

A Numerical Test of Soil Layering Effects on Theoretical and Practical Beerkan Infiltration Runs

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Abstract

With reference to a more compacted and less conductive upper soil layer overlying a less compacted and more conductive subsoil, a simple three-dimensional (3D) infiltration run is expected to yield more representative results of the upper layer than the subsoil. However, there is the need to quantitatively establish what is meant by more representativeness. At this aim, numerically simulated infiltration was investigated for a theoretically unconfined process under a null ponded head of water (d0H0 setup, with d = depth of ring insertion and H = ponded depth of water) and a practical beerkan run (d1H1 setup, $d = H = 1$ cm). The considered layered soils differed by both the layering degree (from weak to strong) and the thickness of the upper soil layer (0.5-3 cm). It was confirmed that water infiltration should be expected to be more representative of the upper soil layer when this layer is the less permeable since, for a 2-h experiment, the instantaneous infiltration rates for the layered soil were 1.0-2.1 times greater than those of the homogeneous low permeable soil and 1.3-20.7 smaller than those of the homogeneous coarser soil that constituted the subsoil. Similarity with the homogeneous fine soil increased as expected as the upper layer became thicker. For a weak layering condition, the layered soil yielded an intermediate infiltration as compared with that of the two homogeneous soils forming the layered system. For a strong layering degree, the layered soil was more similar to the homogeneous fine soil than to the homogeneous coarse soil. Using the practical setup instead of the theoretical one should have a small to moderate effect on the instantaneous infiltration rates since all the calculated percentage differences between the d1H1 and d0H0 setups fell into the relatively narrow range of -18.8% to +17.4%. A sequential analysis procedure appeared usable to detect layering conditions but with some modifications as compared with the originally proposed procedure. The practical setup enhanced the possibility to recognize the time at which the characteristics of the subsoil start to influence the infiltration process. In conclusion, this investigation contributed to better interpret both the theoretical and the practically established 3D infiltration process in a soil composed of a less conductive upper soil layer overlying a more conductive subsoil and it also demonstrated that modifying a recently proposed procedure only using infiltration data could be advisable to determine the time when layering starts to influence the process.

1. INTRODUCTION

Soil hydraulic properties are frequently estimated by establishing a three-dimensional (3D) infiltration process through a circular source into the initially unsaturated soil. The applied experimental methodology and the time window to which the data refer can differ from case to case. In particular, the experiment can be carried out by establishing a ponded head of water or a negative pressure head on the infiltration surface. Both the transient and the steady-state stages of the process can be sampled but the data could also be collected for only one of the two stages. Regardless of these differences, the vast majority of data analysis methods relies on 3D infiltration models that were developed under the assumption that the data are representative of an ideal soil, that is homogeneous, isotropic, rigid and uniformly unsaturated at the beginning of the

experiment (Angulo-Jaramillo et al., 2016). However, ideal soil conditions are rarely, if ever, met in the field and efforts must be made to find a link between theory and reality.

Infiltration experiments can be carried out, sometimes with no or little awareness, on soil volumes that consist of two layers with different hydraulic properties. In particular, a compacted soil layer, more or less thin, overlying a less compacted subsoil is rather common in different environments and conditions and its presence can have a large impact on hydrological processes, soil-water relations and soil physical quality (Lenhard, 1986; Ben-Hur et al., 1987; Ramos et al., 2000; Assouline and Mualem, 2002; Reynolds et al., 2009).

There doesn't seem to be much in the literature with reference to a 3D infiltration process into layered soils (Shan and Stephens, 1995; Wu et al., 1997; Dohnal et al., 2016), but some investigations specifically considering the case of a low permeable layer overlying a more permeable layer can be found. In particular, Lassabatere et al. (2010) and da Silva Ribas et al. (2021) suggested that, when the soil is layered and the upper layer is the less permeable, water infiltration is more representative of the upper layer. Di Prima et al. (2018) tested in the laboratory infiltration in a sealed soil and they recognized that the process was mainly governed by seal formation and also that the final infiltration rates were representative of the hydraulic behavior of the seal layer, irrespective of the upper soil layer thickness. Working in the field in a Mediterranean vineyard, Alagna et al. (2019) also suggested that a simple infiltration experiment is suitable enough to detect the effect of the altered upper layer on flow. Recently, Moret-Fernández et al. (2021) developed a method to analyze the 3D infiltration data collected in layered soils regardless of whether the upper soil layer is finer or coarser than the subsoil. In particular, the time when the infiltration bulb reaches the interface between the upper soil layer and the subsoil can be identified by sequentially fitting a cumulative infiltration model to the data and then examining how the root mean square error, $RMSE$, changes during the run given that some of the properties of the deeper soil layer are expected to also influence the measured infiltration. In the absence of any soil layering, $RMSE$ should remain nearly constant. Instead, it suddenly increases when the infiltration bulb reaches the interface. Moret-Fernández et al. (2021) obtained encouraging results by applying the sequential analysis procedure with the four-term approximation of the cumulative infiltration model by Haverkamp et al. (1994), proposed by Moret-Fernández et al. (2020).

Therefore, with reference to a more compacted and less conductive upper layer overlying a less compacted and more conductive subsoil, the expectation is that a simple 3D infiltration run will yield more representative results for the upper layer than the subsoil (Lassabatere et al., 2010; da Silva Ribas et al., 2021) and that the measured infiltration will contain (Moret-Fernández et al., 2021), but not necessarily in a perceivable manner (Di Prima et al., 2018), some information related to the subsoil. These conclusions lead to the need to carry out further investigations. In particular, it is necessary to better establish what is meant in practice when speaking of an infiltration curve that is more representative of the upper layer and also what factors influence this similarity and how.

BEST (Beerkan Estimation of Soil Transfer parameters) methods of 3D infiltration data analysis (Lassabatere et al., 2006; Yilmaz et al., 2010; Bagarello et al., 2014) are largely used to obtain a complete soil hydraulic characterization (Angulo-Jaramillo et al., 2019). They are based on the theoretical description of the infiltration process with the model by Haverkamp et al. (1994), that was specifically developed to analyze a fully 3D processes under a pressure head not greater than zero. Therefore, the use of this model to deduce the saturated soil hydrodynamic parameters is theoretically limited to the case of a ring insertion depth, d , equal to zero and also a null ponded depth of water on the infiltration surface ($H = 0$). Establishing an unconfined 3D infiltration process under a null ponded depth of water is not easy from an experimental point of view. In particular, it may require performing a relatively complicated experiment with the tension infiltrometer, which is not free from uncertainties and problems such as those concerning the hydraulic contact between the device and the soil surface (Close et al., 1998). More easily, a beerkan infiltration run could be performed since literature suggests that it should be representative of such a kind of infiltration process (Lassabatere et al., 2006). However, the experimental data obtained by this type of run agree only approximately with the theoretical assumptions of the infiltration model by Haverkamp et al. (1994) since

the ring used to confine the infiltration surface has to be inserted into the soil to a short depth to avoid lateral loss of water during the run. In addition, repeatedly pouring a small water volume into the ring or using automated infiltrometers (Di Prima, 2015) determine a small positive pressure head on the soil surface for at least a part of the experiment.

With reference to five ideal soils, Bagarello et al. (2022) recently established a numerical comparison between the theoretical ($d = H = 0$) and the practical ($d = H = 1$ cm) setups. Inserting the ring a little into the soil reduced the instantaneous infiltration rates, i_r , for the entire run and hence cumulative infiltration, I , while establishing a small ponded depth of water on the infiltration surface increased i_r and I . Generally, the two effects, even if they had an opposite sign, did not compensate one with other without a residual. In any case, differences between the two setups, denoted as d0H0 and d1H1 for the theoretical and practical situations, respectively, were small and perhaps negligible in many practical circumstances since they did not exceed a few percentage units (approximately, 10%) for both i_r and I .

Testing setup effects on layered soils is necessary to verify if this conclusion also applies to a more complex scenario. In the case of a layered soil, it can be presumed that, depending on the thickness of the upper layer relative to the ring insertion depth, infiltration into this layer will exclusively be one-dimensional (1D) or 3D after a 1D phase. Similarly, infiltration in the subsoil can exclusively be 3D or 3D after a 1D phase.

Numerical simulation appears an appropriate tool for testing layering and setup effects on the infiltration process since a numerical experiment makes it possible to determine how a specific factor influences the soil hydrodynamic response and it can be performed in fully controlled conditions, without any experimental error (e.g., Wu et al., 1993; Lai and Ren, 2007; Lai et al., 2010; Dohnal et al., 2016; Bagarello et al., 2019, 2022; Reynolds, 2013).

The general objective of this investigation was to verify the impact of a low permeable soil layer overlying a more permeable subsoil on both a theoretically unconfined process under a null ponded depth of water (d0H0 setup) and a practical beerkan run (d1H1 setup). The specific objectives were to: i) determine the impact of the soil layering degree and the thickness of the upper layer on infiltration rates, cumulative infiltration and parameters of an adapted infiltration model; ii) test the applicability of the recently suggested sequential analysis procedure to identify the time when the infiltration front reaches the subsoil; and iii) verifying if and in what situations a theoretical analysis that assumes a fully unconfined process under a null ponded depth of water can be considered appropriate for a beerkan infiltration run performed on a layered soil.

2. MATERIALS AND METHODS

2.1. Soils and numerical experiments

Four soils, namely silt-loam (SIL), loam (L), sandy-loam (SAL) and loamy-sand (LS), were considered in this study. Soil hydraulic properties were expressed according to the van Genuchten–Mualem model (Mualem, 1976; van Genuchten, 1980) with hydraulic parameters adopted from Carsel and Parrish (1988) (**Table 1**). Infiltration experiments were numerically simulated using Hydrus-2D/3D (Šimůnek et al., 2007), which was found to be a robust and reliable tool for simulating water flow in the soil under various conditions (Šimůnek et al., 2016; Varvaris et al., 2021).

Simulations were carried out for the four homogeneous soils (SIL, L, SAL and LS) and for the following nine layered soils: 0.5 cm of SIL soil on L soil (acronym SIL0.5L), 1 cm of SIL on L (SIL1L), 3 cm of SIL on L (SIL3L), 0.5 cm of SIL on SAL (SIL0.5SAL), 1 cm of SIL on SAL (SIL1SAL), 3 cm of SIL on SAL (SIL3SAL), 0.5 cm of SIL on LS (SIL0.5LS), 1 cm of SIL on LS (SIL1LS), and 3 cm of SIL on LS (SIL3LS). Therefore, three layering degrees were considered in this investigation, namely a weakly layered soil (SIL over L), a moderately layered soil (SIL over SAL) and a strongly layered soil (SIL over LS). For each layering degree, three thickness values of the upper soil layer, t_{ul} , were considered.

All simulations were performed with both the d0H0 (null depth of ring insertion, d , and null pressure head on the soil surface, H) and d1H1 ($d = H = 1$ cm) setups. The ring radius was equal to 5 cm and the initial condition was that of a uniform soil water pressure head, $h_i = -9022$ cm. Only this initially dry condition

was considered for a threefold reason: i) the drier the soil the greater the lateral capillarity term, which represents the distinguishing term between 3D and 1D infiltration (Vandervaere et al., 2000); ii) determining soil hydrodynamic properties with a beerkan run and the Haverkamp et al. (1994) model requires that the experiment is performed in an initially relatively dry soil (Lassabatere et al., 2006); and iii) with reference to several homogeneous soils, including those considered in this investigation, and a range of relatively small soil water content values, differences between the infiltration rates for the d0H0 and d1H1 setups were not appreciably affected by the antecedent soil water content (Bagarello et al., 2022). Each infiltration process was simulated for 2 hours, in accordance with Dušek et al. (2009) and also considering that this duration likely represents a plausible time limit for a field run in many circumstances. Instantaneous infiltration rate, i_r (L/T), and cumulative infiltration, $I(L)$, data were stored at 0.2 min time intervals. Infiltration rate was expressive of the process at any particular instant whereas cumulative infiltration integrated infiltration rates until a given instant.

The modeling comprised of three-dimensional axisymmetric simulations. To guarantee an unrestricted flow, the size of the flow domain was 80 cm in X and 100 cm in Z for the L and SIL soils while it was 100 cm in X and 200 cm in Z for the LS and SAL soils. The element size was 0.05 cm for the upper 10 cm of the flow domain and then it gradually increased to a maximum of 5 cm at the bottom. A variable density of element mesh was chosen to ensure the simulation accuracy for a relatively large flow domain. The boundary condition at the bottom was free drainage and no lateral flux was considered for the vertical boundaries of the simulation domain. An upper boundary condition of constant water head was assigned within the ring to simulate infiltration under ponding conditions. Flow was not affected by the established boundaries of the domain during the simulation period. Therefore, the setting of the simulation domain was appropriate to represent the actual infiltration process under the various considered setups.

