# Connecting nonlinear $\$(\backslash$ varpi-F_ $\{\backslash$ mathcal $\{\mathrm{C}\}\}) \$$-contractions and fractional operators in the modelling of novel Coronavirus 2019-nCoV/SARS-CoV-2 

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#### Abstract

There is still a dramatic increase for using mathematical modelling in the study of epidemiology diseases. Mathematical models were developed to predict how infectious diseases advance to explain the potential outcome of an outbreak, and better facilitate initiatives in global policy. In this article, we present new insights of existence and uniqueness solutions of the novel coronavirus 2019-nCoV/SARS-CoV-2 model via fractional derivatives by using $\$(\backslash$ varpi-F- $\{\backslash$ mathcal $\{\mathrm{C}\}\}) \$$-contractions.


# Connecting nonlinear $\left(\omega-F_{\mathcal{C}}\right)$-contractions and fractional operators in the modelling of novel Coronavirus 2019-nCoV/SARS-CoV-2 

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#### Abstract

There is still a dramatic increase for using mathematical modelling in the study of epidemiology diseases. Mathematical models were developed to predict how infectious diseases advance to explain the potential outcome of an outbreak, and better facilitate initiatives in global policy. In this article, we present new insights of existence and uniqueness solutions of the novel coronavirus $2019-\mathrm{nCoV} / \mathrm{SARS}-\mathrm{CoV}-2$ model via fractional derivatives by using $\left(\omega-F_{\mathcal{C}}\right)$-contractions.


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## 1 Introduction

The planet has been facing an extreme assault from a peculiar virus called Cov-19 from more than one year. The breakout began in the city of Wuhan and has now spread to every part of the world. Thousands of people across the globe have been killed by a devastating contagious disease. The disorder is very fatal for elderly people and people with health issues. It is known that, in low temperatures, the spread is more violent, but the evidence behind this has not yet been identified. Though several scientists from many backgrounds have undertaken some preliminary studies on the spread, the surface life span, the strategies that could be used to stop the spread, the etymology of this virus is not researched. Some individuals claim that the virus was born from bats, pangolins, seafood, etc. and others believe that the virus is man-made. Few eradicative vaccines are available to this virus last few months around the world. The Coronavirus spread and death has left many cities around the world empty due to humanity's precautions to flatten the spread. It's indirectly a strong indication that the rules of the nature should be observed by humanity, as breaking these rules could wake up some unseen enemies which could theoretically hurt and paralyze all that human beings, have been constructed in recent decades. Now, Scientists can understand that nature is dynamic and our awareness is not strong enough to threaten the nature. It's highly experienced in the process of vaccine invention. Therefore, when communicating with various beings living within our world, caution must be taken. Several studies have been done to determine the principles, the life span, the mode of propagation, the effect of temperature affluence, age, and many others in the human body. To better understand the distribution, however, mathematicians create a mathematical model using the notion of differentiation and the facts observed. Such data allows them to provide a model for which the solution might represent the real-world problem's future actions as a function of time. For a collection of the data set and constraints, mathematicians use this solution to understand, control, and forecast the behavior of the real world problem.

On the other side, the concept of $F$-contraction was defined and studied by Wardowsky [50]. It has attracted many authors to publish many interesting results in this area. It was sort of topological ideas like Cauchy, completeness, converges and a contraction type mapping of the form:

$$
d(\mathscr{H} a, \mathscr{H} b)>0 \Rightarrow \tau+F(d(\mathscr{H} a, \mathscr{H} b)) \leq F(d(a, b))
$$

for all $a, b \in X$, where $\tau>0, \mathscr{H}: X \rightarrow X$ and $F:(0, \infty) \rightarrow \mathbb{R}$ fulfills the subsequent conditions:
$\left(F_{1}\right) F$ is strictly increasing;
( $F_{2}$ ) $\lim _{n \rightarrow \infty} t_{n}=0$ iff $\lim _{n \rightarrow \infty} F\left(t_{n}\right)=-\infty$;
$\left(F_{3}\right)$ There exists $k \in(0,1)$ such that $\lim _{t \rightarrow 0^{+}} t^{k} F(t)=0$.
Definition 1.1 [43] A mapping $\mathscr{H}: X \rightarrow X$ is $\alpha$-admissible if there exists a function $\alpha: X \times X \rightarrow \mathbb{R}^{+}$such that $a, b \in X, \alpha(a, b) \geq 1 \Rightarrow \alpha(\mathscr{H} a, \mathscr{H} b) \geq 1$.

If we reckon the concepts of $F$-contraction and $\alpha$-admissibility in the concept of fixed points, there is a wide growth in this concern. The ideas used in [50] and [43] attracted several authors, and used these pivotal concepts in their applications as well (see for example $[1,2,3,4,6,7,8,11,12,13,14,15,16,17,18,21,22,23,24,25,27,28,29,30,31$, $32,33,34,35,36,37,38,39,40,41,42,44,45,46,47,48,49,51,52,53])$.

On the mirror side of this section, Czerwik [10] introduced the idea of a $b$-metric space in 1993.

[^0]Definition 1.2 [10] Assume that $X$ be a non empty set and $s \geq 1$. A function $d: X \times X \longrightarrow[0, \infty)$ is called a $b$-metric space provided that the subsequent properties are fulfilled:
(i) $d(a, b)=0$ iff $a=b$;
(ii) $d(a, b)=d(b, a), \forall a, b \in X$;
(iii) $d(a, b) \leq s[d(a, c)+d(c, b)], \forall a, b, c \in X$.

Definition 1.3 [26] Assume that $X$ be a non-empty set and $p: X \times X \rightarrow[1, \infty)$. A function $\mathcal{C}: X \times X \rightarrow[0, \infty)$ is said to be a controlled metric type if $\forall a, b, c \in X$, it the subsequent properties are fulfilled:
(i) $\mathcal{C}(a, b)=0$ iff $a=b$;
(ii) $\mathcal{C}(a, b)=\mathcal{C}(b, a)$;
(iii) $\mathcal{C}(a, b) \leq p(a, c) \mathcal{C}(a, c)+p(c, b) \mathcal{C}(c, b)$.

The pair $\left(X, \mathcal{C}_{b}\right)$ is said to be a controlled metric type space.
Now, we present the idea of controlled $b$-metric space by summing up the above concepts, i.e., a $b$-metric space and a controlled metric type space in the following direction:

Definition 1.4 Assume that $X$ be a non-empty set and $s \geq 1$. Given $p: X \times X \rightarrow[1, \infty)$. A function $\mathfrak{C}_{b}: X \times X \rightarrow$ $[0, \infty)$ is said to be a controlled $b$-metric (simply, a $\mathfrak{C}_{b}$-metric) provided that $\forall a, b, c \in X$,
(i) $\mathcal{C}_{b}(a, b)=0$ iff $a=b$;
(ii) $\mathcal{C}_{b}(a, b)=\mathcal{C}_{b}(b, a)$;
(iii) $\mathfrak{C}_{b}(a, b) \leq s\left[p(a, c) \mathfrak{C}_{b}(a, c)+p(c, b) \mathfrak{C}_{b}(c, b)\right]$.

Example 1.5 Let $X=\{0,1,2\}$. Define $p: X \times X \rightarrow[1, \infty)$ and $\mathcal{C}_{b}: X \times X \rightarrow[0, \infty)$ as $p(a, b)=1+a b$ and

$$
\mathfrak{C}_{b}(2,2)=\mathcal{C}_{b}(0,0)=\mathcal{C}_{b}(1,1)=0 ;
$$

$$
\begin{gathered}
\mathcal{C}_{b}(2,0)=\mathcal{C}_{b}(0,2)=5 ; \mathfrak{C}_{b}(0,1)=\mathcal{C}_{b}(1,0)=10 \\
\mathcal{C}_{b}(2,1)=\mathcal{C}_{b}(1,2)=30 \text { and } s=2
\end{gathered}
$$

Note that (i) and (ii) trivially hold. For (iii), we obtain

$$
\begin{aligned}
& \mathcal{C}_{b}(2,0)=5 ; \quad 2\left[p(2,1) \mathcal{C}_{b}(2,1)+p(1,0) \mathfrak{C}_{b}(1,0)\right]=200 \\
& \mathcal{C}_{b}(1,2)=30 ; \quad 2\left[p(1,0) \mathfrak{C}_{b}(1,0)+p(0,2) \mathcal{C}_{b}(0,2)\right]=30 \\
& \mathfrak{C}_{b}(0,1)=10 ; 2\left[p(0,2) \mathfrak{C}_{b}(0,2)+p(2,1) \mathfrak{C}_{b}(2,1)\right]=190
\end{aligned}
$$

Hence, $\forall a, b, c \in X, \mathfrak{C}_{b}(a, b) \leq s\left[p(a, c) \mathfrak{C}_{b}(a, c)+p(c, b) \mathfrak{C}_{b}(c, b)\right]$. So $\left(X, \mathfrak{C}_{b}\right)$ is a controlled $b$-metric space.
Remark 1.6 A controlled b-metric space is not a controlled metric type space since from above example

$$
\mathcal{C}_{b}(1,2) \not \leq 2\left[p(1,0) \mathcal{C}_{b}(1,0)+p(0,2) \mathcal{C}_{b}(0,2)\right] .
$$

Definition 1.7 Let $\left(X, \mathcal{C}_{b}\right)$ be a $\mathcal{C}_{b}$-metric space and $\left\{a_{n}\right\}$ be a sequence of points of $X$. Then

- $\left\{a_{n}\right\}$ converges to $a$ iff $\forall \epsilon>0, \exists N=N(\epsilon) \in \mathbb{N}$ such that $\mathcal{C}_{b}\left(a_{n}, a\right)<\epsilon \forall n \geq N$. In this situation, we define $\lim _{n \rightarrow \infty} a_{n}=a$;
- $\left\{a_{n}\right\}$ is Cauchy if and only if $\lim _{m, n \rightarrow \infty} \mathcal{C}_{b}\left(a_{m}, a_{n}\right)=0$;
- $\left(X, \mathcal{C}_{b}\right)$ is complete iff every Cauchy sequence $\left\{a_{n}\right\}$ in $X$ converges to $a \in X$.


