The new stochastic solutions to the perturbed NLSE

Hesham Abdelwahed[1] and mahmoud abdelrahman[2]

[1]Mansoura University
[2]department of mathematics-faculty of science-mansoura university

January 30, 2024

Abstract

This article extracts exact solutions for the perturbed nonlinear Schrödinger's equation (PNLSE) with Kerr law nonlinearity forced by multiplicative noise in Itˆo sense by utilizing the unified solver method. The presented solutions involve three types: rational function, trigonometric function and hyperbolic function solutions. These stochastic solutions may be applicable for investigation various complex phenomena in applied science and new physics. We exhibit the influence of multiplicative noise on the solution of the PNLSE forced by multiplicative noise in Itˆo sense. The study and acquired solutions clarify that the unified solver technique is sturdy and efficient. Finally, some 3D proles to some of the gained solutions are also illustrated.

The new stochastic solutions to the perturbed NLSE

H.G. Abdelwahed[1,2] *and Mahmoud A.E. Abdelrahman[3,4] †

[1] *Department of Physics, College of Science and Humanities, Al-Kharj, Prince Sattam bin Abdulaziz University, Al- Kharj11942, Saudi Arabia*

[2] *Theoretical Physics Group, Faculty of Science, Mansoura University, Mansoura, Egypt*

[3] *Department of Mathematics, College of Science, Taibah University, Al-Madinah Al-Munawarah, Saudi Arabia*

[4] *Department of Mathematics, Faculty of Science, Mansoura University, 35516 Mansoura, Egypt*

Abstract This article extracts exact solutions for the perturbed nonlinear Schödinger's equation (PNLSE) with Kerr law nonlinearity forced by multiplicative noise in Itô sense by utilizing the unified solver method. The presented solutions involve three types: rational function, trigonometric function and hyperbolic function solutions. These stochastic solutions may be applicable for investigation various complex phenomena in applied science and new physics. We exhibit the influence of multiplicative noise on the solution of the PNLSE forced by multiplicative noise in Itô sense. The study and acquired solutions clarify that the unified solver technique is sturdy and efficient. Finally, some 3D profiles to some of the gained solutions are also illustrated.

Keywords: Unified solver method, exact solutions, PNLSE, multiplicative noise, physical applications.

Mathematics Subject Classification (2010): 35A22, 35C08, 60H10, 60H15, 35Q40, 35Q55

---

1 Introduction

The structure of various special solitary waves of the nonlinear partial differential equations (NPDEs) is explained in terms of solitons [1–6]. Recently, several techniques for getting exact solutions to NPDEs have been proposed and developed, see [7–16]. Solitary

*h.abdelwahed@psau.edu.sa; hgomaa_eg@mans.edu.eg

†mahmoud.abdelrahman@mans.edu.eg

wave solutions also appear on propagating systems, which are represented by nonlinearity and dissipation. Many complex nonlinear phenomena emerging in various branches of natural sciences, like, plasma physics, quantum mechanics, biology, nonlinear optics, electro magnetic wave propagation, nuclear physics, deep water, optoelectronics and fluid dynamics can be depicted in the form of NLSEs [17–22]. The NLSEs describe the propagation of waves in media with both nonlinear and dispersive response. Considering stochastic effects for the NPDEs is so important and reflect many vital phenomena in natural sciences [23–27].

Let us consider the NPDEs forced by multiplicative noise in Itô sense given by:

$$\Lambda(\mathcal{G}, \mathcal{G}_x, \mathcal{G}_t, \mathcal{G}_{xx}, \mathcal{G}_{xt}, \mathcal{G}_{tt}, ...) = 0. \tag{1.1}$$

Using wave transformation:

$$\mathcal{G}(x,t) = \mathcal{G}(\xi), \qquad \eta = cx - \nu\, t, \tag{1.2}$$

the Eq. (1.1) reduced to SODE:

$$H(\mathcal{G}, \mathcal{G}', \mathcal{G}'', \mathcal{G}''', ...) = 0. \tag{1.3}$$

It is well known that there are various models in new physics and applied science in forms of Eq. (1.1) reduced to the following ODE:

$$L\mathcal{G}'' + M\mathcal{G}^3 + N\mathcal{G} = 0, \tag{1.4}$$

see for example [28–36]. As a result of the importance of the deterministic Eq. (1.3) we demonstrate the powerful unified solver for the wide range of NPDEs [37]. In this work, we can develop this solver to solve Eq.(1.3) in a random case.