This was not the first time that a 3D infiltration experiment was numerically simulated by combining two different homogeneous soils with each other to form a layered soil since the same methodology was recently applied by Moret-Fernández et al. (2021).

2.2. Data analysis

Initially, the data obtained with the d0H0 setup were analyzed according to the following three steps: i) analysis of the infiltration rates; ii) analysis of cumulative infiltration; and iii) fitting of infiltration models to the data.

Concerning the first step, the i_r values of the finest (SIL) homogeneous soil were first compared with those of the homogeneous coarser (L, SAL, LS) soils. Then, the i_r values obtained for each combination between layering degree (weak, moderate, strong) and t_{ul} value (0.5, 1, 3 cm) were compared with those of the two homogeneous soils that were combined with each other to form the layered soil. To define in more detail the link between infiltration in a homogeneous fine soil and infiltration in a layered soil composed of an upper layer of the same fine soil and a coarser subsoil, the infiltration rate ratio, $i_{r\ LH} = i_r(\text{layered soil})/i_r(\text{SIL})$ (L = layered; H = homogeneous), vs. time curves were then examined for the nine layering scenarios. According to Wu et al. (1997), the presence of layering could not be easy to recognize when infiltration rates are plotted against time. Therefore, a check of this previous finding was carried out by considering the following four representations of the infiltration rate vs. time curve: i_r vs. t ; i_r on a log scale vs. t ; i_r vs. t on a log scale; both i_r and t on log scales. The first 0.5 h of infiltration were considered for this check.

With reference to the second step of the analysis, focused on cumulative infiltration, it was preliminarily decided to consider the first 0.5 h of the process. This choice was made for a twofold reason: i) a relatively short infiltration run, allowing to save time and water in intensive field campaigns, has more practical interest than a long run; and ii) with this duration, it was possible to consider different scenarios, ranging from a process that was presumably affected by layering very soon ($t_{ul} = 0.5$ cm) to a process that occurred in the upper soil layer for an appreciable part of its total duration ($t_{ul} = 3$ cm). The cumulative infiltration curves for the layered and the two corresponding homogeneous soils were first compared. The percentage differences between cumulative infiltration of the layered (I_{layered}) and the homogeneous SIL (I_{SIL}) soils, $\Delta I_{\text{LH}} =$

$100 \times (I_{\text{layered}} - I_{\text{SIL}})/I_{\text{SIL}}$, were then calculated at any instant during the run.

Concerning the third step of the analysis for the theoretical setup, two checks were carried out. In particular, the following well-known two-parameter infiltration model was fitted to the I (mm) vs. t (h) data:

$$I = C_1\sqrt{t} + C_2t$$

in which C_1 (mm/h^{0.5}) and C_2 (mm/h) are fitting parameters. These parameters were determined since they can be used to estimate soil hydrodynamic properties from a transient infiltration experiment at a null pressure head on the infiltration surface (Vandervaere et al., 2000). For the four homogeneous soils (SIL, L, SAL, LS) and the nine layered soils (3 layering degrees \times 3 t_{ul} values), a second order equation with a null known term was fitted to the data by setting $x = t^{0.5}$ to determine C_1 and C_2 . A check of the sequential analysis procedure was then carried out. Preliminarily, the methodology by Moret-Fernández et al. (2021) was applied by using the four-term approximation of the model by Haverkamp et al. (1994) (Moret-Fernández et al., 2020) and the “Solver[®]” program built into Microsoft Excel[®]. However, the results were not satisfactory due to the difficulty to obtain valid, i.e. non-null, values of the last two coefficients of the model. Instead, positive, and hence valid, parameters were obtained by considering the infiltration rates instead of cumulative infiltration and fitting the empirical three-term infiltration model by Horton (1940) to the data. Therefore, this last approach was used to verify if the sequential analysis procedure was usable to reveal layering conditions. The use of Horton-type models for the analysis of the 3D infiltration process is not new in the scientific literature (Hussen and Warrick, 1993; Jacques et al., 2002; Iovino et al., 2021).

A comparison between the d0H0 and d1H1 setups was subsequently carried out.

In particular, the relationship between $\Delta i_{r\text{ PT}} = 100 \times [i_r(\text{d1H1}) - i_r(\text{d0H0})]/i_r(\text{d0H0})$ (P = practical; T = theoretical) and t was determined for the nine layered soils and also for the homogeneous SIL soil for comparative purposes. Similar calculations were made for cumulative infiltration. In this last case, the relationship between $\Delta I_{\text{PT}} = 100 \times [I(\text{d1H1}) - I(\text{d0H0})]/I(\text{d0H0})$ and t was determined. A comparison was then performed for both the homogeneous SIL soil and each layered soil between the C_1 and C_2 values obtained with the d1H1 and d0H0 infiltration data. Finally, the effect of the used setup on the results of the sequential analysis procedure was evaluated.

The analysis of the instantaneous infiltration rates was initially performed by considering the simulated data at $\Delta t = 0.2$ min, representing the shortest available time interval. However, in the development of the analysis, it was considered that the cumulative infiltration data, the fitting of a model to the data and the experimental information collected with the d1H1 setup mainly have a practical relevance since the data collected in the field are frequently used to estimate soil hydrodynamic properties. In this case, it is unlikely that infiltration will be measured at very short time intervals, except for limited time periods. Therefore, the cumulative infiltration data and those obtained with the d1H1 setup were analyzed by considering a longer time interval between two subsequent readings, that is $\Delta t = 1$ min. The same choice was made to fit an infiltration model to the data.

3. RESULTS AND DISCUSSION

3.1. Unconfined infiltration under a null ponded depth of water (d0H0 setup)

3.1.1. Infiltration rates

As expected, the infiltration rate at any given instant during the 2-h run increased in the homogeneous soils according to the $LS > SAL > L > SIL$ order. On average, the $i_r(L)/i_r(SIL)$, $i_r(SAL)/i_r(SIL)$ and $i_r(LS)/i_r(SIL)$ ratios were equal to 1.7, 5.3 and 15.5, respectively (means of 600 values for each ratio), and they increased during the run (**Fig.1**). In particular, $i_r(L)/i_r(SIL)$ increased by 1.3 times (from 1.4 at $t = 0.2$ min to 1.8 at $t = 2$ h), $i_r(SAL)/i_r(SIL)$ increased by 2.2 times (2.7-6.0) and $i_r(LS)/i_r(SIL)$ increased by 3.7 times (4.9-18.3). Therefore, comparing more similar soils implied obtaining, as expected, infiltration rate ratios that were closer to one and more stable during the run. The $i_r(L)/i_r(SIL)$, $i_r(SAL)/i_r(SIL)$ and $i_r(LS)/i_r(SIL)$ vs. t relationships were concave downwards, indicating that the i_r ratios increased at a

decreasing rate ast increased and also suggesting that they tended to stabilize at long times, in accordance with theory (Reynolds and Elrick, 1990).

For each layered soil (weakly, moderately, strongly), **Fig.2** shows the infiltration rate curves corresponding to the three thickness values of the upper layer ($t_{ul} = 0.5, 1$ and 3 cm) together with those for the two homogeneous soils that were combined with each other to form the layered soil. Generally, the infiltration rates of the layered soil approached more the i_r -values of the homogeneous SIL soil than those of the homogeneous coarser soil constituting the subsoil in the layered system (L, SAL, LS, depending on the scenario). In particular, for a weak layering (SIL over L), the differences between the layered soil and the coarser soil did not differ very much from those between the layered soil and the finer soil given that two corresponding infiltration rates (layered vs. homogeneous) differed by 1.3-1.8 times during the run in the former case and by 1.0-1.4 times in the latter one. This result occurred since infiltration rates for the homogeneous SIL and L soils were relatively similar (**Figs.1** and **2**). Instead, in the case of a strong layering (SIL over LS), the layered soil yielded i_r values differing by 4.9-20.7 and 1.0-2.1 times as compared with the coarser and the finer soil, respectively. Intermediate results were obtained for the moderately layered soil (SIL over SAL) since, in this case, infiltration rates differed by 2.7-6.3 times from those of the coarser soil and by 1.0-2.0 times as compared with the finer soil.

Therefore, the layered soils showed a greater similarity with the SIL soil than the coarser soils. This result became more and more clear as the thickness of the upper layer increased (**Fig.2**), as expected, but it was also detected with a very thin upper layer (0.5 cm).

After two hours of infiltration, 0.5 cm of SIL induced a decrease of the final infiltration rate, i_{rf} , by 1.3 times when this layer was positioned over the L soil, 3.0 times when the subsoil was a SAL soil and 8.6 times with the LS soil as the subsoil (**Table 2**). Therefore, a thin upper layer of a fine soil had a larger effect on i_{rf} as the subsoil was coarser. Final infiltration rates of the three homogeneous coarse soils (L, SAL, LS) differed by 10.3 times (29.6-304.4 mm/h). The presence of a thin layer of a fine soil on these three soils determined a shift of the range of i_{rf} towards smaller (22.6-35.4 mm/h) and less variable (ratio between the highest and the lowest i_{rf} value = 1.6) values. Conversely, the thin layer of SIL soil over a coarser soil yielded higher i_{rf} values for the layered soil than the homogeneous SIL soil (**Table 2**). The increase of i_{rf} was greater as the layering degree increased (from 1.4 times for the SIL0.5L soil to 2.1 times for the SIL0.5LS soil).

Therefore, for a weak layering degree, the i_{rf} value of the layered soil was almost equally distant from the i_{rf} values of the two associated homogeneous soils (SIL0.5L soil: 1.3 times smaller as compared with the L soil and 1.4 times greater as compared with the SIL soil). A stronger layering enhanced differences between the layered soil and the two associated homogeneous soil but more appreciably with reference to the differences with the coarser soil than the finer soil (SIL0.5LS soil: 8.6 times smaller i_{rf} value as compared with the LS soil and 2.1 times greater i_{rf} value as compared with the SIL soil).

As logical, a greater thickness of the upper SIL layer enhanced similarities between the layered and the homogeneous SIL soils and differences between the layered and the homogeneous coarser soils (**Table 2**).

For each layered soil (SIL over L, SIL over SAL and SIL over LS), **Figs.3a-c** show $i_{r\ LH}$ vs. t for the three t_{ul} values (0.5, 1 and 3 cm). The same data were reported in **Figs.3d-f** by plotting, for each t_{ul} value, $i_{r\ LH}$ vs. t for the three layering degrees. The following sequence of four stages was generally detected for the $i_{r\ LH}$ vs. t relationships: stage 1, $i_{r\ LH} = 1$; stage 2, $i_{r\ LH} < 1$, decreasing as t increased; stage 3, $i_{r\ LH} < 1$, increasing as t increased; stage 4, $i_{r\ LH} > 1$, increasing with time. For $t_{ul} = 3$ cm, the last stage was not detected and stage 3 was only weakly perceived. Evidently, stage 1 corresponded to the early phase of the infiltration process, during which the characteristics of the flow field did not change between the layered and the homogeneous soil. As logical, the duration of stage 1 increased with the thickness of the upper soil layer nearly independently of the underlying coarser soil since this duration was 0.2 min for a thickness of 0.5 cm, 1.6 min for a thickness of 1 cm and 18.8-19.4 min for a thickness of 3 cm (**Table 3**). As infiltration started to occur in the underlying coarser soil, the infiltration rates became smaller as compared with those of the homogeneous SIL soil. This slowdown reached a maximum (smallest $i_{r\ LH}$ value) and then it decreased

since, from a certain instant, $i_{r, LH}$ started to steadily increase. Due to this increase, infiltration later became faster in the layered soil than in the homogeneous SIL soil. The thinner the upper layer, i) the smaller the lowest $i_{r, LH}$ value during stages 2 and 3 ($i_{r, LH} < 1$) (**Figs.3a-c**), ii) the higher the greatest $i_{r, LH}$ value during stage 4 ($i_{r, LH} > 1$), and iii) the shortest the duration of stages 2 and 3 (**Table 2**). For a given thickness of the upper soil layer, the coarser the subsoil, i) the lower the minimum $i_{r, LH}$ value during stages 2 and 3 (**Figs.3d-f**) and ii) the longer the duration of these two stages (**Table 2**).

