Adaptive Anti-disturbance Switching Control for Switched T-S Fuzzy Systems

Ying Zhao[1], Yuxuan Gao[1], Hong Sang[1], and Ben Niu[2]

[1]Dalian Maritime University
[2]Shandong Normal University

April 11, 2023

Abstract

In this paper, an adaptive anti-disturbance switching (AADS) control strategy is proposed for switched Takagi-Sugeno fuzzy systems (ST-SFSs) subject to multi-source disturbances. The disturbances consist of two parts: the available unmodeled disturbance and the disturbance modeled by dynamic neural network. Firstly, a novel adaptive disturbance observer is designed to approximate the dynamic neural network modeled disturbance. Secondly, the attenuation performance from the output to the available disturbance is analyzed by the L 2 gain index. Thirdly, a controller based on the adaptive disturbance observer is constructed for the ST-SFS under the average dwell time switching signal limitation. Further, under the designed adaptive disturbance observer, and the controller, a sufficient condition is established for the ST-SFS to realize multi-source disturbance suppression (DS). Finally, a mass-spring-damping simulation example is given to verify the rationality of the established AADS control scheme.

Received: Added at production | Revised: Added at production | Accepted: Added at production

DOI: xxx/xxxx

ARTICLE TYPE

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Ying Zhao*[1] | Yuxuan Gao[1] | Hong Sang[1] | Ben Niu[2]

[1]The College of Marine Electrical Engineering, Dalian Maritime university, Dalian, Liao Ning, 116026, China
[2]The School of Information Science and Engineering, Shandong Normal University, Jinan, Shandong, 250014,, China

Correspondence
*Ying Zhao, Dalian Maritime university. Email: yingz@dlmu.edu.cn

Summary

In this paper, an adaptive anti-disturbance switching (AADS) control strategy is proposed for switched Takagi-Sugeno fuzzy systems (ST-SFSs) subject to multi-source disturbances. The disturbances consist of two parts: the available unmodeled disturbance and the disturbance modeled by dynamic neural network. Firstly, a novel adaptive disturbance observer is designed to approximate the dynamic neural network modeled disturbance. Secondly, the attenuation performance from the output to the available disturbance is analyzed by the L_2 gain index. Thirdly, a controller based on the adaptive disturbance observer is constructed for the ST-SFS under the average dwell time switching signal limitation. Further, under the designed adaptive disturbance observer, and the controller, a sufficient condition is established for the ST-SFS to realize multi-source disturbance suppression (DS). Finally, a mass-spring-damping simulation example is given to verify the rationality of the established AADS control scheme.

KEYWORDS:
Multi-source disturbances, dynamic neural network, adaptive anti-disturbance, switched Takagi-Sugeno fuzzy systems

1 | INTRODUCTION

As a kind of typical hybrid systems, switched systems are very favorable because they can describe a large number of actual complex processes and provide relatively simple nonlinear control construction methods. Also, switched systems, described as a set of interconnected subsystems that are activated and deactivated by control signals from external sources [1], have become a research hotspot in the field of automatic control [2,3,4,5,6,7]. Due to the important role in switched systems, switching rules have attracted the attention of many scholars [8,9,10,11]. So far, numerous research results have been reported on various switched systems including variable order fractional switched systems [12], networked switched systems [11,13], nonlinear switched systems [14] and so on.

In engineering practice, T-SFSs have been widely concerned because of their simple structure, powerful function and wide application scenarios [15]. Since the fact that T-SFSs can describe complex nonlinear systems by simply weighting linear systems, many mature theories of linear systems can be used to investigate complicated nonlinear systems [16]. Usually, by the virtue of the T-SF method, one can divide a nonlinear dynamic system into a series of linear subsystems by using IF-THEN rules. Up to now, a large number of scholars have studied T-SFSs and made some achievements. Also, numerous interesting control approaches have been developed, such as the dynamic event-triggered security control [17], the networked fault detection control [18], the robust static

[0]**Abbreviations:** AADS, adaptive anti-disturbance switching; ST-SFSs, switched Takagi-Sugeno fuzzy systems; DS, disturbance suppression

output feedback H_∞ control[19], the stability and stabilization control[20]. It is necessary to point out that in[18]–[20], the interaction between linear subsystems divided by IF-THEN rules is not considered. It also does not consider the case that no feasible common matrix can be found when T-SFSs have a large number of IF-THEN rules[21]. Therefore, ST-SFSs which have the advantages of both switched systems and T-SFSs have been widely concerned. To mention several, the asynchronous filtering control[22], the dissipative filtering control[23], the event-triggered control issue[24] and the non-fragile quantized H_∞ filtering control[25].

In practical industrial process and control engineering, a control system is usually encountered by many kinds of disturbances which may seriously degrade system performance and even induce system instability. Limited by the production level and cost, it is very difficult to change the equipment structure of the system to reduce the impact of disturbances. Fortunately, some disturbance attenuation technologies have been developed, such as the H_∞ control strategy[26], model-based AD control scheme[27] and sliding mode technique[28,29]. The disturbance model-based AD control method is an attractive control strategy which has been widely utilized in control systems including the switched systems[30], LPV systems[31], the T-SFSs[32], etc. Compared with other AD control methods, disturbance model-based AD control technique has been widely used in control engineering because of its advantages in simple structure, high control precision and easy combination with other control strategies[33,34]. Unfortunately, most disturbance model-based AD method can only deal with harmonic or constant disturbances, like[35]. Therefore, how to describe and eliminate or reduce the influence of irregular nonlinear disturbances, especially those pre-unknown ones (e.g., attenuated harmonic disturbances, sawtooth wave disturbances) is a meaningful research topic.

The neural networks are powerful tools for identifying complexly high nonlinear systems due to their immediate applicability, inherent approximation ability and parallelism[36,37,38,39]. In practice, dynamic neural networks also have been successfully applied to control systems, which can capture system dynamics through the measured data[40,41] in view of their ability of strong memory and approximation. Thus, these excellent properties of dynamic neural networks have inspired researchers to approximate the unknown external disturbance with a dynamic neural network[42]. Besides, compared with the conventional linearized external disturbance models, such approximate approach is not only more accurate but also can described more kinds of disturbances (e.g., the irregular nonlinear disturbances).

Based on the above discussions, we propose an AADS control method for ST-SFSs with multi-source disturbances. The specific contributions are given below.

i) The introduced dynamic neural network model can approximate the irregular nonlinear disturbances which can not be described by linearized external disturbance models used in the existing results[11,30,31,32]. This expands the types of disturbances estimated by the disturbance observers.

ii) Unlike the exist ST-SFSs affected by a single disturbance in[9,23,43,44], this paper studies the ST-SFSs affected by multi-source disturbances (the available unmodeled disturbance and the unavailable dynamic neural network modeled disturbance). In fact, the coexistence of these two disturbances is more common for practice.

iii) An AADS control scheme is proposed for the ST-SFSs. Under the average dwell time switching strategies, an adaptive disturbance observer and a controller are dual-designed. Accordingly, sufficient conditions are developed to drive the multi-source DS.

Structure. This paper contains five sections. Section 2 described the system and control target . Section 3 gives the design process of the AADS control method. Examples of simulations are given in Section 4. In Section 5, the conclusions are constructed.

The symbols discussed in this paper are provided in Table 1 and are summarized accordingly.

