An intentional random mathematical model of immigration: The case of Spain

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February 20, 2023

Abstract

In this paper a random discrete mathematical population model of immigration is constructed using not only rational factors as the gradient of economy level among the host and issuing countries, the geography, or the regulation laws, but also hidden intentional factors such as political interests of governments, or the business of smuggling of migrants by mafias, which use the immigration practically as a weapon. These non-rational factors are modeled as sudden random arrival flow waves represented by a Poisson distribution. The period of study is short in order to make reliable the economic forecast of the next years. Although the study is focused in Spain, the approach is exportable to other geographic areas by adapting the data. Results are applied to predict the necessary national budget of the host country to afford this social phenomenon.

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Received ; | Revised ; | Accepted

DOI: xxx/xxxx

RESEARCH PAPER

An intentional random mathematical model of immigration: The case of Spain

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Summary

In this paper a random discrete mathematical population model of immigration is constructed using not only rational factors as the gradient of economy level among the host and issuing countries, the geography, or the regulation laws, but also hidden intentional factors such as political interests of governments, or the business of smuggling of migrants by mafias, which use the immigration practically as a weapon. These non-rational factors are modeled as sudden random arrival flow waves represented by a Poisson distribution. The period of study is short in order to make reliable the economic forecast of the next years. Although the study is focused in Spain, the approach is exportable to other geographic areas by adapting the data. Results are applied to predict the necessary national budget of the host country to afford this social phenomenon.

KEYWORDS:
Immigration mathematical modeling; random discrete dynamic; population dynamics; intentional; unaccompanied minor

1 | INTRODUCTION

Movement of human populations is as old as the mankind and also among irrational animals in search of better live conditions. Nomadic way of living maybe voluntary or forced, for instance due to fair to wars consequences. There are two types of human migration, the so called economic to improve live conditions, and the one motivated by the fair to die as a consequence of conflicts among neighbours. Human migration is becoming a viable weapon for many state and non-state actors pursuing unconventional means to increase regional influence and to obtain political or economic objectives. Although some old migration phenomena are susceptible to be regarded as weaponization of the migration, this new strategy is becoming more relevant due to the use of migration as an instrument to obtain benefits of political or economic pawns from the affected countries[1 2 3 4]. Searching these pawns is not only a matter of issuing countries but also it is possible from host countries throughout open doors strategies[5 6].

Since the weaponization of the immigration flows is external to the decision to immigrate by the population itself, and such strategies may be driven by sudden and circumstantial possibilities linked to random events such as wars, or conflicts, it is worthy to introduce a random component in the immigration modelling process. Thus the immigration population phenomena have a deterministic predictable component, at least in a short time period, linked to the behavior of the economy of the host country and the regulation of legal immigration. Historic recent data shows the existence of exceptional immigration waves occurring simultaneously with disputes among neighbour countries.

Weaponized immigration populations may appear as mass population escaping by the fair to war, as the immigration population escaping from the war in Syria, used by Turkey as an instrument to gain social political and economic advantages from

the European Union. Apart from the public political interest of governments, there exists also the private business of smuggling of migrants by mafias that get benefit approaching populations of immigrants from away zones to places close to the boundary of the issuing countries[7]. The fact that several immigration waves occurring in certain times suggests the modelization of the random component as a Poisson distribution. Antecedent of deterministic mathematical immigration models may be found in[8] and references therein.

Population migration has been traditionally treated by the so called gravity models, that are models taking into account physical factors without psychological or intentional influences. In fact, they assume that the probability of a flow of population between two locations decays directly as a function of a distance and the size of populations, see[9 10 11 12 13 14 15 16 17].

More recently, since 2012[18 15 19], the radiation models assume that the probability of a trip decays with the distance of intervening opportunities. Both gravity and radiation models disregard intentionality, emotions, interest and other human factors[20]. Recently appear the Machine Learning models that are able to incorporate any number of exogenous features to predict origin/destination human migration flows[9].

This paper, that way be regarded as a continuation of[8] deals with the construction of a random discrete dynamic mathematical model, taking into account economic, geographical and intentional factors. With respect to the deterministic model[8] we introduced several improvements. Firstly we split the non-adult irregular population into two new subpopulations because young irregular immigrants close to the adulthood have a short way to become regular in Spain and they constitute a relevant immigration population. In addiction, taking into account recent trend of flow waves, we introduce randomness in the model to capture political intention of issuing countries.

