Study on Controllability Results for Semilinear Integrodifferential Evolution System with Non-local Delayed Impulses

RADHAKRISHNAN BHEEMAN[1] and Sathya Thangavel[1]

[1]PSG College of Technology

April 16, 2024

Abstract

The foremost aim of this paper is to reveal the extent of the controllability of the semilinear evolution integrodifferential impulse system with delayed impulses and non local conditions. This paper begins with the grit of the control formula for the same impulse system in Banach space. Moreover, As a result, is extended to controllability. The sufficient conditions are introduced by utilizing the Hausdorff measure of noncompactness, Sadovskii fixed point theorem, and operator semigroups in appropriate dropping compactness of the operator. Sequentially, an example is provided to show our results.

Study on Controllability Results for Semilinear Integrodifferential Evolution System with Non-local Delayed Impulses

B.Radhakrishnan* and T.Sathya [b]

Abstract

The foremost aim of this paper is to reveal the extent of the controllability of the semilinear evolution integrodifferential impulse system with delayed impulses and non local conditions. This paper begins with the grit of the control formula for the same impulse system in Banach space. Moreover, As a result, is extended to controllability. The sufficient conditions are introduced by utilizing the Hausdorff measure of noncompactness, Sadovskii fixed point theorem, and operator semigroups in appropriate dropping compactness of the operator. Sequentially, an example is provided to show our results.

Keywords: Controllability, Impulse control system, Evolution operator, Semigroup theory, Fixed point theorem.

2010 AMS Subject Classification: 93B05, 34A37.

1 Introduction

Recent days, functional differential equations or evolution equations work as an abstract formulations of various partial differential equations which exist in problems related with heat-flow in materials with memory, viscoelasticity, and many other physical phenomena. The theory of differential equations in abstract spaces is a captivating field with significant applications to several areas of analysis and other branches of mathematics. Depending on the nature of the problems, these equations may adopt many forms such as ordinary differential equations, functional differential equations, partial differential equations, and sometimes a mixture of combining systems of ordinary and partial differential equations.

Further control theory is a branch of application-oriented mathematics that contracts with the fundamental sources carrying the analysis and study of control systems. To control an object indicates the importance of its performance so as to achieve the desired goal. In order to complete this rule, practitioners make tools and their communication with the object being controlled is the subject of control theory. Many papers have performed on

*[b] Department of Mathematics, PSG College of Technology, Coimbatore 641004, TamilNadu, India.
*Corresponding author e-mail: brk.maths@psgtech.ac.in

the problem of controllability of semilinear integrodifferential evolution systems in Banach spaces [1–4, 11, 12]. Xue [5] examined the nonlinear differential equation inseparable and uniformly smooth Banach spaces with nonlocal initial conditions and find the solution via the Hausdorff measure of non compactness. The controllability and local controllability of neutral functional differential systems with unbounded delay by using the theory of evolution families and Sadovskii fixed point theorem proved by Fu [6]. By using local Lipschitz continuity of a nonlinear function Surendra kumar at el [10] proved the exact controllability of semilinear systems with a single constant point delay in control. By using semigroup theory and functional analysis methods Sakthivel at el. [22] proved sufficient conditions for approximate controllability of impulsive differential equations with state dependent delay.

The investigation of impulsive systems has grown extra prominent in modern times as various evolutionary methods that happen in physics, chemistry, biology, population dynamics, engineering, information science, etc. [7, 9] are described by the point that, at some moments of times, the state function experiences a sudden change, that is, in the form of impulses. There has been a notable improvement in the impulsive theory in the past three decades. For a full study on impulsive differential equations refer Lakshmikantham et al. [20]. Consequently impulsive control in dynamical systems has gained significant recognition and it has been extensively used in several areas such as dosage supply in economics, mechanics, electronics, medicine and biology, pharmacokinetics, orbital transfer of satellite, ecosystems management, synchronization in chaotic secure communication systems. In numerous cases, a real system may meet some sudden changes at some time moments and cannot be counted continuously. This unexpected change is called the impulsive event, and it has been largely examined based on impulsive differential equations in the earlier days.

Really, the study of control with impulse systems can be followed back to the beginning of modern control theory. Numerous control with impulse systems were strongly promoted under the framework of optimal control and were occasionally called impulsive control. Its necessity and importance lie in that, the main idea of impulsive control is to change the states immediately at some instants. Therefore, impulsive control can reduce control costs and the amount of transmitted information drastically. In addition, in many cases, impulsive control can give an efficient way to deal with systems that cannot endure continuous control inputs. For various control systems in real life, impulses and delays are essential features that do not change their controllability. So that under specific conditions the unexpected changes and delays as disturbances of a system do not modify certain properties such as controllability. In additional words, controllability is sound by looking at the impulses and delays as perturbations.

Further, the Controllability for semilinear impulsive control systems with multiple time delays in control has been studied by Vijayakumar at el [13]. Yuming and Zou [14, 16] discussed the stability of an impulsive control system with impulse time windows and impulsive control of nonlinear systems with impulse time window. Controllability of impulsive system was well studied by George [17]. Controllability of switched time delay systems was studied by Wang [18].Further the detailed study of impulsive control theory refer in [19]. The different faces of control with impulse systems like Lyapunov stability, input-to-state stability, finite-time control, and state-dependent impulses were investigated by Yang at el. [15].

From these points, our main contributions are highlighted as follows:

- Most of the available literature, for the first time semilinear evolution integrodifferential impulse system with non local delayed impulses in abstract spaces, has been reviewed.
- A new set of sufficient conditions are implemented for finding the controllability result of the semilinear system with delayed impulse and non local.
- Sadovskii's fixed point theorem is effectively used to prove the controllability result.

Best of our knowledge, an investigation concerning the semilinear evolution of integrodifferential impulsive systems with delayed impulses and nonlocal Condition in abstract spaces has not been established yet. Thus, it will make an effort to analyze such results in this paper. In this spirit, inspired by the efforts contributed up to here, the necessity and importance of the realization of a work that will contribute to the controllability of non local semilinear evolution integrodifferential system with delayed impulse has been studied. The present work is created as, the basic definitions are discussed in second section. In section 3 control formula for the system (2.1) is studied. Fourth section deals the controllability result. Finally, an example is presented to illustrate our obtained result in fifth section.