2 | PROBLEM DESCRIPTION

2.1 | System description

Consider the following ST-SFS:
IF $A_i^1(t)$ is $B_i^{a1}(t)$ and $\cdots$ and $A_i^b(t)$ is $B_i^{ab}(t)$, **THEN**

$$\begin{aligned} \dot{x}(t) &= M^a_{\sigma(t)}x(t) + N^a_{\sigma(t)}[u(t) + \mathscr{c}(t)] + O_{\sigma(t)}\mathscr{c}_1(t), \\ z(t) &= P^a_{\sigma(t)}x(t) + H_{\sigma(t)}\mathscr{c}_1(t), \end{aligned} \tag{1}$$

where $\sigma(t)$ specifies the switching rule, $i \in Z^+ = \{1, 2, 3, \dots, s\}$, s stands for the number of subsystems, $A_i^1(t)$, $A_i^2(t)$, $\dots$, $A_i^b(t)$ denote the prerequisite variable, $B_i^1(t)$, $B_i^2(t)$, $\dots$, $B_i^b(t)$ represent the fuzzy sets. b denotes the quantity of $\mathbf{IF-THEN}$ rule, $a \in \{1, 2, 3, \dots, L\}$, $x(t) \in \mathscr{R}^{\mathbf{a}}$ denotes system state, $u(t) \in \mathscr{R}^{\mathbf{b}}$ stands for the system control input, $z(t) \in \mathscr{R}^{\mathbf{c}}$ represents

Table 1

Notation	Meaning
$\mathscr{R}^n$	A collection of real numbers
$N(N^+)$	A collection of all non-negative (positive) integers
$A > 0$	Positive symmetric definite matrix
$L_2[0, \infty)$	The space of squared integrable functions on $[0, \infty)$
I	Identity matrix
$v_{max}(A)(v_{min}(A))$	The maximum (minimum) eigenvalue of A
$\|\| * \|\|(\|\| * \|\|_2)(\|\| * \|\|_F)$	Euclidean (2) (F) norm of $*$
$diag\{A\}$	A is a diagonal matrix

control output, respectively, $\mathscr{c}(t) \in \mathscr{R}^{\mathbf{b}}$ and $\mathscr{c}_1(t) \in \mathscr{R}^{\mathbf{d}}$ stand for the disturbance and $\mathscr{c}(t)$ is generated by the following neural network system mode

$$\begin{aligned} \dot{\chi}(t) &= E_{\sigma(t)}\chi(t) - \mathscr{X}^*\alpha(\chi(t)), \\ \mathscr{c}(t) &= F_{\sigma(t)}\chi(t), \end{aligned} \tag{2}$$

where $\chi(t) \in \mathscr{R}^{\mathbf{e}}$ represents the neural network system state, $\alpha(t) \in \mathscr{R}^{\mathbf{f}}$ stands for the activation function of the neural network and $\mathscr{X}^*$ is the optimal weight matrix.

By the **IF − THEN** rules, we can get the global model of the ST-SFS

$$\begin{aligned} \dot{x}(t) &= \sum_{a=1}^{L} \rho^a_{\sigma(t)}(t)\{M^a_{\sigma(t)}x(t) + N^a_{\sigma(t)}[u(t) + \mathscr{c}(t)]\} + O_{\sigma(t)}\mathscr{c}_1(t), \\ z(t) &= \sum_{a=1}^{L} \rho^a_{\sigma(t)}(t)P^a_{\sigma(t)}x(t) + H_{\sigma(t)}\mathscr{c}_1(t), \end{aligned} \tag{3}$$

where

$$\begin{aligned} \rho^a_i(t) &= \mu^a_i(h_i(t)) \Big/ \sum_{a=1}^{L} \mu^a_i(h_i(t)), \\ h_i(t) &= [\, h^1_i(t) \;\; h^2_i(t) \;\; \cdots \;\; h^b_i(t) \,], \\ \mu^a_i(h_i(t)) &= \Pi^b_{l=1} B^{al}_i(h^l_i(t)), \end{aligned}$$

$B^{al}_i(h^l_i(t))$ represents the membership function of $h^l_i(t)$ in B^{al}_i.
Therefore

$$\rho^a_i(t) \geq 0, \sum_{a=1}^{L} \rho^a_{\sigma(t)}(t) = 1.$$

2.2 | Observer design

To eliminate the effect of the unavailable disturbance $\mathscr{c}(t)$, we design the following adaptive disturbance observer

$$\begin{aligned} \hat{\mathscr{c}}(t) &= F_{\sigma(t)}\hat{\chi}(t), \\ \hat{\chi}(t) &= \beta(t) - K_0x(t), \\ \dot{\beta}(t) &= \sum_{a=1}^{L} \rho^a_{\sigma(t)}(t)\{(E_{\sigma(t)} + K_0N^a_{\sigma(t)}F_{\sigma(t)})[\beta(t) - K_0x(t)] - \hat{\mathscr{X}}(t)\alpha(\hat{\chi}(t)) + K_0[M^a_{\sigma(t)}x(t) + N^a_{\sigma(t)}u(t)]\} \end{aligned} \tag{4}$$

where $\hat{\mathscr{c}}(t)$ represents the estimation of $\mathscr{c}(t)$, $\hat{\chi}(t)$ stands for the observed value of the external neural network model, $\beta(t)$ indicates the observer state, K_0 represents the gain of the observer to be solved and $\hat{\mathscr{X}}(t)$ stands for the dynamically adjustable weight.
Define the estimation error

$$e(t) = \chi(t) - \hat{\chi}(t). \tag{5}$$

From (2), (4) and (5), one can infer

$$\begin{aligned}\dot{e}(t) &= \dot{\chi}(t) - \dot{\hat{\chi}}(t)\\ &= E_{\sigma(t)}\chi(t) - \mathscr{X}^*\alpha(\chi(t)) - \dot{\beta}(t) + K_0\dot{x}(t)\\ &= \sum_{a=1}^{L}\rho^a_{\sigma(t)}(t)[E_{\sigma(t)}\chi(t) + K_0N^a_{\sigma(t)}c(t) + K_0O_{\sigma(t)}c_1(t) - (E_{\sigma(t)} + K_0N^a_{\sigma(t)}F_{\sigma(t)})\hat{\chi}(t) + \hat{\mathscr{X}}(t)\alpha(\chi(t)) - \mathscr{X}^*\alpha(\chi(t))].\end{aligned}$$

Thus, we can get the estimation error dynamics

$$\dot{e}(t) = \sum_{a=1}^{L}\rho^a_{\sigma(t)}(t)(E_{\sigma(t)} + K_0N^a_{\sigma(t)}F_{\sigma(t)})e(t) + K_0O_{\sigma(t)}c_1(t) + \hat{\mathscr{X}}(t)\alpha(\hat{\chi}(t)) - \mathscr{X}^*\alpha(\chi(t)). \tag{6}$$

2.3 | Controller design

Under the same fuzzy rule of the ST-SFS (1), we design the controller as follows

$$u(t) = K^a_{\sigma(t)}x(t) - \hat{c}(t)$$

where K^a_i, $i \in Z^+$ represents the controller gain to be designed.

Accordingly, it is not difficult to conclude that the global model of the controller as

$$u(t) = \sum_{a=1}^{L}\rho^a_{\sigma(t)}(t)K^a_{\sigma(t)}x(t) - \hat{c}(t). \tag{7}$$

From (3) and (7), one has

$$\begin{aligned}\dot{x}(t) &= \sum_{a=1}^{L}\sum_{l=1}^{L}\rho^a_i(t)\rho^l_{\sigma(t)}(t)[(M^a_{\sigma(t)} + N^a_{\sigma(t)}K^l_{\sigma(t)})x(t) + N^a_{\sigma(t)}F_{\sigma(t)}e(t)] + O_{\sigma(t)}c_1(t),\\ z(t) &= \sum_{a=1}^{L}\rho^a_{\sigma(t)}(t)P^a_{\sigma(t)}x(t).\end{aligned} \tag{8}$$

Combining (6) and (8), one can derive the following augmented system

$$\begin{aligned}\dot{\xi}(t) &= \sum_{a=1}^{L}\sum_{l=1}^{L}\rho^a_{\sigma(t)}(t)\rho^l_{\sigma(t)}(t)\tilde{M}^{al}_{\sigma(t)}\xi(t) + \tilde{O}_{\sigma(t)}c_1(t) + \tilde{N}[\hat{\mathscr{X}}(t)\alpha(\chi(t)) - \mathscr{X}^*\alpha(\chi(t))],\\ z(t) &= \sum_{a=1}^{L}\rho^a_{\sigma(t)}(t)\tilde{P}^a_{\sigma(t)}\xi(t) + \tilde{H}_{\sigma(t)}c_1(t),\end{aligned} \tag{9}$$

where

$$\xi(t) = \begin{bmatrix} x(t)\\ e(t)\end{bmatrix}, \tilde{O}_i = \begin{bmatrix} O_i\\ 0\end{bmatrix},$$

$$\tilde{M}^{al}_i = \begin{bmatrix} M^a_i + N^a_iK^l_i & N^a_iF_i\\ 0 & E_i + K_0N^a_iF_i\end{bmatrix},$$

$$\tilde{N} = \begin{bmatrix} 0\\ I\end{bmatrix}, \tilde{P}^a_i = \begin{bmatrix} P^a_i & 0\end{bmatrix}.$$

2.4 | Control objective

The control objective is twofold.