This paper is organized as follows. The core section 2 deals with the model construction including historic trends, Poisson modeling of immigration floods, and the statement of the vector mathematical model. Results, robustness and social budget applications are included in Section 3. Conclusions are drawn in Section 4.

2 | MODEL CONSTRUCTION

War conflicts and sudden transition to dictatorship in neighbour countries are natural sources of immigration flows apart form the traditional legal (regular) immigration flow due to economic reasons. However, new strategies are coming from non democratic countries by using the immigration as a political pressure against boundary countries in order to get advantages of political or economic type[1 21].

Well known cases to the European case are the examples of Turkey or the Republic of Belarus, and related to Spain, mainly Morocco and Algeria. This coercive political factor is somewhat unpredictable since it depends of the timely decision of Governments trying to get advantages usually derived from tensions of diplomatic relationships among countries. The regular immigration is closely related to economic behavior of the host country that is measured using the unemployment rate as the economic indicator. In this paper we split the immigration population into four categories:unaccompanied minor under fourteen (CH), unaccompanied minor aged in $[14, 18[$ (M), adult irregular immigrant (I) and regular immigrant (L).

We consider an intentional short time period of study in order to improve the hypothesis of the next becoming times related to economic indicator estimations but also the possible relevant changes about conflicts or tensions among boundary countries. The period of study is $[2019, 2027]$ and it is split early discretely from $n = 0$ corresponding to 2019 and the final time step $n = 8$. This the relevant population vector $Z(n)$ takes the form

$$Z(n) = [\, CH(n),\ M(n),\ I(n),\ L(n)\,]^T\,, \tag{1}$$

where $CH(n)$ represents the amount of unaccompanied minor under fourteen, $[9, 14[$ at the end or year n; $M(n)$ the amount of unaccompanied minors aged in $[14, 18[$; $I(n)$ denotes the amount of irregular adults at the end of year n, and $L(n)$ means the amount of regular immigrants at the end of year n. The population transit from year n to the $(n+1)$-th depends on some relevant factors such as the economy of the host country, in this case Spain, measured using the unemployment rate.

Another important factor has a political nature and it depends strongly of the issuing countries, in our case, mainly Morocco and Algeria. This political factor has a deterministic historic trend, but in addition it has also a random component derived of the coercive use of immigration as a strategic weapon to get advantages in the political relationships among the boundary countries.

From the host country point of view there also regulations laws which depend on the issuing countries political agreements, but there are also random events such as neighbour war conflicts as the one of Ukraine, worsening the living conditions in big areas, including the risk of famine, apart from the growth of prices of energy and overall inflation. These exceptional factors,

having a random character produce immigration sudden waves that should be taken into account in the model. Each population category or entry of the vector $Z(n)$ given in eqrefvector Z depends on the factors in a different way, but always follow the pattern

$$Z(n+1) = Z(n) + \Delta[Z(n), Z(n+1)], \tag{2}$$

where $\Delta[Z(n), Z(n+1)]$ denotes the transit of the population from $Z(n)$ to $Z(n+1)$.