## 2 Banach Contraction Principle

Now we state and prove the first result.
Theorem 2.1 Assume that $\left(X, \mathcal{C}_{b}\right)$ be a complete controlled $b$-metric space. Let $\mathscr{H}: X \rightarrow X$ satisfy

$$
\begin{equation*}
\mathfrak{C}_{b}(\mathscr{H} a, \mathscr{H} b) \leq k \bigodot_{b}(a, b), \tag{2.1}
\end{equation*}
$$

$\forall a, b \in X$ and $k \in(0,1)$. For $a_{0} \in X$, take $a_{n}=\mathscr{H}^{n} a_{0}=\mathscr{H} a_{n-1}$. Suppose that

$$
\begin{equation*}
\sup _{m \geq 1} \lim _{i \rightarrow \infty} s \frac{p\left(a_{i+1}, a_{i+2}\right)}{p\left(a_{i}, a_{i+1}\right)} p\left(a_{i+1}, a_{m}\right)<\frac{1}{k} . \tag{2.2}
\end{equation*}
$$

In addition, let us consider that for every $a \in X$, we obtain

$$
\begin{equation*}
\lim _{n \rightarrow \infty} p\left(a_{n}, a\right) \text { and } \lim _{n \rightarrow \infty} p\left(a, a_{n}\right) \text { exist and are finite. } \tag{2.3}
\end{equation*}
$$

Then $\mathscr{H}$ has a unique fixed point.
Proof: Initially, we have to verify the uniqueness. On contrary, let $\mathscr{H}$ has two fixed points, say $u$ and $v$. Thus

$$
\mathfrak{C}_{b}(u, v)=\mathfrak{C}_{b}(\mathscr{H} u, \mathscr{H} v) \leq k \mathfrak{C}_{b}(u, v)
$$

so

$$
(1-k) \mathfrak{C}_{b}(u, v) \leq 0
$$

that is,

$$
1-k=0
$$

which gives a contradiction result. Therefore, $\mathscr{H}$ has a unique fixed point.
Assume that $a_{0} \in X$ be arbitrary. Define the iterative sequence $\left\{a_{n}\right\}$ by $a_{n}=\mathscr{H}^{n} a_{0}$. By using (2.1), we get

$$
\mathcal{C}_{b}\left(a_{n}, a_{n+1}\right) \leq k^{n} \mathcal{C}_{b}\left(a_{0}, a_{1}\right) ; \forall n \geq 0 .
$$

For all $n, m \in \mathbb{N}(n<m)$, we have

$$
\begin{aligned}
& \mathcal{C}_{b}\left(a_{n}, a_{m}\right) \leq s\left[p\left(a_{n}, a_{n+1}\right) \mathcal{C}_{b}\left(a_{n}, a_{n+1}\right)+p\left(a_{n+1}, a_{m}\right) \mathcal{C}_{b}\left(a_{n+1}, a_{m}\right)\right] \\
& \leq s p\left(a_{n}, a_{n+1}\right) \mathfrak{C}_{b}\left(a_{n}, a_{n+1}\right)+s^{2} p\left(a_{n+1}, a_{m}\right) p\left(a_{n+1}, a_{n+2}\right) \mathfrak{C}_{b}\left(a_{n+1}, a_{n+2}\right) \\
& +s^{2} p\left(a_{n+1}, a_{m}\right) p\left(a_{n+2}, a_{m}\right) \mathcal{C}_{b}\left(a_{n+2}, a_{m}\right) \\
& \leq s p\left(a_{n}, a_{n+1}\right) \mathcal{C}_{b}\left(a_{n}, x_{n+1}\right)+s^{2} p\left(a_{n+1}, a_{m}\right) p\left(a_{n+1}, a_{n+2}\right) \mathcal{C}_{b}\left(a_{n+1}, a_{n+2}\right) \\
& +s^{3} p\left(a_{n+1}, a_{m}\right) p\left(a_{n+2}, a_{m}\right) p\left(a_{n+2}, a_{n+3}\right) \mathcal{C}_{b}\left(a_{n+2}, a_{n+3}\right) \\
& +s^{3} p\left(a_{n+1}, a_{m}\right) p\left(a_{n+2}, a_{m}\right) p\left(a_{n+3}, a_{m}\right) \mathcal{C}_{b}\left(a_{n+3}, a_{m}\right) \\
& \leq s p\left(a_{n}, a_{n+1}\right) \mathfrak{C}_{b}\left(a_{n}, a_{n+1}\right)+s^{2} p\left(a_{n+1}, a_{m}\right) p\left(a_{n+1}, a_{n+2}\right) \mathfrak{C}_{b}\left(a_{n+1}, a_{n+2}\right) \\
& +s^{3} p\left(a_{n+1}, a_{m}\right) p\left(a_{n+2}, a_{m}\right) p\left(a_{n+2}, a_{n+3}\right) \mathcal{C}_{b}\left(a_{n+2}, a_{n+3}\right) \\
& +s^{4} p\left(a_{n+1}, a_{m}\right) p\left(a_{n+2}, a_{m}\right) p\left(a_{n+3}, a_{m}\right) p\left(a_{n+3}, a_{n+4}\right) \mathcal{C}_{b}\left(a_{n+3}, a_{n+4}\right) \\
& +s^{4} p\left(a_{n+1}, a_{m}\right) p\left(a_{n+2}, a_{m}\right) p\left(a_{n+3}, a_{m}\right) p\left(a_{n+4}, a_{m}\right) \mathfrak{C}_{b}\left(a_{n+4}, a_{m}\right) \\
& \vdots \\
& \leq s p\left(a_{n}, a_{n+1}\right) \mathfrak{C}_{b}\left(a_{n}, a_{n+1}\right)+s^{2} p\left(a_{n+1}, a_{m}\right) p\left(a_{n+1}, a_{n+2}\right) \mathfrak{C}_{b}\left(a_{n+1}, a_{n+2}\right) \\
& +s^{3} p\left(a_{n+1}, a_{m}\right) p\left(a_{n+2}, a_{m}\right) p\left(a_{n+2}, a_{n+3}\right) \mathcal{C}_{b}\left(a_{n+2}, a_{n+3}\right) \\
& +s^{4} p\left(a_{n+1}, a_{m}\right) p\left(a_{n+2}, a_{m}\right) p\left(a_{n+3}, a_{m}\right) p\left(a_{n+3}, a_{n+4}\right) \mathcal{C}_{b}\left(a_{n+3}, a_{n+4}\right) \\
& +s^{4} p\left(a_{n+1}, a_{m}\right) p\left(a_{n+2}, a_{m}\right) p\left(a_{n+3}, a_{m}\right) \\
& \vdots \\
& +s^{i} p\left(a_{n+1}, a_{m}\right) p\left(a_{n+2}, a_{m}\right) p\left(a_{n+3}, a_{m}\right) \ldots . p\left(a_{n+i}, a_{m}\right) \mathfrak{C}_{b}\left(a_{n+i}, a_{m}\right)
\end{aligned}
$$

Therefore,

$$
\begin{align*}
& \mathfrak{C}_{b}\left(a_{n}, a_{m}\right) \leq s p\left(a_{n}, a_{n+1}\right) \mathfrak{C}_{b}\left(a_{n}, a_{n+1}\right)+\prod_{j=n+1}^{n+1} s p\left(a_{j}, a_{m}\right) s p\left(a_{n+1}, a_{n+2}\right) \mathfrak{C}_{b}\left(a_{n+1}, a_{n+2}\right) \\
& +\prod_{j=n+1}^{n+2}\left(s p\left(a_{j}, a_{m}\right)\right) s p\left(a_{n+2}, a_{n+3}\right) \mathcal{C}_{b}\left(a_{n+2}, a_{n+3}\right) \\
& +\prod_{j=n+1}^{n+3}\left(s p\left(a_{j}, a_{m}\right)\right) s p\left(a_{n+3}, a_{n+4}\right) \mathfrak{C}_{b}\left(a_{n+3}, a_{n+4}\right) \\
& +\prod_{j=n+1}^{n+4}\left(s p\left(a_{j}, a_{m}\right)\right) s p\left(a_{n+4}, a_{n+5}\right) \mathcal{C}_{b}\left(a_{n+4}, a_{n+5}\right) \\
& \vdots \\
& +\prod_{j=n+1}^{m-2}\left(s p\left(a_{j}, a_{m}\right)\right) s p\left(a_{m-2}, a_{m-1}\right) \mathcal{C}_{b}\left(a_{m-2}, a_{m-1}\right) \\
& \left.+\prod_{i=n+1}^{m-1}\left(s p\left(a_{i}, a_{m}\right)\right) \mathfrak{C}_{b}\left(a_{m-1}, a_{m}\right)\right) \\
& \leq \operatorname{sp}\left(a_{n}, a_{n+1}\right) \mathfrak{C}_{b}\left(a_{n}, a_{n+1}\right) \\
& +\sum_{i=n+1}^{m-2}\left(\prod_{j=n+1}^{i}\left(s p\left(a_{j}, a_{m}\right)\right)\right) s p\left(a_{i}, a_{i+1}\right) \mathcal{C}_{b}\left(a_{i}, a_{i+1}\right) \\
& \left.+\prod_{i=n+1}^{m-1}\left(s p\left(a_{i}, a_{m}\right)\right) \mathcal{C}_{b}\left(a_{m-1}, a_{m}\right)\right)  \tag{2.4}\\
& \leq \operatorname{sp}\left(a_{n}, a_{n+1}\right) k^{n} \mathcal{C}_{b}\left(a_{0}, a_{1}\right) \\
& +\sum_{i=n+1}^{m-2}\left(\prod_{j=n+1}^{i}\left(s p\left(a_{j}, a_{m}\right)\right)\right) s p\left(a_{i}, a_{i+1}\right) k^{i} \mathcal{C}_{b}\left(a_{0}, a_{1}\right) \\
& \left.+\prod_{i=n+1}^{m-1}\left(s p\left(a_{i}, a_{m}\right)\right) k^{m-1} \mathcal{C}_{b}\left(a_{0}, a_{1}\right)\right) \\
& \leq \operatorname{sp}\left(a_{n}, a_{n+1}\right) k^{n} \mathcal{C}_{b}\left(a_{0}, a_{1}\right) \\
& +\sum_{i=n+1}^{m-2}\left(\prod_{j=n+1}^{i}\left(s p\left(a_{j}, a_{m}\right)\right)\right) s p\left(a_{i}, a_{i+1}\right) k^{i} \mathcal{C}_{b}\left(a_{0}, a_{1}\right) \\
& \left.+\left(\prod_{i=n+1}^{m-1}\left(s p\left(a_{i}, a_{m}\right)\right)\right) s k^{m-1} p\left(a_{m-1}, a_{m}\right) \mathfrak{C}_{b}\left(a_{0}, a_{1}\right)\right) \\
& \leq s p\left(a_{n}, a_{n+1}\right) k^{n} \mathcal{C}_{b}\left(a_{0}, a_{1}\right) \\
& +\sum_{i=n+1}^{m-1}\left(\prod_{j=n+1}^{i}\left(s p\left(a_{j}, a_{m}\right)\right)\right) s p\left(a_{i}, a_{i+1}\right) k^{i} \bigotimes_{b}\left(a_{0}, a_{1}\right) \\
& \leq s p\left(a_{n}, a_{n+1}\right) k^{n} \mathcal{C}_{b}\left(a_{0}, a_{1}\right) \\
& +\sum_{i=n+1}^{m-1}\left(\prod_{j=0}^{i}\left(s p\left(a_{j}, a_{m}\right)\right)\right) s p\left(a_{i}, a_{i+1}\right) k^{i} \mathfrak{C}_{b}\left(a_{0}, a_{1}\right) .
\end{align*}
$$

Let us consider the sum

$$
\mathcal{S}_{l}=\sum_{i=0}^{l}\left(\prod_{j=0}^{i}\left(s p\left(a_{j}, a_{m}\right)\right)\right) s p\left(a_{i}, a_{i+1}\right) k^{i}
$$

From (2.4), we get

$$
\begin{equation*}
\mathcal{C}_{b}\left(a_{n}, a_{m}\right) \leq \mathcal{C}_{b}\left(a_{0}, a_{1}\right)\left[k^{n} s p\left(a_{n}, a_{n+1}\right)+\left(\mathcal{S}_{m-1}-\mathcal{S}_{n}\right)\right] . \tag{2.5}
\end{equation*}
$$

Since $p(a, b) \geq 1, s \geq 1$, and by applying ratio test, $\lim _{n \rightarrow \infty} \mathcal{S}_{n}$ exists and therefore the real sequence $\left\{\mathscr{S}_{n}\right\}$ is Cauchy.