For the system (1) with the neural network disturbance model (2), suppose that there exist the AADS control method ensuring the following relations:

i) When $c_1(t) \equiv 0$, the system state $\xi(t)$ of the augmented system (9) satisfies practically stable.

ii) When $\mathscr{c}_1(t) \neq 0$, the system output $z(t)$ suggests

$$\int_0^{\infty} e^{-\delta \wp} z^T(\wp) z(\wp) d\wp \leq \gamma^2 \int_0^{\infty} \mathscr{c}_1{}^T(\wp) \mathscr{c}_1(\wp) d\wp + \varpi, \tag{10}$$

where γ denotes a designated positive constant as the L_2-gain index, the constant $\varpi > 0$.

Then, the problem of the AADS control for the system (3) is solvable. Also, the controller (7) can solve the AADS control strategy of the system (1). Now, the configuration of the AADS control strategy is exhibited in Fig. 1.

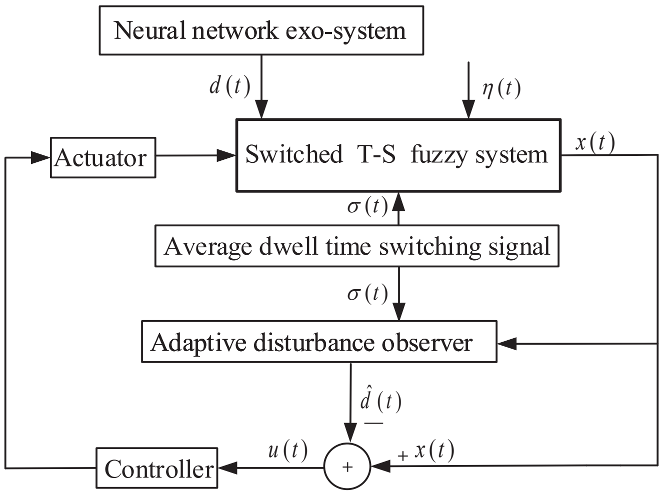

Figure 1 Configuration of the AADS control.

A definition and an assumption are provided before forming the main result.

Definition 1. [45] For the switching signal $\sigma(t)$ and any time $t > \wp > 0$, $H_\sigma(\wp, t)$ denotes the amount of switching in the time range $(\wp, t)$, if

$$H_\sigma(\wp, t) \leq H_0 + \frac{t - \wp}{\wp_d} \tag{11}$$

and $\wp_d > 0$ hold, then H_0 and $\wp_d$ are known as the chattering bound of $\sigma(t)$ and average dwell-time, respectively.

Assumption 1. The neural network activation function $\alpha(\chi(t))$ meets the following Lipschitz condition

$$[\alpha(\chi(t)) - \alpha(\hat{\chi}(t))]^T [\alpha(\chi(t)) - \alpha(\hat{\chi}(t))] \leq e^T(t) U_e^T U_e e(t)$$

where U_e stands for a given matrix.

3 | MAIN RESULTS

In this section, we first focus on how to obtain the stability and multi-source DS performance of system (1) by designing the AADS control strategy. Then, the sufficient conditions are given to ensure the adaptive disturbance observer, switching rule, adaptive law and controller can solve the AADS control issue of the system (1).

First, the following theorem will discuss the bound of $\hat{\mathscr{X}}(t)$ and give the corresponding adaptive adjustment dynamics.

Theorem 1. Consider the adaptive disturbance observer (4). For the pre-arranged constant $n > 0$, if there exists the matrix $Y_2 > 0$ forcing

$$\dot{\hat{\mathscr{X}}}(t) = nY_2 \hat{\chi}(t) \alpha^T(\hat{\chi}(t)) - \|\hat{\chi}(t)\| \hat{\mathscr{X}}(t), \tag{12}$$

then $\hat{\mathscr{X}}(t) \in \Sigma_{\hat{\mathscr{X}}}$ and $\hat{\mathscr{X}}(t) \in L_\infty$, where $\hat{\mathscr{X}}(0) \in \Sigma_{\hat{\mathscr{X}}} = \{\hat{\mathscr{X}} : \left\|\hat{\mathscr{X}}(t)\right\|_F \le n\sqrt{n_\zeta}\left\|Y_2\right\|\}$.

Proof. Choose the following Lyapunov function

$$V_1(t) = \frac{1}{2}tr\{\hat{\mathscr{X}}^T(t)n^{-1}\hat{\mathscr{X}}(t)\}. \tag{13}$$

Taking the derivative of (13), one can get

$$\begin{aligned}\dot{V}_1(t) &= tr\{\alpha^T(\hat{\chi}(t))\hat{\chi}^T(t)Y_2\hat{\mathscr{X}}(t)\} - n^{-1}\left\|\hat{\chi}(t)\right\|\left\|\hat{\mathscr{X}}(t)\right\|_F^2 \\ &= \left\|\hat{\chi}^T(t)Y_2\hat{\mathscr{X}}(t)\alpha^T(\hat{\chi}(t))\right\| - n^{-1}\left\|\hat{\chi}(t)\right\|\left\|\hat{\mathscr{X}}(t)\right\|_F^2.\end{aligned}$$

Inspired by [36] and [46], we choose the neural network activation function

$$\alpha(\chi(t)) = [\ \tfrac{1}{1+e^{-n_1\chi_1}},\ \cdots,\ \tfrac{1}{1+e^{-n_1\chi_{n_\chi-1}}},\ 1\ ]^T, \tag{14}$$

where $n_1 > 0$ is a constant. The middle term of the activation function is

$$\frac{1}{1+e^{-n_1\chi_k}},$$

$k \in \{1, 2, 3, \ldots, n_\chi - 1\}$.
Thus, it is easy to verify that

$$\frac{1}{1+e^{-n_1\chi_k}} \le 1.$$

Then, we can deduce that $\left\|\alpha(\chi(t))\right\| \le \sqrt{n_\chi}$ and rewrite equation (14) to get

$$\begin{aligned}\dot{V}_1(t) &\le \sqrt{n_\chi}\left\|\hat{\chi}^T(t)\right\|\left\|Y_2\right\|\left\|\hat{\mathscr{X}}(t)\right\|_F - n^{-1}\left\|\hat{\chi}(t)\right\|\left\|\hat{\mathscr{X}}(t)\right\|_F^2 \\ &= \left\|\hat{\chi}(t)\right\|\left\|\hat{\mathscr{X}}(t)\right\|_F(\sqrt{n_\chi}\left\|Y_2\right\| - n^{-1}\left\|\hat{\mathscr{X}}(t)\right\|_F).\end{aligned} \tag{15}$$

This means that if

$$\left\|\hat{\mathscr{X}}(t)\right\|_F > n\sqrt{n_\chi}\left\|Y_2\right\|,$$

then $\dot{V}_1(t) \le 0$.
Thus, if $\mathscr{X}(0) \in \alpha_{\hat{\mathscr{X}}}$, then one have $\mathscr{X}(t) \in \alpha_{\hat{\mathscr{X}}}$ and $\hat{\mathscr{X}}(t) \in L_\infty$.