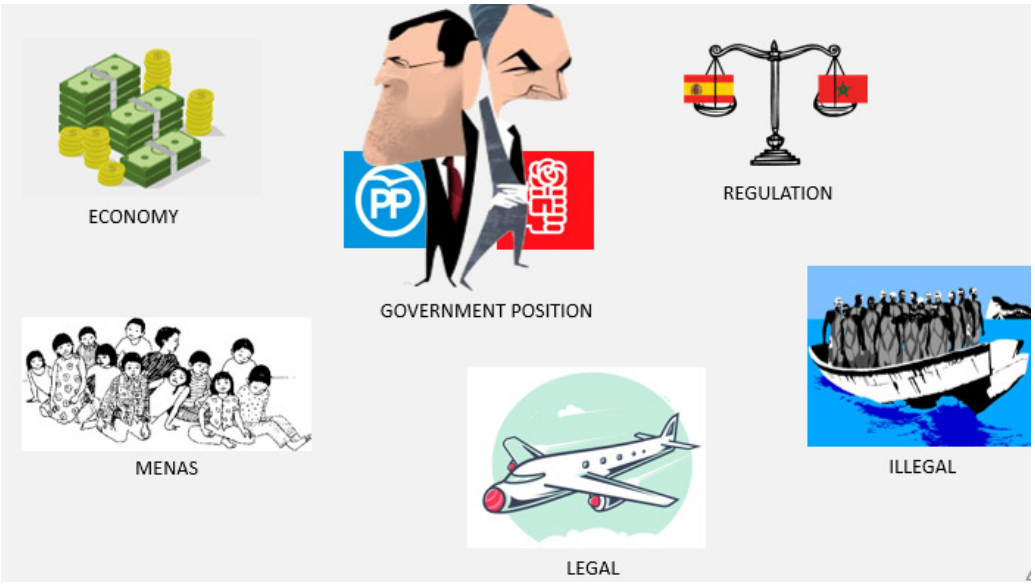

FIGURE 1 Main factors of the immigration process.

2.1 | Deterministic historic trends

Table 1 collects yearly historic data of irregular immigration during the period [2010, 2021][22]. Our hypothesis of the model assumes some characteristics matching the historic data. There is a double influence in the arrival of irregular immigration. One is predictable and deterministic depending on the political sign of the Spanish Government, because of the differences between the main parties PSOE (socialist) and PP (conservative) with respect to the immigration phenomenon. These differences are observable from the data: the immigration grows more with PSOE governments than with PP governments. Thus, the red rows of table 1 correspond to socialist governments and the blue rows are linked to conservative governments. The non predictable irregular immigration has two different sources, both political but depending on the host government and the issuing government countries. In fact, the so called "call effect" corresponds with a relaxation in the irregular immigration controls. This effect induces a stable tendency of the increase of the arrivals of irregular immigration. Once the call effect occurs, the irregular business of the transportation and approaching of irregular immigrants follows the rules of the market: when there is expectation of benefit the offer grows. For instance, with the arrival of the PSOE party to the government, after a censure motion in May 2018, one produced a call effect, and the amount of irregular immigrants in Spain jumped from 21,971 in 2017 to 57,498 in 2018. This motivates the appearance of an outlier data in table 1 corresponding to 2018. The second non predictable or random source comes from the political interest of issuing countries to get political or economic advantages, using the immigration process as a political weapon[1 21]. Thus our model assumes a random component of the incoming immigration process due to random events like sudden changes in the political relationships between Spain and Morocco, or Algeria; the black swan of the inflation; wars of political changes in neighbours countries. That is the case of year 2020 and 2021, corresponding to the invasive flood of immigrants arriving to Canary Islands in November 2020[23]; and the invasive flood of unregulated immigrants in Ceuta in May 2021[24] motivated by an "open door" strategy of Morocco government. This is the reason why we split the amount of irregular immigrants for there two years in two columns in table 1 .

Year	Steady Irregular Immigration	Flood Immigration	Overall entries
2013	3,237	–	3,237
2014	4,552	–	4,552
2015	5,312	–	5,312
2016	8,162	–	8,162
2017	21,971	–	21,971
2018	–	–	57,498
2019	26,103	–	26,103
2020	31,949	8,157	40,106
2021	40,100	8,000	48,100

TABLE 1 Historic incoming Irregular Immigration in Spain.

Now we address the modeling of the deterministic component of the incoming immigration process based on a linear regression function. In order to capture this predictable tendency we delete the outliers data corresponding to years 2018 and 2020 in table 1 . Let $f(t)$ be the least squares regression line

$$y = f(t) = a(t - 2019) + b\,, \tag{3}$$

where t represents the time in the period $[2013, 2021]$ and y denotes the steady incoming irregular immigration population. The resulting values a and b according with the data are

$$a = 4,768.34; \quad b = 27,805.98 \tag{4}$$

with the reliable correlation coefficient $R = 0.97$. The next figure 2 shows the linear correlation of the arrival immigration population yearly distributed during the period $[2013, 2021]$.

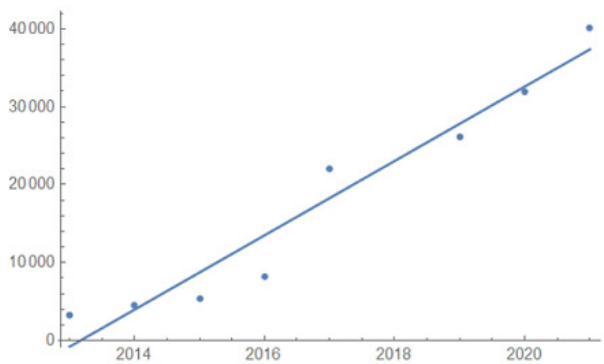

FIGURE 2 Least squares regression line of the Incoming Irregular Immigration.