2 System Description and Basic concepts

Let $(E, \|\cdot\|)$ be a real Banach space and $\mathbb{C}([0,\infty), E)$ be the space of continuous functions with the norm $\|x\|_\infty = \sup\{\|x(t)\| : t \in [0,\infty)\}$. Consider

$$\left.\begin{aligned} \frac{dv}{dt} &= \mathcal{G}(t)x(t) + Bu(t) + \mathcal{F}\Big(t, v(t), \int_0^t C(t, v(s))ds\Big), \quad t \neq t_n, \\ v(t_0^+) &= v_0 + a(x), \quad t_0 \geq 0, \\ \Delta v(t) &= \mathcal{I}_n(t, v(t-\tau)) \quad t = t_n, \end{aligned}\right\} \tag{2.1}$$

where the state variable $v(\cdot)$ takes values in the Banach space E with norm $\|\cdot\|$ and the control function $u(\cdot)$ is given in $\mathcal{L}^2(\mathcal{J},\mathcal{U})$, a Banach space of admissible control functions with $\mathcal{U}$ as a Banach space and $\mathcal{J} = \{(t,s) : 0 \leq s \leq t \leq b\}$. B is a bounded linear operator from $\mathcal{U}$ into E and $\mathcal{G}(t)$ is the closed bounded linear operator defined on the common domain $D(E)$ which is dense in E. The nonlinear operators $\mathcal{F} : \mathcal{J} \times E \times E \to E$ and $C : \mathcal{J} \times E \to E$ are continuous. The non local function $a : \mathbb{PC}([0,b],E) \to E$ is a given function. $\mathcal{I}_n : \mathcal{J} \times E \to E$, $\Delta v(t) = v(t^+) - v(t^-)$, where $v(t^+) = \lim_{h\to 0} v(t+h), v(t^-) = \lim_{h\to 0} v(t-h)$. The impulse times $[t_k, k \in \mathbb{Z}_+]$ satisfy $0 \leq t_0 < t_1 < ... < t_n \to +\infty$ as $n \to +\infty$.

Consider the linear non-autonomous system

$$z' = A(t)z(t)$$
$$z(s) = x \in E$$

has associated evolution family of operators $U(t,s) : 0 \leq s \leq t \leq b$. In the following definition, $L(E)$ is a space of bounded linear operators from into E endowed with the uniform convergence topology.

Definition 2.1 *[8] A two parameter family of bounded linear operator $U(t,s)$, $0 \leq s \leq t \leq b$ on E is called a evolution operator of* (2.1) *if the following conditions are satisfied,*

(a) $U(t,t)x = x$, for every $t \in [0,b]$ and $U(t,s)U(t,\tau) = U(t,\tau)$, for every $s \leq \tau \leq t$ and all $x \in E$.

(b) For each $x \in E$, the functions for $(t,s) \to U(t,s)x$ is continuous and $U(t,s) \in \mathcal{L}(E)$ for every $t \geq s$ and

(c) For $0 \leq s \leq t \leq b$, the function $t \to U(t,s)x$ is continuous, for $(s,t] \in \mathcal{L}(E)$ is differentiable with

$$\frac{\partial U(t,s)}{\partial t} = A(t)U(t,s).$$

Definition 2.2 *A solution $v(\cdot) \in \mathbb{PC}([0,b],E)$ is said to be a mild solution of the system* (2.1) *if the following integral equation*

$$\begin{aligned} v(t) \;=\; & U(t,t_0)\prod_{j=0}^{m-1}(\mathcal{I} + c_{m-j}E_{m-j}^{(\tau)})[v_0 + a(x)] + \prod_{j=0}^{m-1}(\mathcal{I} + c_{m-j}E_{m-j}^{(\tau)})\int_{t_0}^{t_1-\tau} U(t,s)Bu(s)ds \\ & + \prod_{j=0}^{m-1}(\mathcal{I} + c_{m-j}E_{m-j}^{(\tau)})\int_{t_0}^{t_1-\tau} U(t,s)\mathcal{F}\Big(t, v(t), \int_0^s C(r,v(r))dr\Big)ds \\ & + \sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I} + c_{m-j}E_{m-j}^{(\tau)})\int_{t_i-\tau}^{t_{i+1}-\tau} U(t,s)Bu(s)ds + \int_{t_m-\tau}^{t} U(t,s)Bu(s)ds \end{aligned}$$

$$+\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})\int_{t_i-\tau}^{t_{i+1}-\tau}U(t,s)\mathfrak{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)ds$$
$$+\int_{t_m-\tau}^{t}U(t,s)\mathfrak{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)ds$$

is satisfied.

To prove the controllability result via the fixed point of a condensing operator to recall Kuratowskii's measure of non-compactness which will be used in the following section. The measure for a bounded set D with norm $\|\cdot\|$ of a Banach space E or Kuratowskii's measure of non compactness is defined as

$$\alpha(\mathbb{D})=\inf\{d>0|\ \ \mathbb{D}\}$$

can be covered with a finite number of sets of diameter small than d. Throughout this work consider the impulse times $[t_k, k\in\mathbb{Z}_+]$ on the interval satisfy $t_0<t_1<...<t_m<t_b<t_{m+1}$, $m>0$ is an integer.

The following operators are needed to prove the controllability result

(H1) The linear operators $W_{m-1}^{\tau}u, W_m^{\tau}u, W_0^{\tau}u:\mathcal{L}^2(J,U)\to X$ is defined by

$$\begin{aligned}W_0^{(\tau)}u &= \int_{t_0}^{t_1-\tau}U(b,s)Bu(s)ds.\\ W_{m-1}^{(\tau)}u &= \int_{t_m-\tau}^{t_{m+1}-\tau}U(b,s)Bu(s)ds,\\ W_m^{(\tau)}u &= \int_{t_m-\tau}^{b}U(b,s)Bu(s)ds.\end{aligned}$$

has an inverse operator $(W_e^{(\tau)})^{-1}, e=0,m-1,m$. Which takes values in $\mathcal{L}^2(\mathcal{J},U)/kerW$ and there exists a positive constant.