Remark 1. Theorem 1 offers the adaptive law of the dynamic adjustable weight matrix $\hat{\mathscr{X}}(t)$ in the adaptive disturbance observe (4).

Next, the stability and L_2 performance of the augmented system (9) will be analyzed.

Theorem 2. Recall the system (9). If we have the matrices $Y_{1i} > 0, Y_2 > 0$, K_i^l, K_0, $\bar{\mathscr{X}}$, U_e, and the positive scalars $\delta_0, \delta, n, n_\chi, \mu_2, \mu_3$, scalars $\bar{\mu} \ge 1$ and $\chi_{ij} \le 0$ satisfy the following constraints

$$\begin{bmatrix}\Omega_{11}^{ial} & \Omega_{12}^{i} & \Omega_{13}^{ia} \\ * & \Omega_{22}^{ia} & 0 \\ * & * & \Omega_{33}^{i}\end{bmatrix} < 0, \tag{16}$$

$$Y_{1i} \le \bar{\mu}Y_{1j}, i \ne j, i, j \in Z^+, \tag{17}$$

where

$$\begin{aligned}\Omega_{11}^{ial} &= sym(Y_{1i}M_i^a + Y_{1i}N_i^aK_i^l) + 2\delta_0Y_{1i} + P_i^{aT}P_i^a, \\ \Omega_{12}^{i} &= Y_{1i}N_i^aF_i, \\ \Omega_{13}^{ia} &= Y_{1i}O_i + P_i^{aT}H_i, \\ \Omega_{22}^{ia} &= sym(Y_2E_i + Y_2K_0N_i^aF_i) + \mu_2^{-2}Y_2K_0O_iO_i^TK_0^TY_2 + Y_2\bar{\mathscr{X}}Y_2 \\ &\quad + U_e^TU_e + \mu_3^{-2}I + 2\delta_0Y_2 + (n^{-1}+1)I,\end{aligned}$$

$$\Omega_{33}^{i} = \mu_2^2 I + H_i^T H_i - \delta I,$$
$$\Xi_{11}^{i} = -\lambda_i^2 I + \sum_{j=1, j\neq i}^{Z^+} \chi_{ij}(Y_{1i} - Y_{1j}),$$
$$\Xi_{12}^{ia} = (K_i^a - K^{a*})^T,$$
$$\Xi_{13}^{il} = (K_i^l - K^{l*})^T,$$

then, under the average dell-time switching regulation

$$\wp_d > \wp_d^* = \frac{\ln \bar{\mu}}{\bar{\delta}}, \bar{\delta} \in (0, \delta_0), \tag{18}$$

the state $x(t)$ of the system (3) and the observer estimation $e(t)$ of the observer (6) are ensured to converge into a compact set $\mathbf{p}(\xi)$ defined by

$$\mathbf{p}(\xi) = \{\xi(t) : \|\xi(t)\| \leq \max_{i\in Z^+}\{\sqrt{\frac{p}{2\delta_0 \nu_{\min}(Y_i)}}\}\}.$$

Proof. For the i-th subsystem, the following Lyapunov function is selected

$$\Psi_i(t) = \Psi_{1i}(t) + \Psi_2(t)$$

with

$$\begin{aligned} \Psi_{1i}(t) &= x^T(t) Y_{1i} x(t), \\ \Psi_2(t) &= e^T(t) Y_2 e(t) + tr\{\tilde{\mathcal{X}}^T(t) n^{-1} \tilde{\mathcal{X}}(t)\}. \end{aligned} \tag{19}$$

From (19), one can deduce

$$\begin{aligned} \dot{\Psi}_{1i}(t) =& \sum_{a=1}^{L}\sum_{l=1}^{L} \rho_i^a(t)\rho_i^l(t)[x^T(t) sym(Y_{1i}M_i^a + Y_{1i}N_i^a K_i^l)x(t) + 2x^T(t)Y_{1i}N_i^a F_i e(t)] \\ &+ 2x^T(t)Y_{1i}O_i \varepsilon_1(t) \\ \leq& \sum_{a=1}^{L}\sum_{l=1}^{L} \rho_i^a(t)\rho_i^l(t)[x^T(t) sym(Y_{1i}M_i^a + Y_{1i}N_i^a K_i^l)x(t) + 2x^T(t)Y_{1i}N_i^a F_i e(t)] \\ &+ \mu_1^2 \varepsilon_1{}^T(t)\varepsilon_1(t) + \mu_1^{-2} x^T(t) Y_{1i} O_i O_i^T Y_{1i} x(t). \end{aligned} \tag{20}$$

If the adaptive rule is taken in the way of (12), one can get

$$\begin{aligned} \dot{\Psi}_2(t) =& \sum_{a=1}^{L} \rho_i^a(t)[e^T(t) sym(Y_2 E_i + Y_2 K_0 N_i^a F)e(t)] + 2e^T(t)Y_2K_0O_i\varepsilon_1(t) + 2e^T(t)Y_2\tilde{\mathcal{X}}(t)\alpha(\hat{\chi}(t)) \\ &+ 2e^T(t)Y_2\mathcal{X}^*[\alpha(\hat{\chi}(t)) - \alpha(\chi(t))] + 2tr\{\dot{\tilde{\mathcal{X}}}(t)n^{-1}\tilde{\mathcal{X}}(t)\} \\ \leq& \sum_{a=1}^{L} \rho_i^a(t)[e^T(t) sym(Y_2 E_i + Y_2 K_0 N_i^a F)e(t)] + \mu_2^2\varepsilon_1{}^T(t)\varepsilon_1(t) + \mu_2^{-2}e^T(t)Y_2K_0O_iO_i^TK_0^TYe(t) \\ &+ e^T(t)U_e^TU_e e(t) + 2\chi^T(t)Y_2\tilde{\mathcal{X}}(t)\alpha(\hat{\chi}(t)) - 2\hat{\chi}^T(t)Y_2\tilde{\mathcal{X}}(t)\alpha(\hat{\chi}(t)) + 2tr\{\dot{\tilde{\mathcal{X}}}(t)n^{-1}\tilde{\mathcal{X}}(t)\} \\ \leq& \sum_{a=1}^{L} \rho_i^a(t)[e^T(t) sym(Y_2 E_i + Y_2 K_0 N_i^a F)e(t)] + \mu_2^2\varepsilon_1{}^T(t)\varepsilon_1(t) + \mu_2^{-2}e^T(t)Y_2K_0O_iO_i^TK_0^TYe(t) \\ &+ e^T(t)U_e^TU_e e(t) + 2\chi^T(t)Y_2\tilde{\mathcal{X}}(t)\alpha(\hat{\chi}(t)) + 2\|\hat{\chi}(t)\| tr\{\tilde{\mathcal{X}}(t)n^{-1}\hat{\mathcal{X}}(t)\}. \end{aligned}$$