Another predictable trend of the immigration process is the variation of the regular immigration flow with respect to the behaviour of the economy of the host country, Spain, modeled by the unemployment rate. As the flow may be in two senses, incoming and exit of foreigners, the next analysis describes the net balance of the regular immigration versus the unemployment rate. Table 2 shows the net balance of the foreign regular migration population flow, according with the data from Spanish National Statistic Institute[25] during the period $[2013, 2021]$. Table 2 also shows the yearly unemployment rate according with the official data coming from he statistics web page[22].

We assume that the net balance of the regular migration changes linearly with the unemployment rate. This assumption will be confirmed with the Pearson correlation coefficient, R. Let $g(x)$ be the least squares regression line

$$z = g(x) = Ax + B, \tag{5}$$

Year	Unemployment %	Net balance
2013	25.73	-210,936
2014	23.70	-64,802
2015	20.90	383,18
2016	18.63	112,666
2017	16.55	174,231
2018	15.25	330,197
2019	13.90	444,587
2020	16.30	230,026
2021	13.30	153,094

TABLE 2 Regular Migration Net Balance vs Unemployment Rate.

where x represents the Spanish percentage unemployment rate, and $z = g(x)$ quantifies the expected regular migration net balance. According with data of table 2 , one gets,

$$A = -40,905, \quad B = 880,723, \tag{6}$$

with Pearson correlation coefficient $R = 0.9$. The linear correlation is shown in figure 3 .

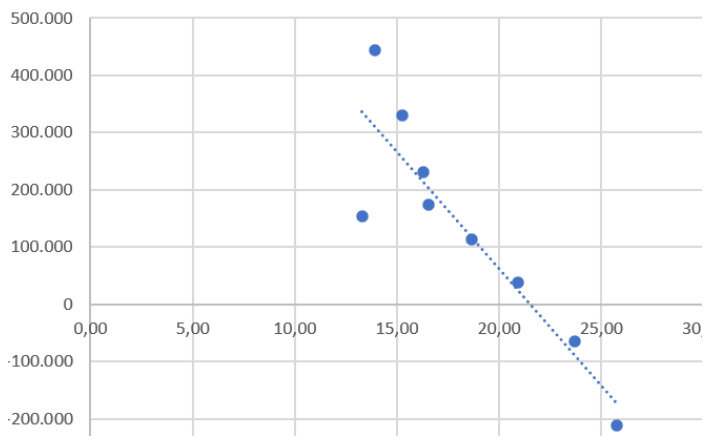

FIGURE 3 Regular Migration Net Balance vs Unemployment Rate.

The linear correlation trends described in (3) and (5) will be used to forecast in the future period [2022, 2027] the deterministic (predictable) evolution of the irregular and regular immigration population, versus time and unemployment rate, respectively. In the case of (5), the future unemployment rates will be obtained from reliable short-time predictions of the Spanish unemployment rate[22].

2.2 | Poisson type random immigration floods

In this subsection we model the unpredictable human flood events occurring in the unregulated immigration process, as we have checked that they occur when governments of issuing countries use the immigration as a political weapon. We assume that this unpredictable immigration component follows a truncated Poisson distribution up to four events, $J = 4$, and expected rate λ. Thus the global arrival immigration population $B(n)$ can be decomposed in two terms,

$$B(n, \lambda) = B_1(n) + B_2(\lambda), \tag{7}$$

where $B_1(n) = an + b$ is the predictable irregular immigration arrival and $B_2(\lambda)$ represents the expected incoming irregular immigration of the right truncated Poisson distribution[26 27]. Here n denotes the year after 2019. As the study period is [2020, 2027], n takes the values $n = 1, 2, 3, 4, 5, 6, 7, 8$ corresponding $n = 0$ to 2019. Taking into account historic date we assume up to three possible scenarios for λ ; $\lambda = 0$, $\lambda = 1$, $\lambda = 2$. For instance, $\lambda = 0$ means that there is no flood event in a year, see the years from 2013 to 2018 in table 1 . In this case $B(n, 0) = B_1(n)$ and all the arrivals are predictable. The probability of having k floods in a year, with expected rate λ is

$$P(k, \lambda) = \frac{\lambda^k}{k! \sum_{j=0}^{J} \frac{\lambda^j}{j!}}, \quad k \in \{0, 1, 2, 3, 4\}, \ J = 4. \tag{8}$$

The greater is the value k of the number of floods, the minor is the jump intensity of the arrival immigration. We assume this relationship among the number of events (floods) in a year and intensities:

Number of events k	Jump intensity $N(k)$	Overall incoming population $kN(k)$
1	6,000	6,000
2	4,000	8,000
3	3,000	9,000
4	2,000	8,000

TABLE 3 Distribution hypothesis among number of floods k in a year and amount of immigrants per flood.