In this article we consider the 1-D stochastic PNLSE with Kerr law non-linearity for propagation of light in single-mode fibers; existence of attenuation, dispersion and nonlinear effects [38–41],

$$i\chi_t + \chi_{xx} + \alpha \mid \chi \mid^2 \chi + i\left(\delta_1\chi_{xxx} + \delta_2 \mid \chi \mid^2 \chi_x + \delta_3(\mid \chi \mid^2)_x\chi\right) + \sigma\,\chi\beta_t = 0\,, \qquad i = \sqrt{-1}, \tag{1.5}$$

where $\delta_1, \delta_2, \delta_3$ indicate, third order dispersion, nonlinear dispersion, another version of nonlinear dispersion and σ is the noise strength. The noise β_t is the time derivative of the brownian motion $\beta(t)$. This equation describes the propagation of optical solitons in fibers that displays a Kerr law non-linearity forced by multiplicative noise in Itô sense. Eq. (1.5) has vital applications in the field of theoretical physics.

We aim to present closed-form structures of solutions for the widely used families of NPDEs with noise term in Itô sense. This structure will be utilized as a box solver to the engineers, physicists and mathematicians. The main characteristic of the presented solver, is that it present various new solutions with additional free parameters. These solutions are very important to clarify pivotal complex phenomena in natural sciences. The presented technique is simple, functional and powerful. One of the main feature for the proposed solver is that it gives various new solutions such rational solutions, solitons, dissipative, breathers, rough or periodic. The type of these solutions depend on the physical parameters in dispersion & nonlinear coefficients. The given solutions of the perturbed NLSE with Kerr law nonlinearity are vital in various fields of natural sciences, such as optical fibers communications, superfluids, semiconductor materials, plasma physics, telecommunications experiments and femtosecond pulse [42–46].

The rest of the paper is arranged as follows. Sec. 2 gives the closed-form wave structures for $L\mathcal{G}'' + M\mathcal{G}^3 + N\mathcal{G} = 0$. Sec. 3 introduces some new stochastic solutions for the perturbed NLSE forced by multiplicative noise in Itô sense. Sec. 4 presents the explantation for the presented solutions of the SPNLSE with Kerr law nonlinearity. Finally, conclusion is reported in Sec. 5.

2 The closed-form solutions

The closed-form solutions of the following equation [37]:

$$L\mathcal{G}'' + M\mathcal{G}^3 + N\mathcal{G} = 0, \tag{2.1}$$

given by :

Rational function solutions: (when $N = 0$**)**

$$\mathcal{G}_{1,2}(x,t) = \left(\mp\sqrt{\frac{-M}{2L}}\,(\eta + \varpi) \right)^{-1}. \tag{2.2}$$

Trigonometric function solutions: (when $\frac{N}{L} < 0$**)**

$$\mathcal{G}_{3,4}(x,t) = \pm\sqrt{\frac{N}{M}}\; tan\left(\sqrt{\frac{-N}{2L}}\,(\eta + \varpi) \right) \tag{2.3}$$

and

$$\mathcal{G}_{5,6}(x,t) = \pm\sqrt{\frac{N}{M}}\; cot\left(\sqrt{\frac{-N}{2L}}\,(\eta + \varpi) \right). \tag{2.4}$$

Hyperbolic function solutions: (when $\frac{N}{L} > 0$**)**

$$\mathcal{G}_{7,8}(x,t) = \pm\sqrt{\frac{-N}{M}}\, tanh\left(\sqrt{\frac{N}{2L}}\,(\eta+\varpi)\right) \tag{2.5}$$

and

$$\mathcal{G}_{9,10}(x,t) = \pm\sqrt{\frac{-N}{M}}\, coth\left(\sqrt{\frac{N}{2L}}\,(\eta+\varpi)\right). \tag{2.6}$$

Here ϖ is an arbitrary constant.

