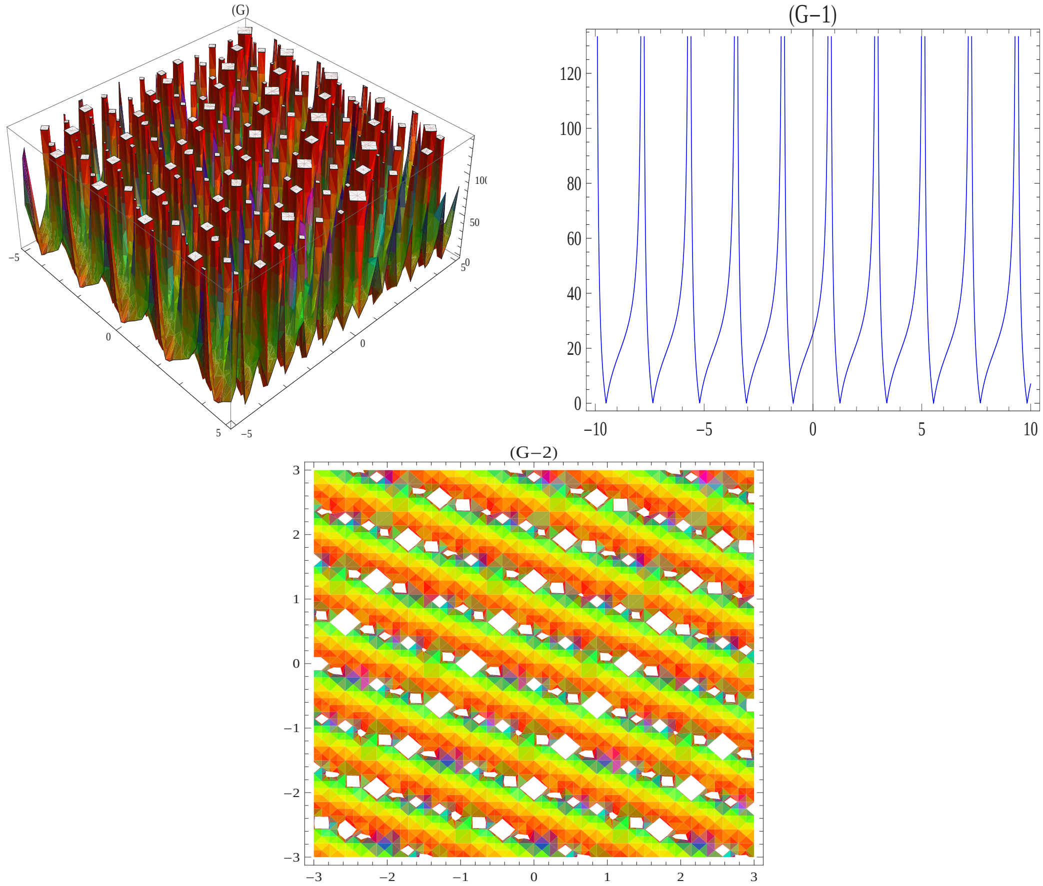

Figure 7: (G) 3D graph of (71) with $\theta = 1.5,\ \ \lambda = 2.2,\ \ \mu = 2.9,\ \ \Theta = 1.7,\ \ p = 0.98,\ \ q = 0.95,\ \ k = 2.2,\ \ A = 2.9,\ \ \delta = 2,\ \ \varsigma = 2.1,\ \ \ c = 3.1,\ \ y = 2.7,\ \ l = 3.8$. (G-1) 2D plot of (71) with $t = 1$.(G-2) Contour graph of (71).

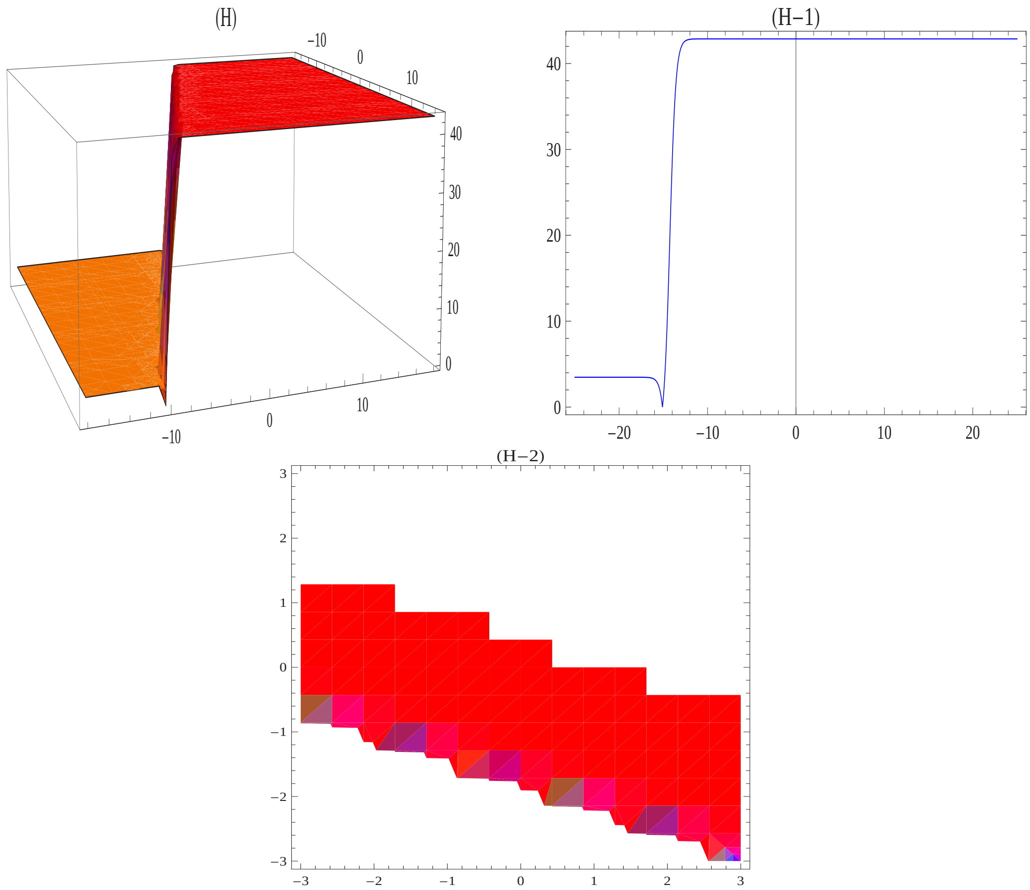

Figure 8: (H) 3D graph of (82) with $\theta = 2.4, \quad \lambda = 2.2, \quad \mu = 1.2, \quad \Theta = 1.7, \quad p = 0.98, \quad q = 0.95, \quad k = 2.4, \quad A = 2.1, \quad \delta = 2, \quad \varsigma = 2, \quad c = 3.3, \quad y = 2.8, \quad l = 1.1.$ (H-1) 2D plot of (82) with $t = 1$.(H-2) Contour graph of (82).

5 Conclusion

In this paper, we constructed new solitons solutions for two different models via the new EDAM, in the form of bright, dark, mixed bright-dark solitons as well as hyperbolic and trigonometric functions solutions. By choosing the suitable values of parameters and to better understand the physical structures of the solutions, 3-d and 2-d graphs have been plotted. From the acquired results and figures, it is observed that all solutions demonstrated wave behavior. Also, these solutions yield traveling dark wave behaviors to the considered models, physically.

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