In the end, if we consider the limit in (2.5) when $n, m \rightarrow \infty$, we infer that

$$
\begin{equation*}
\lim _{n, m \rightarrow \infty} \mathcal{C}_{b}\left(a_{n}, a_{m}\right)=0 \tag{2.6}
\end{equation*}
$$

Thus, $\left\{a_{n}\right\}$ is a Cauchy sequence in the complete controlled $b$-metric space $\left(X, \mathcal{C}_{b}\right)$. So $\exists \rho \in X \ni$,

$$
\begin{equation*}
\lim _{n \rightarrow \infty} \mathcal{C}_{b}\left(a_{n}, \rho\right)=0 \text {, i.e., } a_{n} \rightarrow \rho \text { when } n \rightarrow \infty \tag{2.7}
\end{equation*}
$$

Now, we have to verify $\rho$ is a fixed point of $\mathscr{H}$.
From condition (iii) in Definition 1.4,

$$
\mathfrak{C}_{b}\left(\rho, a_{n+1}\right) \leq s\left[p\left(\rho, a_{n}\right) \mathfrak{C}_{b}\left(\rho, a_{n}\right)+p\left(a_{n}, a_{n+1}\right) \mathcal{C}_{b}\left(a_{n}, a_{n+1}\right)\right]
$$

By using (2.2), (2.3), (2.6) and (2.7), we can deduce that

$$
\begin{equation*}
\lim _{n \rightarrow \infty} \mathcal{C}_{b}\left(\rho, a_{n+1}\right)=0 \tag{2.8}
\end{equation*}
$$

Again, by using the condition (iii) in Definition 1.4 and (2.1)

$$
\begin{aligned}
\mathfrak{C}_{b}(\rho, \mathscr{H} \rho) & \leq s\left[p\left(\rho, a_{n+1}\right) \mathfrak{C}_{b}\left(\rho, a_{n+1}\right)+p\left(a_{n+1}, \mathscr{H} \rho\right) \mathcal{C}_{b}\left(a_{n+1}, \mathscr{H} \rho\right)\right] \\
& \leq s\left[p\left(\rho, a_{n+1}\right) \mathfrak{C}_{b}\left(\rho, a_{n+1}\right)+p\left(a_{n+1}, \mathscr{H} \rho\right) k \mathfrak{C}_{b}\left(a_{n}, \rho\right)\right]
\end{aligned}
$$

Allowing $n \rightarrow \infty$ and applying (2.3), (2.7) and (2.8), we can easily deduce that $\mathcal{C}_{b}(\rho, \mathscr{H} \rho)=0$. This yields that $\rho=\mathscr{H} \rho$. We conclude that the proof is finished.

We illustrate the above theorem by the subsequent example.
Example 2.2 Assume $X=\{0,1,2\}$. Determine $p: X \times X \rightarrow[1, \infty)$ and $\mathcal{C}_{b}: X \times X \rightarrow[1, \infty)$ when $p(a, b)=1+a b$ and

$$
\begin{gathered}
\mathfrak{C}_{b}(2,2)=\mathfrak{C}_{b}(0,0)=\mathfrak{C}_{b}(1,1)=0 \\
\mathfrak{C}_{b}(2,0)=\mathfrak{C}_{b}(0,2)=5 ; \mathfrak{C}_{b}(0,1)=\mathfrak{C}_{b}(1,0)=10 \\
\mathfrak{C}_{b}(2,1)=\mathfrak{C}_{b}(1,2)=30 \text { and } s=2
\end{gathered}
$$

Now, define $\mathscr{H}: X \rightarrow X$ by

$$
\mathscr{H} a= \begin{cases}0, & \text { if } a \in\{0,2\} \\ 2, & \text { if } a=1\end{cases}
$$

and choose $k=0.9$.
Case I: If $a=0, b=1$, we have

$$
\mathcal{C}_{b}(\mathscr{H} a, \mathscr{H} b)=\mathcal{C}_{b}(\mathscr{H} 0, \mathscr{H} 1)=\mathcal{C}_{b}(0,2)=5
$$

Thus,

$$
\mathfrak{C}_{b}(\mathscr{H} a, \mathscr{H} b) \leq k \mathfrak{C}_{b}(a, b)
$$

Case II: If $a=0, b=2$, we have

$$
\mathcal{C}_{b}(\mathscr{H} a, \mathscr{H} b)=\mathcal{C}_{b}(\mathscr{H} 0, \mathscr{H} 2)=\mathcal{C}_{b}(0,0)=0
$$

So

$$
\mathfrak{C}_{b}(\mathscr{H} a, \mathscr{H} b) \leq k \mathfrak{C}_{b}(a, b)
$$

Case III: If $a=1, b=2$, we get

$$
\mathcal{C}_{b}(\mathscr{H} a, \mathscr{H} b)=\mathcal{C}_{b}(\mathscr{H} 1, \mathscr{H} 2)=\mathcal{C}_{b}(2,0)=5
$$

that is,

$$
\mathcal{C}_{b}(\mathscr{H} a, \mathscr{H} b) \leq k \mathfrak{C}_{b}(a, b)
$$

Case IV: If $a=b=0$ or $a=b=1$ or $a=b=2$, we get

$$
\mathcal{C}_{b}(\mathscr{H} a, \mathscr{H} b)=0 .
$$

Consequently,

$$
\mathcal{C}_{b}(\mathscr{H} a, \mathscr{H} b) \leq k \mathfrak{C}_{b}(a, b)
$$

$\forall a, b \in X$. Therefore, all the requirements of above theorem are fulfilled and $\mathscr{H}$ has a unique fixed point, which is, $a=0$.

We may discuss many facts as particular cases of Banach contraction by arranging the below different consecutive values in Definition 1.4.

## Special Cases:

- Suppose we assume $s=1$ in Theorem, then we can get Theorem 1 of Mlaiki et al. [26].
- If we take $p(a, c)=p(c, b)$ and $s=1$ in theorem, then we get Theorem 2 of Kamran et al. [20] (since we omit the strong hypothesis concerning the continuity of the extended metric imposed in [20] and it is replaced by a weak hypothesis, as condition (3)).
- If we take $p(a, c)=p(c, b)=1$ in theorem, then we can get Theorem 1 of Czerwik [10].
- If we take $p(a, c)=p(c, b)=s=1$ in above theorem, then it reducing to standard complete metric space.


## 3 On $\left(\omega-F_{\alpha}\right)$-contractions

We now present the being next definition for proving the main results:
Definition 3.1 Assume that $\left(X, \mathcal{C}_{b}\right)$ be a $\mathfrak{C}_{b}$-metric space. A mapping $\mathscr{H}: X \rightarrow X$ is called a $\left(\omega-F_{\mathcal{C}}\right)$-contraction on $X$ provided that $\exists \alpha: X \times X \rightarrow \mathbb{R}^{+}$and $\omega:(0, \infty) \rightarrow(0, \infty)$ such that
$\left(\mathcal{H}_{1}\right) \quad F_{\mathcal{C}}$ satisfies $\left(F_{1}\right),\left(F_{2}\right)$ and $\left(F_{3}\right)$;
$\left(\mathcal{H}_{2}\right) \quad \lim \inf _{s \rightarrow t^{+}} \boldsymbol{\omega}(s)>0$ for all $t \geq 0$;
$\left(\mathcal{H}_{3}\right) \omega(\mathcal{P}(a, b))+\alpha(a, b) F_{\mathcal{C}}\left(\mathcal{C}_{b}(\mathscr{H} a, \mathscr{H} b)\right) \leq F_{\mathcal{C}}(\mathcal{P}(a, b))$, where

$$
\mathcal{P}(a, b)=\max \left\{\mathfrak{C}_{b}(a, b), \mathfrak{C}_{b}(a, \mathscr{H} a), \mathcal{C}_{b}(b, \mathscr{H} b), \frac{\mathcal{C}_{b}(a, \mathscr{H} a) \cdot \mathfrak{C}_{b}(b, \mathscr{H} b)}{1+\mathcal{C}_{b}(a, b)}\right\}
$$

$\forall a, b \in X$ with $\mathcal{C}_{b}(\mathscr{H} a, \mathscr{H} b)>0$.
Theorem 3.2 Assume that $\left(X, \mathcal{C}_{b}\right)$ be a complete $\mathcal{C}_{b}$-metric space and $\mathscr{H}: X \rightarrow X$ be a $\left(\omega-F_{\mathfrak{C}}\right)$-contraction fulfilling the subsequent properties:
(I) $\mathscr{H}$ is $\alpha$-admissible;
(II) $\exists a_{0} \in X \ni \alpha\left(a_{0}, \mathscr{H} a_{0}\right) \geq 1$;
(III) $\mathscr{H}$ is continuous;
(IV) $\sup _{p \geq 0} \lim _{i \rightarrow \infty}\left\{\frac{\mathfrak{C}_{b}\left(a_{i+1}, a_{i+2}\right) \mathfrak{C}_{b}\left(a_{i+1}, a_{n+p}\right)}{\mathcal{C}_{b}\left(a_{i}, a_{i+1}\right)}\right\}<1$.

Then $\mathscr{H}$ has a fixed point.
Proof: Assume that $a_{0} \in X \ni \alpha\left(a_{0}, \mathscr{H} a_{0}\right) \geq 1$. We determine $\left\{a_{n}\right\}$ in $X$ by $a_{n+1}=\mathscr{H} a_{n} \forall n \in \mathbb{N}$. Clearly, if $\exists n_{0} \in \mathbb{N}$ for which $a_{n_{0}+1}=a_{n_{0}}$, then $\mathscr{H} a_{n_{0}}=a_{n_{0}}$ and the proof is finished. Hence, we assume $a_{n+1} \neq a_{n}$ for $n \in \mathbb{N}$. By applying (I) and (II), clearly

$$
\alpha\left(\mathscr{H} a_{n}, \mathscr{H} a_{n+1}\right) \geq 1 \forall n \in \mathbb{N} .
$$

It gives that

$$
\begin{aligned}
F_{\mathcal{C}}\left(\mathcal{C}_{b}\left(a_{n+1}, a_{n}\right)\right) & =F_{\mathcal{C}}\left(\mathcal{C}_{b}\left(\mathscr{H} a_{n}, \mathscr{H} a_{n-1}\right)\right) \\
& \leq \alpha\left(a_{n}, a_{n-1}\right) F_{\mathcal{C}}\left(\mathcal{C}_{b}\left(\mathscr{H} a_{n}, \mathscr{H} a_{n-1}\right)\right) .
\end{aligned}
$$