Therefore, in comparison with the infiltration rates of a homogeneous fine soil, those of a layered soil composed of an upper fine soil layer overlying a coarser soil evolved according to the following sequence: i) identical; ii) smaller and decreasing; iii) smaller and increasing; iv) greater and increasing. This trend had an impact on cumulative infiltration, I , as shown in the example of **Fig.4** for a thickness of the upper fine soil layer equal to 1 cm. In particular, the I values of the homogeneous and layered soils coincided during stage 1 and also later, but only at a single instant in this case. At this particular instant, there was a balance between the decreased infiltration rates during stages 2 and 3 and the increased infiltration rates during stage 4. The coarser the soil underlying the upper layer, the later this instantaneous condition of equality of I was reached.

Soil layering appeared more clearly detectable on the i_r vs. t curve as the layering degree increased, since it was nearly inappreciable when the coarse underlying soil was the L soil (**Fig.5a**) and most appreciable with the LS soil as the deeper layer (**Fig.5b**). Even in this case, however, the layering effect was rather clear for an intermediate thickness of the upper layer (1 cm) whereas it was invisible for the smallest thickness (0.5 cm) and very hard to identify for the largest thickness (3 cm). Plotting time on a log axis appeared to make the change of slope of the i_r vs. t curve a little more easily perceivable.

According to this investigation, the conclusion by Lassabatere et al. (2010) and da Silva Ribas et al. (2021) can be formulated in a little greater detail. In particular, the suggestion by these authors was supported but it is also necessary to specify that, for a little permeable upper layer, the similarity between the layered soil and the homogeneous fine soil depends on both the thickness of the upper layer and the nature of the subsoil. Similarity generally increases with a thicker upper layer. In the case of a weak layering, the layered soil is expected to yield an intermediate infiltration as compared with that for the two homogeneous soils that form the layered system. If the layering degree is strong, the layered soil appears more similar to the homogeneous fine soil than to the homogeneous coarse soil. The presence of layering is not clearly indicated by the infiltration rate curve when the two soil layers do not differ substantially and also when layering influences either the very early or late stages of the infiltration process. Instead, it is perceivable if it occurs in a strongly layered soil and for a thickness of the upper soil layer that is small but not very small.

3.1.2. Cumulative infiltration

For each SIL-subsoil scenario, **Fig.6** compares the cumulative infiltration curves for the two homogeneous soils that were combined with each other to form the layered soil with those for the three layered soils ($t_{ul} = 0.5, 1$ and 3 cm). Cumulative infiltration was obviously greater in the homogeneous coarse soils (L, SAL, LS) than the homogeneous SIL soil. As expected, differences between two homogeneous soils were relatively small for the L-SIL comparison (ratio between I at $t = 0.5$ h equal to 1.5), intermediate for the SAL-SIL comparison (ratio = 3.9) and relatively high in the case of the LS-SIL comparison (ratio = 10.2).

Regardless of both the degree of layering and the thickness of the upper SIL layer, the cumulative infiltration curves of the layered soils were closer to that of the homogeneous SIL soil than to the cumulative infiltration curve of the homogeneous soil constituting the coarser subsoil in the layered system. In the representations of **Fig.6**, the overlap of the infiltration curves of the homogeneous SIL soil and the layered soils appeared more complete for $t_{ul} = 1$ and 3 cm than $t_{ul} = 0.5$ cm. In this last case, with the exclusion of the early stage of the run, more water infiltrated in the layered soil than in the homogeneous SIL soil.

Fig.7 shows the nine ΔI_{LH} vs. t relationships. Similarity between the layered soils and the SIL soil generally increased with the thickness of the upper layer, as expected, since $-11.1\% < \Delta I_{LH} < +29.5\%$, $-9.7\% < \Delta I_{LH} < +2.4\%$ and $-1.4\% < \Delta I_{LH} < 0$ was obtained for $t_{ul} = 0.5, 1$ and 3 cm, respectively. With a small

thickness of the upper layer (0.5 cm), $I_{\text{layered}} > I_{\text{SIL}}$ (positive ΔI_{LH} values) was the most frequent result during the run. The opposite result ($I_{\text{layered}} < I_{\text{SIL}}$ for the largest part of the run) was obtained with an intermediate thickness (1 cm), especially with reference to a moderate and strong layering condition. Finally, $I_{\text{layered}} = I_{\text{SIL}}$ prevailed with the thickest upper layer (3 cm). Weaker layering conditions generally made infiltration for the layered soil more similar to that of the homogeneous SIL soil, regardless of the thickness of the upper layer. A thicker upper soil generally made infiltration in layered soil more similar to that of the homogeneous SIL soil, regardless of the layering degree.

3.1.3. Adapting infiltration models

The fitting quality of the two-parameter infiltration model was good or very good in all cases since the coefficient of determination, R^2 , was not smaller than 0.9978 (SIL1LS soil, **Fig.8**). Discontinuities in the infiltration process were not perceived, which was expected in accordance with Vandervaere et al. (2000).

The C_1 values obtained on the layered soils were always closer to the C_1 value of the homogeneous SIL soil (C_1 values differing by 1.0 to 1.7 times, depending on the layering degree and the thickness of the upper layer) than to the C_1 values of the homogeneous coarser soils that constituted the subsoil in the layered systems (C_1 values differing by 1.3 to 4.7 times) (**Fig.9**). Similar results were generally obtained for C_2 since the C_2 values for the layered soils differed by 1.0 to 2.9 times from those of the homogeneous SIL soil and by 1.3 to 29.6 times from the C_2 values of the homogeneous coarser soils. In this case, the only exception was that the C_2 value of the SIL0.5L soil was closer to that of the L soil than the SIL soil (estimates differing by 1.3 and 1.6 times, respectively). Therefore, the C_1 and C_2 values of the layered soils were closer to those of the upper SIL layer than those of the coarser subsoil in almost all cases.

To further compare the layered soils with the homogeneous SIL soil with reference to both parameters of the infiltration model, the percentage differences between the C_1 and C_2 estimates obtained for the layered soil and those for the SIL soil (ΔC_1 and ΔC_2 , respectively) were plotted against t_{ul} for each layered soil (**Fig.10**). Regardless of the degree of layering, $\Delta C_1 < 0$ was obtained for $t_{ul} = 0.5$ and 1 cm and the ΔC_1 values were closer to zero as the degree of layering decreased. For $t_{ul} = 3$ cm, ΔC_1 was positive but close to zero (0.3-1.9%; highest percentage difference in the strong layering case). For the two smaller t_{ul} values, $\Delta C_2 > 0$ was obtained in all cases with larger differences between layered and homogeneous soils for the stronger layering degrees than the weak one. A negative ΔC_2 value was obtained for $t_{ul} = 3$ cm but differences were overall rather small also in this case (from -6.9% for the SIL3LS soil to -1.1% for the SIL3L soil).

Therefore, when the upper layer of a layered soil was relatively thick (3 cm), the C_1 and C_2 values for the layered soil were close to those of the homogeneous SIL soil. In the case of a thin upper layer (0.5 cm), layering yielded smaller C_1 values and larger C_2 values than the homogeneous SIL soil with differences that, for both parameters, decreased as the layering degree became weaker. In comparison with a thin upper layer, a layer of intermediate thickness (1 cm) increased the similarity between the layered and the homogeneous soil for both parameters and all layering degrees, but more appreciably in the case of a moderate or a strong layering than in that of a weak layering. In summary, as compared with a homogeneous fine soil, a layered soil consisting of a fine-textured layer placed on top of a coarser layer should be expected to yield similar C_1 and C_2 values if the thickness of the upper layer is high, which is a rather obvious result. Otherwise, smaller C_1 values and larger C_2 values should be expected, with differences that overall increase as the thickness of the upper layer decreases and the layering degree becomes stronger.

With reference to the test of the sequential analysis procedure, the infiltration model was adapted to the data for $t > 0.17$ h, i.e. by considering at least 10 data points for the fitting, which yielded 21 *RMSE* values for a run. Calculations of *RMSE* were performed for the homogeneous SIL soil and the SIL3L, SIL3SAL and SIL3LS layered soils. The sequential analysis was not applied for small t_{ul} values (< 1 cm) since, in this case, the wetting front reached the interface between the two layers very soon (< 1.6 min, **Table 2**), meaning that *RMSE* values representative of the upper layer alone could not be obtained. Instead, for $t_{ul} = 3$ cm, the first 17 *RMSE* values ($t < 0.283$ h) were obtained in the upper layer whereas the last 13 values represented infiltration in the two-layered system.

For the homogeneous SIL soil, the $RMSE$ vs. t relationship described a curve concave downward, denoting that $RMSE$ increased regularly with time at a decreasing rate as t increased (**Fig.11a**). An increasing $RMSE$ vs. t relationship was also detected in the presence of the subsoil. However, approximately starting from the time at which infiltration also occurred in the subsoil, $RMSE$ became higher for the layered soils than the homogeneous SIL soil, as expected (Moret-Fernández et al., 2021). This increase of $RMSE$ was minimal in the case of a weakly layered soil (SIL3L) and more appreciable as the layering degree increased.

Despite the detected differences between the homogeneous and the layered soils, the usability of the $RMSE$ vs. t relationship to determine the time when the infiltration bulb reaches the interface between the upper soil layer and the subsoil appeared somehow questionable for a twofold reason: i) the general shape of the $RMSE(t)$ curve did not change substantially between homogeneous and layered soils, and ii) field data are unavoidably more noised than those used for this analysis. Therefore, layering could be difficult to be detected in practice from the $RMSE$ vs. t relationship.

However, analyzing the same data, that is $RMSE$ and t , in an alternative manner was found to make detection of soil layering conditions easier. In particular, **Fig.11b** shows $\Delta RMSE / \Delta t$ plotted against the duration of the infiltration run. In the case of the homogeneous SIL soil, $\Delta RMSE / \Delta t$ decreased monotonically and the relationship between this ratio and t was concave upward. For both a moderately and a strongly layered soil, after a decreasing stage denoting infiltration in the upper layer, $\Delta RMSE / \Delta t$ increased rather abruptly and then it started to decrease again after reaching a maximum. The time at which the increase of $\Delta RMSE / \Delta t$ became detectable was very close to the time when infiltration started to occur in the subsoil (difference by no more than 1-2 min), suggesting that $\Delta RMSE / \Delta t$ started to increase because the subsoil started to influence infiltration. The results were less clear for a weakly layered soil due to the lack of a well-defined peak in the $\Delta RMSE / \Delta t$ vs. t plot. However, even in this case, a departure of the $\Delta RMSE / \Delta t$ vs. t relationship from that obtained for the homogeneous SIL soil appeared perceivable.

Therefore, the sequential analysis procedure first introduced by Moret-Fernández et al. (2021) appears usable to detect layering conditions. However, according to this investigation, infiltration rates should be considered, the Horton (1940) infiltration model should be fitted to the data and the relationship between $\Delta RMSE / \Delta t$ and t should be examined. A concave upward relationship between these two variables is expected for a homogeneous soil. A moderate or strong layering condition is signaled by a sharp increase of the $\Delta RMSE / \Delta t$ vs. t relationship after a decreasing stage. For a weakly layered soil condition, an alteration of the regularity of the $\Delta RMSE / \Delta t$ vs. t curve could represent all that it is possible to detect. Said in other words, the effectiveness of the suggested procedure appears to depend on the layering degree, being not surprisingly highest in the case of strong layering degrees.

3.2. Comparing the d0H0 and d1H1 setups

3.2.1. Infiltration rates

With reference to a 2 h infiltration run sampled at $\Delta t = 1$ min time intervals, **Fig. 12** shows, for the three layered soils and the three t_{ul} values for each layering degree, the relationship between $\Delta i_{r\ PT}$, i.e. the percentage difference between the i_r values for the d1H1 and d0H0 setups, and t . The $\Delta i_{r\ PT}$ values obtained for the homogeneous SIL soil were also reported in the figure. The individual $\Delta i_{r\ PT}$ values varied from a minimum of -18.8% (SIL3LS soil) to a maximum of +17.4% (SIL0.5LS soil) and the means of $\Delta i_{r\ PT}$ varied from -10.4% (SIL soil), or -9.9% (SIL3L soil) by only considering the layered soils, to +16.1% (SIL0.5LS soil). Therefore, the effect of the setup on i_r was overall small or moderate.