It follows from Theorem 1 that

$$2\|\hat{\chi}(t)\| tr\{\tilde{\mathcal{X}}(t)n^{-1}\hat{\mathcal{X}}(t) \geq \|\tilde{\mathcal{X}}(t)\|_F^2 - \|\mathcal{X}^*\|_F^2, \tag{21}$$

$$\begin{aligned} &2\chi^T(t)Y_2\tilde{\mathcal{X}}(t)\alpha(\hat{\chi}(t)) \\ &\leq \sqrt{2n_\chi}\,\|\chi(t)\|\,\|Y_2\|\,\|\tilde{\mathcal{X}}(t)\|_F^2\,\|\mathcal{X}^*\|_F^2 \\ &\leq \sqrt{\frac{2n_\eta n_\chi}{\upsilon_{\min}(F_i^T F_i)}}\,\|Y_2\|\,(n\sqrt{n_\chi} + \sqrt{tr\{\bar{\mathcal{X}}\}}), \end{aligned} \tag{22}$$

and

$$\begin{aligned} 2\,\|\hat{\chi}(t)\|\,\|\mathcal{X}^*\|_F^2 &\le 2\,\|\chi(t)\|\,\|\mathcal{X}^*\|_F^2 + 2\,\|e(t)\|\,\|\mathcal{X}^*\|_F^2 \\ &\le \mu_3^{-2} e^T(t)e(t) + \sqrt{\frac{4n_\eta}{\upsilon_{\min}(F_i^T F_i)}} tr\{\bar{\mathcal{X}}\} + \mu_3^2 (tr\{\bar{\mathcal{X}}\})^2, \end{aligned} \tag{23}$$

where $\bar{\mathcal{X}} \ge \mathcal{X}^{*T}\mathcal{X}^*$ is the positive upper bound of $\mathcal{X}^*$.
Combing (21), (22) and (23) yields

$$\begin{aligned} \dot{\Psi}_2(t) \le& \sum_{a=1}^{L} \rho_i^a(t)[e^T(t)sym(Y_2E_i + Y_2K_0N_i^aF)e(t)] + \mu_3^{-2}e^T(t)e(t) \\ &+ \mu_2^2 \mathcal{C}_1^T(t)\mathcal{C}_1(t) + \mu_2^{-2}e^T(t)Y_2K_0O_iO_i^TK_0^TYe(t) + e^T(t)U_e^TU_ee(t) + e^T(t)Y_2\bar{\mathcal{X}}Y_2e(t) \\ &+ \sqrt{\frac{4n_\eta}{\upsilon_{\min}(F_i^TF_i)}}tr\{\bar{\mathcal{X}}\} + \mu_3^2(tr\{\bar{\mathcal{X}}\})^2 + \sqrt{\frac{2n_\eta n_\chi}{\upsilon_{\min}(F_i^TF_i)}}\,\|Y_2\|\,(n\sqrt{n_\chi} + \sqrt{tr\{\bar{\mathcal{X}}\}}). \end{aligned} \tag{24}$$

By means of (20), (24), we can get

$$\begin{aligned} &\dot{\Psi}_{1i}(t) + \dot{\Psi}_2(t) + 2\delta_0(\Psi_{1i}(t) + \Psi_2(t)) + z^T(t)z(t) - \delta\mathcal{C}_1^T(t)\mathcal{C}_1(t) \\ &\le \sum_{a=1}^{L}\sum_{l=1}^{L} \rho_i^a(t)\rho_i^l(t)[x^T(t)sym(Y_{1i}M_i^a + Y_{1i}N_i^aK_i^l)x(t) + 2x^T(t)Y_{1i}N_i^aF_ie(t)] \\ &+ \mu_1^2\mathcal{C}_1^T(t)\mathcal{C}_1(t) + \mu_1^{-2}x^T(t)Y_{1i}O_iO_i^TY_{1i}x(t) + \sum_{a=1}^{L}\rho_i^a(t)[e^T(t)sym(Y_2E_i + Y_2K_0N_i^aF)e(t)] \\ &+ \mu_2^2\mathcal{C}_1^T(t)\mathcal{C}_1(t) + \mu_2^{-2}e^T(t)Y_2K_0O_iO_i^TK_0^TYe(t) + e^T(t)U_e^TU_ee(t) + e^T(t)Y_2\bar{\mathcal{X}}Y_2e(t) \\ &+ \mu_3^{-2}e^T(t)e(t) + 2\delta_0 tr\{\tilde{\mathcal{X}}(t)n^{-1}\tilde{\mathcal{X}}(t)\} + \sqrt{\frac{4n_\eta}{\upsilon_{\min}(F_i^TF_i)}}tr\{\bar{\mathcal{X}}\} + \mu_3^2(tr\{\bar{\mathcal{X}}\})^2 \\ &+ \sqrt{\frac{2n_\eta n_\chi}{\upsilon_{\min}(F_i^TF_i)}}\,\|Y_2\|\,(n\sqrt{n_\chi} + \sqrt{tr\{\bar{\mathcal{X}}\}}) + 2\delta_0x^T(t)Y_{1i}x(t) + 2\delta_0e^T(t)Y_2e(t) \\ &+ \mathcal{C}_1^T(t)H_i^TH_i\mathcal{C}_1(t) + x^T(t)\sum_{a=1}^{L}\rho_i^a(t)P_i^{aT}P_i^ax(t) + 2x^T(t)\sum_{a=1}^{L}\rho_i^a(t)P_i^{aT}H_i\mathcal{C}_1(t) - \delta\mathcal{C}_1^T(t)\mathcal{C}_1(t) \\ &= \begin{bmatrix} x(t) \\ e(t) \\ \mathcal{C}_1(t) \end{bmatrix}^T \begin{bmatrix} \Omega_{11}^{ial} & \Omega_{12}^{i} & \Omega_{13}^{ia} \\ * & \Omega_{22}^{ia} & 0 \\ * & * & \Omega_{33} \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \\ \mathcal{C}_1(t) \end{bmatrix} + p, \end{aligned}$$

where

$$p = \sqrt{\frac{4n_\eta}{\upsilon_{\min}(F_i^TF_i)}}tr\{\bar{\mathcal{X}}\} + \mu_3^2(tr\{\bar{\mathcal{X}}\})^2 + \sqrt{\frac{2n_\eta n_\chi}{\upsilon_{\min}(F_i^TF_i)}}\,\|Y_2\|\,(n\sqrt{n_\chi} + \sqrt{tr\{\bar{\mathcal{X}}\}}) + 8\delta_0\sqrt{n_\chi}\|Y_2\|.$$

From (13), we can infer

$$\dot{\Psi}_{1i}(t) + \dot{\Psi}_2(t) + 2\delta_0(\Psi_{1i}(t) + \Psi_2(t)) + z^T(t)z(t) - \delta\mathcal{C}_1^T(t)\mathcal{C}_1(t) \le p. \tag{25}$$

When $\mathcal{C}_1(t) = 0$, it is straightforward to deduce that

$$\dot{\Psi}_{1i}(t) + \dot{\Psi}_2(t) \le 2\delta_0(\Psi_{1i}(t) + \Psi_2(t)) + p. \tag{26}$$

According to (17), one can obtain

$$\Psi_{1i}(t) \le \bar{\mu}\Psi_{1j}(t). \tag{27}$$

Applying (26) and (27) generates that

$$\begin{aligned} \Psi_{1i}(t) + \Psi_2(t) &\le \bar{\mu}^{H_\sigma(0,t)}e^{-2\delta_0 t}[\Psi_{1i}(0) + \Psi_2(0)] + \frac{p}{2\delta_0}(1 - e^{-2\delta_0 t}) \\ &= e^{H_\sigma(0,t)\ln\bar{\mu} - 2\delta_0 t}[\Psi_{1i}(0) + \Psi_2(0)] + \frac{p}{2\delta_0}(1 - e^{-2\delta_0 t}). \end{aligned} \tag{28}$$

Further, it follows from (11) and $H_\sigma(0,t) \le t \big/ \wp_d^*$ that

$$H_\sigma(0,\wp)\ln\bar{\mu} \le \bar{\delta}t. \tag{29}$$

Thus, one can deduce

$$\Psi_{1i}(t)+\Psi_2(t) \le e^{-(2\delta_0-\bar{\delta})t}[\Psi_{1i}(0)+\Psi_2(0)] + \frac{p}{2\delta_0}(1-e^{-2\delta_0 t}). \tag{30}$$

This means that the state $\xi(t)$ of the system (9) can be guaranteed to converge into the following region

$$\mathbf{p}(\xi) = \max_{i\in Z^+}\{\xi(t)\in\mathscr{R}^{\mathbf{m}+c_1} : \xi^T(t)Y_i\xi(t) \le \frac{p}{2\delta_0}\},$$

where $Y_i = diag\{Y_{1i}, Y_2\}$.