3 The solutions for PNLSE

We use the traveling wave solution of the form [38]:

$$\chi(x,t) = e^{i(kx-rt+\sigma\,\beta(t))}q(\eta), \qquad \eta = x - v\,t, \tag{3.1}$$

where k, r and v are constants and σ is the noise strength. Superseding Eq. (3.1) into Eq. (1.5) gives

$$i\left(\delta_1 q''' - 3\delta_1 k^2 q' + \delta_2 q^2 q' + 2\delta_3 q^2 q' - v\,q' + 2k\,q'\right) + \left(ru + q'' - k^2 q + \alpha\,q^3 + 3\delta_1 k q'' + \delta_1 k^3 q - \delta_2 k u^3\right) = 0. \tag{3.2}$$

Then we have [38]:

$$\delta_1 k^2\,q'' + \frac{1}{3}(\delta_2 + 2\delta_3)\,q^3 + (2k - v - 3\delta_1 k^2)q = 0\,, \tag{3.3}$$

In the light of the above closed-form structure, Eq. (1.5) has the following solutions:

Family I:

The rational solutions are

$$q_{1,2}(x,t) = \left(\mp\sqrt{\frac{-(\delta_2+\delta_3)}{6\delta_1 k^2}}\,(x - v\,t + \varpi)\right)^{-1}. \tag{3.4}$$

Thus the stochastic solutions of the Eq. (1.5) are

$$\chi_{1,2}(x,t) = e^{i(kx-rt+\sigma\,\beta(t))}\left(\mp\sqrt{\frac{-(\delta_2+\delta_3)}{6\delta_1 k^2}}\,(x - v\,t + \varpi)\right)^{-1}. \tag{3.5}$$

Family II :

The trigonometric solutions are

$$q_{3,4}(x,t) = \pm\sqrt{\frac{3(2k - v - 3\delta_1 k^2)}{\delta_2 + 2\delta_3}}\; tan\left(\sqrt{\frac{v + 3\delta_1 k^2 - 2k}{2\delta_1 k^2}}\,(x - v\,t + \varpi)\right) \tag{3.6}$$

and

$$q_{5,6}(x,t) = \pm\sqrt{\frac{3(2k - v - 3\delta_1 k^2)}{\delta_2 + 2\delta_3}}\; cot\left(\sqrt{\frac{v + 3\delta_1 k^2 - 2k}{2\delta_1 k^2}}\,(x - v\,t + \varpi)\right). \tag{3.7}$$

Thus the stochastic solutions of the Eq. (1.5) are

$$\chi_{3,4}(x,t) = \pm e^{i(kx - rt + \sigma\,\beta(t))}\sqrt{\frac{3(2k - v - 3\delta_1 k^2)}{\delta_2 + 2\delta_3}}\; tan\left(\sqrt{\frac{v + 3\delta_1 k^2 - 2k}{2\delta_1 k^2}}\,(x - v\,t + \varpi)\right) \tag{3.8}$$

and

$$\chi_{5,6}(x,t) = \pm e^{i(kx - rt + \sigma\,\beta(t))}\sqrt{\frac{3(2k - v - 3\delta_1 k^2)}{\delta_2 + 2\delta_3}}\; cot\left(\sqrt{\frac{v + 3\delta_1 k^2 - 2k}{2\delta_1 k^2}}\,(x - v\,t + \varpi)\right). \tag{3.9}$$

Family III:

The hyperbolic solutions are

$$q_{7,8}(x,t) = \pm\sqrt{\frac{3(v + 3\delta_1 k^2 - 2k)}{\delta_2 + 2\delta_3}}\; tanh\left(\sqrt{\frac{2k - v - 3\delta_1 k^2}{2\delta_1 k^2}}\,(x - v\,t + \varpi)\right) \tag{3.10}$$

and

$$q_{9,10}(x,t) = \pm\sqrt{\frac{3(v + 3\delta_1 k^2 - 2k)}{\delta_2 + 2\delta_3}}\; coth\left(\sqrt{\frac{2k - v - 3\delta_1 k^2}{2\delta_1 k^2}}\,(x - v\,t + \varpi)\right). \tag{3.11}$$

Thus the stochastic solutions of the Eq. (1.5) are

$$\chi_{7,8}(x,t) = \pm e^{i(kx - rt + \sigma\,\beta(t))}\sqrt{\frac{3(v + 3\delta_1 k^2 - 2k)}{\delta_2 + 2\delta_3}}\; tanh\left(\sqrt{\frac{2k - v - 3\delta_1 k^2}{2\delta_1 k^2}}\,(x - v\,t + \mu)\right) \tag{3.12}$$

and

$$\chi_{9,10}(x,t) = \pm e^{i(kx - rt + \sigma\,\beta(t))}\sqrt{\frac{3(v + 3\delta_1 k^2 - 2k)}{\delta_2 + 2\delta_3}}\; coth\left(\sqrt{\frac{2k - v - 3\delta_1 k^2}{2\delta_1 k^2}}\,(x - v\,t + \mu)\right). \tag{3.13}$$