3 Control formula

Theorem 3.1 *For $x_b\in X$, define the control*

$$\left.\begin{aligned}u(t) \;=\; & (W_e^{(\tau)})^{-1}\Bigg\{v_b-U(b,t_0)\prod_{j=0}^{m-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})[v_0+a(x)]+\prod_{j=0}^{m-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})\\ &\times\int_{t_0}^{t_1-\tau}U(b,s)\mathfrak{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)ds+\sum_{i=1}^{m-1}\prod_{j=0}^{m-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})\\ &\times\int_{t_i-\tau}^{t_{i+1}-\tau}U(b,s)\mathfrak{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)ds\\ &+\int_{t_m-\tau}^{t}U(b,s)\mathfrak{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)ds\Bigg\}\quad e=m-1,m,0.\end{aligned}\right\}\quad(3.1)$$

transfers initial state v_0 to

$$\left.\begin{array}{rcl} v(t) &=& U(t,t_0)\prod_{j=0}^{m-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})[v_0+a(x)]+\prod_{j=0}^{m-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})\int_{t_0}^{t_1-\tau}U(t,s) \\ && \times Bu(s)ds+\prod_{j=0}^{m-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})\int_{t_0}^{t_1-\tau}U(t,s)\mathfrak{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)ds \\ && +\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})\int_{t_i-\tau}^{t_{i+1}-\tau}U(t,s)Bu(s)ds+\int_{t_m-\tau}^{t}U(t,s) \\ && Bu(s)ds+\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})\int_{t_i-\tau}^{t_{i+1}-\tau}U(t,s) \\ && \times\mathfrak{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)ds+\int_{t_m-\tau}^{t}U(t,s)\mathfrak{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)ds \end{array}\right\}(3.2)$$

final state v_b.

Proof : Let $\mathcal{I}_1=c_1v(t_1-\tau)$ and $t_n-t_{n-1}>\tau$. Suppose that there exists an $l\in 0,1,...,m$ then $(W_e^{\tau})^{-1}$ is invertible. The following proof is divided into three cases. By substituting this control (3.1) in equation (3.2), the following equations are obtained at b.

Case 1: If $l_1\in 1,...,m-1$, then

$$\left.\begin{array}{rcl} v(t) &=& U(t,t_0)\prod_{j=0}^{m-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})[v_0+a(x)]+\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)}) \\ && \times\int_{t_i-\tau}^{t_{i+1}-\tau}U(t,s)Bu(s)ds+\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})\int_{t_i-\tau}^{t_{i+1}-\tau}U(t,s) \\ && \times\mathfrak{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)ds \end{array}\right\}(3.3)$$

the control law is designed as,

$$\left.\begin{array}{rcl} u(t) &=& \Big(\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(I+c_{m-j}E_{m-j}^{(\tau)})\Big)^{-1}(W_{l_2}^{(\tau)})^{-1}\Big[v(b) \\ && -U(t,t_0)\prod_{j=0}^{m-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})[v_0+a(x)]-\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)}) \\ && \times\int_{t_i-\tau}^{t_{i+1}-\tau}U(t_0,s)\mathfrak{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)ds\Big]. \end{array}\right\}(3.4)$$

Now substitute (3.4) in (3.3),

$$\begin{array}{rcl} v(b) &=& U(t,t_0)\prod_{j=0}^{m-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})[v_0+a(x)]+\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)}) \\ && \times\int_{t_i-\tau}^{t_{i+1}-\tau}U(t,s)B\Big(\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(I+c_{m-j}E_{m-j}^{(\tau)})\Big)^{-1}(W_{l_1}^{(\tau)})^{-1}\Big[v(b) \end{array}$$

$$-U(b,s)\prod_{j=0}^{m-1}(\mathcal{I}+c_{m-j}E^{(\tau)}_{m-j})[v_0+a(x)]-\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I}+c_{m-j}E^{(\tau)}_{m-j})$$
$$\times\int_{t_i-\tau}^{t_{i+1}-\tau}U(b,s)\mathfrak{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)\Big](s)ds+\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I}+c_{m-j}E^{(\tau)}_{m-j})$$
$$\times\int_{t_i-\tau}^{t_{i+1}-\tau}U(t,s)\mathfrak{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)ds$$

$$\begin{aligned}v(b) &= U(t,t_0)\prod_{j=0}^{m-1}(\mathcal{I}+c_{m-j}E^{(\tau)}_{m-j})[v_0+a(x)]+\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I}+c_{m-j}E^{(\tau)}_{m-j})\\
&\times(W^{(\tau)}_{l_1})\Big(\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(I+c_{m-j}E^{(\tau)}_{m-j})\Big)^{-1}(W^{(\tau)}_{l_1})^{-1}\\
&\times\Big[v(b)-U(b,t_0)\prod_{j=0}^{m-1}(\mathcal{I}+c_{m-j}E^{(\tau)}_{m-j})[v_0+a(x)]-\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I}+c_{m-j}E^{(\tau)}_{m-j})\\
&\times\int_{t_i-\tau}^{t_{i+1}-\tau}U(b,s)\mathfrak{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)\Big](s)ds+\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I}+c_{m-j}E^{(\tau)}_{m-j})\\
&\times\int_{t_i-\tau}^{t_{i+1}-\tau}U(t,s)\mathfrak{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)ds=v_b.\end{aligned}$$

Case 2: If $l_2\in 1,...,m$, then

$$\left.\begin{aligned}v(t) &= U(t,t_0)\prod_{j=0}^{m-1}(\mathcal{I}+c_{m-j}E^{(\tau)}_{m-j})[v_0+a(x)]+\int_{t_m-\tau}^{t}U(t,s)Bu(s)ds\\
&+\int_{t_m-\tau}^{t}U(t,s)\mathfrak{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)ds\end{aligned}\right\}\tag{3.5}$$

the control law is designed as,

$$\left.\begin{aligned}u(t) &= (W^{(\tau)}_{l_2})^{-1}\Big[v(b)-U(b,s)\prod_{j=0}^{m-1}(I+c_{m-j}E^{(\tau)}_{m-j})[v_0+a(x)]\\
&-\int_{t_m-\tau}^{t}U(b,s)\mathfrak{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)\Big]ds.\end{aligned}\right\}\tag{3.6}$$

Now substitute (3.6) in (3.5),

$$\begin{aligned}v(b) &= U(b,t_0)\prod_{j=0}^{m-1}(I+c_{m-j}E^{(\tau)}_{m-j})[v_0+a(x)]+(W^{(\tau)}_{l_2})(W^{(\tau)}_{l_2})^{-1}\Big[v_b-U(b,t_0)\\
&\times\prod_{j=0}^{m-1}(I+c_{m-j}E^{(\tau)}_{m-j})v_0-\int_{t_m-\tau}^{t}U(t_0,s)\mathfrak{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)ds\Big](s)ds\\
&+\int_{t_m-\tau}^{t}U(t_0,s)\mathfrak{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)ds=v_b.\end{aligned}$$