When $h_1(t)\neq 0$, from (25) and (27), we deduce

$$\begin{aligned}
&\Psi_{1i}(t)+\Psi_2(t)\\
&\le [\Psi_{1i}x(t_{H_\sigma(0,t)}) + \Psi_2(t_{H_\sigma(0,t)},t)]e^{-\delta_0(2t-H_\sigma(0,t))} + \int_{t_{H_\sigma(0,t)}}^{t} e^{-\delta_0(2t-\wp)}\Gamma(\wp)d\wp\\
&\le \bar{\mu}[\Psi_{1i}(x(t^-_{H_\sigma(0,t)}) + \Psi_2(t^-_{H_\sigma(0,t)})]e^{-\delta_0(2t-t_{H_\sigma(0,t)})} + \int_{t_{H_\sigma(0,t)}}^{t} e^{-\delta_0(2t-\wp)}\Gamma(\wp)d\wp\\
&\le \bar{\mu}\{\Psi_{1i}(x(t_{H_\sigma(0,t)-1}) + \Psi_2(t_{H_\partial(0,t)-1})]e^{-\delta_0(2t_{H_\sigma(0,t)}-t_{H_\sigma(0,t)-1})}\\
&\quad + \int_{t_{H_\sigma(0,t)-1}}^{t_{H_\sigma(0,t)}} e^{-\delta_0(2t_{H_\sigma(0,t)}-\wp)}\Gamma(\wp)d\wp\}e^{-\delta_0(t-2t_{H_\sigma(0,t)})} + \int_{t_{H_\sigma(0,t)}}^{t} e^{-\delta_0(2t-\wp)}\Gamma(\wp)d\wp\\
&\le \cdots\cdots\\
&\le \bar{\mu}^{H_\partial(0,t)}e^{-\delta_0(2t-t_0)}[\Psi_{1i}(0)+\Psi_2(0)] + \bar{\mu}^{H_\sigma(0,t)}\int_{t_0}^{t_1} e^{-\delta_0(2t-\wp)}\Gamma(\wp)d\wp\\
&\quad + \bar{\mu}^{H_\sigma(0,t)-1}\int_{t_1}^{t_2} e^{-\delta_0(2t-\wp)}\Gamma(\wp)d\wp + \cdots + \bar{\mu}^0\int_{t_{\mathrm{H}_\sigma(0,t)}}^{t} e^{-\delta_0(2t-\wp)}\Gamma(\wp)d\wp\\
&= \bar{\mu}^{H_\sigma(t_0,t)}e^{-\delta_0(2t-t_0)}[\Psi_{1i}(0)+\Psi_2(0)] + \int_{t_0}^{t}\bar{\mu}^{H_\sigma(\wp,t)}e^{-\delta_0(2t-\wp)}\Gamma(\wp)d\wp\\
&= e^{-\delta_0(2t-t_0)+H_\sigma(t_0,t)\ln\bar{\mu}}[\Psi_{1i}(0)+\Psi_2(0)] + \int_{t_0}^{t} e^{-\delta_0(2t-\wp)+H_\sigma(\wp,t)\ln\bar{\mu}}\Gamma(\wp)d\wp,
\end{aligned} \tag{31}$$

where $\Gamma(\wp) = -z^T(\wp)z(\wp) + \gamma^2 c_1{}^T(\wp)c_1(\wp) + p$.

Multiplying two sides of (31) by $e^{-H_\sigma(0,t)\ln\bar{\mu}}$, we can deduce

$$\int_0^t e^{-\delta_0(t-\wp)-H_\sigma(0,\wp)\ln\bar{\mu}}(-p)d\wp \le e^{-\delta_0(2t-t_0)}[\Psi_{1i}(0)+\Psi_2(0)] + \gamma^2\int_0^t e^{-\delta_0(2t-\wp)}c_1{}^T(\wp)c_1(\wp)d\wp. \tag{32}$$

By means of (29), one can get

$$\int_0^t e^{-\delta_0(2t-\wp)-\bar{\delta}\wp}(z^T(\wp)z(\wp)-p)d\wp \le \gamma^2\int_0^t e^{-\delta_0(2t-\wp)}[c_1{}^T(\wp)c_1(\wp)]d\wp + e^{-2\delta_0 t}[\Psi_{1i}(0)+\Psi_2(0)]. \tag{33}$$

Integrating (33) from $t : 0 \to \infty$ on both sides yields

$$\int_0^\infty e^{-\delta \wp} z^T(\wp) z(\wp) d\wp \le \gamma^2 \int_0^\infty \varepsilon_1^T(\wp)\varepsilon_1(\wp) d\wp + \varpi,$$

where $\varpi \ge \frac{p}{\delta_0} + \Psi_{1i}(0) + \Psi_2(0)$ is a positive constant. Therefore, we can easily deduce the L_2-gain property (10) when $t \to \infty$.

Remark 2. Theorem 2 establishes the sufficient condition through which the issue of AADS control issue for the system (1) is solvable. If the disturbance $\varepsilon_1(t) \equiv 0$, we can obtain from Theorem 2 that the ST-SFS (1) is practical stable, while if $\varepsilon_1(t) \neq 0$, the L_2-gain property (10) can be easily deduced when $t \to \infty$.

Further, the construction process of the controller (7) and observer (4) will be provided.

Theorem 3. Consider the system (9). Assume that matrices $\bar{Y}_{1i} > 0, Y_2 > 0, \Lambda_i^l, \Lambda, K_0, \bar{\mathscr{X}}, U_e$ and the positive scalars $\delta_0, \delta, \mu_2, \mu_3$ can be searched to satisfy the following constraints

$$\begin{bmatrix} \Delta_{11}^{ial} & \Delta_{12}^{ia} & \Delta_{13}^{ia} & 0 & 0 & \bar{Y}_{1i}P_i^{aT} \\ * & \Delta_{22}^{ia} & 0 & Y_2 & \Lambda O_i & 0 \\ * & * & \Delta_{33}^{i} & 0 & 0 & 0 \\ * & * & * & -\bar{\mathscr{X}}^{-1} I & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{34}$$

$$Y_{1i} \le \bar{\mu} Y_{1j}, i \neq j, i, j \in Z^+, \tag{35}$$

where

$$\begin{aligned} \Delta_{11}^{ial} &= sym(\bar{Y}_{1i} M_i^{aT} + N_i^a \Lambda_i^l) + 2\delta_0 \bar{Y}_{1i}, \\ \Delta_{12}^{ia} &= N_i^a F_i, \\ \Delta_{13}^{ia} &= O_i + \bar{Y}_{1i} P_i^{aT} H_i, \\ \Delta_{22}^{ia} &= sym(E_i^T Y_2 + F_i^T N_i^{aT} \Lambda^T) + U_e^T U_e + \mu_3^{-2} I + 2\delta_0 Y_2 + (n^{-1}+1) I, \\ \Delta_{33}^{i} &= \mu_3^2 I + H_i^T H_i - \delta I, \end{aligned}$$

then, we can claim that the observe (4) and the controller (7) can deal with AADS control problem for the system (1). Further, the gains can be formulated by $K_i^l = \Lambda_i^l \bar{Y}_{1i}^{-1}$ and $K_0 = Y_2^{-1}\Lambda$, $i \in Z^+$.