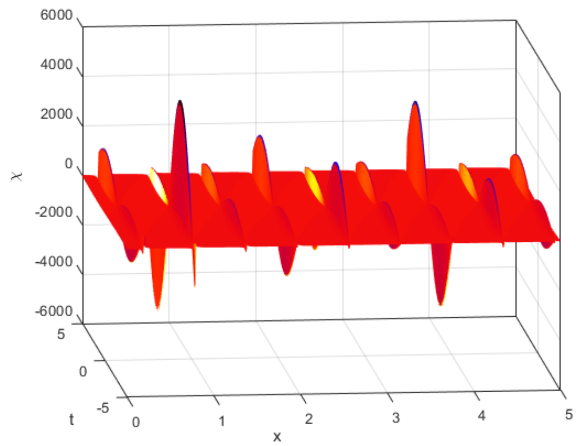

Figure 1: Real of χ_1 with $\delta_1 = 1.5, \delta_2 = 2.3, \delta_3 = 2.1, \varpi = 1, k = 2.8, r = 0.6, \sigma = 1$.

4 Results and discussion

We have implemented the unified solver for extracting explicit exact solutions to PNLSE with Kerr law nonlinearity forced by multiplicative noise in Itô sense. The unified solver technique has been efficiently introduced to construct many new stochastic solutions. The presented stochastic solutions including rational, trigonometric, hyperbolic functions. These solutions exhibit some vital complex phenomena in applied science and new physics, such as solid mechanics, optical fibers communications, modeling of deep water, plasma physics, semiconductor materials and magneto-static spin waves. To our knowledge, the proposed results in this work have not been presented in the literature. The attitude of these solutions are soliton, rough, breather, periodic, explosive, shock or dissipative, is based on the physical parameters in the PNLSE with Kerr law nonlinearity. For example, the behaviour of wave varies at critical points from compressive to rarefactive and stability regions become unstable regions at certain values of wave number named critical values [47–49]. The dependence of the presented solutions features based on coefficients of dispersion $\delta_1, \delta_2, \delta_3$ and nonlinearity α has an important role in the stable & unstable regime and wave amplitude modulations on the optical explosive excitations. The results illustrate the efficiency and reliability of the proposed technique for finding stochastic solutions of some complicated NPDEs. In Figs. 1-4, we show the behaviour of the solutions for the PNLSE forced by multiplicative noise in Itô sense.

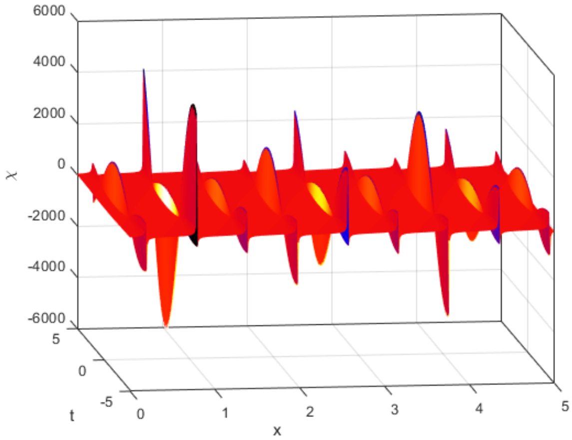

Figure 2: Imaginary of χ_1 with $\delta_1 = 1.5, \delta_2 = 2.3, \delta_3 = 2.1, \varpi = 1, k = 2.8, r = 0.6, \sigma = 1$.

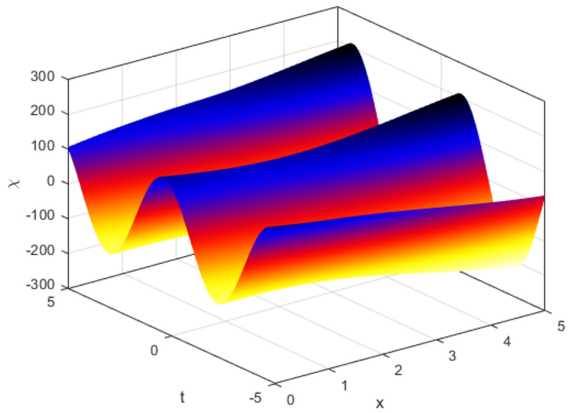

Figure 3: Real of χ_7 with $\delta_1 = -1.6, \delta_2 = 1.8, \delta_3 = 0.5, \varpi = 0, k = -0.6, r = 1.2, \sigma = 1$.