For the homogeneous SIL soil, $\Delta i_{r\ PT}$ was < 0 for the entire run duration (from -15.0% to -4.4%), meaning that i_r (d1H1) was steadily smaller than i_r (d0H0). In other terms, the d1H1 setup determined a slower infiltration process than the d0H0 setup, as already reported by Bagarello et al. (2022).

Although $\Delta i_{r\ PT}$ fell into a not very broad range of values, the layering degree influenced the $\Delta i_{r\ PT}$ calculations. In particular, $-15.0\% < \Delta i_{r\ PT} < +2.6\%$, $-16.7\% < \Delta i_{r\ PT} < +12.9\%$ and $-18.8\% < \Delta i_{r\ PT} < +17.4\%$ was obtained for the SIL-L, SIL-SAL and SIL-LS soils, respectively, denoting a wider range

for a stronger layering. As expected, the greatest similarity between the $\Delta i_{r, PT}$ values calculated for the homogeneous SIL soil and those obtained for the layered soils was detected in the case of the thickest upper layer ($t_{ul} = 3$ cm), regardless of the layering degree. In particular, the overlap of the two $\Delta i_{r, PT}(t)$ curves was initially complete, which was obvious since flow fields initially coincided. Then, an effect of the subsoil was detected since the overlap of the two curves persisted in the case of a weak layering (SIL3L soil) but not, or at least less clearly, in that of a moderate and strong layering (SIL3SAL and SIL3LS soils, respectively). The decrease of t_{ul} made the difference between the results for the layered and the homogeneous SIL soils more appreciable since it generally determined an increase of $\Delta i_{r, PT}$ for the layered soils that became less negative or even positive, depending on the layering degree. In particular, a thin upper layer ($t_{ul} = 0.5$ cm) yielded a mean of $\Delta i_{r, PT}$ nearly null (-0.2%) and individual $\Delta i_{r, PT}$ values close to or not very different from zero (from -5.7% to +2.6%) for a weak layering. For a moderate and a strong layering, the individual $\Delta i_{r, PT}$ values and the corresponding means were > 0 (from +0.3% to +12.9% and +10.9% in the former case; from +2.0% to +17.4% and +16.1% in the latter one).

For the homogeneous SIL soil, the final infiltration rate, i_{rf} , obtained with the practical setup (d1H1) was smaller by 7.4% than the corresponding value obtained with the theoretical setup (d0H0). With reference to the layered soils, $\Delta i_{rf, PT}$, that is the percentage difference between corresponding i_r values with the two setups at the end of the 2 h run, varied from -6.5% to +17.4%. In particular, the conditions determining the greatest differences between the d0H0 and d1H1 setups ($\Delta i_{rf, PT} > +13.0\%$) were those of a moderately or strongly layered soil and a thin (0.5 cm) upper layer. The best similarity between the two setups ($-0.4\% < \Delta i_{rf, PT} < +2.6\%$) was detected for a weakly layered soil and a thin or relatively thin (0.5-1 cm) upper layer.

Therefore, this analysis suggested that, with a thick upper layer, the results should not be expected to differ greatly between the layered soil and the homogeneous SIL soil, regardless of the layering degree. In particular, smaller infiltration rates are expected with the practical setup (d1H1) as compared with the theoretical one (d0H0). If the upper layer is thin, the impact of the layering degree becomes more appreciable. For a weak layering degree, i_r (d1H1) [?] i_r (d0H0) appears a plausible assumption. For moderate or strong degrees of layering, i_r (d1H1) $>$ i_r (d0H0) appears more likely. In other words, layering may make theoretical (d0H0) and practical (d1H1) infiltration rates similar to each other. This situation is expected to occur in the case of a weak layering and a thin upper layer.

3.2.2. Cumulative infiltration

Fig.13 shows the percentage differences, ΔI_{PT} , between the cumulative infiltration values obtained with the d1H1 and d0H0 setups during the 0.5 h run. The ΔI_{PT} values varied in the rather narrow range of $-12.2\% < \Delta I_{PT} < +11.8\%$, denoting an overall limited effect of the used setup on cumulative infiltration. As expected in accordance with the infiltration rate calculations (**Fig. 12**), the setup effect for a thick upper layer ($t_{ul} = 3$ cm) was similar to that observed for the homogeneous SIL soil, meaning that, regardless of the layering degree, cumulative infiltration for the d1H1 setup was up to about 11-12% lower than that obtained with the d0H0 setup. A layering degree effect was instead perceived with a thin upper layer ($t_{ul} = 0.5$ cm). In particular, a weak layering implied detecting I (d1H1) $<$ I (d0H0) albeit by only a little given that ΔI_{PT} was not smaller than -4.0%. Instead, I (d1H1) $>$ I (d0H0) was obtained with a moderate and a strong layering but only by a little also in these cases since ΔI_{PT} did not exceed +6.4% in the former case and +11.8% in the latter one. With an upper layer of an intermediate thickness ($t_{ul} = 1$ cm), $\Delta I_{PT} < 0$ was obtained. In particular, for a weak layering, ΔI_{PT} stabilized around a value of nearly -8.0% as the duration of the run increased up to 0.5 h. For a moderate and a strong layering, ΔI_{PT} reached a minimum of nearly -7.0% and then it started to increase, remaining negative.

Therefore, the correspondence between the two setups differed depending on both the layering degree and the thickness of the upper layer. Less cumulative infiltration occurred with the d1H1 setup than the d0H0 one for an upper layer of intermediate to high thickness and also for a thin upper layer but only in the case of a weak layering degree. Otherwise, the practical setup yielded more cumulative infiltration than the theoretical one.

3.2.3. Adapting infiltration models

Even with the d1H1 setup, the fitting quality of the two-parameter infiltration model to the data was good or very good in general since $R^2 > 0.9959$ was obtained and signs of discontinuities in the infiltration processes were not noticed.

For the homogeneous SIL soil, using the d1H1 setup instead of the d0H0 one yielded ΔC_1 and ΔC_2 values, that is percentage differences between the estimated parameters with the two setups, equal to -1.4% and -35.9%, respectively (**Fig. 14**), denoting that the d1H1 setup yielded a perhaps marginally smaller C_1 value but also an appreciably smaller C_2 value than the d0H0 setup, in accordance with Bagarello et al. (2022).

Similar results were obtained with a thick upper layer ($t_{ul} = 3$ cm) since ΔC_1 and ΔC_2 varied from -1.1% to 0 and from -44.3% to -37.4%, respectively, depending on the layering degree. Moreover, the weaker the layering degree, the greater the similarity between the layered soil and the homogeneous SIL soil.

A decrease of the thickness of the upper layer implied detecting non-monotonically decreasing (more negative) ΔC_1 values and also monotonically increasing ΔC_2 values that remained negative for a weak layering but were positive with a moderate or strong layering. A certain similarity between the two considered setups (d0H0 and d1H1) was noticed when the upper layer of the weakly layered soil was thin ($t_{ul} = 0.5$ cm) since in this case ΔC_1 and ΔC_2 were equal to -1.6% and -7.2%, respectively. A stronger layering degree implied detecting smaller (more negative) ΔC_1 values and positive ΔC_2 values of +14.3% or more. With a slightly thicker upper layer ($t_{ul} = 1$ cm), ΔC_1 was closest to zero (-4.7%) for the SIL-L soil whereas ΔC_2 was closest to zero (+2.7%) for the SIL-SAL soil.

Therefore, this analysis suggested that, under particular circumstances, that is weakly layered soil and thin upper layer, the d0H0 and d1H1 setups could be expected to yield relatively similar estimates of both C_1 and C_2 . Instead, a thick upper layer implies a similarity of the setup effects between the layered soil and the homogeneous SIL soil. With an upper layer of intermediate thickness, the level of similarity between the two setups depends on the layering degree and it changes with the considered parameter.

Fig. 15 compares the $\Delta RMSE / \Delta t$ curves for the d0H0 and d1H1 setups. The general shape of this curve did not change between these two setups. However, the increasing phase of the $\Delta RMSE / \Delta t$ vs. t relationship was detected earlier with the practical setup than the theoretical one. This result was consistent with the circumstance that the infiltration rates for the layered and the homogeneous SIL soils started to diverge a little earlier with the d1H1 setup (approximately after 16-17 min) than with the d0H0 setup (after nearly 19 min). The increase of the curve appeared sharper for the practical setup than the theoretical one and the largest vertical amplitude of the $\Delta RMSE / \Delta t$ vs. t curves, defined as the largest distance between the $\Delta RMSE / \Delta t$ vs. t curves for the layered and the homogeneous SIL soils, increased from the d0H0 setup (0.6-3.5 units, depending on the layered soil) to the d1H1 one (0.7-4.5 units). Therefore, the practical setup induced in general a more clearly detectable departure of the $\Delta RMSE / \Delta t$ vs. t curve from the one corresponding to the homogeneous SIL soil. Said in other words, the practical setup enhanced the possibility to recognize the time at which the characteristics of the subsoil started to influence the infiltration process.

4. CONCLUSIONS

In this investigation, infiltration in a layered soil with a less permeable upper layer and a more permeable subsoil was numerically studied for both a theoretical process with null depth of ring insertion, d , and ponded head of water, H (d0H0 setup), and an approximation of this process, obtained with $d = H = 1$ cm (practical setup, d1H1). The considered layered soils differed by both the layering degree (from weak to strong) and the thickness of the upper soil layer (0.5-3 cm).

With the theoretical setup (d0H0), the main conclusions were the following:

- it was confirmed that water infiltration is more representative of the upper soil layer when this layer

is the less permeable. A quantitative meaning was attributed in this investigation to the qualitative representativeness concept. In particular, for a 2-h experiment, more representative means that the instantaneous infiltration rates for the layered soil are 1.0-2.1 times greater than those of the homogeneous little permeable soil and 1.3-20.7 smaller than those of the homogeneous coarser soil that constitutes the subsoil. Similarity with the homogeneous fine soil increases as the upper layer becomes thicker. Moreover, for a weak layering condition, the layered soil yields an intermediate infiltration as compared with that of the two homogeneous soils forming the layered system. For a strong layering degree, the layered soil is more similar to the homogeneous fine soil than to the homogeneous coarse soil.

- as compared with a homogeneous fine soil, the layered soil should be expected to yield similar C_1 and C_2 values if the thickness of the upper layer is high (percentage differences varying from -7% to +2% depending on the parameter and the layering degree). Otherwise, smaller C_1 values (up to -42%) and larger C_2 values (up to +185%) can be obtained, with differences that overall increase as the thickness of the upper layer decreases and the layering degree becomes stronger.
- a sequential analysis procedure appears usable to detect layering conditions but with some modifications as compared with the originally proposed procedure. The effectiveness of the method seems to depend on the layering degree, being not surprisingly highest in the case of strong layering degrees.

The comparison between the practical (d1H1) and theoretical (d0H0) setups led instead to the following main conclusions:

using the practical setup instead of the theoretical one should have a small to moderate effect on the instantaneous infiltration rates since all the calculated percentage differences between the two setups fell into the relatively narrow range of -18.8% to +17.4%. With a thick upper layer, smaller infiltration rates are expected with the practical setup as compared with the theoretical one, which is a similar result to that expected for the homogeneous fine soil. If the upper layer is thin, the impact of the layering degree becomes appreciable. For a weak layering degree, the two setups yield similar infiltration rates. For moderate or strong layering degrees, higher infiltration rates are obtained with the practical setup than the theoretical one.

The practical setup appears to enhance the possibility to recognize the time at which the characteristics of the subsoil start to influence the infiltration process with the modified sequential analysis procedure.

