A robust beamforming for MIMO radar against virtual array steering vector mismatch

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Abstract

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This letter considers the problem of beamforming in multiple-input multiple-output (MIMO) radar. The mismatch phenomenon of MIMO radar virtual array steering vector is addressed and a new robust beamforming method for MIMO radar is proposed. The object function of this robust MIMO radar beamformer is constructed from an infinite norm of the output data, which is solved by linear programming. The performance of the proposed beamformer is verified by simulation results. Numerical results illustrate that proposed beamformer exhibits good performance improvement in virtual array steering vector mismatch compared to conventional methods.

Introduction: Multiple-input multiple-output (MIMO) radars have attracted more attentions due to its remarkable advantages over phased-based radar [1–4], such as high spatial resolution, accurate target detection and strong interference suppression. The performance benefits of MIMO radar come from the transmitting and receiving multiple orthogonal signals, and larger virtual array. In order to improve the signal to interference noise ratio (SINR) of target detection, a beamformer is usually designed to gather the echo beam energy of MIMO radar [5–10]. However, there are various factors that will degrade the performance of beamformers in practice. For a MIMO radar, either the array error at the transmitter or at the receiver will significantly affect the performance of beamforming. Conventional MIMO beamforming mitigate the performance degradations caused by the transmit/receive array error to a certain degree. However, they are still very sensitive to virtual array steering vector error, and even a small virtual array steering vector error will degrade the performance greatly.

In this letter, we propose a robust beamforming for MIMO radar against the virtual array steering vector mismatch. Regardless of the error at the transmitter or receiver, we integrates its modeling as a virtual aperture steering vector mismatch. The object function of this robust MIMO radar beamformer is constructed from an infinite norm of the output data, which is solved by linear programming. Numerical results shows the proposed beamformer outperforms conventional counterparts. In addition, the influence of different MIMO radar arrays on the performance is discussed in this letter.

Notations: We mark the conjugate, transpose, and conjugate transpose as $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$, respectively. The $|\cdot|_p$ and $\|\cdot\|_p$ are the ℓ_p-modulus and ℓ_p-Norm. Note that $\mathrm{Re}[\cdot]$ and $\mathrm{Im}[\cdot]$ denote the real and imaginary parts of a complex matrix. In addition, the $\otimes$ represents the Kronecker product.

MIMO radar signal model: Assume that the transmit and receive arrays are both one-dimensional equidistant linear arrays which include N_t transmit antenna elements and N_r receive anttenna elements. Let d_t and d_r denote the transmitting array spacing and the receiving array spaceing, then the transmit steering vector can be expressed as follows:

$$\boldsymbol{a}_t(\theta) = [1, e^{-j2\pi d_t \sin(\theta)/\lambda}, \cdots, e^{-j2\pi(N_t-1)d_t \sin(\theta)/\lambda}]^T \quad (1)$$

and the receive steering vector can be expressed as follows:

$$\boldsymbol{a}_r(\theta) = [1, e^{-j2\pi d_r \sin(\theta)/\lambda}, \cdots, e^{-j2\pi(N_r-1)d_r \sin(\theta)/\lambda}]^T \quad (2)$$

where θ denotes the angle away from the MIMO radar array normal, $\lambda = c/f_0$ denotes the MIMO radar transmitted signal wavelength, and f_0 is the MIMO radar transmitted signal carrier frequency.

Consider there is a target at θ_0 and K interferences at $\theta_k(k = 1, 2, \cdots, K)$ in the distance gate of interest. Then, the MIMO radar received signal at time t can be given by

$$\boldsymbol{x}(t) = \alpha_0 \boldsymbol{a}_t(\theta_0) \otimes \boldsymbol{a}_r(\theta_0) s(t) + \sum_{k=1}^{K} \alpha_k \boldsymbol{a}_t(\theta_k) \otimes \boldsymbol{a}_r(\theta_k) s_i(t) + \boldsymbol{N} \quad (3)$$

where α_0 and $\alpha_k(k = 1, 2, \cdots, K)$ are scattering coefficients of the target and the interferences, $\boldsymbol{N}$ is the additive noise. Each element of $\boldsymbol{N}$ satisfies the Gaussian distribution of zero mean and variance σ_e^2. $s(t)$ is the transmitted signal

$$s(t) = \boldsymbol{\phi}(t) e^{-j2\pi f_0 t} \quad (4)$$

and $s_i(t)$ are the interferences

$$s_i(t) = \boldsymbol{\phi}(t) e^{-j2\pi f_i t}, (i = 1, 2, \cdots, K) \quad (5)$$

where f_i denotes the ith interference carrier frequency, and $\boldsymbol{\phi}(t) = [\phi_1(t), \phi_2(t), \cdots, \phi_{N_t}(t)]^T$. Here $\phi_i(t)$ are mutually orthogonal, and $\phi_i(t)\phi_i^H(t) = 1$.