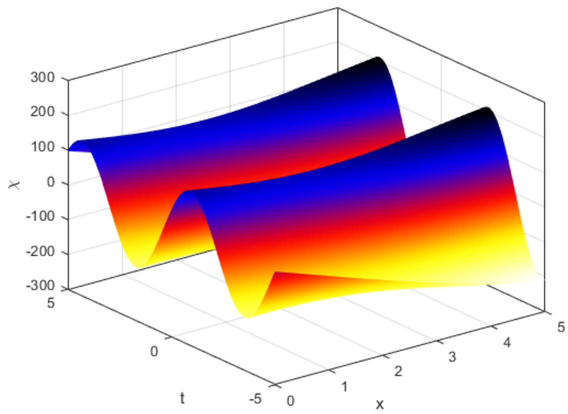

Figure 4: Imaginary of χ_7 with $\delta_1 = 1.5, \delta_2 = 2.3, \delta_3 = 2.1, \varpi = 1, k = 2.8, r = 0.6, \sigma = 1$.

5 Conclusions

We have given some new stochastic solutions for the PNLSE with Kerr law nonlinearity in presence of noise term in Itô sense. We have employed the unified solver method, which offers closed form of solutions. Indeed, the acquired solutions clearly exhibit the reliability of the presented technique. We deduced that the proposed approach can be prolonged to solve various equations of NPDEs arising in different fields of natural sciences, such as biology, fluid mechanics, engineering, physics, chemistry and other more.

References

[1] M. Younis, S. Ali and S.A. Mahmood, Solitons for compound KdV Burgers equation with variable coefficients and power law nonlinearity. Nonlinear Dyn. 81 (2015), 1191-1196.

[2] S.Z. Hassan, N.A. Alyamani and M.A.E. Abdelrahman, A construction of new traveling wave solutions for the 2D Ginzburg-Landau equation, Eur. Phys. J. Plus, (2019) 134: 425.

[3] M.A.E. Abdelrahman, A note on Riccati-Bernoulli sub-ODE method combined with complex transform method applied to fractional differential equations, Nonlinear Engineering, 7(4) (2018), 279–285.

[4] C.K. Kuo and B. Ghanbari, Resonant multi-soliton solutions to new (3+1)-dimensional Jimbo-Miwa equations by applying the linear superposition principle, Nonlinear Dyn 96(2019), 459–464.

[5] H.G. Abdelwahed, E.K. El-Shewy and M.A.E. Abdelrahman, Positron superthermality effects on the solitonic, dissipative, periodic waveforms for M-Kadomstev-Petviashvili-plasma-equation, Physica Scripta, 95(10) (2020) 105204.

[6] H.G. Abdelwahed and M.A.E. Abdelrahman, New nonlinear periodic, solitonic, dissipative waveforms for modified-Kadomstev-Petviashvili-equation in nonthermal positron plasma, Results in Physics, 19 (2020) 103393.

[7] A.M. Wazwaz, Exact solutions to the double sinh-Gordon equation by the tanh method and a variable separated ODE. method, Comput. Math. Appl., 50 (2005), 1685-1696.

[8] M.L. Wang, J.L. Zhang and X.Z. Li, The $(\frac{G'}{G})$- expansion method and travelling wave solutions of nonlinear evolutions equations in mathematical physics, Phys. Lett. A 372 (2008), 417-423.

[9] H. Aminikhad, H. Moosaei and M. Hajipour, Exact solutions for nonlinear partial differential equations via Exp-function method, Numer. Methods Partial Differ. Equations, 26 (2009), 1427-1433.

[10] M. Mirzazadeh, M. Eslami and A. Biswas, 1-Soliton solution of KdV equation. Nonlinear Dyn. 80(1-2) (2015), 387-396.

[11] X.F. Yang, Z.C. Deng and Y. Wei, A Riccati-Bernoulli sub-ODE method for nonlinear partial differential equations and its application, Adv. Diff. Equa. 1 (2015), 117-133.