This investigation tried to take a small and incomplete step forward as regards the transition from the ideal conditions to the real ones. In this context, this investigation could represent the starting point of additional developments in the study of 3D infiltration into a soil composed of a little permeable upper layer and a coarser subsoil. In particular, layering and setup effects should be studied in the future by considering, in addition to the infiltration process, soil heterogeneity. Numerical simulation could be an appropriate tool at this purpose since different relevant data, such as those on water content of the wetted soil volume, lateral expansion of this volume and pressure heads in different points of the wetted soil, can be obtained with a great detail. It should also be considered that, in the field, a single soil layer is not homogeneous, rigid and isotropic. Therefore, numerical simulations should be performed by improving soil description, maybe step by step, in an attempt to get as close as possible to a realistic description of the porous medium. Finally, another objective that perhaps should be pursued is trying to better define potential and limitations of the sequential analysis procedure, at least in the perspective to detect the presence of layering with a simple infiltration experiment.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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Fig. 1. Ratio between the infiltration rates, i_r , for the L (loam), SAL (sandy-loam) and LS (loamy-sand) homogeneous soils and those for the homogeneous SIL (silt-loam) soil plotted against time, t

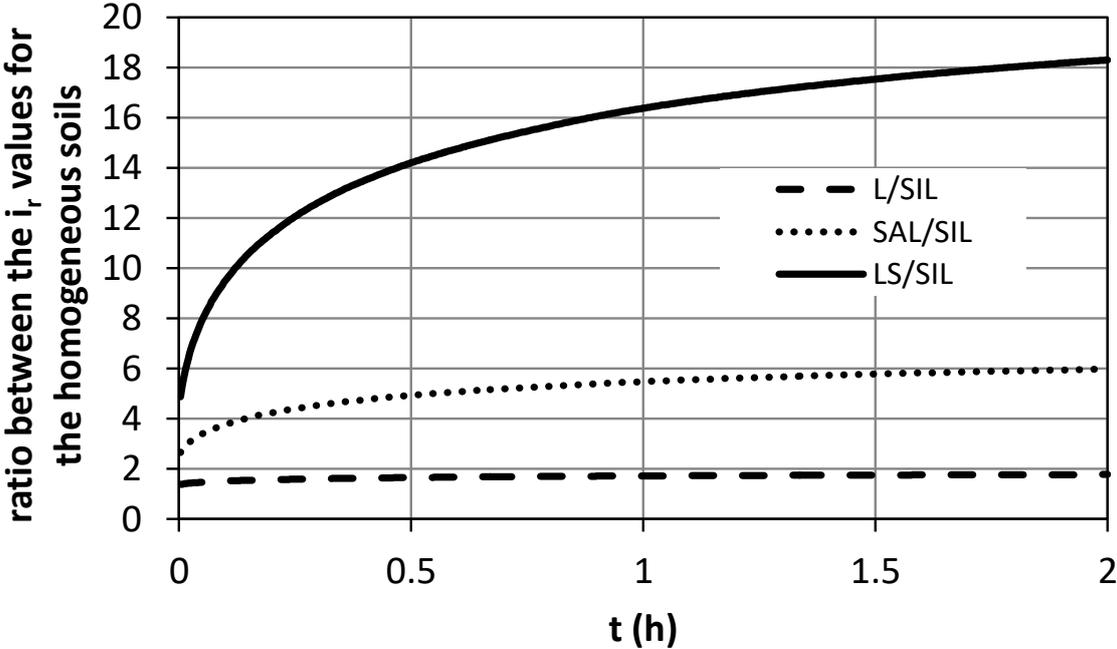


Fig. 2. Infiltration rates, i_r , for the homogeneous SIL, L, SAL and LS soils and the layered soils plotted against time, t : a) SIL overlying L (weakly layered soil); b) SIL overlying SAL (moderately layered soil); c) SIL overlying LS (strongly layered soil) (SIL: silt-loam; L: loam; SAL: sandy-loam; LS: loamy sand)

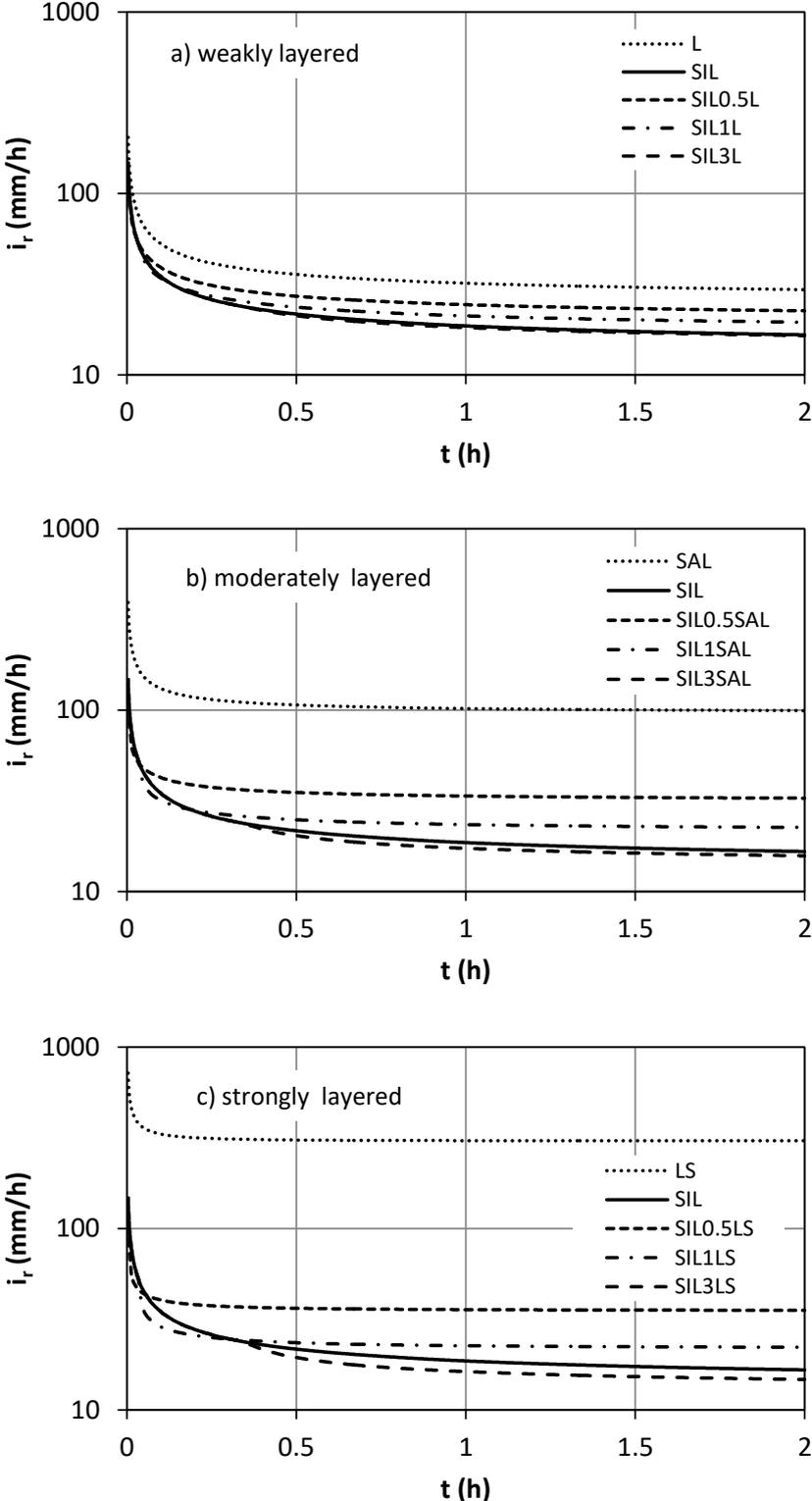


Fig. 3. Infiltration rate ratios, i_{rLH} (layered/homogeneous SIL soil), plotted against time, t , for a weakly (a), moderately (b) and strongly (c) layered soil and for a thickness of the upper layer equal to 0.5 cm (d), 1 cm (e) and 3 cm (f) (SIL: silt-loam; L: loam; SAL: sandy-loam; LS: loamy sand)

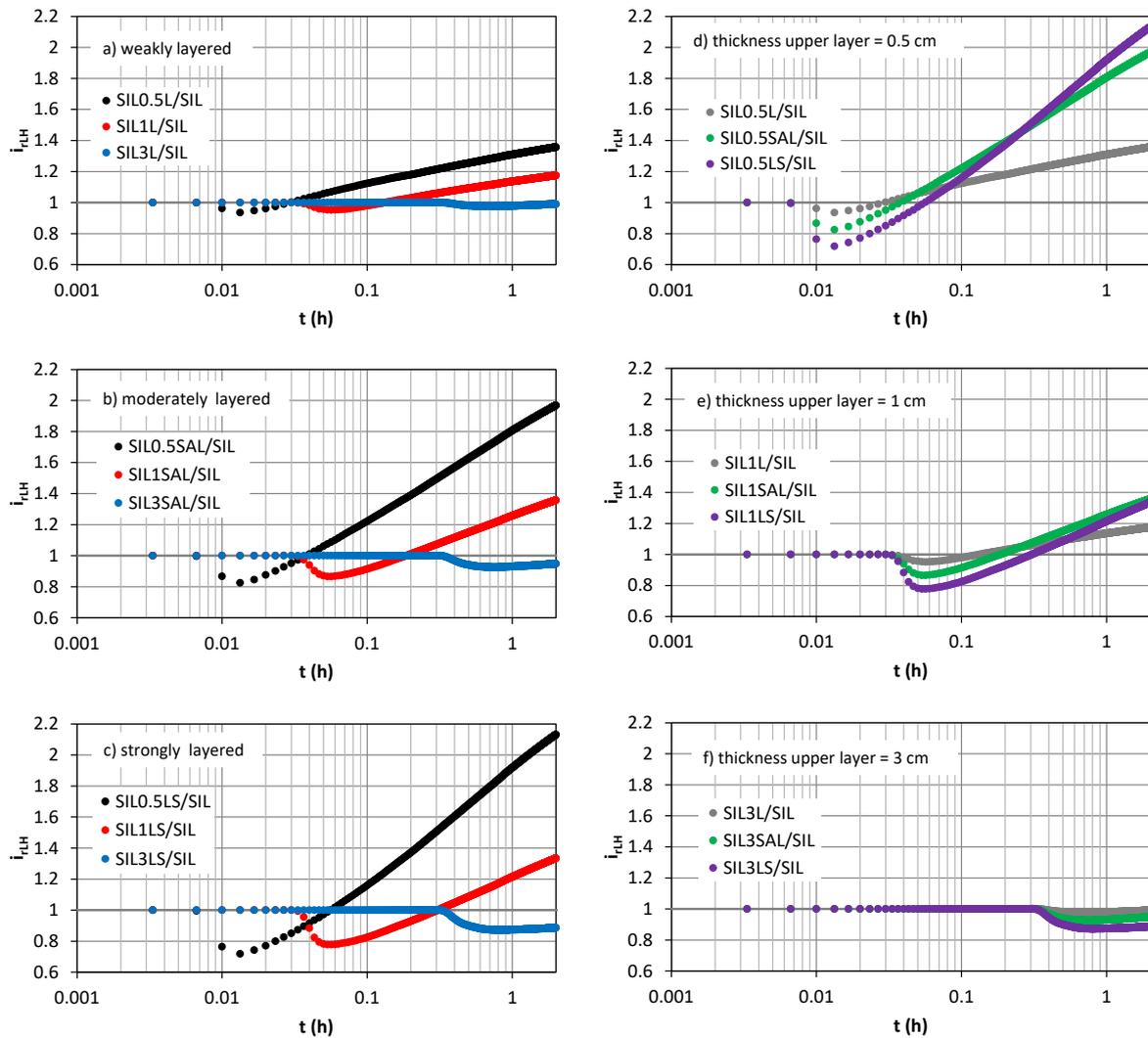


Fig. 4. Ratio between the cumulative infiltration of the layered soil and that of the homogeneous SIL soil plotted against time, t (SIL: silt-loam; L: loam; SAL: sandy-loam; LS: loamy sand)