From (8) and the hypothesis compiled in table 3 , we have

$$B_2(\lambda) = \sum_{k=0}^{4} kN(k)P(k, \lambda), \quad \lambda = 1, \ \lambda = 2. \tag{9}$$

Thus, we have $B_2(0) = 0$, and

$$\begin{aligned} B_2(1) &= 4,369\,, \\ B_2(2) &= 6,476\,. \end{aligned} \tag{10}$$

Summarizing the next table 4 shows the global incoming irregular immigration $B(n, \lambda)$ for each scenario, $\lambda = 0$, $\lambda = 1$ and $\lambda = 2$.

Year n	$B(n, \lambda = 0) = B_1(n)$	$B(n, \lambda = 1)$	$B(n, \lambda = 2)$
2020	32,574	36,943	39,050
2021	37,343	41,712	43,819
2022	42,111	46,480	48,587
2023	46,879	51,248	53,355
2024	51,648	56,017	58,124
2025	56,416	60,785	62,892
2026	61,184	65,553	67,660
2027	65,952	70,321	72,428

TABLE 4 Overall Incoming Irregular Immigration Scenarios.

2.3 | Immigration mathematical model

In order to model the dynamics of the immigration population, symbolically represented by (2), it is important to pay attention to the age distribution of immigrant people, as well as the regulation laws changes of the host country, in this case, Spain. The first relevant transit coefficient β_1 is the proportion of the unaccompanied minor immigrants of age belonging to the interval [9, 14[denoted by $CH(n)$, becoming fourteen years old and consequently moving to the subpopulation $M(n+1)$. The next coefficient β_2 is the proportion of people in $M(n)$ achieving the adulthood the year $n+1$. This coefficient β_2 is particularly relevant because the recent Spanish regulation law,[28] provides available regularization of immigrants, allowing residence and labor rights. Both coefficients β_1 and β_2 in reality change yearly, because they depend on the age distribution of people inside the own category $CH(n)$ and $M(n)$, respectively. As it occurs in the study of phenomena linked to violence, delinquency, or legality there is an impossibility to know the real data because there is a willing of misleading, for instance to achieve official adulthood as soon as possible. Thus we assume as a reasonable hypothesis that the age distribution inside these population categories is uniform or uniformly distributed. This hypothesis means that coefficients β_1 and β_2 are constant, independent of n, in fact, parameters of the model. As the possible different ages of people in category $CH(n)$ are five, we assume that β_1 is the twenty percent, so

$$\beta_1 = 0.2\,. \tag{11}$$

In an analogous way, as we have four possible different ages inside category $M(n)$, we assume

$$\beta_2 = 0.25\,. \tag{12}$$

From (1)-(2) and (11)-(12) we can write

$$CH(n+1) = CH(n) - \beta_1 CH(n) + \alpha_1 B(n,\lambda)\,, \tag{13}$$

$$M(n+1) = M(n) - \beta_2 M(n) + \beta_1 CH(n) + \alpha_2 B(n,\lambda)\,, \tag{14}$$

where α_1 and α_2 are the proportions of the incoming immigration population $B(n,\lambda)$ belonging to the category CH and M, respectively. Like in the case of coefficients β_1 and β_2, these coefficients α_1 and α_2 are not available, and although lightly, they change with n. In agreement with[22] and[8] we assume that $\alpha_1 + \alpha_2 = 0.4$. In addition, we assume that only 10% of arrival unaccompanied minor belongs to $CH(n)$ and the age of the 90%, belongs to the interval [14, 18[corresponding to the category $M(n)$, see[25]. Thus,

$$\alpha_1 = 0.04; \quad \alpha_2 = 0.36\,. \tag{15}$$

Apart from the access to the regular status due to the adulthood, irregular immigrants may achieve the legal status due to social roots, labour or family roots and political asylum criteria[25]. The new Spanish regulation law, introduced in operating since August 2022[29], makes easier and quicker the access to the regular status, incorporating training experience. Thus, unlike to[8] we assume

$$\beta_3 = 0.03\,. \tag{16}$$

From (1)-(2) and (16) the dynamics of subpopulation $I(n)$ of adult unregulated immigrants can be described as

$$I(n+1) = I(n) - \beta_3 I(n) + \left(1 - \alpha_1 - \alpha_2\right) B(n,\lambda)\,. \tag{17}$$