Case 3: If $l = 0$, then

$$\left.\begin{aligned} v(t) &= U(t,t_0)\prod_{j=0}^{m-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})[v_0+a(x)] + \prod_{j=0}^{m-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})\int_{t_0}^{t_1-\tau}U(t,s) \\ &\quad \times Bu(s)ds + \prod_{j=0}^{m-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})\int_{t_0}^{t_1-\tau}U(t,s)\mathfrak{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)ds \end{aligned}\right\} \quad (3.7)$$

the control law is designed as,

$$\left.\begin{aligned} u(t) &= \Big(\prod_{j=0}^{m-1}(I+c_{m-j}E_{m-j}^{(\tau)})\Big)^{-1}(W_0^{(\tau)})^{-1}\Big[v(b)-U(b,t_0) \\ &\quad \times \prod_{j=0}^{m-1}(I+c_{m-j}E_{m-j}^{(\tau)})[v_0+a(x)] - \prod_{j=0}^{m-1}(I+c_{m-j}E_{m-j}^{(\tau)})\int_{t_0}^{t_1-\tau}U(b,s) \\ &\quad \times \mathfrak{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)ds\Big]. \end{aligned}\right\} \quad (3.8)$$

Now substitute (3.8) in (3.7),

$$\begin{aligned} v(b) &= U(b,t_0)\prod_{j=0}^{m-1}(I+c_{m-j}E_{m-j}^{(\tau)})x_0 + \Big(\prod_{j=0}^{m-1}(I+c_{m-j}E_{m-j}^{(\tau)})\Big)(W_0^{(\tau)}) \\ &\quad \times \Big(\prod_{j=0}^{m-1}(I+c_{m-j}E_{m-j}^{(\tau)})\Big)^{-1}(W_0^{(\tau)})^{-1}\Big[v(b)-U(b,t_0)\prod_{j=0}^{m-1}(I+c_{m-j}E_{m-j}^{(\tau)} \\ &\quad \times [v_0+a(x)] - \prod_{j=0}^{m-1}(I+c_{m-j}E_{m-j}^{(\tau)})\int_{t_0}^{t_1-\tau}U(b,s) \\ &\quad \times \mathfrak{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)ds\Big] \\ &\quad + \prod_{j=0}^{m-1}(I+c_{m-j}E_{m-j}^{(\tau)})\int_{t_0}^{t_1-\tau}U(t_0,s)\mathfrak{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)ds \end{aligned}$$

$$\begin{aligned} v(b) &= U(b,t_0)\prod_{j=0}^{m-1}(I+c_{m-j}E_{m-j}^{(\tau)})[v_0+a(v)] + \prod_{j=0}^{m-1}(I+c_{m-j}E_{m-j}^{(\tau)})W_0^{(\tau)} \\ &\quad \times \Big(\prod_{j=0}^{m-1}(I+c_{m-j}E_{m-j}^{(\tau)})\Big)^{-1}(W_0^{(\tau)})^{-1}\Big[v_b-U(b,t_0) \\ &\quad \times \prod_{j=0}^{m-1}(I+c_{m-j}E_{m-j}^{(\tau)})[v_0+a(v)] - \prod_{j=0}^{m-1}(I+c_{m-j}E_{m-j}^{(\tau)})\int_{t_0}^{t_1-\tau}U(b,s) \\ &\quad \times \mathfrak{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)ds\Big] + \prod_{j=0}^{m-1}(I+c_{m-j}E_{m-j}^{(\tau)}) \\ &\quad \times \int_{t_0}^{t_1-\tau}U(t_0,s)\int_0^s C(r,v(r))drds = v_b. \end{aligned}$$

The proof is complete.

Definition 3.2 *The given system* (2.1) *is said to be controllable on the interval* $\mathcal{J}$*, if for every initial function* $v_0 \in E$ *and* $v_1 \in E$*, there exists a control* $u \in L^2(\mathcal{J}, U)$ *such that the solution* $v(\cdot)$ *to* (2.1) *satisfies* $v(b) = v_1$.

4 Controllability Result

To establish our results, the following assumptions are introduced on system (2.1). Consider the interval $J' = [t_m - \tau, t_{m+1} - \tau]$.

(H2) $\mathcal{G}(t)$ generates family of the strongly continuous semi group of bounded linear operators $U(t,s)$ is compact when $t > s > 0$ and there exist constants $M_1 > 0$ such that $\|U(t,s)\| \leq M_1$.

(H3) The linear operator $W_{m-1}^{(\tau)} : \mathcal{L}(J', \mathcal{U}) \to E$ defined by

$$W_{m-1}^{(\tau)} u = \int_0^b U(b,s)Bu(s)ds, t \in J',$$

has an inverse operator $(W_{m-1}^{(\tau)})^{-1}$ which takes values in $\mathcal{L}^2(J', \mathcal{U})/kerW$ and there exist positive constants $M_2, M_3 > 0$ such that $\|(W_{m-1}^{(\tau)})^{-1}\| \leq M_2$ and $\|B\| \leq M_3$.

(H4) For each $t, s \in J'$, the function $C(t,s,\cdot) : E \to E$ is continuous and for each $x \in E$, the function $C(\cdot,\cdot,v) : \Lambda \to E$ is strongly measurable.

(H5) The function $C(\cdot) : E \to E$ is continuous and there exist a constants $M_4 > 0$ such that there exists an integrable function $\mathcal{K}_c : \mathcal{J}' \times \mathcal{J}' \to [0,\infty)$

$$\|C(t,\psi)\| \leq M_4,$$

$$\|C(t,\eta_1) - C(t,\eta_2)\| \leq \mathcal{K}_c(t,s)\|\eta_1 - \eta_2\|, \qquad t,s \in J', \eta_1, \eta_2 \in E.$$

(H6) For each $t \in \mathcal{J}'$, the function function $\mathcal{F}(t,\cdot,\cdot) : E \times E \to E$ is continuous and for each $(\theta, v) \in E \times E$, the function $\mathcal{F}(\cdot, \eta_1, \eta_2) : J' \to E$ is strongly measurable.

(H7) There exists a function $\mathcal{K}_f(\cdot) \in \mathcal{L}^1(J', \mathbb{R}^+)$ such that

$$\|\mathcal{F}(t, v_1, q_1) - \mathcal{F}(t, v_2, q_2)\| \leq \mathcal{K}_f(\|v_1 - v_2\| + \|q_1 - q_2),$$

for any $t \in J', v_1 v_2, q_1, q_2 \in E$. (ii) The function $\mathcal{F} : J' \times E \times E \to E$ is compact.