[12] A. Yusuf, F. Tchier and M. Inc, New interaction and combined multi-wave solutions for the Heisenberg ferromagnetic spin chain equation. The European Physical Journal Plus, 135(6)(2020), 416.

[13] M.A. Akinlar, F. Tchier and M. Inc, Chaos control and solutions of fractionalorder Malkus waterwheel model. Chaos, Solitons & Fractals, 135(2020), 109746.

[14] W. Gao, B. Ghanbari, H. Günerhan and H.M. Baskonus, Some mixed trigonometric complex soliton solutions to the perturbed nonlinear Schrödinger equation, Modern Physics Letters B 2020; 34(03): 2050034.

[15] B. Ghanbari and C. Kuo, New exact wave solutions of the variable-coefficient $(1 + 1)$-dimensional Benjamin-Bona-Mahony and $(2 + 1)$-dimensional asymmetric Nizhnik-Novikov-Veselov equations via the generalized exponential rational function method. Eur. Phys. J. Plus 134, 334 (2019).

[16] C.K. Kuo and B. Ghanbari, Resonant multi-soliton solutions to new (3+1)-dimensional Jimbo-Miwa equations by applying the linear superposition principle, Nonlinear Dyn 96 (2019), 459–464.

[17] M. Eslami, Trial solution technique to chiral nonlinear Schrödinger's equation in(1+2)-dimensions, Nonlinear Dyn. 85 (2) (2016), 813-816.

[18] H. Triki and A.M. Wazwaz, New solitons and periodic wave solutions for the (2+1)-dimensional Heisenberg ferromagnetic spin chain equation, J. Electromagn. Waves Appl. 30(6) (2016), 788-794.

[19] M.A.E. Abdelrahman and M.A. Sohaly, Solitary waves for the nonlinear Schrödinger problem with the probability distribution function in stochastic input case. Eur. Phys. J. Plus 132, 339 (2017).

[20] M.A.E. Abdelrahman and N.F. Abdo, On the nonlinear new wave solutions in unstable dispersive environments, Physica Scripta, 95(4) (2020), 045220.

[21] Y.F. Alharbi, M.A.E. Abdelrahman, M.A. Sohaly and M. Inc, Stochastic treatment of the solutions for the resonant nonlinear Schrödinger equation with spatio-temporal dispersions and inter-modal using beta distribution, The European Physical Journal Plus volume 135 (2020) 368.

[22] A. Houwe, S. Abbagari, G. Betchewe, M. Inc, S.Y. Doka, K.T. Crpin, D. Baleanu and B. Almohsen, Exact optical solitons of the perturbed nonlinear Schrödinger-Hirota equation with Kerr law nonlinearity in nonlinear fiber optics. Open Physics, 18(1)(2020), 526-534.

[23] M.A.E. Abdelrahman and M.A. Sohaly, Solitary waves for the nonlinear Schrödinger problem with the probability distribution function in stochastic input case. Eur. Phys. J. Plus 132, 339 (2017).

[24] M.A.E. Abdelrahman and M.A. Sohaly, The development of the deterministic nonlinear PDEs in particle physics to stochastic case, Results in Physics, 9 (2018), 344-350.

[25] M.A.E. Abdelrahman and M.A. Sohaly, On the new wave solutions to the MCH equation, Indian Journal of Physics, 93(2019), 903-911.

[26] Y.F. Alharbi, M.A.E. Abdelrahman, M.A. Sohaly and S.I. Ammar, Disturbance solutions for the long–short-wave interaction system using bi-random Riccati-Bernoulli sub-ODE method, Journal of Taibah University for Science, 14(1) (2020), 500-506.

[27] Y.F. Alharbi, M.A.E. Abdelrahman, M.A. Sohaly and M. Inc, Stochastic treatment of the solutions for the resonant nonlinear Schrödinger equation with spatio-temporal dispersions and inter-modal using beta distribution, The European Physical Journal Plus volume 135 (2020) 368.

[28] M.A.E. Abdelrahman, S.Z. Hassan and M. Inc, The coupled nonlinear Schrödinger-type equations. Modern Physics Letters B, 34(6) (2020) 2050078.

[29] A.M. Wazwaz, A sine-cosine method for handling nonlinear wave equations, Math. Comput. Modelling, 40 (2004), 499-508.