Since $\mathscr{H}$ is a $\left(\omega-F_{\alpha}\right)$-contraction, for every $n \in \mathbb{N}$, we can write

$$
\begin{align*}
& \omega\left(\mathcal{C}_{b}\left(a_{n}, a_{n-1}\right)\right)+F_{\mathcal{C}}\left(\mathcal{C}_{b}\left(a_{n+1}, a_{n}\right)\right) \\
& \leq \boldsymbol{\omega}\left(\mathcal{C}_{b}\left(a_{n}, a_{n-1}\right)\right)+\mathcal{C}_{b}\left(a_{n}, a_{n-1}\right) F_{\mathbb{C}}\left(\mathcal{C}_{b}\left(\mathscr{H} a_{n}, \mathscr{H} a_{n-1}\right)\right) \\
& \leq F_{\mathcal{C}}\left(\mathcal{P}\left(a_{n}, a_{n-1}\right)\right) \\
& \leq F_{\mathcal{C}}\left(\max \left\{\mathcal{C}_{b}\left(a_{n}, a_{n-1}\right), \mathfrak{C}_{b}\left(a_{n}, \mathscr{H} a_{n}\right), \mathcal{C}_{b}\left(a_{n-1}, \mathscr{H} a_{n-1}\right), \frac{\mathcal{C}_{b}\left(a_{n}, \mathscr{H} a_{n}\right) \cdot \mathcal{C}_{b}\left(a_{n-1}, \mathscr{H} a_{n-1}\right)}{1+\mathcal{C}_{b}\left(a_{n}, a_{n-1}\right)}\right\}\right)  \tag{3.1}\\
& =F_{\mathcal{C}}\left(\max \left\{\mathfrak{C}_{b}\left(a_{n}, a_{n-1}\right), \mathfrak{C}_{b}\left(a_{n}, a_{n+1}\right), \mathfrak{C}_{b}\left(a_{n-1}, a_{n}\right), \frac{\mathfrak{C}_{b}\left(a_{n}, a_{n+1}\right) \cdot \mathfrak{C}_{b}\left(a_{n-1}, a_{n}\right)}{1+\mathfrak{C}_{b}\left(a_{n}, a_{n-1}\right)}\right\}\right) \\
& =F_{\mathcal{C}}\left(\max \left\{\mathcal{C}_{b}\left(a_{n}, a_{n-1}\right), \mathcal{C}_{b}\left(a_{n}, a_{n+1}\right)\right\}\right) .
\end{align*}
$$

If there exists $n \in \mathbb{N}$ such that $\max \left\{\mathcal{C}_{b}\left(a_{n}, a_{n-1}\right), \mathfrak{C}_{b}\left(a_{n}, a_{n+1}\right)\right\}=\mathcal{C}_{b}\left(a_{n}, a_{n+1}\right)$, then (3.1) becomes

$$
\begin{equation*}
\omega\left(\mathcal{C}_{b}\left(a_{n}, a_{n-1}\right)\right)+F_{\mathcal{C}}\left(\mathcal{C}_{b}\left(a_{n+1}, a_{n}\right)\right) \leq F_{\mathcal{C}}\left(\mathcal{C}_{b}\left(a_{n}, a_{n+1}\right)\right), \tag{3.2}
\end{equation*}
$$

which is a contradiction due to $\left(\mathcal{H}_{2}\right)$. Therefore $\max \left\{\mathcal{C}_{b}\left(a_{n}, a_{n-1}\right), \mathfrak{C}_{b}\left(a_{n}, a_{n+1}\right)\right\}=\mathcal{C}_{b}\left(a_{n}, a_{n-1}\right)$ for all $n \in \mathbb{N}$. Thus from (3.1), we get

$$
\omega\left(\mathcal{C}_{b}\left(a_{n}, a_{n-1}\right)\right)+F_{\mathcal{C}}\left(\mathcal{C}_{b}\left(a_{n+1}, a_{n}\right)\right) \leq F_{\mathcal{C}}\left(\mathcal{C}_{b}\left(a_{n}, a_{n-1}\right)\right) \text { for all } n \in \mathbb{N} .
$$

Thus,

$$
F_{\mathcal{C}}\left(\mathfrak{C}_{b}\left(a_{n+1}, a_{n}\right)\right) \leq F_{\mathcal{C}}\left(\mathfrak{C}_{b}\left(a_{n}, a_{n-1}\right)\right)-\omega\left(\mathcal{C}_{b}\left(a_{n}, a_{n-1}\right)\right) \forall n \in \mathbb{N} .
$$

By referring (3.2) and $\left(F_{1}\right)$, we obtain $\mathcal{C}_{b}\left(a_{n}, a_{n+1}\right)$ is decreasing and therefore $\mathcal{C}_{b}\left(a_{n}, a_{n+1}\right) \searrow t, t \geq 0$. In view of $\left(\mathcal{H}_{2}\right), \exists e>0$ and $n_{0} \in \mathbb{N} \ni \omega\left(\mathcal{C}_{b}\left(a_{n}, a_{n+1}\right)\right)>e$ for each $n \geq n_{0}$. Consider

$$
\begin{align*}
& F_{\mathcal{C}}\left(\mathcal{C}_{b}\left(a_{n}, a_{n+1}\right)\right) \leq F_{\mathcal{C}}\left(\mathcal{C}_{b}\left(a_{n-1}, a_{n}\right)\right)-\omega\left(\mathcal{C}_{b}\left(a_{n-1}, a_{n}\right)\right) \\
& \leq F_{\mathcal{C}}\left(\mathcal{C}_{b}\left(a_{n-2}, a_{n-1}\right)\right)-\omega\left(\mathcal{C}_{b}\left(a_{n-2}, a_{n-1}\right)\right)-\omega\left(\mathcal{C}_{b}\left(a_{n-1}, a_{n}\right)\right) \\
& \leq F_{\mathcal{C}}\left(\mathcal{C}_{b}\left(a_{n-3}, a_{n-2}\right)\right)-\omega\left(\mathcal{C}_{b}\left(a_{n-3}, a_{n-2}\right)\right)-\omega\left(\mathcal{C}_{b}\left(a_{n-2}, a_{n-1}\right)\right)-\omega\left(\mathcal{C}_{b}\left(a_{n-1}, a_{n}\right)\right) \\
& \vdots  \tag{3.3}\\
& \leq F_{\mathcal{C}}\left(\mathcal{C}_{b}\left(a_{0}, a_{1}\right)\right)-\omega\left(\mathcal{C}_{b}\left(a_{0}, a_{1}\right)\right)-\omega\left(\mathcal{C}_{b}\left(a_{1}, a_{2}\right)\right)-\ldots . .-\omega\left(\mathcal{C}_{b}\left(a_{n-1}, a_{n}\right)\right) \\
& =F_{\mathfrak{C}}\left(\mathfrak{C}_{b}\left(a_{0}, a_{1}\right)\right)-\sum_{i=1}^{n} \omega\left(\mathfrak{C}_{b}\left(a_{i-1}, a_{i}\right)\right) .
\end{align*}
$$

From (3.3), we get

$$
\begin{equation*}
F_{\mathcal{C}}\left(\mathcal{C}_{b}\left(a_{n}, a_{n+1}\right)\right) \leq F_{\mathcal{C}}\left(\mathcal{C}_{b}\left(a_{0}, a_{1}\right)\right)-n e . \tag{3.4}
\end{equation*}
$$

Letting $n \rightarrow \infty$ in (3.4), we get

$$
F_{\mathcal{C}}\left(\mathcal{C}_{b}\left(a_{n}, a_{n+1}\right)\right) \rightarrow-\infty .
$$

By $\left(F_{2}\right)$,

$$
\begin{equation*}
\mathcal{C}_{b}\left(a_{n}, a_{n+1}\right) \rightarrow 0, \text { as } n \rightarrow \infty . \tag{3.5}
\end{equation*}
$$

To show that $\left\{a_{n}\right\}$ is a Cauchy sequence, from $\left(F_{3}\right), \exists k \in(0,1) \ni$,

$$
\begin{equation*}
\lim _{n \rightarrow \infty}\left(\mathcal{C}_{b}\left(a_{n}, a_{n+1}\right)\right)^{k} F_{\mathcal{C}}\left(\mathcal{C}_{b}\left(a_{n}, a_{n+1}\right)\right)=0 . \tag{3.6}
\end{equation*}
$$

By (3.4), the being next holds $\forall n \in \mathbb{N}$.

$$
\begin{align*}
& \left(\mathcal { C } _ { b } \left(a_{n},\right.\right. \\
& \left.\left.\quad a_{n+1}\right)\right)^{k} F_{\mathcal{C}}\left(\mathcal{C}_{b}\left(a_{n}, a_{n+1}\right)\right)-\left(\mathcal{C}_{b}\left(a_{n}, a_{n+1}\right)\right)^{k} F_{\mathcal{C}}\left(\mathfrak{C}_{b}\left(a_{0}, a_{1}\right)\right)  \tag{3.7}\\
& \quad \leq\left(\mathfrak{C}_{b}\left(a_{n}, a_{n+1}\right)\right)^{k}\left(F_{\mathcal{C}}\left(\mathcal{C}_{b}\left(a_{0}, a_{1}\right)\right)-n e\right)-\left(\mathfrak{C}_{b}\left(a_{n}, a_{n+1}\right)\right)^{k} F_{\mathcal{C}}\left(\mathfrak{C}_{b}\left(a_{0}, a_{1}\right)\right) \\
& \quad=-\left(\mathfrak{C}_{b}\left(a_{n}, a_{n+1}\right)\right)^{k} n e \\
& \quad \leq 0 .
\end{align*}
$$

Letting $n \rightarrow \infty$ in (3.7) and using (3.5) and (3.6), we obtain

$$
\begin{equation*}
\lim _{n \rightarrow \infty} n\left(\mathcal{C}_{b}\left(a_{n}, a_{n+1}\right)\right)^{k}=0 \tag{3.8}
\end{equation*}
$$

By referring (3.8), $\exists n_{1} \in \mathbb{N} \ni n\left(\mathcal{C}_{b}\left(a_{n}, a_{n+1}\right)\right)^{k} \leq 1 \forall n \geq n_{1}$. Consequently, we have

$$
\mathfrak{C}_{b}\left(a_{n}, a_{n+1}\right) \leq \frac{1}{n^{\frac{1}{k}}} ; \text { for all } n \geq n_{1} .
$$