In order to describe the dynamics of $L(n)$ it is important to point out that there is a possible flow from both $M(n)$ and $I(n)$ to $L(n+1)$, apart from the regular incoming immigration arrival. Thus we can write

$$L(n+1) = L(n) + \beta_2 M(n) + \beta_3 I(n) + G(n)\,, \tag{18}$$

where $G(n)$ denotes the amount of regular incoming immigration arrival obtained from (5). From (1), (2), (13), (17) and 18 the model can be written in vector form as

$$Z(n+1) = A\,Z(n) \;+\; C(n,\lambda)\,, \tag{19}$$

where

$$A(n) = \begin{bmatrix} 1-\beta_1 & 0 & 0 & 0 \\ \beta_1 & 1-\beta_2 & 0 & 0 \\ 0 & 0 & 1-\beta_3 & 0 \\ 0 & \beta_2 & \beta_3 & 1 \end{bmatrix}, \tag{20}$$

$$C(n,\lambda) = \begin{bmatrix} \alpha_1 B(n,\lambda) \\ \alpha_2 B(n,\lambda) \\ \left(1-\alpha_1-\alpha_2\right) B(n,\lambda) \\ G(n) \end{bmatrix}. \tag{21}$$

Since the random behavior affects to all subpopulations throughout the external term $C(n,\lambda)$, see (19), we have a random discrete mathematical model with Poisson jumps, where $Z(n) = Z(n,\lambda)$ is a vector population process depending on the year n and the expected rate λ of the Poisson flood distribution. Thus, finally we have

$$Z(n+1,\lambda) = A\,Z(n,\lambda) \;+\; C(n,\lambda), \tag{22}$$

where the subpopulations become $CH(n,\lambda)$, $M(n,\lambda)$, $I(n,\lambda)$ and $L(n,\lambda)$ assuming the expected rate $\lambda \in \{0,1,2\}$.

3 | RESULTS, ROBUSTNESS AND SOCIAL BUDGET APPLICATIONS

Note that the explicit vector difference equation has a closed form solution that can be written as

$$Z(n,\lambda) = A^n Z(0,\lambda) + \sum_{j=0}^{n-1} A^j C(n-1-j,\lambda), \quad n > 0, \tag{23}$$

Tables 5 , 6 and 7 contain the values of each entry of $Z(n,\lambda)$ for $\lambda = 0$, $\lambda = 1$ and $\lambda = 2$, respectively. In addition figures 4 , 5 , 6 and 7 represent the change of each subpopulation for the different values of the expected rate λ of the truncated Poisson distribution.

Year	n	$CH(n)$	$M(n)$	$I(n)$	$L(n)$
2020	1	2,184	19,346	779,899	5,526,954
2021	2	3,050	26,673	776,046	5,769,159
2022	3	3,934	34,058	775,170	6,135,795
2023	4	4,832	41,491	777,182	6,535,989
2024	5	5,740	48,961	781,994	6,836,825
2025	6	6,658	56,462	789,523	7,160,126
2026	7	7,583	63,988	799,687	7,493,708
2027	8	8,514	71,534	812,407	7,821,296

TABLE 5 Immigrant Population for $\lambda = 0$.

The next table 8 shows the effect of the Poisson immigration flood modelling at the end of the period of study for the different subpopulations. As it can be checked in the table, the Poisson effect affects in the same way to the two irregular minor subpopulations $CH(n)$ and $M(n)$ and about four times more than adults irregular immigrants $I(n)$ and it is practically negligible for the legal immigrant $L(n)$. Entries of table 8 are obtained from the expressions:

$$\%\Delta P_i = \frac{P_i(8,\lambda) - P_i(8,0)}{P_i(8,0)} \times 100 \tag{24}$$

where $P_i \in \{CH, M, I, L\}$.