(H8) The function $a : PC(J', E) \to E$ is Lipschitz continuous in the following sense: there exists a constants $\mathcal{K}_a > 0$ such that

$$\|a(x) - a(y)\| \leq \mathcal{K}_a \|x - y\|, \quad \text{for } x,\ y \in E.$$

For convenience take, $\prod_{j=0}^{m-1}(I+c_{m-j}E^{(\tau)}_{m-j})=C_1$,

$M_1C_1\|x_0\|+C_1M_1M_3C_2M_2t+[\|x_b\|-M_1C_1\|x_0\|]]+C_1M_1M_2[(M_5b-M_5(s)]<1.$

To prove the controllability result via the fixed points of a condensing operator Kuratowskiis measure of non-compactness will be used in the following section. Kuratowskiis measure of non compactness or the measure for a bounded set D of a Banach space E with norm $\|\cdot\|$ is defined as

$$(\mathcal{G})=\inf\{\alpha>0\},$$

$\mathcal{G}$ can be covered with a finite number of sets of diameter small than α.

Lemma 4.1 *(Sadovskii fixed point theorem) [21] Let $\mathcal{T}$ be the condensing operator on a Banach space E, that is $\mathcal{T}$ is continuous and takes bounded sets into bounded sets and $\alpha(\mathcal{T}(D))<\alpha(D)$, for every bounded set D of E with $\alpha(D)>0$. If $\mathcal{T}(\mathcal{S})\subset\mathcal{S}$ for a convex, closed and bounded set $\mathcal{S}$ of E, then $\mathfrak{T}$ has a fixed point in $\mathcal{S}$.*

Let us consider the case 1 for this section.

Theorem 4.2 *Assume that the impulse times $\{t_i, i\in\mathbb{Z}_+\}$ on the interval J'. If the assumption $(H1)-(H4)$ are satisfied then the system (2.1) is controllable on E.*

Proof. Using $(H3)$ for an arbitrary function $v(\cdot)\in PC(J',E)$, define the control

$$\begin{aligned}u(t) &= \Big(\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(I+c_{m-j}E^{(\tau)}_{m-j})\Big)^{-1}(W^{(\tau)}_{l_2})^{-1}\Big[v(b)\\ &\quad -U(t,t_0)\prod_{j=0}^{m-1}(\mathcal{I}+c_{m-j}E^{(\tau)}_{m-j})[v_0+a(x)]-\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I}+c_{m-j}E^{(\tau)}_{m-j})\\ &\quad\times\int_{t_i-\tau}^{t_{i+1}-\tau}U(t_0,s)\mathcal{F}\Big(t,v(t),\int_0^s C(r,v(r))dr\Big)ds\Big].\end{aligned}$$

Consider the Banach space $\mathbb{Y}=\mathcal{PC}(J',E)$ with the norm $\|v\|=\sup\{\|v(t)\|:t\in J'\}$. Using $(H2)$ for an arbitrary function $v(\cdot)\in\mathcal{C}(J',E)$, and define an operator $\Phi:\mathbb{Y}\to\mathbb{Y}$.

$$\begin{aligned}(\Phi v)(t) &= U(t,t_0)\prod_{j=0}^{m-1}(\mathcal{I}+c_{m-j}E^{(\tau)}_{m-j})[v_0+a(x)]+\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I}+c_{m-j}E^{(\tau)}_{m-j})\\ &\quad\times\int_{t_i-\tau}^{t_{i+1}-\tau}U(t,s)B(s)\Big(\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(I+c_{m-j}E^{(\tau)}_{m-j})\Big)^{-1}(W^{(\tau)}_{l_1})^{-1}\Big[v(b)\\ &\quad -U(b,s)\prod_{j=0}^{m-1}(\mathcal{I}+c_{m-j}E^{(\tau)}_{m-j})[v_0+a(x)]-\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I}+c_{m-j}E^{(\tau)}_{m-j})\end{aligned}$$

$$\times \int_{t_i-\tau}^{t_{i+1}-\tau} U(b,s)\mathcal{F}\bigg(s, v(s), \int_0^s C(r, v(r))dr\bigg)\bigg](s)ds + \sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I} + c_{m-j}E_{m-j}^{(\tau)})$$
$$\times \int_{t_i-\tau}^{t_{i+1}-\tau} U(t,s)\mathcal{F}\bigg(s, v(s), \int_0^s C(r, v(r))dr\bigg)ds$$

has a fixed point $X(\cdot)$. This fixed point is the mild solution to system 2.1, which is implies that the system is controllable on J'. To prove that operator Φ is a completely continuous operator. Set $\mathcal{B}_\rho = \{v \in \mathcal{C}[0,b] : \|v\|_{\mathcal{C}} \le \rho\}$ for some $\rho \ge 1$. For each ρ, $\mathcal{B}_\rho$ is a bounded closed convex set in $\mathbb{Y}$.

Step 1. To claim that there exists a positive constant r such that $\Phi(\mathcal{B}_\rho) \subset \mathcal{B}_\rho$. If this is not true, then for each positive number ρ, there exists a function $v^\rho \in \mathcal{B}_\rho$, does not belong to $\mathcal{B}_\rho$, that is $\|(\Phi v^\rho)(t)\| > \rho$ for some $t \in J^{'}$. Next we have

$$\begin{aligned}
\rho &< \|(\Phi v^\rho)(t)\| \\
&\le \bigg\|U(t,t_0)\prod_{j=0}^{m-1}(\mathcal{I} + c_{m-j}E_{m-j}^{(\tau)})[v_0 + a(v^\rho)] + \sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I} + c_{m-j}E_{m-j}^{(\tau)}) \\
&\quad \times \int_{t_i-\tau}^{t_{i+1}-\tau} U(t,s)B(s)\bigg[v(b) - U(b,s)\prod_{j=0}^{m-1}(\mathcal{I} + c_{m-j}E_{m-j}^{(\tau)})[v_0 + a(v^\rho)] \\
&\quad - \sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I} + c_{m-j}E_{m-j}^{(\tau)}) \\
&\quad \times \int_{t_i-\tau}^{t_{i+1}-\tau} U(b,s)\mathcal{F}\bigg(s, v^\rho(s), \int_0^s C(r, v^\rho(r))dr\bigg)\bigg](s)ds + \sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I} + c_{m-j}E_{m-j}^{(\tau)}) \\
&\quad \times \int_{t_i-\tau}^{t_{i+1}-\tau} U(t,s)\mathcal{F}\bigg(s, v^\rho(s), \int_0^s C(r, v^\rho(r))dr\bigg)ds\bigg\| \\
&\le M_1C_1\|v_0\|N_1 + C_1M_1M_2(t_{i+1}-t_i)\bigg[\|v_b\| - M_1C_1\|v_0\|N_1 - C_1M_1 \\
&\quad \times \int_{t_i-\tau}^{t_{i+1}-\tau} \mathcal{F}\bigg(s, v^\rho(s), \int_0^s C(r, v^\rho(r))dr\bigg)\bigg] \\
&\quad + C_1\int_{t_i-\tau}^{t_{i+1}-\tau} M_1\|\mathcal{F}\bigg(s, v^\rho(s), \int_0^s C(r, v^\rho(r))dr\bigg)\|ds
\end{aligned}$$