[30] H.M. Baskonus and H. Bulut, New wave behaviors of the system of equations for the ion sound and Langmuir Waves, Waves Random Complex Media, 26 (2016), 613-625.

[31] C. Liu, Exact solutions for the higher-order nonlinear Schrödinger equation in nonlinear optical fibres, Chaos, Solitons and Fractals, 23 (2005), 949-955.

[32] S. Zhang, Exp-function method for solving Maccari's system, Phys. Lett. A. 371 (2007), 65-71.

[33] K. Hosseini, D. Kumar, M. Kaplan and E.Y. Bejarbaneh, New exact traveling wave solutions of the unstable nonlinear Schrödinger equations. Commun. Theor. Phys., 68 (2017c), 761-767.

[34] H. Bulut, T.A. Sulaiman and H.M. Baskonus, Optical solitons to the resonant nonlinear Schrödinger equation with both spatio-temporal and inter-modal dispersions under Kerr law nonlinearity. Optik, 163 (2018), 49-55.

[35] H.G. Abdelwahed, E.K. El-Shewy, M.A.E. Abdelrahman and R. Sabry, New super waveforms for modified Korteweg-de-Veries-equation, Results in Physics 19 (2020) 103420.

[36] H.G.Abdelwahed, E.K.El-Shewy, Mahmoud A.E.Abdelrahman, A.F.Alsarhana, On the physical nonlinear (n+1)-dimensional Schrödinger equation applications, Results in Physics, 21, (2021), 103798.

[37] M.A.E. Abdelrahman and H. AlKhidhr, A robust and accurate solver for some nonlinear partial differential equations and tow applications, Physica Scripta, 95 (2020) 065212.

[38] Z.Y. Zhang, Z.H. Liu, X.J. Miao and Y.Z. Chen, New exact solutions to the perturbed nonlinear Schrödinger's equation with Kerr law nonlinearity. Appl. Math. Comput., 216(10)(2010), 3064-3072.

[39] H. Moosaei, M. Mirzazadeh and A.Yildirim, Exact solutions to the perturbed nonlinear Schrödinger's equation with Kerr law nonlinearity by using the first integral method, Nonlinear Anal. Model. Control, 16(3) (2011), 332-339.

[40] M. Eslami, Solitary wave solutions for perturbed equation nonlinear Schrödinger's with Kerr law nonlinearity under the DAM, Optik, 126 (2015), 1312-1317.

[41] N. Ahmed, A. Irshad, S.T. Mohyud-Din and U. Khan, Exact solutions of perturbed nonlinear Schrödinger's equation with Kerr law nonlinearity by improved $tan(\frac{\phi(\xi)}{2})$-expansion method, Opt Quant Electron, 50, 45 (2018).

[42] M. Shats, H. Punzmann and H. Xia, Capillary Rogue Waves, Phys. Rev. Lett. 104(2010), 104503.

[43] H. Bailung, S. K. Sharma, and Y. Nakamura, Observation of Peregrine Solitons in a Multicomponent Plasma with Negative Ions, Phys. Rev. Lett. 107(2011), 255005.

[44] B. Kibler, J. Fatome, C. Finot, G. Millot, F. Dias, G. Genty, N. Akhmediev and J. M. Dudley, The Peregrine soliton in nonlinear fibre optics, Nature Phys 6 (2010), 790-795.

[45] A. Chabchoub, N. P. Hoffmann and N. Akhmediev, Rogue Wave Observation in a Water Wave Tank, Phys. Rev. Lett. 106 (2011), 204502.

[46] P. Pathak, S.K. Sharma, Y.N. akamura and H. Bailung, Observation of ion acoustic multi-Peregrine solitons in multicomponent plasma with negative ions, Phys. Lett. A381 (2017), 4011-4018.

[47] W. Watanabe, Self-modulation of a nonlinear ion wave packet, J. Plasma Phys. 17, 487 (1977).

[48] H. Ikezi, K. Schwarzenegger, A. L. Simsons, Y. Ohsawa and T. Kamimura, Nonlinear self-modulation of ion-acoustic waves, Phys. Fluids 21, 239 (1978).

[49] V.E. Zakharov and L.A. Ostrovsky, Modulation instability: The beginning, Physica D: Nonlinear Phenomena 238(5) (2009), 540-548.