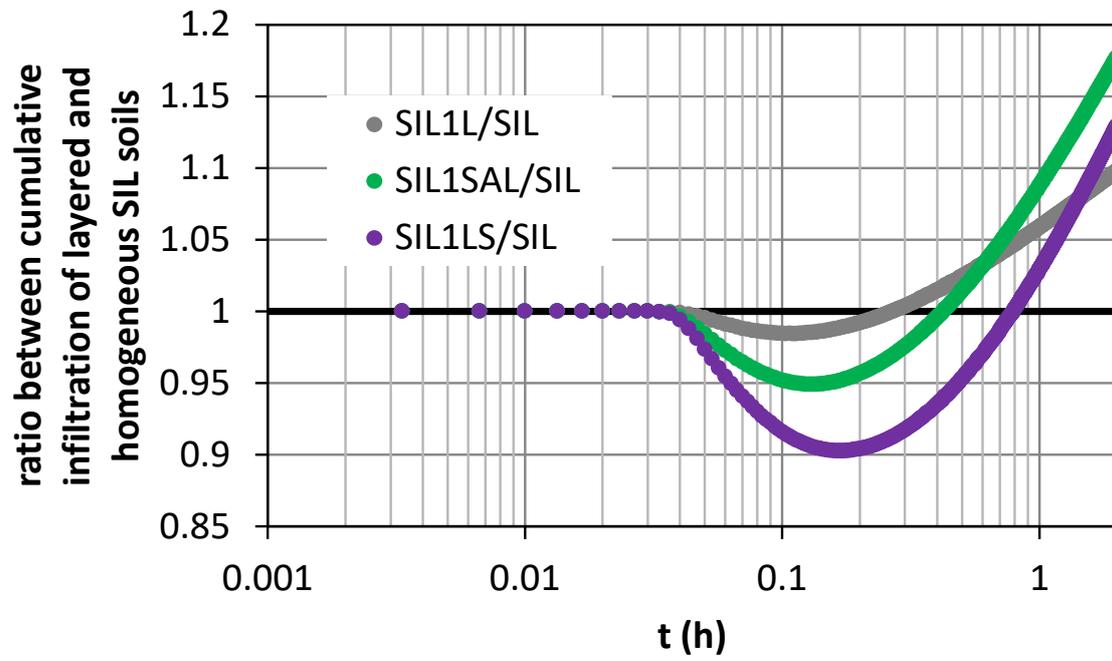


Fig. 5. Infiltration rates, i_r , plotted against time, t , for a) a weakly layered soil and b) a strongly layered soil considering different thickness values of the upper soil layer ($t_{ul} = 0.5, 1, 3$ cm) and using normal or logarithmic scale for the x- and y-axis (SIL: silt-loam; L: loam; LS: loamy sand). The dashed vertical line denotes the time when the i_r values of the layered soil started to become smaller than those of the homogeneous SIL soil

a)

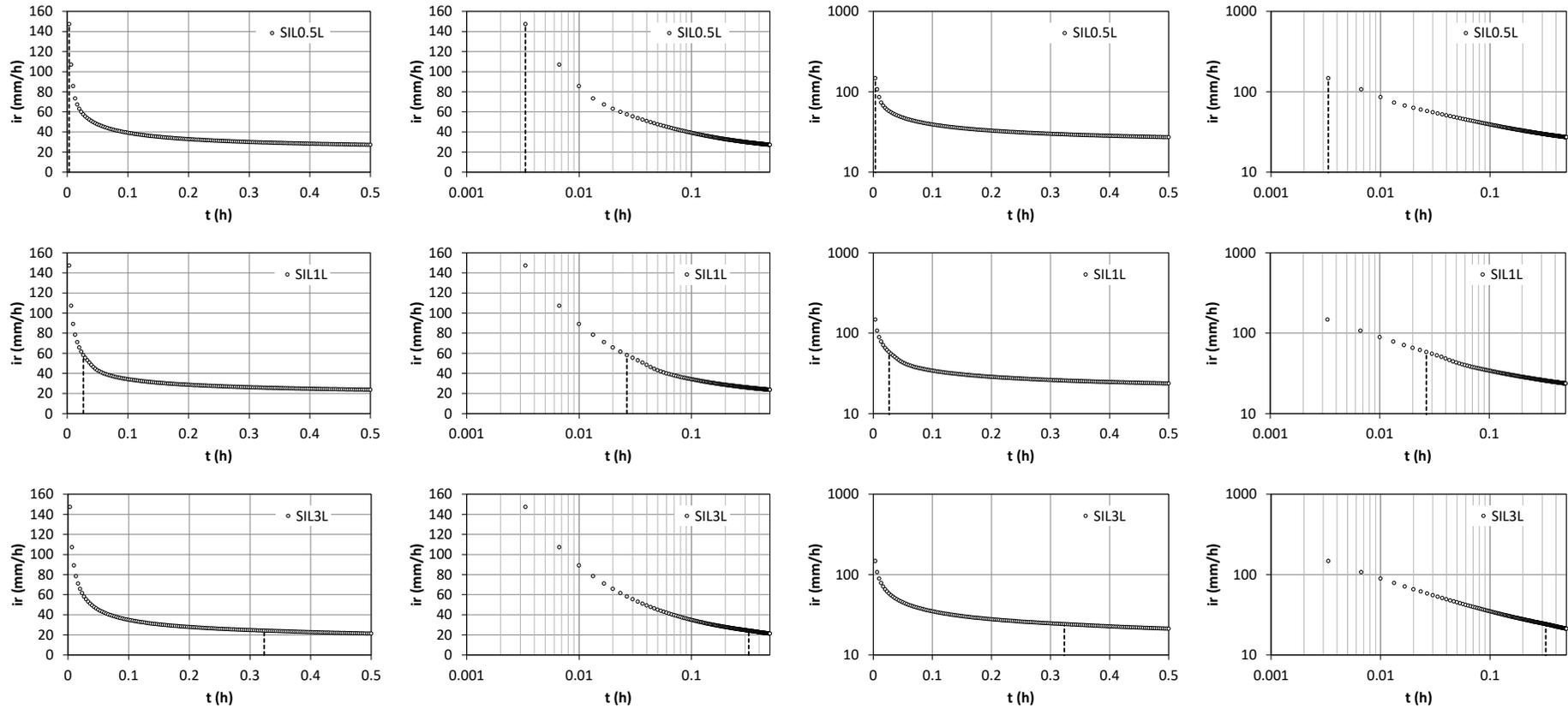


Fig. 6. Cumulative infiltration, I , for the homogeneous SIL, L, SAL and LS soils and the layered soils plotted against time, t : a) SIL overlying L (weakly layered soil); b) SIL overlying SAL (moderately layered soil); c) SIL overlying LS (strongly layered soil) (SIL: silt-loam; L: loam; SAL: sandy-loam; LS: loamy sand)

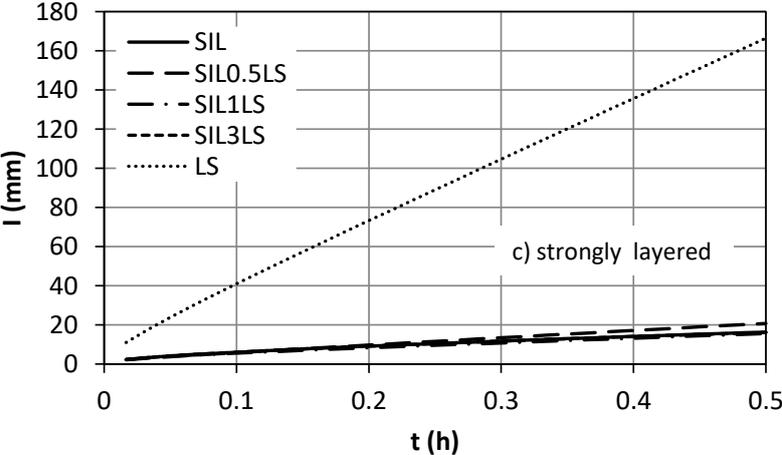
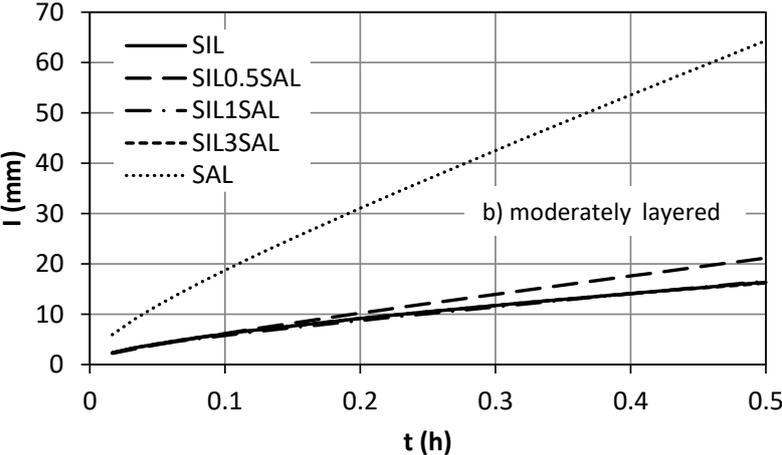
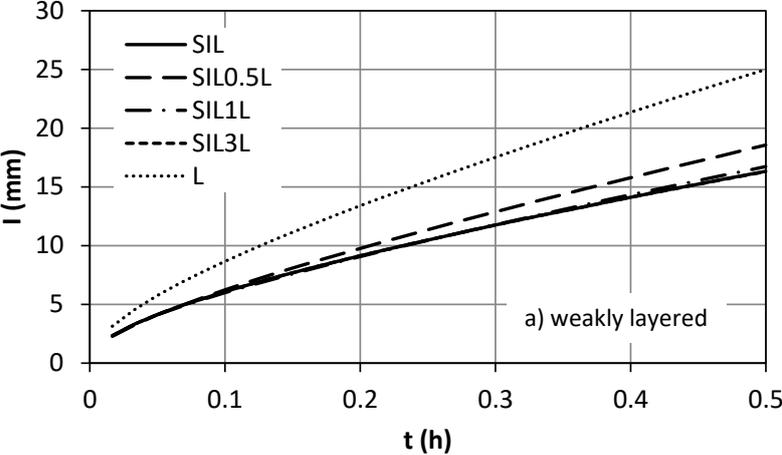


Fig. 7. Percentage difference between cumulative infiltration of the layered and homogeneous SIL soils, ΔI_{LH} , plotted against time, t , for the different layering degrees and thickness values of the upper soil layer (SIL: silt-loam; L: loam; SAL: sandy-loam; LS: loamy sand)

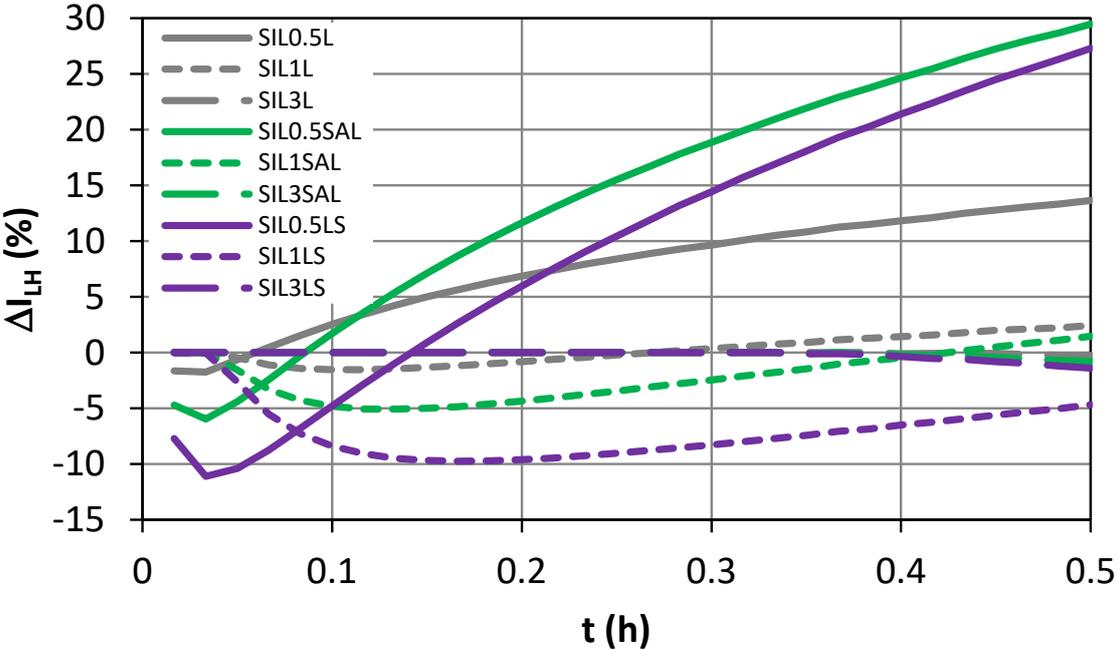


Fig. 8. Example of the fitting of the two-parameter infiltration model to the cumulative infiltration, I , vs. time, t , data