Proof. Pre- and post-multiplying (16) by $diag\{Y_{1i}^{-1}, I\}$ generates

$$\begin{bmatrix} \bar{\Omega}_{11}^{ial} & \bar{\Omega}_{12}^{i} & \bar{\Omega}_{13}^{ia} \\ * & \Omega_{22}^{ia} & 0 \\ * & * & \Omega_{33}^{i} \end{bmatrix} < 0, \tag{36}$$

where

$$\begin{aligned} \bar{\Omega}_{11}^{ial} &= sym(M_i^a Y_{1i}^{-1} + N_i^a K_i^l Y_{1i}^{-1}) + 2\delta_0 Y_{1i}^{-1} + Y_{1i}^{-1} P_i^{aT} P_i^a Y_{1i}^{-1}, \\ \bar{\Omega}_{12}^{i} &= N_i^a F_i, \\ \bar{\Omega}_{13}^{ia} &= O_i + Y_{1i}^{-1} P_i^{aT} H_i. \end{aligned}$$

Defining $\bar{Y}_{1i} = Y_{1i}^{-1}, \Lambda_a^l = \bar{Y}_{1i} K_i^{lT}, \Lambda = Y_2 K_0$ and adopting Complement Lemma to (36), one can derive

$$\begin{bmatrix} \Delta_{11}^{ial} & \Delta_{12}^{ia} & \Delta_{13}^{ia} & 0 & 0 \\ * & \Delta_{22}^{ia} & 0 & Y_2 & \Lambda O_i \\ * & * & \Delta_{33}^{i} & 0 & 0 \\ * & * & * & -\bar{\mathscr{X}}^{-1} I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \tag{37}$$

where

$$\bar{\Delta}_{11}^{ial} = sym(\bar{Y}_{1i} M_i^{aT} + \Lambda_i^l N_i^a) + 2\delta_0 \bar{Y}_{1i} + \bar{Y}_{1i} P_i^{aT} P_i^a \bar{Y}_{1i},$$

Adopting Schur complement lemma again to (37) produces (34).

4 | VERIFICATION

To demonstrate the validity of the designed AADS control strategy, a mass-spring-damping case study depicted as in Fig. 2 is given for simulation research.

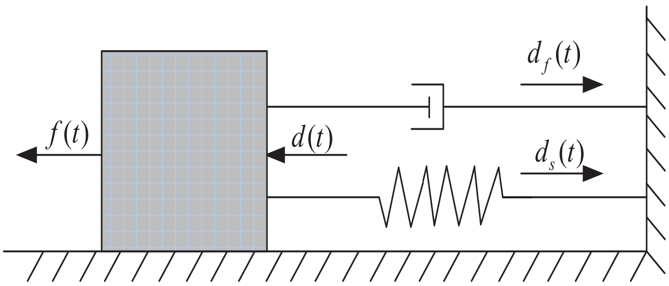

Figure 2 Schematic diagram of the mass-spring-damper model.

Similar to[47] and[48], we give the following mass-spring-damping dynamics

$$\begin{aligned} m\ddot{f}(t) + d_f(t) + d_s(t) &= s(t), \\ d_f(t) &= p_1 \dot{f}(t)^3, \\ d_s(t) &= p_2 f(t) + p_3 f(t)^3, \end{aligned} \tag{38}$$

where $m > 0$, $p_l > 0$ with $l \in \{1, 2, 3\}$. The notations in the mass-spring-damping model (38) are listed as follows Given the

Notation	Meaning
m	Mass quality
$f(t)$	Mass displacement
$s(t)$	Spring pushing force
$d_f(t)$	The friction of the damper
$d_s(t)$	The restoring force of the spring

constants value

$$m = 1, p_1 = 0.2, p_2 = 0.03, p_3 = 0.56.$$

Then, one can deduce that

$$\ddot{f}(t) = -0.56f(t)^3 - 0.2\dot{f}(t)^3 - 0.03f(t) + s(t).$$

Choose

$$\begin{bmatrix} \Delta\pi^T(t) & \Delta\varpi^T(t) \end{bmatrix}^T = \begin{bmatrix} f^T(t) & \dot{f}^T(t) \end{bmatrix}^T, u(t) = s(t),$$

thus, one can deduce

$$\begin{bmatrix} \Delta\dot{\pi}(t) \\ \Delta\dot{\varpi}(t) \end{bmatrix} = \begin{bmatrix} \varpi(t) \\ -0.2\varpi^3(t) - 0.03\pi(t) - 0.56\pi^3(t) + u(t) \end{bmatrix}.$$

Notice that $-0.2\varpi^3(t)$ and $-0.56\pi^3(t)$ are nonlinear terms which fulfill the requirements

$$\begin{bmatrix} \Delta\pi^T(t) & \Delta\varpi^T(t) \end{bmatrix}^T \in [-1.5, 1.5],$$

$$\begin{bmatrix} \Delta\dot{\pi}^T(t) & \Delta\dot{\varpi}^T(t) \end{bmatrix}^T \in [-1.5, 1.5].$$

Defining

$$\vartheta = \begin{bmatrix} \Delta\pi^T(t) & \Delta\varpi^T(t) \end{bmatrix}^T$$

and using the upper and lower bounds to represent the nonlinear terms $-0.2\varpi^3(t)$ and $-0.56\pi^3(t)$ yields

$$
\begin{aligned}
-0.56\vartheta^3(t) &= h_1^1(t) * 0 * \vartheta(t) - (1 - h_1^1(t)) * 1.5075\vartheta(t),\\
-0.2\dot{\vartheta}^3(t) &= h_2^1(t) * 0 * \dot{\vartheta}(t) - (1 - h_2^1(t)) * 0.224\dot{\vartheta}(t).
\end{aligned}
$$

Thus, we can deduce that

$$
\begin{aligned}
h_1^1(t) = -\vartheta^2(t)/\mathscr{g} + 1, h_2^1(t) = -\dot{\vartheta}^2(t)/\mathscr{g} + 1,\\
z_1^2(t) = \vartheta^2(t)/\mathscr{g}, h_2^2(t) = \dot{\vartheta}^2(t)/\mathscr{g}, \mathscr{g} = 2.25.
\end{aligned}
$$

When the nonlinear terms $-0.2\varpi^3(t)$ and $-0.56\pi^3(t)$ reach the maximum or minimum value the switch occurs. Thus, the membership function is chosen as follows

$$
\begin{aligned}
\rho_1^1(t) = \rho_2^1(t) &= \frac{h_1(t)[1 - h_1(t)]}{h_1(t)[1 - h_1(t)] + h_2(t)[1 - h_2(t)]},\\
\rho_1^2(t) = \rho_2^2(t) &= 1 - \rho_1^1(t).
\end{aligned}
\tag{39}
$$

Under the membership function (39), mass-spring-damping system (38) is represented by

$$
\begin{aligned}
\begin{bmatrix} \Delta\dot{\pi}(t) \\ \Delta\dot{\varpi}(t) \end{bmatrix} &= \sum_{a=1}^{2} \rho_{\sigma(t)}^a(t)\{M_{\sigma(t)}^a \begin{bmatrix} \Delta\pi(t) \\ \Delta\varpi(t) \end{bmatrix} + N_{\sigma(t)}^a [u(t) + c(t)]\} + O_{\sigma(t)} c_1(t),\\
z(t) &= \sum_{a=1}^{2} \rho_{\sigma(t)}^a(t) P_{\sigma(t)}^a \begin{bmatrix} \Delta\pi(t) \\ \Delta\varpi(t) \end{bmatrix} + H_{\sigma(t)} c_1(t).
\end{aligned}
\tag{40}
$$

The parameters in the mass-spring-damping model (40) are provided as

$$
\begin{aligned}
&M_1^1 = \begin{bmatrix} 0 & -0.5 \\ -0.5 & -1.5 \end{bmatrix}, M_2^1 = \begin{bmatrix} 0 & -0.61 \\ -0.52 & -1.5 \end{bmatrix},\\
&M_1^2 = \begin{bmatrix} 0 & -0.43 \\ -0.44 & -1.5 \end{bmatrix}, M_2^2 = \begin{bmatrix} 0 & -0.71 \\ -0.53 & -1.5 \end{bmatrix},\\
&N_1^1 = \begin{bmatrix} 0 & 2.01 \end{bmatrix}, N_2^1 = \begin{bmatrix} 0 & 1.02 \end{bmatrix},\\
&N_1^2 = \begin{bmatrix} 0 & 3.00 \end{bmatrix}, N_2^2 = \begin{bmatrix} 0 & 1.03 \end{bmatrix},\\
&O_1 = \begin{bmatrix} 2.2 \\ -2.3 \end{bmatrix}, O_2 = \begin{bmatrix} 2.4 \\ -3.3 \end{bmatrix},\\
&P_1 = \begin{bmatrix} 3.17 & 0 \end{bmatrix}, P_2 = \begin{bmatrix} 0.96 & 1.03 \end{bmatrix},\\
&H_1 = 0.2, H_2 = 2.3, c_1(t) = 2.9e^{-1.3t} sin(5t).
\end{aligned}
$$