We will verify $\left\{a_{n}\right\}_{n \in \mathbb{N}}$ is a Cauchy sequence. Consider the triangle inequality for $q \geq 1$,

$$
\begin{aligned}
& \mathcal{C}_{b}\left(a_{n}, a_{n+q}\right) \leq s\left[p\left(a_{n}, a_{n+1}\right) \mathfrak{C}_{b}\left(a_{n}, a_{n+1}\right)+p\left(a_{n+1}, a_{n+q}\right) \mathfrak{C}_{b}\left(a_{n+1}, a_{n+q}\right)\right] \\
& \leq s p\left(a_{n}, a_{n+1}\right) \mathcal{C}_{b}\left(a_{n}, a_{n+1}\right)+s p\left(a_{n+1}, a_{n+q}\right) \mathcal{C}_{b}\left(a_{n+1}, a_{n+q}\right) \\
& \leq s p\left(a_{n}, a_{n+1}\right) \mathcal{C}_{b}\left(a_{n}, a_{n+1}\right)+s p\left(a_{n+1}, a_{n+q}\right)\left[s \left\{p\left(a_{n+1}, a_{n+2}\right) \mathfrak{C}_{b}\left(a_{n+1}, a_{n+2}\right)\right.\right. \\
& \left.\left.+p\left(a_{n+2}, a_{n+q}\right) \mathcal{C}_{b}\left(a_{n+2}, a_{n+q}\right)\right\}\right] \\
& \leq s p\left(a_{n}, a_{n+1}\right) \mathcal{C}_{b}\left(a_{n}, a_{n+1}\right)+s^{2} p\left(a_{n+1}, a_{n+q}\right) p\left(a_{n+1}, a_{n+2}\right) \mathcal{C}_{b}\left(a_{n+1}, a_{n+2}\right) \\
& +s^{2} p\left(a_{n+1}, a_{n+q}\right) p\left(a_{n+2}, a_{n+q}\right) \mathcal{C}_{b}\left(a_{n+2}, a_{n+p}\right) \\
& \leq s p\left(a_{n}, a_{n+1}\right) \mathfrak{C}_{b}\left(a_{n}, a_{n+1}\right)+s^{2} p\left(a_{n+1}, a_{n+q}\right) p\left(a_{n+1}, a_{n+2}\right) \mathfrak{C}_{b}\left(a_{n+1}, a_{n+2}\right) \\
& +s^{3} p\left(a_{n+1}, a_{n+q}\right) p\left(a_{n+2}, a_{n+q}\right) p\left(a_{n+3}, a_{n+q}\right) \mathfrak{C}_{b}\left(a_{n+3}, a_{n+p}\right)+\ldots \\
& \leq s p\left(a_{n}, a_{n+1}\right) \mathfrak{C}_{b}\left(a_{n}, a_{n+1}\right)+s^{2} p\left(a_{n+1}, a_{m}\right) p\left(a_{n+1}, a_{n+2}\right) \mathfrak{C}_{b}\left(a_{n+1}, a_{n+2}\right) \\
& +s^{3} p\left(a_{n+1}, a_{m}\right) p\left(a_{n+2}, a_{m}\right) p\left(a_{n+2}, a_{n+3}\right) \mathcal{C}_{b}\left(a_{n+2}, a_{n+3}\right) \\
& +s^{4} p\left(a_{n+1}, a_{m}\right) p\left(a_{n+2}, a_{m}\right) p\left(a_{n+3}, a_{m}\right) p\left(a_{n+3}, a_{n+4}\right) \mathfrak{C}_{b}\left(a_{n+3}, a_{n+4}\right) \\
& +s^{4} p\left(a_{n+1}, a_{m}\right) p\left(a_{n+2}, a_{m}\right) p\left(a_{n+3}, a_{m}\right) p\left(a_{n+4}, a_{m}\right) \mathfrak{C}_{b}\left(a_{n+4}, a_{m}\right) \\
& \vdots \\
& \leq s p\left(a_{n}, a_{n+1}\right) \mathfrak{C}_{b}\left(a_{n}, a_{n+1}\right)+s^{2} p\left(a_{n+1}, a_{m}\right) p\left(a_{n+1}, a_{n+2}\right) \mathcal{C}_{b}\left(a_{n+1}, a_{n+2}\right) \\
& +s^{3} p\left(a_{n+1}, a_{m}\right) p\left(a_{n+2}, a_{m}\right) p\left(a_{n+2}, a_{n+3}\right) \mathfrak{C}_{b}\left(a_{n+2}, a_{n+3}\right) \\
& +s^{4} p\left(a_{n+1}, a_{m}\right) p\left(a_{n+2}, a_{m}\right) p\left(a_{n+3}, a_{m}\right) p\left(a_{n+3}, a_{n+4}\right) \mathcal{C}_{b}\left(a_{n+3}, a_{n+4}\right) \\
& +s^{4} p\left(a_{n+1}, a_{m}\right) p\left(a_{n+2}, a_{m}\right) p\left(a_{n+3}, a_{m}\right) \\
& \vdots \\
& +s^{i} p\left(a_{n+1}, a_{m}\right) p\left(a_{n+2}, a_{m}\right) p\left(a_{n+3}, a_{m}\right) \ldots . p\left(a_{n+i}, a_{m}\right) \mathfrak{C}_{b}\left(a_{n+i}, a_{m}\right) \\
& \leq s p\left(a_{n}, a_{n+1}\right) \mathcal{C}_{b}\left(a_{n}, a_{n+1}\right)+\left(\prod_{j=n+1}^{n+1} s p\left(a_{j}, a_{n+p}\right)\right)\left(s p\left(a_{n+1}, a_{n+2}\right) \mathcal{C}_{b}\left(a_{n+1}, a_{n+2}\right)\right) \\
& +\left(\prod_{j=n+1}^{n+2} s p\left(a_{j}, a_{n+q}\right)\right)\left(s p\left(a_{n+2}, a_{n+3}\right) \mathcal{C}_{b}\left(a_{n+2}, a_{n+3}\right)\right) \\
& +\left(\prod_{j=n+1}^{n+3} s p\left(a_{j}, a_{n+q}\right)\right)\left(s p\left(a_{n+3}, a_{n+4}\right) \mathcal{C}_{b}\left(a_{n+3}, a_{n+4}\right)\right)+\ldots \\
& +\left(\prod_{j=n+1}^{n+q-2} s p\left(a_{j}, a_{n+q}\right)\right)\left(s p\left(a_{n+q-2}, a_{n+q-1}\right) \mathfrak{C}_{b}\left(a_{n+q-2}, a_{n+q-1}\right)\right) \\
& \left.+\left(\prod_{i=n+1}^{n+q-1} s p\left(a_{i}, a_{n+q}\right)\right) \mathcal{C}_{b}\left(a_{n+q-1}, a_{n+q}\right)\right) \\
& \leq s p\left(a_{n}, a_{n+1}\right) \mathcal{C}_{b}\left(a_{n}, a_{n+1}\right)+\sum_{i=n+1}^{n+q-2}\left(\prod_{j=n+1}^{i} s p\left(a_{j}, a_{n+q}\right)\right)\left(s p\left(a_{i}, a_{i+1}\right) \mathcal{C}_{b}\left(a_{i}, a_{i+1}\right)\right) \\
& \left.+\prod_{i=n+1}^{n+q-1} s p\left(a_{i}, a_{n+q}\right) \mathfrak{C}_{b}\left(a_{n+q-1}, a_{n+q}\right)\right) \\
& \leq s p\left(a_{n}, a_{n+1}\right) \mathcal{C}_{b}\left(a_{n}, a_{n+1}\right)+\sum_{i=n+1}^{n+q-2}\left(\prod_{j=n+1}^{i} s p\left(a_{j}, a_{n+q}\right)\right)\left(s p\left(a_{i}, a_{i+1}\right) \mathcal{C}_{b}\left(a_{i}, a_{i+1}\right)\right) \\
& \left.+\left(\prod_{i=n+1}^{n+q-1} s p\left(a_{i}, a_{n+q}\right)\right) \mathfrak{C}_{b}\left(a_{n+q-1}, a_{n+q}\right)\right) .
\end{aligned}
$$

Thus,

$$
\begin{align*}
\mathcal{C}_{b}\left(a_{n}, a_{n+q}\right) \leq & s p\left(a_{n}, a_{n+1}\right) \mathfrak{C}_{b}\left(a_{n}, a_{n+1}\right)+\sum_{i=n+1}^{n+q-2}\left(\prod_{j=n+1}^{i} s p\left(a_{j}, a_{n+q}\right)\right)\left(s p\left(a_{i}, a_{i+1}\right) \mathcal{C}_{b}\left(a_{i}, a_{i+1}\right)\right) \\
& +\left(\prod_{i=n+1}^{n+q-1} s p\left(a_{i}, a_{n+q}\right)\right)\left(s p\left(a_{n+q-1}, a_{n+q}\right) \mathfrak{C}_{b}\left(a_{n+q-1}, a_{n+q}\right)\right) \\
\leq & s p\left(a_{n}, a_{n+1}\right) \mathfrak{C}_{b}\left(a_{n}, a_{n+1}\right)+\sum_{i=n+1}^{n+q-1}\left(\prod_{j=n+1}^{i} s p\left(a_{j}, a_{n+q}\right)\right)\left(s p\left(a_{i}, a_{i+1}\right) \mathcal{C}_{b}\left(a_{i}, a_{i+1}\right)\right)  \tag{3.9}\\
\leq & s p\left(a_{n}, a_{n+1}\right) \mathfrak{C}_{b}\left(a_{n}, a_{n+1}\right)+\sum_{i=n+1}^{n+q-1}\left(\prod_{j=0}^{i} s p\left(a_{j}, a_{n+q}\right)\right) s p\left(a_{i}, a_{i+1}\right) \mathfrak{C}_{b}\left(a_{i}, a_{i+1}\right) \\
\leq & s p\left(a_{n}, a_{n+1}\right) \mathfrak{C}_{b}\left(a_{n}, a_{n+1}\right)+\sum_{i=n+1}^{n+q-1}\left(\prod_{j=0}^{i} s p\left(a_{j}, a_{n+q}\right)\right) s p\left(a_{i}, a_{i+1}\right) \frac{1}{i^{\frac{1}{k}}} .
\end{align*}
$$

Now, consider

$$
\begin{aligned}
\sum_{i=n+1}^{n+q-1}\left(\prod_{j=0}^{i} s p\left(a_{j}, a_{n+q}\right)\right) & s p\left(a_{i}, a_{i+1}\right) \frac{1}{i^{\frac{1}{k}}} \\
& =\sum_{i=n+1}^{n+q-1} \frac{1}{i^{\frac{1}{k}}}\left(\prod_{j=0}^{i} s p\left(a_{j}, a_{n+q}\right)\right) s p\left(a_{i}, a_{i+1}\right) \\
& =\sum_{i=n+1}^{\infty} \frac{1}{i^{\frac{1}{k}}}\left(\prod_{j=0}^{i} s p\left(a_{j}, a_{n+q}\right)\right) s p\left(a_{i}, a_{i+1}\right) \\
& =\sum_{i=n+1}^{\infty} U_{i} V_{i}
\end{aligned}
$$

where

$$
U_{i}=\frac{1}{i^{\frac{1}{k}}}
$$

and

$$
V_{i}=\left(\prod_{j=0}^{i} s p\left(a_{j}, a_{n+q}\right)\right) s p\left(a_{i}, a_{i+1}\right)
$$

Because $\frac{1}{k}>1, \sum_{i=n+1}^{\infty} \frac{1}{i^{\frac{1}{k}}}$ converges and additionally $V_{i}=\left(\prod_{j=0}^{i} s p\left(a_{j}, a_{n+q}\right)\right) s p\left(a_{i}, a_{i+1}\right)$ is increasing and is bounded above. Therefore, $\lim _{i \rightarrow \infty}\left\{V_{i}\right\}=\sup \left(V_{i}\right)$, which is non zero and exists. Therefore, the product $\prod_{j=0}^{i} s p\left(a_{j}, a_{n+q}\right) s p\left(a_{i}, a_{i+1}\right)$ converges.