Since

$$\begin{aligned}
&\le \int_{t_i-\tau}^{t_{i+1}-\tau} \|\mathcal{F}(s, v^\rho(s), \int_0^s C(r, v^\rho(r))dr\|ds \\
&\le \int_{t_i-\tau}^{t_{i+1}-\tau} [\|\mathcal{F}(s, v^\rho(s), \int_0^s C(r, v^\rho(r))dr) - \mathcal{F}(s,0,0)\| + \|\mathcal{F}(s,0,0)\|]ds \\
&\le \int_{t_i-\tau}^{t_{i+1}-\tau} [\mathcal{K}_f(\|v^r(s)\| + \|\int_0^s C(r, v^\rho(r))dr)\| + \|\mathcal{F}(s,0,0)\|)]ds
\end{aligned}$$

$$
\begin{aligned}
&\leq \int_{t_i-\tau}^{t_{i+1}-\tau}[\mathcal{K}_f(\|v^r(s)\|+\|\int_0^s\{\|C(r,v^\rho(r))-C(r,\rho,0))\|+\|C(r,\rho,0)\|\}dr+\|\mathcal{F}(s,0,0)\|)]ds\\
&\leq \int_{t_i-\tau}^{t_{i+1}-\tau}\left[\mathcal{K}_f\left(\|v^r(s)\|+\int_0^s[\mathcal{K}_c\|v^\rho(s)\|+\|C(r,\rho,0)\|]dr+\|\mathcal{F}(s,0,0)\|\right)\right]ds\\
&\leq \mathcal{K}_f(t_{i+1}-t_i)[\left(\|v^r(s)\|+s[\mathcal{K}_c\|v^\rho(s)\|+\|C(r,\rho,0)\|]+\|\mathcal{F}(s,0,0)\|\right)]\\
&\leq M_1C_1d_1M_2M_3\big[\|v_0\|+\|v(b)\|+M_1C_1\|v_0\|+M_1\|\mathcal{K}_c\|_{L^1}\|\|C(r,\rho,0)\|\\
&\quad +M_1C_1\|\mathcal{F}(s,0,0)\|\big]+M_1C_1\mathcal{K}_fd_2[\|C(r,\rho,0)\|+\|\mathcal{F}(s,0,0)\|]\\
&\quad +\|\rho\|\Big[M_1C_1(1+M_1C_1d_1M_2M_3)+M_1C_1d_1\mathcal{K}_f(M_2M_3M_1C_1d_2\mathcal{K}_c+1)\Big].
\end{aligned}
$$

Dividing both side of ρ and taking the limit as $\rho\to\infty$, we have

$$
\Big[M_1C_1(1+M_1C_1d_1M_2M_3)+M_1C_1d_1\mathcal{K}_f(1+M_2M_3M_1C_1d_2\mathcal{K}_c)\Big]\geq 1.
$$

This contradicts to our assumption $[H(5)]$. Hence for some positive number ρ, $\Phi(\mathcal{B})\subseteq\mathcal{B}_\rho$.

Now to prove that the operator Φ is a condensing operator,and introduce the decomposition $\Phi=\Phi_1+\Phi_2$, where

$$
\begin{aligned}
(\Phi_1v)(t) &= U(t,t_0)\prod_{j=0}^{m-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})[v_0+a(x)]+\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})\\
&\quad\times\int_{t_i-\tau}^{t_{i+1}-\tau}U(t,s)B(s)\left(\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(I+c_{m-j}E_{m-j}^{(\tau)})\right)^{-1}(W_{l_1}^{(\tau)})^{-1}\Bigg[v(b)\\
&\quad -U(b,s)\prod_{j=0}^{m-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})[v_0+a(x)]-\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})\\
&\quad\times\int_{t_i-\tau}^{t_{i+1}-\tau}U(b,s)\mathcal{F}\left(s,v(s),\int_0^sC(r,v(r))dr\right)\Bigg](s)ds\\
(\Phi_2v)(t) &= \sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})\int_{t_i-\tau}^{t_{i+1}-\tau}U(t,s)\\
&\quad\times[\mathcal{F}\left(s,x(s),\int_0^sC(r,v(r))dr-\mathcal{F}\left(s,ys),\int_0^sC(r,v(r))dr\right)\right]ds.
\end{aligned}
$$

Now to show that when using this control $u(t)$, the operator $\Phi=\Phi_1+\Phi_2$ has a fixed point $x(\cdot)$. This fixed point is the solution to system (2.1), in implying that the system is controllable.

Step 2. Now $t\in J'$, $x_1,y_1\in\mathcal{B}_\rho$, then

$$
\begin{aligned}
&\|(\Phi_1x_1)(t)-(\Phi_1y_1)(t)\|\\
&\quad\leq \|U(t,t_0)\|\|\prod_{j=0}^{m-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})\|a(x_1)-a(y_1)\|
\end{aligned}
$$

$$
\begin{aligned}
&+\|\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})\|\int_{t_i-\tau}^{t_{i+1}-\tau}\|U(t,s)\| \\
&\times\|B(s)\|\|\Big(\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(I+c_{m-j}E_{m-j}^{(\tau)})\Big)^{-1}\| \\
&\times\|(W_{l_1}^{(\tau)})^{-1}\|\Big[\|U(b,s)\|\|\prod_{j=0}^{m-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})\|[a(x_1)-a(y_1)\|] \\
&-\|\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})\|\int_{t_i-\tau}^{t_{i+1}-\tau}\|U(b,s)\| \\
&\times\|\mathcal{F}\Big(s,x_1(s),\int_0^s C(r,v(r))dr-\mathcal{F}\Big(s,y_1(s),\int_0^s C(r,v(r))dr\Big)\Big](s)\|ds \\
\leq\ & \|x_1-y_1\|[M_1C_1\mathcal{K}_a+c_1(t_{i+1}-t_i)M_1M_3M_1C_!\mathcal{K}_a-C_1(t_{i+1}-t_i)M_1\mathcal{K}_f+\mathcal{K}_c].
\end{aligned}
$$

Therefore $\Phi_1(\cdot)$ is a contraction on $\mathcal{B}_\rho$. Next to show that Φ_2 is completely continuous from $\mathcal{B}_\rho$ into $\mathcal{B}_\rho$.