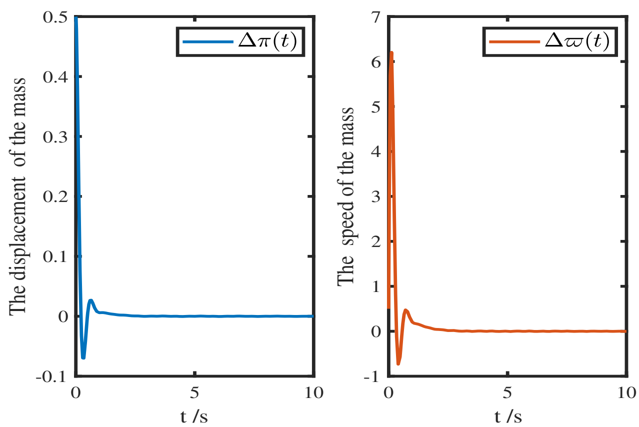

Figure 3 The displacement and speed of the mass.

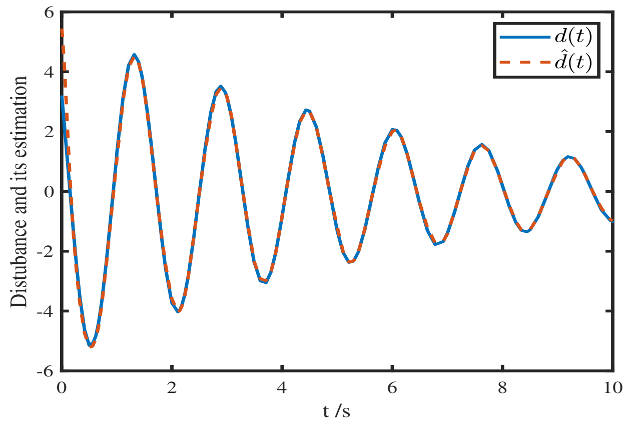

Figure 4 The disturbance $\mathcal{e}(t)$ and its estimation $\hat{\mathcal{e}}(t)$.

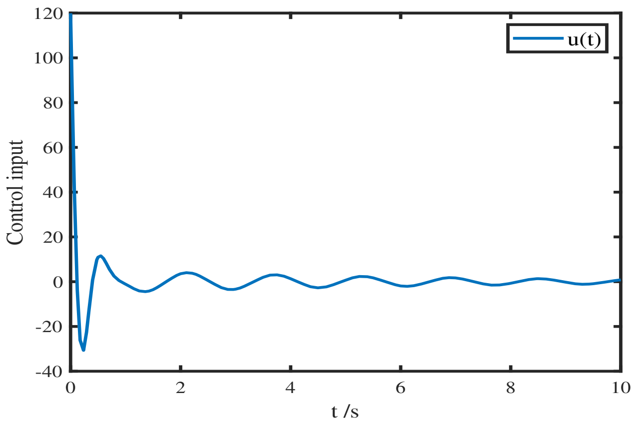

Figure 5 The spring pushing force $u(t)$.

Let's consider the case that the mass-spring-damping system (40) affected by the harmonic disturbance. The parameters of the disturbance $\mathcal{e}(t)$ established by the neural network model are

$$E_1 = \begin{bmatrix} 0 & -4.1 \\ 4 & -0.30 \end{bmatrix}, E_2 = \begin{bmatrix} 0 & -4.0 \\ 4.1 & -0.30 \end{bmatrix},$$
$$F_1 = \begin{bmatrix} 0.501 & -0.10 \end{bmatrix}, F_2 = \begin{bmatrix} 0.502 & -0.11 \end{bmatrix},$$
$$\mathcal{X}^* = \begin{bmatrix} -0.30 & -0.05 \\ 0.40 & 1.45 \end{bmatrix}, \sigma(t) \in \{1, 2\}.$$

The constants are selected as

$$\delta_0 = 2, \delta = 3.4, \mu_1 = 2.1, \mu_2 = 0.5,$$
$$\mu_3 = 2.3, \bar{\mu} = 2.6, n = 0.5.$$

The control task is to regulate the thrust adjustment of the mass-spring-damping model (40) with the multi-source disturbance.

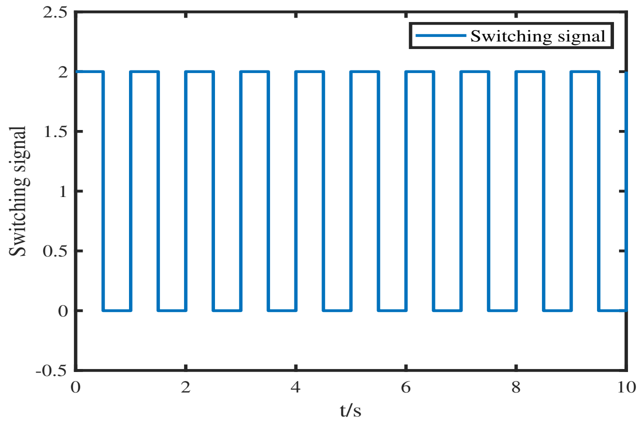

Figure 6 The switching signal.

By solving the linear matrix inequalities (16) and (17), we can get the controller gains K_i^l, observer gain K_0

$$
\begin{aligned}
K_1^1 &= \begin{bmatrix} 62.6659 & -3.9291 \end{bmatrix}, \\
K_1^2 &= \begin{bmatrix} 117.0423 & -5.9501 \end{bmatrix}, \\
K_2^1 &= \begin{bmatrix} 88.3698 & -7.8824 \end{bmatrix}, \\
K_2^2 &= \begin{bmatrix} 35.6628 & -1.2549 \end{bmatrix}, \\
K_0 &= \begin{bmatrix} 2.4798 & -8.2909 \\ -8.2909 & 47.8926 \end{bmatrix},
\end{aligned}
$$

and the L_2-gain performance level γ =3.21.

The simulation results are depicted by Figs. 3-6. The mass displacement $\Delta\pi(t)$ and speed $\Delta\varpi(t)$ are exhibited in Fig. 3. The harmonic attenuated disturbance $\mathscr{c}(t)$ and its estimation $\hat{\mathscr{c}}(t)$ are shown in Fig. 4. The control input $u(t)$ is depicted in Fig. 5. Fig. 6 exhibits the switching signal $\sigma(t)$. Quickly, from Fig. 3, we can observe that the displacement and speed $\Delta\pi(t)$ and $\Delta\varpi(t)$ are bounded. We can obtain from Fig. 4 that the adaptive disturbance observer can track irregular disturbance $\mathscr{c}(t)$. From Fig. 5, we can see that the control input obtained by the AADS control approach is bounded. As a result, we can claim that the proposed AADS control method has a specific application prospect.

5 | CONCLUSIONS

The AADS control issue has been studied for the ST-SFSs encountered by the unavailable neural network modeled disturbance and available unmodeled disturbance by the multiple Lyapunov functions technique. A new adaptive disturbance observer has been introduced to observe neural network modeled disturbance. An AD feedback controller based on adaptive disturbance observer has been established. Then, the AADS control method has been designed, which provides theoretical conditions for solving the AADS control problem of the ST-SFSs. Further, the practical stability, multi-source DS performance with the L_2 index have been acquired. Finally, a simulation of the mass-spring-damping system with nonlinear irregular disturbances.

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