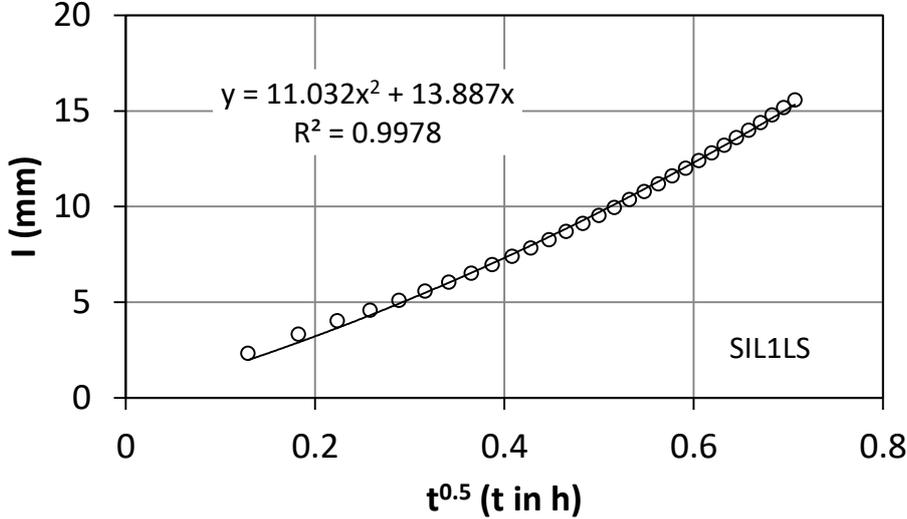


Fig. 9. Values of the C_1 and C_2 parameters of the two-term infiltration model for the homogeneous and the layered soils differing by both the layering degree (weak, moderate, strong) and the thickness of the upper soil layer (0.5, 1, 3 cm) (SIL: silt-loam; L: loam; SAL: sandy-loam; LS: loamy sand)

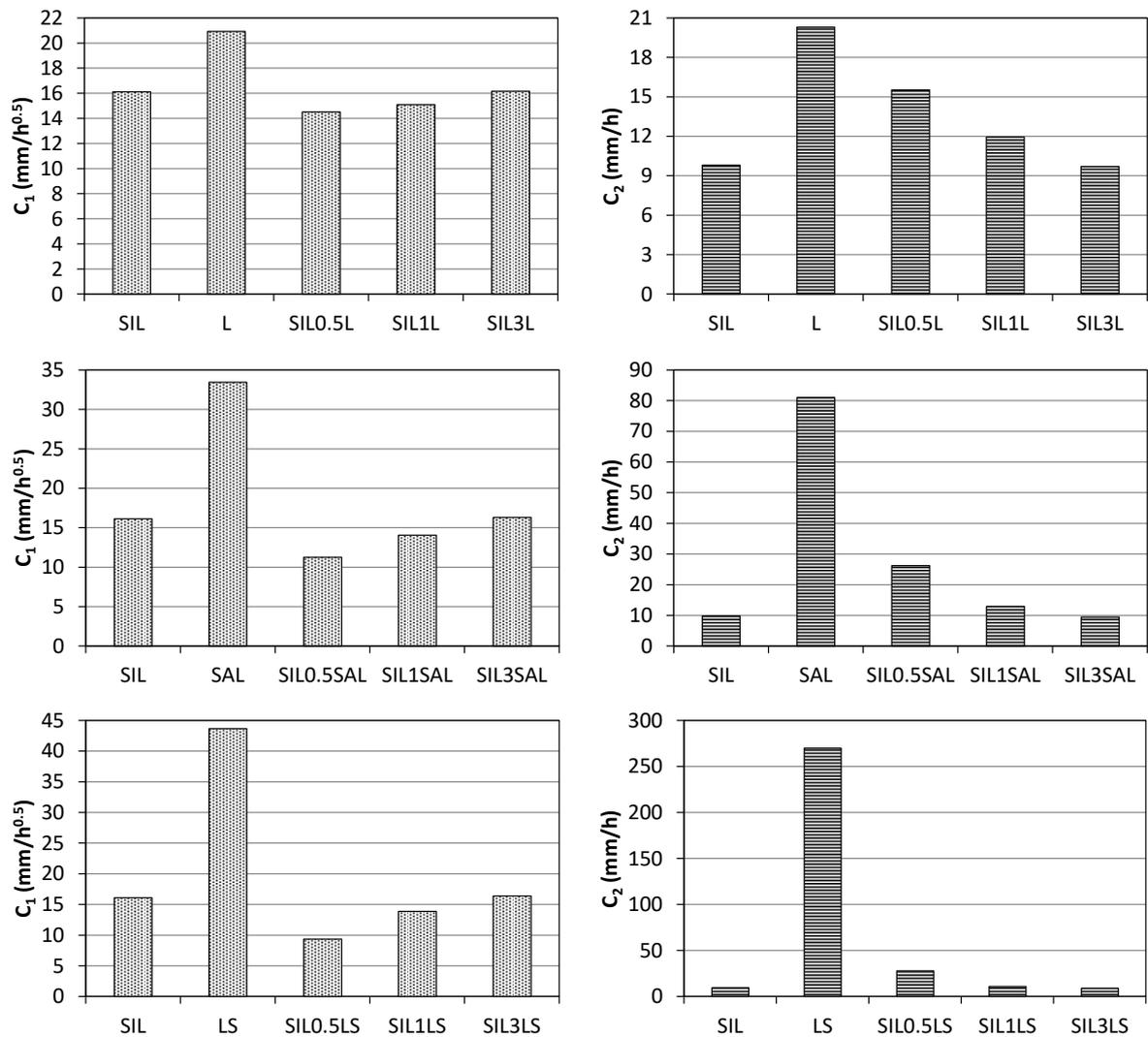


Fig. 10. Percentage differences between the C_1 (ΔC_1) and C_2 (ΔC_2) parameters obtained in the layered soils and those of the homogeneous SIL soil plotted against the thickness of the upper soil layer in the layered soil (SIL: silt-loam; L: loam; SAL: sandy-loam; LS: loamy sand)

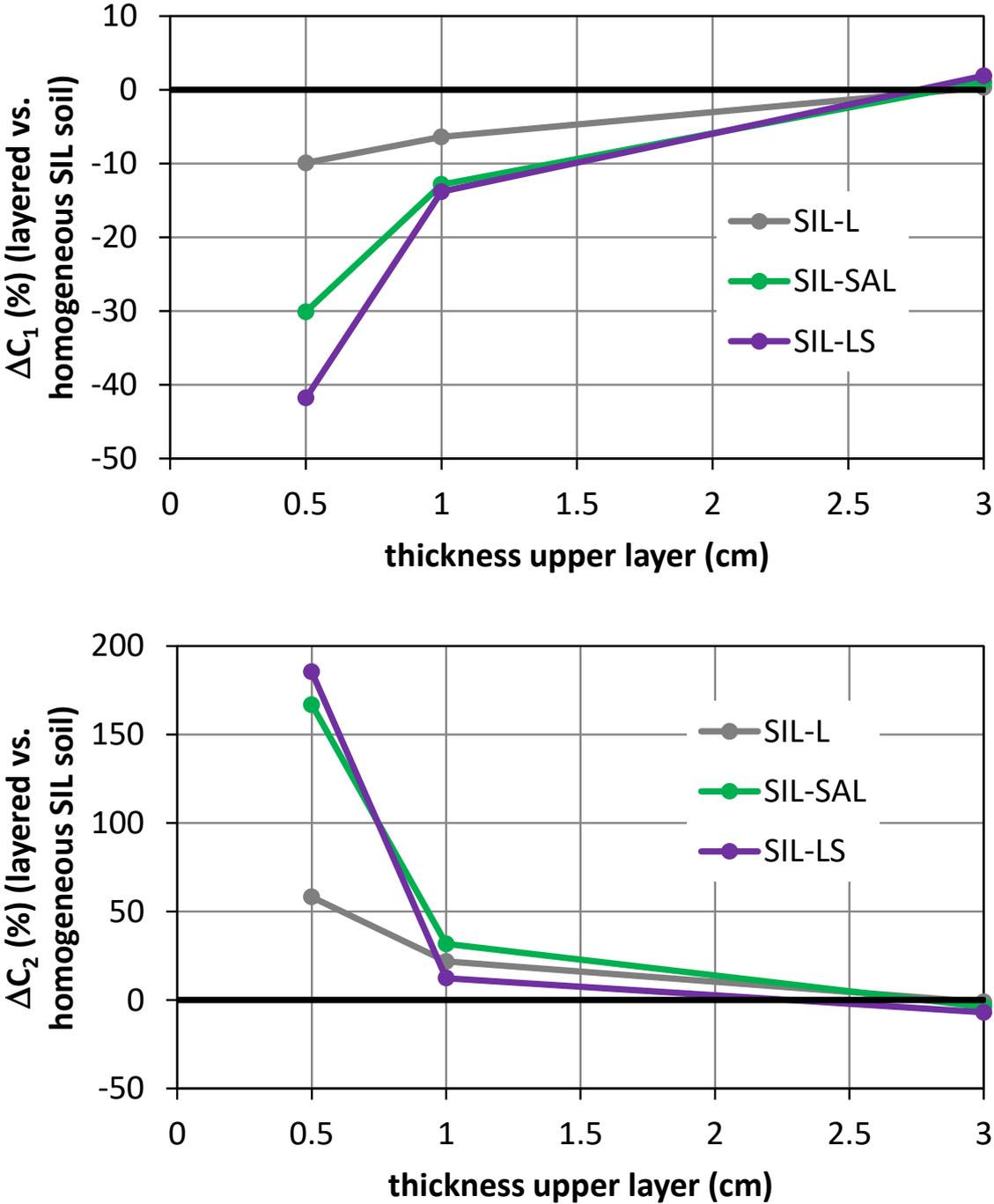


Fig. 11. Plot of the root mean square, $RMSE$ (mm/h), vs. time, t , data according a) to Moret-Fernández et al. (2021) and b) the modified sequential analysis procedure

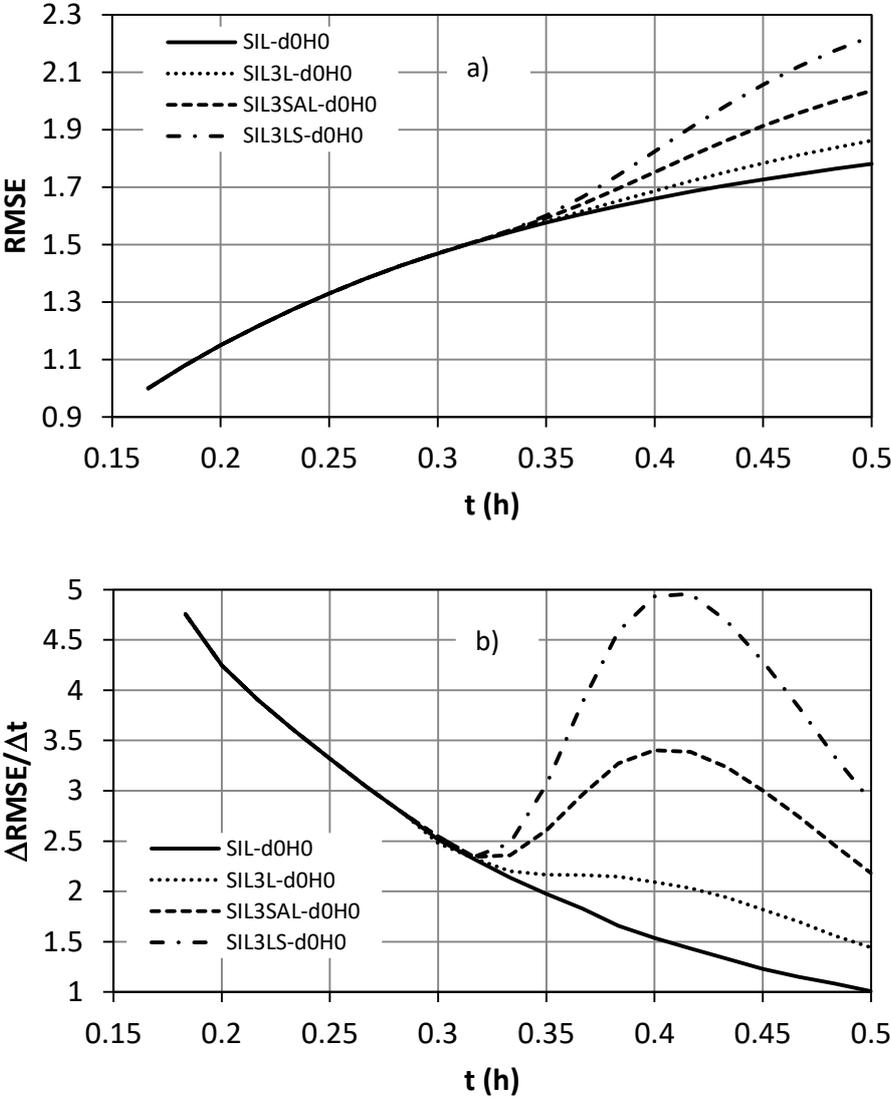


Fig. 12. Percentage difference between infiltration rate for the practical (d1H1) and the theoretical (d0H0) setup, Δi_{rPT} , plotted against time, t , for the different layering degrees and thicknesses of the upper soil layer (SIL: silt-loam; L: loam; SAL: sandy-loam; LS: loamy sand)