Therefore, $\sum_{i=n+1}^{\infty} U_{i} V_{i}$ converges.
Assume that the partial sum

$$
\mathcal{S}_{q}=\sum_{i=0}^{q}\left(\prod_{j=0}^{i} s p\left(a_{j}, a_{n+q}\right)\right) s p\left(a_{i}, a_{i+1}\right) \frac{1}{i^{\frac{1}{k}}} .
$$

Now, from (3.9),

$$
\begin{equation*}
\mathcal{C}_{b}\left(a_{n}, a_{n+q}\right) \leq s p\left(a_{n}, a_{n+1}\right) \mathcal{C}_{b}\left(a_{n}, a_{n+1}\right)+\left(\mathcal{S}_{n+q-1}-\mathcal{S}_{n}\right) . \tag{3.10}
\end{equation*}
$$

Letting $n \rightarrow \infty$ in (3.10) and using (3.5), we get

$$
\lim _{n \rightarrow \infty} \mathcal{C}_{b}\left(a_{n}, a_{n+q}\right)=0
$$

Thus, $\left\{a_{n}\right\}$ is a Cauchy sequence. Since $X$ is a complete $\mathfrak{C}_{b}$-metric space, $\exists \lambda \in X \ni$,

$$
\lim _{n \rightarrow \infty} \mathcal{C}_{b}\left(a_{n}, \lambda\right)=0
$$

In view of the assumptions that the mapping $T$ and the controlled $b$-metric are continuous, since $a_{n} \rightarrow \lambda$, we have that $\mathscr{H} a_{n} \rightarrow \mathscr{H} \lambda$ and hence we have

$$
\lim _{n \rightarrow \infty} \mathcal{C}_{b}\left(\mathscr{H} a_{n}, \mathscr{H} \lambda\right)=0=\lim _{n \rightarrow \infty} \mathcal{C}_{b}\left(a_{n+1}, \mathscr{H} \lambda\right)=\mathcal{C}_{b}(\lambda, \mathscr{H} \lambda),
$$

and hence $\mathscr{H} \lambda=\lambda$. Thus $\lambda$ is a fixed point of $\mathscr{H}$.
We may discuss many facts as particular cases on $\left(\omega-F_{\mathcal{C}}\right)$-contractions by arranging the following various consecutive values in Definition 1.4.

## Special Cases:

- If we take $s=1$ in Theorem 3.2, then Theorem 3.2 reduces to a controlled metric type space, as in [26].
- If we take $p(a, c)=p(c, b)$ and $s=1$ in Theorem 3.2, then Theorem 3.2 reduces to an extended $b$-metric space, as in [20].
- If we take $p(a, c)=p(c, b)=1$ in Theorem 3.2, then Theorem 3.2 reduces to a $b$-metric space.


## 4 Applications to our results in the pursuit of 2019-nCoV modelling

There is regularly a compromise between oversimplified or key models in the mathematical displaying of ailment transmission, as in most different regions of numerical demonstrating, which exclude a large portion of the points of interest and are planned principally to show general subjective conduct, and mind-boggling or strategic models, normally intended for specific conditions, including transient quantitative expectations. All in all, definite models are troublesome or difficult to logically unravel and their handiness is consequently restricted for hypothetical purposes, in spite of the fact that their key worth might be high. For example, extremely oversimplified pestilence models anticipate that after some time, a plague will cease to exist, leaving a segment of the populace unaffected by contamination, and this is likewise valid for models that include insurance measures. This factual hypothesis isn't, without anyone else, extremely fruitful in surveying what well-being measures in a given situation will be the best, yet it implies that an exhaustive model might be valuable for general well-being experts to report the reason as correctly as could reasonably be expected. Specialist based models, which partition the network into people or gatherings of individuals with comparable activities, are the benchmark in thorough models.

One of the significant reasons for death overall keeps on being irresistible illnesses (adding to 26 percent of worldwide mortality in 2001; WHO, 2002). With the unexpected ascent in SARS-CoV episodes in 2003, MersCoV in 2012, Ebola in 2014, and now SARS-CoV-2: bringing up issues about regular mental fighting as well as developing worries about natural animosity, sickness demonstration has gotten applicable to assume a part in network well being methodology making. Mathematics and additionally logical models of irresistible maladies may help us to comprehend the idea of ailments and the pace of transmission. To build up and break down various mediation techniques to evade or fortify defilement and to all the more likely appropriate available assets (for instance, choosing the objective populace, time for intercession, and area), models regularly empower us to reauthorize the spread of ailments in different viewpoints and angles. Note that the utilization of mathematical modeling in the evaluation of epidemiological illnesses is developing drastically. In this interest, we present the presence of answers for the new 2019-nCoV /SARS-CoV-2 novel Coronavirus fractional order models.

### 4.1 Existence of the solution under the setting of complete metric space:

In this section, the mathematical model capable of depicting the spread of the Coronavirus-19 was recently suggested by Baleanu et al. [9], which model has considered several variables in the setting of Caputo-Fabrizio fractional derivative in the following way:

$$
\left.\begin{array}{l}
\mathscr{C F} \mathcal{D}^{\wp} \mathcal{S}_{p}=\Pi_{p}-a_{p} \mathcal{S}_{p}-\frac{b_{p} \mathcal{S}_{p}\left(\mathcal{J}_{p}+\Psi \mathcal{A}_{p}\right)}{\mathcal{N}_{p}}-b_{w} \mathcal{S}_{p} \mathcal{M} ;  \tag{4.1}\\
\mathscr{C} \mathscr{F} \mathcal{D}^{\wp} \mathcal{E}_{p}=\frac{b_{p} \mathcal{S}_{p}\left(\mathcal{J}_{p}+\Psi \mathcal{A}_{p}\right)}{\mathcal{N}_{p}}+b_{w} \mathcal{S}_{p} \mathcal{M}-\left(1-\mathrm{Y}_{p}\right) \omega_{p} \mathcal{E}_{p}-\mathrm{Y}_{p} \xi_{p} \varepsilon_{p}-a_{p} \varepsilon_{p} ; \\
\mathscr{C} \mathscr{F} \mathcal{D}^{\wp} \mathcal{J}_{p}=\left(1-\mathrm{Y}_{p}\right) \omega_{p} \mathcal{E}_{p}-\left(\tau_{p}+a_{p}\right) \mathcal{J}_{p} ; \\
\mathscr{C} \mathscr{F} \mathcal{D}^{\wp} \mathcal{A}_{p}=\mathrm{Y}_{p} \xi_{p} \mathcal{\varepsilon}_{p}-\left(\tau_{\vartheta p}+a_{p}\right) \mathcal{A}_{p} ; \\
\mathscr{C} \mathscr{F} \mathcal{D}^{\wp} \mathcal{R}_{p}=\tau_{p} \mathcal{J}_{p}+\tau_{\vartheta p} \mathcal{A}_{p}-a_{p} \mathcal{R}_{p} ; \\
\mathscr{C} \mathscr{F} \mathcal{D}^{\wp \mathcal{M}}=c_{p} \mathcal{J}_{p}+e_{p} \mathcal{A}_{p}-\pi \mathcal{N} .
\end{array}\right\} .
$$

By applying fractional integral operator, we convert model(4.1) into below integral form:

$$
\begin{align*}
& \mathcal{S}_{p}(t)-\mathcal{S}_{p}(0)=\mathscr{C} \mathscr{F} \mathscr{\mathscr { F }} \wp\left[\Pi_{p}-a_{p} \mathcal{S}_{p}-\frac{b_{p} \mathcal{S}_{p}\left(\mathcal{J}_{p}+\Psi \mathcal{A}_{p}\right)}{\mathcal{N}_{p}}-b_{w} \mathcal{S}_{p} \mathcal{N}\right] ; \\
& \varepsilon_{p}(t)-\varepsilon_{p}(0)={ }^{\mathscr{C} F} \mathscr{I} \wp\left[\frac{b_{p} \mathcal{S}_{p}\left(\mathcal{J}_{p}+\Psi \mathcal{A}_{p}\right)}{\mathcal{N}_{p}}+b_{w} \delta_{p} \mathcal{M}-\left(1-\mathrm{Y}_{p}\right) \omega_{p} \varepsilon_{p}-\mathrm{Y}_{p} \xi_{p} \varepsilon_{p}-a_{p} \varepsilon_{p}\right] ; \\
& \mathcal{J}_{p}(t)-\mathcal{J}_{p}(0)=\mathscr{C} \mathscr{F} \mathscr{I} \mathscr{J}^{\wp}\left[\left(1-\mathrm{Y}_{p}\right) \omega_{p} \varepsilon_{p}-\left(\tau_{p}+a_{p}\right) \mathcal{J}_{p}\right] ; \\
& \mathcal{A}_{p}(t)-\mathcal{A}_{p}(0)={ }^{\mathscr{C} \mathscr{F}} \mathscr{I} \mathscr{I}^{\wp}\left[\mathrm{Y}_{p} \xi_{p} \mathcal{E}_{p}-\left(\tau_{\vartheta p}+a_{p}\right) \mathcal{A}_{p}\right] ;  \tag{4.2}\\
& \mathcal{R}_{p}(t)-\mathcal{R}_{p}(0)={ }^{\mathscr{C} \mathscr{F}} \mathscr{I} \wp\left[\tau_{p} \mathcal{J}_{p}+\tau_{\vartheta p} \mathcal{A}_{p}-a_{p} \mathcal{R}_{p}\right] ; \\
& \mathcal{M}_{p}(t)-\mathcal{M}_{p}(0)=\mathscr{C} \mathscr{F} \quad \mathscr{I} \wp\left[c_{p} \mathcal{J}_{p}+e_{p} \mathcal{A}_{p}-\pi \mathcal{M}\right] .
\end{align*}
$$

Let $S=\{\mathbb{A} \in \mathcal{C}(I, \mathbb{R}): \mathbb{A}(\theta)>0$ for all $\theta \in I=[0, \mathscr{H}], \mathscr{H}>0\}$.
Define a mapping $\mathscr{D}: \mathrm{S} \times \mathrm{S} \rightarrow[0, \infty)$ as $\mathscr{D}(u, v)=|u-v|$ for all $u, v \in \mathrm{~S}$. Then $(\mathrm{S}, \mathscr{D})$ is a complete metric space. Define a mapping $\mathscr{H}: S \rightarrow \mathrm{~S}$ by,

$$
\left.\begin{array}{l}
\mathscr{H} \mathcal{S}_{p}(\theta)=\frac{2-2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)} \mathbb{A}\left(\theta, \mathcal{S}_{p}(\theta)\right)+\frac{2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)} \int_{0}^{\theta} \mathbb{A}\left(v, \mathcal{S}_{p}(v)\right) d v \\
\mathscr{H} \varepsilon_{p}(\theta)=\frac{2-2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)} \mathbb{B}\left(\theta, \mathcal{E}_{p}(\theta)\right)+\frac{2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)} \int_{0}^{\theta} \mathbb{B}\left(v, \varepsilon_{p}(v)\right) d v \\
\mathscr{H} \mathcal{J}_{p}(\theta)=\frac{2-2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)} \mathbb{C}\left(\theta, \mathcal{J}_{p}(\theta)\right)+\frac{2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)} \int_{0}^{\theta} \mathbb{C}\left(v, \mathcal{J}_{p}(v)\right) d v \\
\mathscr{H} \mathcal{A}_{p}(\theta)=\frac{2-2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)} \mathbb{D}\left(\theta, \mathcal{A}_{p}(\theta)\right)+\frac{2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)} \int_{0}^{\theta} \mathbb{D}\left(v, \mathcal{A}_{p}(v)\right) d v  \tag{4.3}\\
\mathscr{H} \mathcal{R}_{p}(\theta)=\frac{2-2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)} \mathbb{E}\left(\theta, \mathcal{R}_{p}(\theta)\right)+\frac{2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)} \int_{0}^{\theta} \mathbb{E}\left(v, \mathcal{R}_{p}(v)\right) d v \\
\mathscr{H} \mathcal{M}_{p}(\theta)=\frac{2-2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)} \mathbb{F}\left(\theta, \mathcal{M}_{p}(\theta)\right)+\frac{2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)} \int_{0}^{\theta} \mathbb{F}\left(v, \mathcal{M}_{p}(v)\right) d v
\end{array}\right\}
$$

Then, we see that $\mathscr{H}$ is continuous. For each $u, v \in \mathrm{~S}$ and $\theta \in I$, we have,

$$
\mathscr{H} S_{p}(\theta)=\frac{2-2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)} \mathbb{A}\left(\theta, S_{p}(\theta)\right)+\frac{2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)} \int_{0}^{\theta} \mathbb{A}\left(v, S_{p}(v)\right) d v .
$$

We now discuss the existence and uniqueness of solutions of (4.3) provided the following conditions satisfied:

1. $\left|\mathbb{A}\left(\theta, \mathcal{S}_{p_{1}}(\theta)\right)-\mathbb{A}\left(\theta, \mathcal{S}_{p_{2}}(\theta)\right)\right| \leq\left|\mathcal{S}_{p_{1}}(\theta)-\mathcal{S}_{p_{2}}(\theta)\right|$
2. $\frac{2}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)}<\zeta$, where $0<\zeta<1$.