Year	n	$CH(n)$	$M(n)$	$I(n)$	$L(n)$
2020	1	2,184	19,346	779,899	5,526,954
2021	2	3,225	28,246	778,667	5,769,159
2022	3	4,248	36,846	780,335	6,136,267
2023	4	5,258	45,217	784,813	6,537,313
2024	5	6,256	53,414	792,017	6,839,309
2025	6	7,246	61,478	801,867	7,164,024
2026	7	8,228	69,440	814,282	7,499,230
2027	8	9,205	77,325	829,185	7,828,619

TABLE 6 Immigrant Population for $\lambda = 1$.

Year	n	$CH(n)$	$M(n)$	$I(n)$	$L(n)$
2020	1	2,184	19,346	779,899	5,526,954
2021	2	3,309	29,005	779,932	5,769,159
2022	3	4,400	38,190	782,825	6,136,495
2023	4	5,464	47,014	788,493	6,537,951
2024	5	6,505	55,561	796,851	6,840,507
2025	6	7,529	63,896	807,820	7,165,903
2026	7	8,539	72,069	821,320	7,501,894
2027	8	9,538	80,117	837,277	7,832,151

TABLE 7 Immigrant Population for $\lambda = 2$.

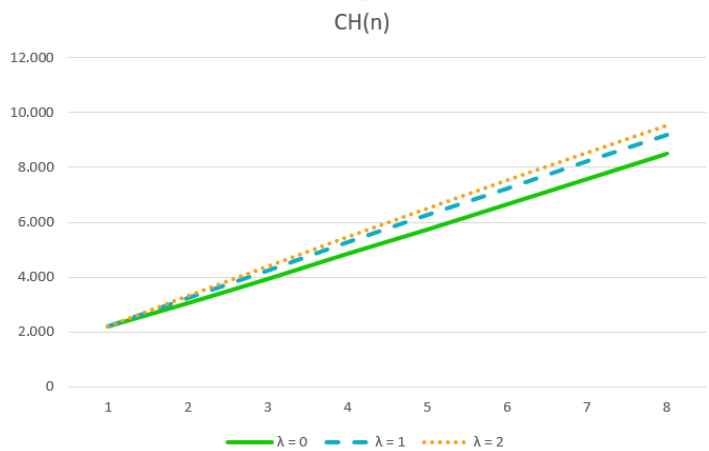

FIGURE 4 $CH(n)$ Population.

The subpopulation of legal immigrants is unaffected by the Poisson effect because these is no direct flood effect of legal subpopulation because this is driven by the economy. The explanation of the size of change of adult irregular immigrants versus the corresponding of minors is that when unaccompanied minors become adults they transit directly to legality, and that coefficients, β_1 and β_2 are similar, see (11) and (12)

One utility of the computation of the immigration populations is to estimate the social budget for the host country, in this case Spain, mainly concerning with the subpopulation of adolescent unaccompanied $M(n)$. As the cost of living in Spain varies strongly from the expensive of big cities as Madrid or Barcelona to the cheaper one as Andalucía or Extremadura regions, we approximate the cost of the assistance (accommodation, nutrition, medical services) of an average Spanish region as Aragón. In

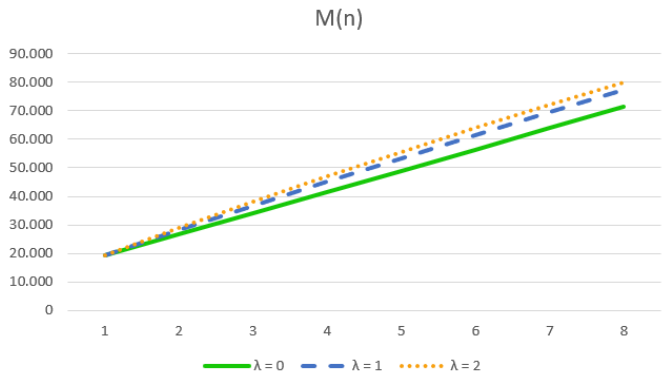

FIGURE 5 $M(n)$ Population.