Step 3. To prove that Φ_2 is completely continuous. First, to prove that $\Phi_2(\cdot)$ is continuous on $\mathcal{B}_\rho$. Let $v_n(t)_0^\infty\subset\mathcal{B}_\rho$, with $v_n\to v$ in $\mathcal{B}_\rho$. Then there exists a number $\rho>0$ such that $\|v_n(t)\|\leq\rho$ for all n and a.e. $t\in J'$, so $v_n\in\mathcal{B}_\rho$. From the dominated convergence theorem, we obtain

$$
\begin{aligned}
\|(\Phi_2v_n)(t)-\Phi_2v(t)\| &\leq \|\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})\int_{t_i-\tau}^{t_{i+1}-\tau}\|U(t,s)\| \\
&\quad\times\|\mathcal{F}\big(s,v_n(s),\int_0^s C(r,v_n(r))dr\big)ds-\mathcal{F}\big(s,v(s),\int_0^s C(r,v(r))dr\big)ds\| \\
&\leq C_1M_1(t_{i+1}-t_i)(\mathcal{K}_f\|v_n-v\|[1+\mathcal{K}_c\|v_n-v\|]) \\
&\quad\to 0 \quad\text{as}\quad n\to\infty.
\end{aligned}
$$

Hence Φ_2 is continuous on $\mathcal{B}_\rho$. Next to prove that Φ_2 is relatively compact as well as equicontinuous on E. From the Ascoli-Arzela theorem, to show that the compactness of Φ_2. We need to prove that $\Phi_2(\mathcal{B}_\rho)\subset\mathbb{PC}(J',E)$ is equi continuous and $\Phi_2(\mathcal{B}_\rho)(t)$ is precompact. For any $v\in\mathcal{B}_\rho$ with $t+h\in J'$, we have

$$
\begin{aligned}
&\|(\Phi_2v_n)(t+h)-\Phi_2v(t)\| \\
\leq\ & \|\sum_{i=1}^{m-1}\prod_{j=0}^{m-i-1}(\mathcal{I}+c_{m-j}E_{m-j}^{(\tau)})\int^{t_{i+1}+h-\tau}-t_i-\tau\|U(t+h,s)-U(t,s)\| \\
\times\ & \|\mathcal{F}\Big(s,v_n(s),\int_0^s C(r,v_n(r))dr\Big)ds-\mathcal{F}\Big(s,v(s),\int_0^s C(r,v(r))dr\Big)ds\| \\
\leq\ & C_1M_1[U(t+h,t)-I]\|\mathcal{F}\Big(s,v_n(s),\int_0^s C(r,v_n(r))dr\Big)ds \\
&-\mathcal{F}\Big(s,v(s),\int_0^s C(r,v(r))dr\Big)ds.
\end{aligned}
$$

Since $\mathcal{F}$ is compact,

$$[U(t+h,t)-I]\|\mathcal{F}\bigg(s, v(s), \int_0^s C(r,v(r))dr\bigg)ds \to 0 \quad \text{as} \quad h \to 0$$

Finally, Φ_2 maps $(\mathcal{B}_\rho)(t)$ into precompact set in E as

$$\Phi_2(\mathcal{B}_\rho)(t) \subset t \ \ \overline{conv}\{U(t,s)\mathcal{F}\bigg(s, v_n(s), \int_0^s C(r,v_n(r))drds\}, \text{ for all } t \in J'.$$

Hence by the above steps $1-3$, we conclude that $\Phi = \Phi_1 + \Phi_2$ is a condensing operator on $(\mathcal{B}_\rho)$. By Lemma 4.1, there exists a fixed point $v(\cdot) \in (\mathcal{B}_\rho)$ such that $(\Phi v)(t) = v(t)$ and this point $v(\cdot)$ is a mild solution to system (2.1). Clearly, $(\Phi v)(b) = v(b) = v(b)$, which implies that system (2.1) is controllable. Hence the proof.

5 Application

Example Consider the partial differential equation of the form

$$\left.\begin{aligned}
\frac{\partial}{\partial t}e(t,w) &= \alpha_1(t,w)\frac{\partial^2}{\partial w^2}e(t,w) + \mu(t,w) + \alpha_2(t,e(t,w)) + \int_0^t \alpha_3(t,s,e(t,w))ds,\\
e(t,0) &= e(t,\pi) = 0, t \geq 0, t \in \mathcal{J},\\
e(t,w) &= e_0(w) + \textstyle\sum_{i=1}^n h_i\phi(s_i,w), \quad \phi \in \mathbb{PC}(\mathcal{J}', E),\\
\Delta e = \mathcal{I}_n &= 0.1,
\end{aligned}\right\}(5.1)$$

where $e_t - \alpha_1(t,w)e_{ww}$ is a uniform parabolic differential operator with $\alpha_1(t,w)$ continuous on $0 \leq w \leq \pi$, $0 \leq t \leq b$ and is uniformly Holder continuous in t, and constant h_i is small and α_2, α_3 are continuous. Let us take $E = \mathcal{U} = \mathcal{L}^2[0,\pi]$ endowed with the usual norm $|\cdot|^2_{\mathcal{L}}$. Put $v(t) = e(t,w)$ and $u(t) = \mu(t,w)$ where $\lambda(t,w) : \mathcal{J} \times [0,\pi]$ is continuous. Define the operators $\mathcal{F}, \mathbb{I}_i$ by

$$\mathcal{F}(t,\lambda_1,\lambda_2)(w) = \alpha_1(t,\lambda_1(t,w)) + \mathbb{K}(\lambda_2).$$

where

$$\mathbb{K}(\lambda_2)(w) = \int_0^t \alpha_3(t,s,\lambda_2(t,w))ds$$

and

$$\mathbb{I}_i(\lambda)(w) = c_i v(t_i - \tau).$$

In particular, set $E = \mathbb{R}^+$, $\mathcal{J}' = [0,1]$,

$$\mathcal{F}(t,v) \quad = \quad \alpha_1 = \frac{x}{15+t^3}, (t,v) \in \mathcal{J}' \times E,$$

$$
\begin{aligned}
\mathcal{G}x(t) &= \int_0^t \alpha_1(t,s,v)ds = \alpha_3 = \int_0^t e^{\frac{-1}{18}}x(s)ds, \\
a(v) &= \sum_{t=1}^{n} h_i\phi = \frac{2}{18}sinx, \quad x \in E.
\end{aligned}
$$