Now consider,

$$
\begin{aligned}
& \mathscr{H} \mathcal{S}_{p_{1}}(\theta)-\mathscr{H} \mathcal{S}_{p_{2}}(\theta)|=| \frac{2-2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)} \mathbb{A}\left(\theta, S_{p_{1}}(\theta)\right)+\frac{2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)} \int_{0}^{\theta} \mathbb{A}\left(v, S_{p_{1}}(v)\right) d v \\
& \left.-\frac{2-2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)} \mathbb{A}\left(\theta, \mathcal{S}_{p_{2}}(\theta)\right)-\frac{2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)} \int_{0}^{\theta} \mathbb{A}\left(v, \mathcal{S}_{p_{2}}(v)\right) d v \right\rvert\, \\
& =\left\lvert\, \frac{2-2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)}\left(\mathbb{A}\left(\theta, \varsigma_{p_{1}}(\theta)\right)-\mathbb{A}\left(\theta, \varsigma_{p_{2}}(\theta)\right)\right)\right. \\
& \left.+\frac{2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)} \int_{0}^{\theta}\left(\mathbb{A}\left(v, \mathcal{S}_{p_{1}}(v)\right) d v-\mathbb{A}\left(v, \mathcal{S}_{p_{2}}(v)\right)\right) d v \right\rvert\, \\
& \leq\left|\frac{2-2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)}\right|\left|\left(\mathbb{A}\left(\theta, \mathcal{S}_{p_{1}}(\theta)\right)-\mathbb{A}\left(\theta, \mathcal{S}_{p_{2}}(\theta)\right)\right)\right| \\
& +\left|\frac{2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)}\right|\left|\int_{0}^{\theta}\left(\mathbb{A}\left(v, \mathcal{S}_{p_{1}}(v)\right) d v-\mathbb{A}\left(v, \mathcal{S}_{p_{2}}(v)\right)\right) d v\right| \\
& \leq\left|\frac{2-2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)}\right|\left|\left(\mathbb{A}\left(\theta, S_{p_{1}}(\theta)\right)-\mathbb{A}\left(\theta, S_{p_{2}}(\theta)\right)\right)\right| \\
& +\left|\frac{2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)}\right|\left|\left(\mathbb{A}\left(\theta, \mathcal{S}_{p_{1}}(\theta)\right)-\mathbb{A}\left(\theta, \mathcal{S}_{p_{2}}(\theta)\right)\right)\right| \\
& \leq\left|\frac{2-2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)}+\frac{2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)}\right|\left|\left(\mathbb{A}\left(\theta, \mathcal{S}_{p_{1}}(\theta)\right)-\mathbb{A}\left(\theta, S_{p_{2}}(\theta)\right)\right)\right| \\
& \leq\left|\frac{2}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)}\right|\left|S_{p_{1}}(\theta)-S_{p_{2}}(\theta)\right| \\
& \leq \zeta\left|S_{p_{1}}(\theta)-S_{p_{2}}(\theta)\right| \text { where } 0<\zeta<1 \text {. }
\end{aligned}
$$

Therefore,

$$
\begin{equation*}
\left|\mathscr{H} S_{p_{1}}(\theta)-\mathscr{H} S_{p_{2}}(\theta)\right| \leq \zeta\left|\delta_{p_{1}}(\theta)-S_{p_{2}}(\theta)\right| . \tag{4.4}
\end{equation*}
$$

Using the above method we discussed above, one can get

$$
\begin{align*}
& \left|\mathscr{H} \mathcal{E}_{p_{1}}(\theta)-\mathscr{H} \mathcal{E}_{p_{2}}(\theta)\right| \leq \zeta\left|\mathcal{E}_{p_{1}}(\theta)-\mathcal{E}_{p_{2}}(\theta)\right| ; \\
& \left|\mathscr{H} \mathcal{J}_{p_{1}}(\theta)-\mathscr{H} \mathcal{J}_{p_{2}}(\theta)\right| \leq \zeta\left|\mathcal{J}_{p_{1}}(\theta)-\mathcal{J}_{p_{2}}(\theta)\right| ; \\
& \left|\mathscr{H} \mathcal{A}_{p_{1}}(\theta)-\mathscr{H} \mathcal{A}_{p_{2}}(\theta)\right| \leq \zeta\left|\mathcal{A}_{p_{1}}(\theta)-\mathcal{A}_{p_{2}}(\theta)\right| ;  \tag{4.5}\\
& \left|\mathscr{H} \mathcal{R}_{p_{1}}(\theta)-\mathscr{H} \mathcal{R}_{p_{2}}(\theta)\right| \leq \zeta\left|\mathcal{R}_{p_{1}}(\theta)-\mathcal{R}_{p_{2}}(\theta)\right| ; \\
& \left|\mathscr{H} \mathcal{M}_{p_{1}}(\theta)-\mathscr{H} \mathcal{M}_{p_{2}}(\theta)\right| \leq \zeta\left|\mathcal{M}_{p_{1}}(\theta)-\mathcal{M}_{p_{2}}(\theta)\right| .
\end{align*}
$$

From Eq.(4.4) we can write, $\mathscr{D}\left(\mathscr{H} \mathcal{S}_{p_{1}}(\theta), \mathscr{H} S_{p_{2}}(\theta)\right) \leq \zeta \mathscr{D}\left(\mathcal{S}_{p_{1}}(\theta), \mathcal{S}_{p_{2}}(\theta)\right)$. Similarly,

$$
\begin{aligned}
& \mathscr{D}\left(\mathscr{H} \mathcal{E}_{p_{1}}(\theta), \mathscr{H} \mathcal{E}_{p_{2}}(\theta)\right) \leq \zeta \mathscr{D}\left(\varepsilon_{p_{1}}(\theta), \mathcal{E}_{p_{2}}(\theta)\right) ; \\
& \mathscr{D}\left(\mathscr{H} \mathcal{J}_{p_{1}}(\theta), \mathscr{H} \mathcal{J}_{p_{2}}(\theta)\right) \leq \zeta \mathscr{D}\left(\mathcal{J}_{p_{1}}(\theta), \mathcal{J}_{p_{2}}(\theta)\right) ; \\
& \mathscr{D}\left(\mathscr{H} \mathcal{A}_{p_{1}}(\theta), \mathscr{H} \mathcal{A}_{p_{2}}(\theta)\right) \leq \zeta \mathscr{D}\left(\mathcal{A}_{p_{1}}(\theta), \mathcal{A}_{p_{2}}(\theta)\right) ; \\
& \mathscr{D}\left(\mathscr{H} \mathcal{R}_{p_{1}}(\theta), \mathscr{H} \mathcal{R}_{p_{2}}(\theta)\right) \leq \zeta \mathscr{D}\left(\mathcal{R}_{p_{1}}(\theta), \mathcal{R}_{p_{2}}(\theta)\right) ; \\
& \mathscr{D}\left(\mathscr{H} \mathcal{M}_{p_{1}}(\theta), \mathscr{H} \mathcal{M}_{p_{2}}(\theta)\right) \leq \zeta \mathscr{D}\left(\mathcal{M}_{p_{1}}(\theta), \mathcal{M}_{p_{2}}(\theta)\right) .
\end{aligned}
$$

Hence the system of novel Coronavirus model gratified all the assertions of special case for standard metric space. Hence Eq. (4.3) has unique fixed point. Thus Eq.(4.3) has a unique solution.

### 4.2 Existence of solution under the setting of complete extended $b$-metric space:

Let $S=\{\mathbb{A} \in \mathcal{C}(I, \mathbb{R}): \mathbb{A}(\theta)>0$ for all $\theta \in I=[0, \mathscr{H}], \mathscr{H}>0\}$.
Define a mapping $\mathscr{D}: \mathrm{S} \times \mathrm{S} \rightarrow[0, \infty)$ and $p: \mathrm{S} \times \mathrm{S} \rightarrow[1, \infty)$ as $\mathscr{D}(u, v)=|u-v|^{2}$ and

$$
p(u, v)=\left\{\begin{array}{l}
|u-v|^{2}, \text { if } u \neq v  \tag{4.6}\\
1, \quad \text { if } u=v
\end{array}\right.
$$

for all $u, v \in \mathrm{~S}$. Then $(\mathrm{S}, \mathscr{D})$ is a complete extended $b$-metric space. Define a mapping $\mathscr{H}: \mathrm{S} \rightarrow \mathrm{S}$ by,

$$
\left.\begin{array}{l}
\mathscr{H} \mathcal{S}_{p}(\theta)=\mathcal{S}_{p}(0)+¥(\sigma) \mathbb{A}\left(\theta, \mathcal{S}_{p}\right)+£(\sigma) \int_{0}^{\theta} \mathbb{A}\left(u, \mathcal{S}_{p}\right) d u \\
\mathscr{H} \mathcal{E}_{p}(\theta)=\mathcal{E}_{p}(0)+¥(\sigma) \mathbb{B}\left(\theta, \mathcal{E}_{p}\right)+£(\sigma) \int_{0}^{\theta} \mathbb{B}\left(u, \varepsilon_{p}\right) d u \\
\mathscr{H} \mathcal{J}_{p}(\theta)=\mathcal{J}_{p}(0)+¥(\sigma) \mathbb{C}\left(\theta, \mathcal{J}_{p}\right)+£(\sigma) \int_{0}^{\theta} \mathbb{C}\left(u, \mathcal{J}_{p}\right) d u \\
\mathscr{H} \mathcal{A}_{p}(\theta)=\mathcal{A}_{p}(0)+¥(\sigma) \mathbb{D}\left(\theta, \mathcal{A}_{p}\right)+£(\sigma) \int_{0}^{\theta} \mathbb{D}\left(u, \mathcal{A}_{p}\right) d u  \tag{4.7}\\
\mathscr{H} \mathcal{R}_{p}(\theta)=\mathcal{R}_{p}(0)+¥(\sigma) \mathbb{E}\left(\theta, \mathcal{R}_{p}\right)+£(\sigma) \int_{0}^{\theta} \mathbb{E}\left(u, \mathcal{R}_{p}\right) d u \\
\mathscr{H} \mathcal{M}_{p}(\theta)=\mathcal{M}_{p}(0)+¥(\sigma) \mathbb{F}\left(\theta, \mathcal{M}_{p}\right)+£(\sigma) \int_{0}^{\theta} \mathbb{F}\left(u, \mathcal{M}_{p}\right) d u
\end{array}\right\},
$$

where $¥(\sigma)=\frac{2-2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)}$ and $£(\sigma)=\frac{2 \sigma}{2 \mathscr{B}(\sigma)-\sigma \mathscr{B}(\sigma)}$.
Then, we see that $\mathscr{H}$ is continuous. For each $u, v \in \mathrm{~S}$ and $\theta \in I$, we have,

$$
\mathscr{H} \mathcal{S}_{p}(\theta)=\mathcal{S}_{p}(0)+¥(\sigma) \mathbb{A}\left(\theta, S_{p}\right)+£(\sigma) \int_{0}^{\theta} \mathbb{A}\left(u, \mathcal{S}_{p}\right) d u
$$