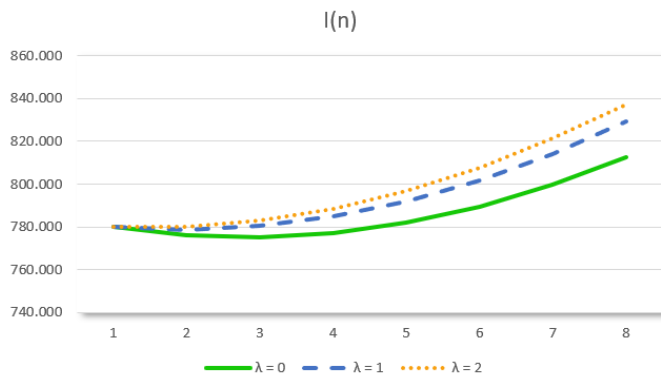

FIGURE 6 $I(n)$ Population.

λ	$\%\Delta CH(8,\lambda)$	$\%\Delta M(8,\lambda)$	$\%\Delta I(8,\lambda)$	$\%\Delta L(8,\lambda)$
1	8.1	8.1	2.1	0.01
2	12	12	3.1	0.1

TABLE 8 Percentage of change along the period.

accordance with the data coming from the Social Affairs Institute of Aragón, the daily cost per capita of each member of category $M(n)$ is about 75 euros. Thus knowing the amount of the population $M(n)$ for each year, we estimate the approximate social budget of assistance of this population for the incoming years. From tables 5 6 7 one gets for the tree different scenarios, the corresponding yearly budget shown in next tables 9 10 11 corresponding to the values of the respective expected rate λ.

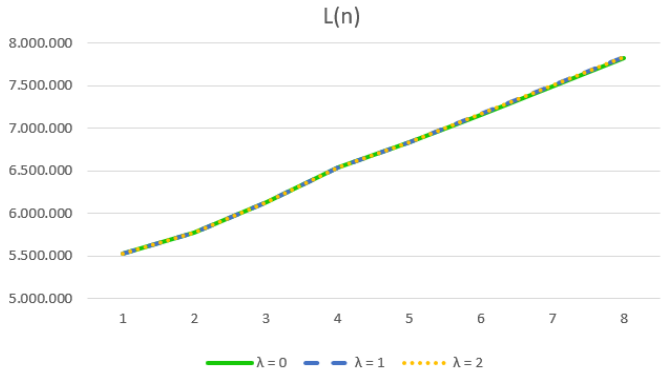

FIGURE 7 $L(n)$ Population.

Year	n	$M(n)$	millions of €
2020	1	19,346	529,605 M
2021	2	26,673	730,178 M
2022	3	34,058	932,349 M
2023	4	41,491	1,135,804 M
2024	5	48,961	1,340,298 M
2025	6	56,462	1,545,643 M
2026	7	63,988	1,751,666 M
2027	8	71,534	1,958,236 M

TABLE 9 Unaccompanied minor budget for $\lambda = 0$.

Year	n	$M(n)$	millions of €
2020	1	19,346	529,605 M
2021	2	28,246	773,235 M
2022	3	36,845	1,008,655 M
2023	4	45,216	1,237,812 M
2024	5	53,413	1,462,195 M
2025	6	61,478	1,682,947 M
2026	7	69,440	1,900,917 M
2027	8	77,325	2,116,761 M

TABLE 10 Unaccompanied minor budget for $\lambda = 1$.

Year	n	$M(n)$	millions of €
2020	1	19,346	529,605 M
2021	2	29,005	793,999 M
2022	3	38,190	1,045,454 M
2023	4	47,014	1,287,006 M
2024	5	55,561	1,520,981 M
2025	6	63,896	1,749,164 M
2026	7	72,069	1,972,895 M
2027	8	80,117	2,193,211 M

TABLE 11 Unaccompanied minor budget for $\lambda = 2$.

4 | CONCLUSIONS

In this paper an intentional random discrete dynamic mathematical model is constructed by incorporating human and hidden intentional variables having influence into the migration phenomenon. This approach is freely different of other physical models as gravity or radiation models disregarding hidden human intentions performed by governments or mafias. Modeling this uncertain behaviors requires some historic data, and it is developed using a Poisson distribution capturing the random flow waves of immigrants temporized at times were such decisions are taken. The amount of population of immigrants is computed and used to estimate the necessary social budget that the host country needs to attend the arrival immigration population, mainly the unaccompanied minor and adolescents. As we have estimated in Section 3, the budget of social assistante of the population $M(n)$ in the last year 2027 is about two thousand millions of euros, that is about 60% of the present of the national social affairs budget attending all the Spanish citizens in situation of dependence. The case of study is Spain, but the model is applicable to other places knowing historic recent data of immigration flows.

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