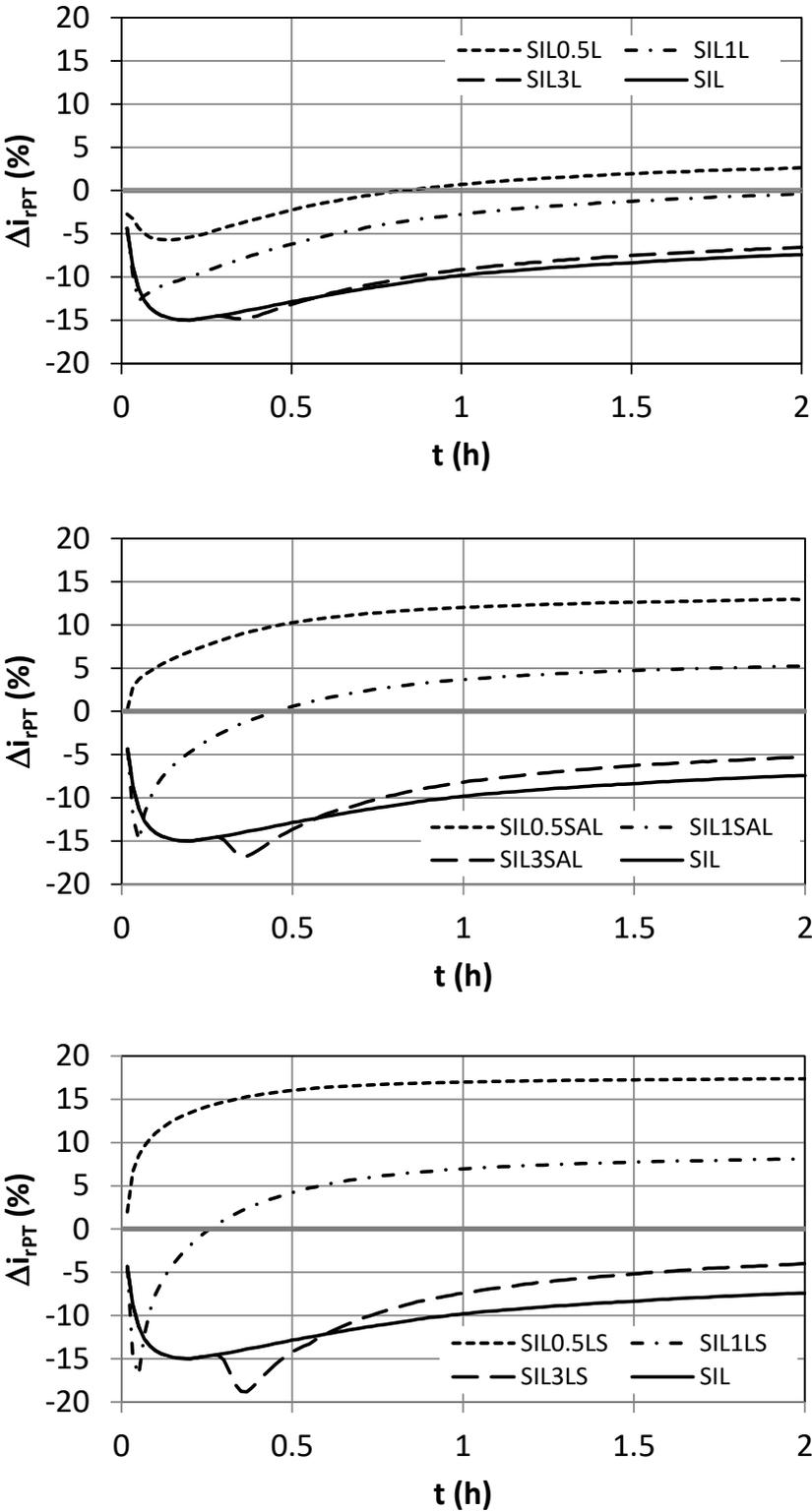


Fig. 13. Percentage difference between cumulative infiltration for the practical (d1H1) and the theoretical (d0H0) setup, ΔI_{PT} , plotted against time, t , for the different layering degrees and thicknesses of the upper soil layer (SIL: silt-loam; L: loam; SAL: sandy-loam; LS: loamy sand)

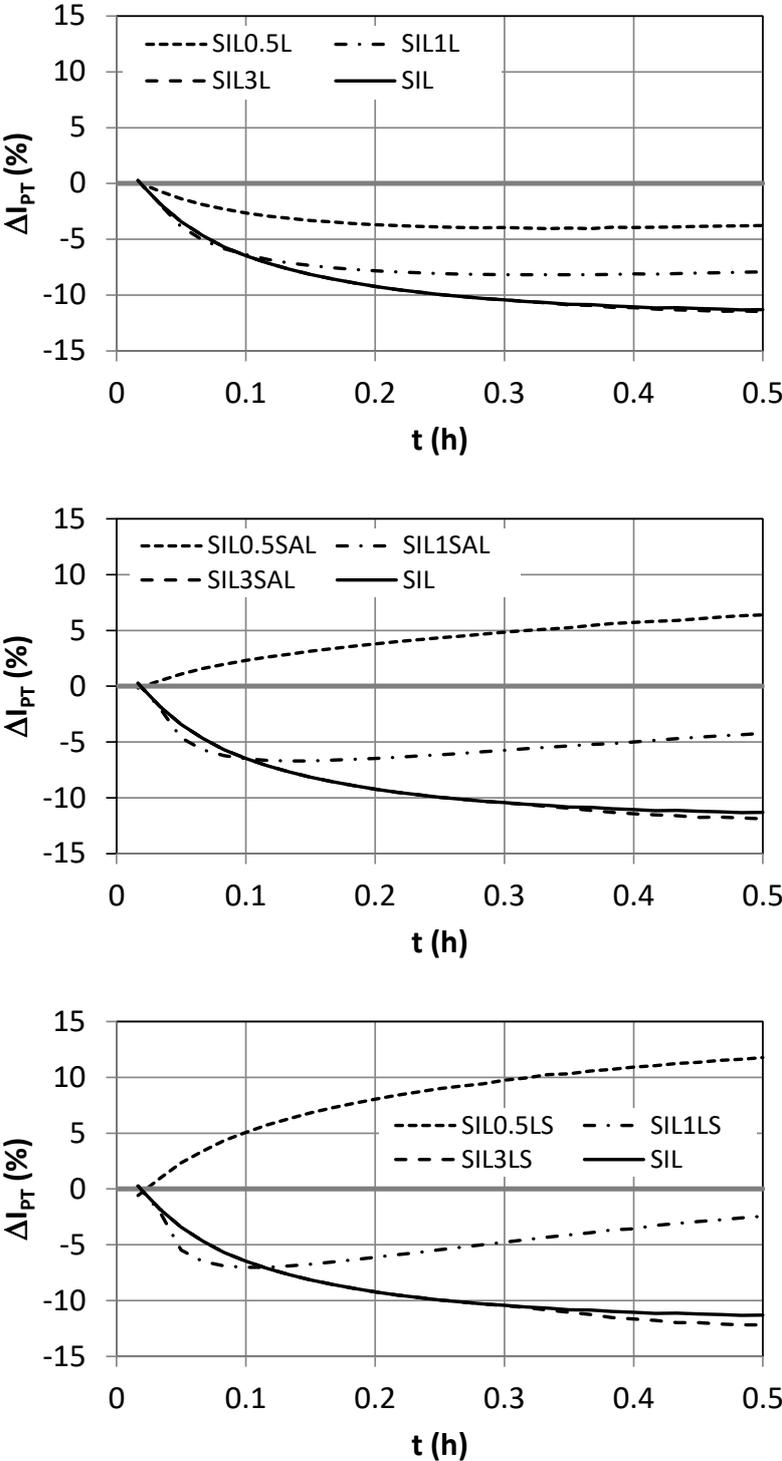


Fig. 14. Percentage differences between the C_1 (ΔC_1) and C_2 (ΔC_2) parameters obtained with the practical (d1H1) and the theoretical (d0H0) setups in the layered soils and in the homogeneous SIL soil plotted against the thickness of the upper soil layer in the layered soil (SIL: silt-loam; L: loam; SAL: sandy-loam; LS: loamy sand)

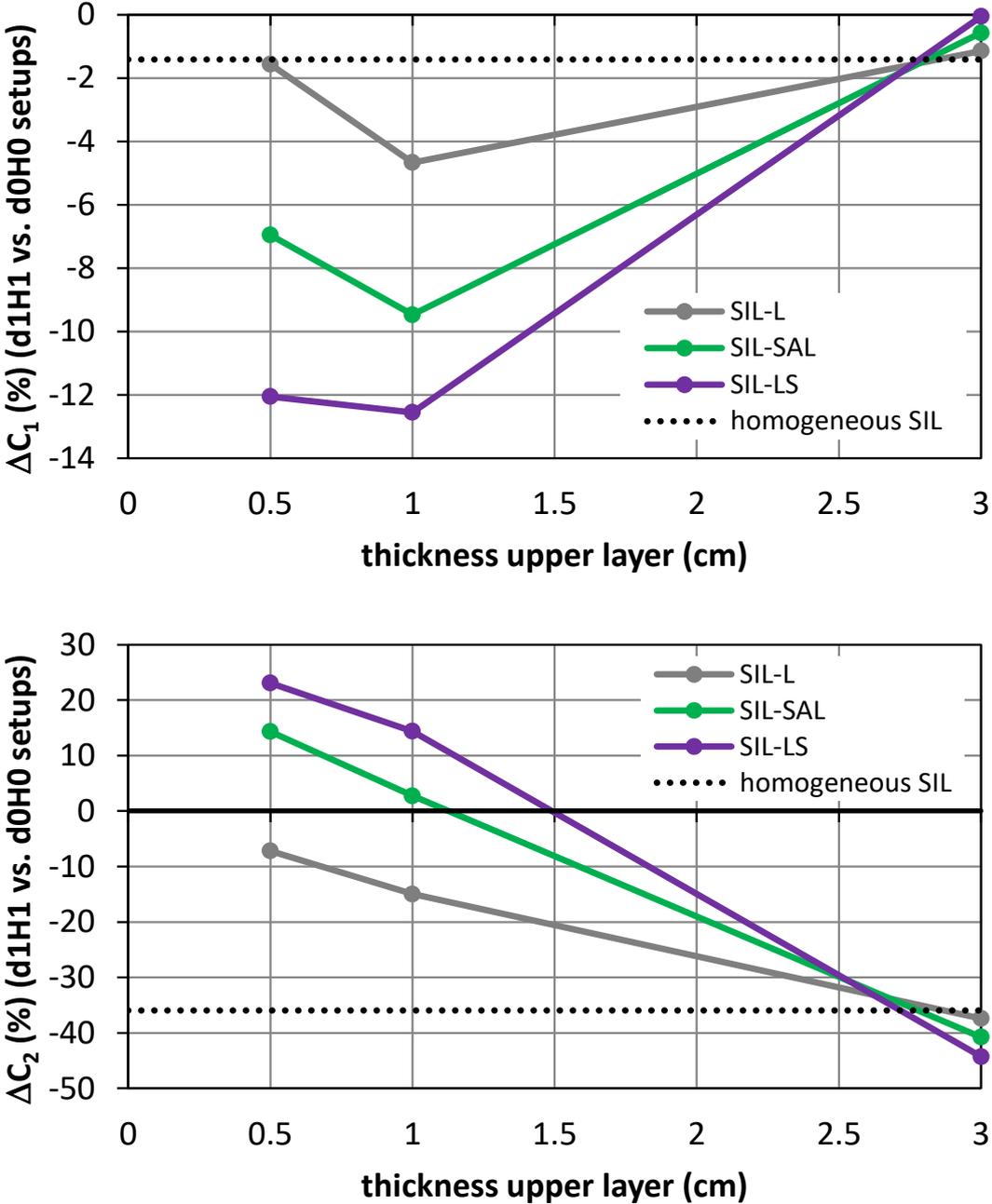


Fig. 15. Comparison of the $\Delta RMSE/\Delta t$ vs. t ($RMSE$ = root mean square error in mm/h; t = time in h) relationships for the practical (d1H1) and the theoretical (d0H0) setups