With this choice $\mathcal{G}(t)$, $\mathcal{F}$ and $B = \mathbb{I}$ the identity operator, we see that (5.1) can be written in the abstract formulation of (2.1). Let $p_1, p_2 \in E$ and $t \in \mathcal{J}'$. Then

$$
\begin{aligned}
\|\mathcal{G}p_1(t) - \mathcal{G}p_2(t)\| &= \|\int_0^t e^{\frac{-1}{18}}p_1(s)ds - \int_0^t e^{\frac{-1}{18}}p_2(s)ds\| \\
&\leq \frac{-1}{18}\|p_1 - p_2\|.
\end{aligned}
$$

Hence the condition $(H5)$ holds with $\mathcal{K}_c = \frac{-1}{18}$. Let $q_1, q_2 \in E$ and $t \in \mathcal{J}'$. Then

$$
\begin{aligned}
\|a(q_1) - a(q_2)\| &= \|\frac{2}{18}\sin(q_1) - \frac{2}{18}\sin(q_2)\| \\
&\leq \frac{2}{18}\|q_1 - q_2\|.
\end{aligned}
$$

Hence, the condition $(H4)$ holds with $\mathcal{K}_a = \frac{2}{18}$. Let $p_3, p_4 \in\in E$ and $t \in \mathcal{J}'$. Then

$$
\begin{aligned}
\|\mathcal{F}p_3 - \mathcal{F}p_4\| &= \|\frac{p_3}{15+t^3} - \frac{p_4}{15+t^3}\| \\
&\leq \frac{1}{15}\|p_3 - p_4\|.
\end{aligned}
$$

Hence, the condition $(H6)$ holds with $\mathcal{K}_f = \frac{1}{15}$. Therefore, all the conditions of the Theorem 4.2 is satisfied. Hence the given system (5.1) is controllable.

References

[1] Balachandran K, Sakthivel R. Controllability of functional semilinear integrodifferential systems in Banach spaces. *Journal of Mathematical Analysis and Applications.* 2001; 255: 447-457.

[2] Balachandran K, Sakthivel R, Dauer J P. Controllability of neutral functional integrodifferential systems in Banach spaces. *International Journal of Computers Mathematics with Applications.*2000; 39: 117-126.

[3] Yan Z. Controllability of semilinear integrodifferential systems with nonlocal conditions. *International Journal of Computational and Applied Mathematics.* 2007; 2: 221-236.

[4] X. Zhang, Exact controllability of semilinear evolution systems and its application, Journal of Optimization Theory and Applications 107 (2000) 415-432.

[5] Xue X. Nonlocal nonlinear differential equations with measure of noncompactness in Banach spaces. *Nonlinear Analysis: Theory Methods and Applications*. 2009; 70: 2593-2601.

[6] Fu X. Controllability of neutral functional differential systems in abstract space. *Applied Mathematics and Computation*. 2003; 141: 281-296.

[7] Bainov D, Simeonov P. *Impulsive Differential Equations: Periodic Solutions and Applications*. Longman Scientific Technical; Essex, U.K: 1993.

[8] Pazy A. *Semigroups of Linear Operators and Applications to Partial Differential Equations.* New York: Springer-Verlag; 1983.

[9] Hernandez E, Henriquez HR. Impulsive partial neutral differential equations. *Applied Mathematical Letters*. 2006; 19: 215-222.

[10] Surendra Kumar, Sukavanam N. Controllability of semilinear systems with fixed delay in control. *Opuscula Mathematics*. 2015: 35; 71-83

[11] Klamka J, Czornik A, Mich, Niezabitowski, Artur Babiarz. Trajectory Controllability of Semilinear Systems with Delay. *Applied Mechanics and Materials*. 2015; 790: 1043-10491.

[12] Gorniewicz L, Ntouyas SK, Oregan D. Controllability of semilinear differential equations and inclusions via semigroup theory in Banach spaces. *Reports on Mathematical Physics*. 2005; 56: 437-470.

[13] Vijayakumar V, Muni, George Raju K. Controllability of semilinear impulsive control systems with multiple time delays in control. *IMA Journal of Mathematical Control and Information*. 2018; 36: 1-31.

[14] Limin Zou, Yang Peng, Yuming Feng, Zhengwen Tu. Impulsive control of nonlinear systems with impulse time window and bounded gain error. *Nonlinear Analysis: Modeling and Control*. 2018; 23: 40-49.

[15] Xueyan Yang, Dongxue Peng, Xiaoxiao Lv, Xiaodi Li. Recent progress in impulsive control systems. *Mathematics and Computers in Simulation*. 2019; 155: 244-268.

[16] Yuming Feng, Chuandong Lia. Comparison system of impulsive control system with impulse time windows. *Journal of Intelligent and Fuzzy Systems*. 2017; 32: 4197-4204.

[17] George RK , Nandakumaran AK. A Note on Controllability of Impulsive Systems. *Journal of Mathematical Analysis and Applications*. 2000; 241: 276-283.

[18] Y. Wang, G. Xie, L. Wang, Controllability of switched time-delay systems under constrained switching. *Journal of Mathematical Analysis and Applications.* 2003; 286: 397-421.

[19] Yang T. *Impulsive Control Theory.* Springer, Berlin: 2001.

[20] Lakshmikantham V, Banov D, Simeonov P. *Theory of Impulsive Differential Equations.* World Scientific; 1989.

[21] Sadaovskii B. On a fixed point principle. *Functional Analysis and Applications.* 1967; 2: 151-153.

[22] Sakthivel R, Anandhi E. Approximate controllability of impulsive differential equations with state-dependent delay. *International Journal of Control.* 2010; 83: 387-393.

♣♣♣♣