We now verify the existence and uniqueness of solutions of (4.7) provided the following conditions satisfied:

1. $\left|\mathbb{A}\left(\theta, \mathcal{S}_{p_{1}}\right)-\mathbb{A}\left(\theta, \mathcal{S}_{p_{2}}\right)\right|^{2} \leq\left|\mathcal{S}_{p_{1}}(\theta)-\mathcal{S}_{p_{2}}(\theta)\right|^{2}$
2. $|¥(\sigma)+£(\sigma) \theta|^{2}<\xi$, where $0<\xi<1$.

Now consider,

$$
\begin{aligned}
\left|\mathscr{H} \mathcal{S}_{p_{1}}(\theta)-\mathscr{H} \mathcal{S}_{p_{2}}(\theta)\right|^{2}= & \mid ¥(\sigma) \mathbb{A}\left(\theta, \mathcal{S}_{p_{1}}\right)+£(\sigma) \int_{0}^{\theta} \mathbb{A}\left(u, \mathcal{S}_{p_{1}}\right) d u \\
& -¥(\sigma) \mathbb{A}\left(\theta, \mathcal{S}_{p_{2}}\right)-\left.£(\sigma) \int_{0}^{\theta} \mathbb{A}\left(u, \mathcal{S}_{p_{2}}\right) d u\right|^{2} \\
= & \mid ¥(\sigma)\left(\mathbb{A}\left(\theta, \mathcal{S}_{p_{1}}\right)-\mathbb{A}\left(\theta, \mathcal{S}_{p_{2}}\right)\right) \\
& +\left.£(\sigma)\left[\int_{0}^{\theta}\left[\mathbb{A}\left(u, \mathcal{S}_{p_{1}}\right)-\mathbb{A}\left(u, \mathcal{S}_{p_{2}}\right)\right] d u\right]\right|^{2} \\
= & \left|¥(\sigma)\left(\mathbb{A}\left(\theta, \mathcal{S}_{p_{1}}\right)-\mathbb{A}\left(\theta, \mathcal{S}_{p_{2}}\right)\right)\right|^{2} \\
& +\left|£(\sigma)\left[\int_{0}^{\theta}\left[\mathbb{A}\left(u, \mathcal{S}_{p_{1}}\right)-\mathbb{A}\left(u, \mathcal{S}_{p_{2}}\right)\right] d u\right]\right|^{2} \\
& +2\left|¥(\sigma)\left(\mathbb{A}\left(\theta, \mathcal{S}_{p_{1}}\right)-\mathbb{A}\left(\theta, \mathcal{S}_{p_{2}}\right)\right)\right| \\
& \left|£(\sigma)\left[\int_{0}^{\theta}\left[\mathbb{A}\left(u, \mathcal{S}_{p_{1}}\right)-\mathbb{A}\left(u, \mathcal{S}_{p_{2}}\right)\right] d u\right]\right| \\
= & |¥(\sigma)|^{2}\left|\left(\mathbb{A}\left(\theta, \mathcal{S}_{p_{1}}\right)-\mathbb{A}\left(\theta, \mathcal{S}_{p_{2}}\right)\right)\right|^{2} \\
& +|£(\sigma)|^{2}\left|\mathbb{A}\left(u, \mathcal{S}_{p_{1}}\right)-\mathbb{A}\left(u, \mathcal{S}_{p_{2}}\right)\right|^{2}\left|\int_{0}^{\theta} d u\right|^{2} \\
& +2|¥(\sigma)|\left|\mathbb{A}\left(\theta, \mathcal{S}_{p_{1}}\right)-\mathbb{A}\left(\theta, \mathcal{S}_{p_{2}}\right)\right| \\
& |£(\sigma)|\left|\mathbb{A}\left(u, \mathcal{S}_{p_{1}}\right)-\mathbb{A}\left(u, \mathcal{S}_{p_{2}}\right)\right|\left|\int_{0}^{\theta} d u\right| \\
= & \left|\mathbb{A}\left(\theta, \mathcal{S}_{p_{1}}\right)-\mathbb{A}\left(\theta, \mathcal{S}_{p_{2}}\right)\right|^{2}\left\{|¥(\sigma)|^{2}+|£(\sigma)|^{2} \theta^{2}+2 ¥(\sigma) £(\sigma)|\theta|\right\} \\
= & \left|\mathbb{A}\left(\theta, \mathcal{S}_{p_{1}}\right)-\mathbb{A}\left(\theta, \mathcal{S}_{p_{2}}\right)\right|^{2}|¥(\sigma)+£(\sigma) \theta|^{2} \\
\leq & \xi\left|\mathcal{S}_{p_{1}}(\theta)-\mathcal{S}_{p_{2}}(\theta)\right|^{2}
\end{aligned}
$$

Therefore,

$$
\begin{equation*}
\left|\mathscr{H} \mathcal{S}_{p_{1}}(\theta)-\mathscr{H} S_{p_{2}}(\theta)\right| \leq \xi\left|\mathcal{S}_{p_{1}}(\theta)-\mathcal{S}_{p_{2}}(\theta)\right| . \tag{4.8}
\end{equation*}
$$

Using the above method we discussed above, one can get

$$
\begin{align*}
& \left|\mathscr{H} \mathcal{E}_{p_{1}}(\theta)-\mathscr{H} \mathcal{E}_{p_{2}}(\theta)\right| \leq \xi\left|\mathcal{E}_{p_{1}}(\theta)-\mathcal{E}_{p_{2}}(\theta)\right| ; \\
& \left|\mathscr{H} \mathcal{J}_{p_{1}}(\theta)-\mathscr{H} \mathcal{J}_{p_{2}}(\theta)\right| \leq \xi\left|\mathcal{J}_{p_{1}}(\theta)-\mathcal{J}_{p_{2}}(\theta)\right| ; \\
& \left|\mathscr{H} \mathcal{A}_{p_{1}}(\theta)-\mathscr{H} \mathcal{A}_{p_{2}}(\theta)\right| \leq \xi\left|\mathcal{A}_{p_{1}}(\theta)-\mathcal{A}_{p_{2}}(\theta)\right| ;  \tag{4.9}\\
& \left|\mathscr{H} \mathcal{R}_{p_{1}}(\theta)-\mathscr{H} \mathcal{R}_{p_{2}}(\theta)\right| \leq \xi\left|\mathcal{R}_{p_{1}}(\theta)-\mathcal{R}_{p_{2}}(\theta)\right| ; \\
& \left|\mathscr{H} \mathcal{M}_{p_{1}}(\theta)-\mathscr{H} \mathcal{M}_{p_{2}}(\theta)\right| \leq \xi\left|\mathcal{M}_{p_{1}}(\theta)-\mathcal{M}_{p_{2}}(\theta)\right| .
\end{align*}
$$

From Eq.(4.8) we can write, $\mathscr{D}\left(\mathscr{H} S_{p_{1}}(\theta), \mathscr{H} S_{p_{2}}(\theta)\right) \leq \xi \mathscr{D}\left(S_{p_{1}}(\theta), S_{p_{2}}(\theta)\right)$. Similarly,

$$
\begin{aligned}
& \mathscr{D}\left(\mathscr{H} \mathcal{E}_{p_{1}}(\theta), \mathscr{H} \mathcal{E}_{p_{2}}(\theta)\right) \leq \xi \mathscr{D}\left(\varepsilon_{p_{1}}(\theta), \mathcal{E}_{p_{2}}(\theta)\right) ; \\
& \mathscr{D}\left(\mathscr{H} \mathcal{J}_{p_{1}}(\theta), \mathscr{H} \mathcal{J}_{p_{2}}(\theta)\right) \leq \xi \mathscr{D}\left(\mathcal{J}_{p_{1}}(\theta), \mathcal{J}_{p_{2}}(\theta)\right) ; \\
& \mathscr{D}\left(\mathscr{H} \mathcal{A}_{p_{1}}(\theta), \mathscr{H} \mathcal{A}_{p_{2}}(\theta)\right) \leq \xi \mathscr{D}\left(\mathcal{A}_{p_{1}}(\theta), \mathcal{A}_{p_{2}}(\theta)\right) ; \\
& \mathscr{D}\left(\mathscr{H} \mathcal{R}_{p_{1}}(\theta), \mathscr{H} \mathcal{R}_{p_{2}}(\theta)\right) \leq \xi \mathscr{D}\left(\mathcal{R}_{p_{1}}(\theta), \mathcal{R}_{p_{2}}(\theta)\right) ; \\
& \mathscr{D}\left(\mathscr{H} \mathcal{M}_{p_{1}}(\theta), \mathscr{H} \mathcal{M}_{p_{2}}(\theta)\right) \leq \xi \mathscr{D}\left(\mathcal{M}_{p_{1}}(\theta), \mathcal{M}_{p_{2}}(\theta)\right) .
\end{aligned}
$$

Hence the system of novel Coronavirus model gratified all the assertions of Theorem 2.1 in the setting of extended $b$-metric space. Hence Eq.(4.7) has unique fixed point. Thus Eq.(4.7) has a unique solution.

## 5 Conclusion

There is still a dramatic increase for using mathematical modelling in the study of epidemiology diseases. Mathematical models were developed to predict how infectious diseases advance to explain the potential outcome of an outbreak, and better facilitate initiatives in global policy. We too propose a solution for existence and uniqueness solutions of the novel Coronavirus $2019-\mathrm{nCoV} / \mathrm{SARS}-\mathrm{CoV}-2$ model via fractional derivatives and nonlinear ( $\omega-F_{\mathcal{C}}$ )-contractions. A few words about possible extensions of the preceding conclusions:

- Correlation between the weather conditions and 2019-nCoV model of type SEIARM [19] in India via nonlinear $\left(\omega-F_{\mathcal{C}}\right)$-contractions.
- Correlation between the weather conditions and 2019-nCoV model of type SIDARTHE [13] in Italy via nonlinear $\left(\omega-F_{\mathfrak{C}}\right)$-contractions.
- Correlation between the weather conditions and 2019-nCoV model of type SCIRD [5] in Spain via nonlinear $\left(\omega-F_{\mathcal{C}}\right)$-contractions.


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