After matched filtering, the signal at time t is

$$\boldsymbol{x}'(t) = \alpha_0 \boldsymbol{a}_t(\theta_0) \otimes \boldsymbol{a}_r(\theta_0) e^{-j2\pi f_0 t} + \sum_{k=1}^{K} \alpha_k \boldsymbol{a}_t(\theta_k) \otimes \boldsymbol{a}_r(\theta_k) e^{-j2\pi f_i t} + \boldsymbol{N} \quad (6)$$

Then the output of the MIMO radar beamformer can be expressed as follows:

$$\boldsymbol{y}(t) = \boldsymbol{\omega}^H \boldsymbol{x}'(t) \quad (7)$$

where $\boldsymbol{\omega}$ is the weight vector of MIMO radar beamformer which determines the effect of beamformer. For MIMO radar beamformers, there are many methods to solve the $\boldsymbol{\omega}$, such as SMI [11], LSMI [12], and LCMV [13]. However, conventional MIMO radar beamformers are sensitive to the MIMO radar virtual steering vector mismatch.

The proposed MIMO radar beamformer: In this paper, we propose a robust beamforming for MIMO radar against the virtual array steering vector mismatch. The output SINR of the proposed robust MIMO radar beamformer is defined as

$$\mathrm{SINR} = \frac{E\{|\boldsymbol{\omega}^H \boldsymbol{v} s'(t)|\}}{E\{|\boldsymbol{\omega}^H(i'(t) + \boldsymbol{N})|\}} = \frac{\sigma_s^2 |\boldsymbol{\omega}^H \boldsymbol{v}|}{\boldsymbol{\omega}^H \boldsymbol{R}_{i+n} \boldsymbol{\omega}} \quad (8)$$

where $\sigma_s^2 = E\{|s'(t)|^2\}$ is the average power of target signal, $\boldsymbol{R}_{i+n}$ is the interferences-plus-noise covariance matrix, $\boldsymbol{v} = \boldsymbol{a}_t(\theta) \otimes \boldsymbol{a}_r(\theta)$ is the virtual array steering vector of the target, $s'(t) = \alpha_0 e^{-j2\pi f_0 t}$, $i'(t) = \sum_{k=1}^{K} \alpha_k \boldsymbol{v}_{i_k} e^{-j2\pi f_0 t}$, and $\boldsymbol{v}_{i_k} = \boldsymbol{a}_t(\theta_k) \otimes \boldsymbol{a}_r(\theta_k)$ is the virtual array steering vector of the interferences. SINR is a standard to measure the quality of the MIMO radar beamformer. The higher SINR is, the better MIMO radar beamformer is.

Resorting to the higher order statistics, we consider minimzing the infinite norm of output vector $\boldsymbol{y}^* = \boldsymbol{X}'^H \boldsymbol{\omega}$, namely

$$\min_{\omega} \|\boldsymbol{X}'^H \boldsymbol{\omega}\|_\infty \quad (9)$$

where $\boldsymbol{X}' = [\boldsymbol{x}'(t_1), \cdots, \boldsymbol{x}'(t_N)]$, N is the number of snapshots.

Denoting

$$\bar{\boldsymbol{X}}' = \begin{bmatrix} \boldsymbol{X}'_R & -\boldsymbol{X}'_I \\ \boldsymbol{X}'_I & \boldsymbol{X}'_R \end{bmatrix}, \quad \bar{\omega} = \begin{bmatrix} \omega_R \\ \omega_I \end{bmatrix} \quad (10)$$

where $\boldsymbol{X}'_R = \mathrm{Re}[\boldsymbol{X}']$, $\boldsymbol{X}'_I = \mathrm{Im}[\boldsymbol{X}']$, and $\boldsymbol{\omega}_R = \mathrm{Re}[\boldsymbol{\omega}]$, $\boldsymbol{\omega}_I = \mathrm{Im}[\boldsymbol{\omega}]$. Then, we have

$$\|\boldsymbol{X}'^H \boldsymbol{\omega}\|_\infty = \|\bar{\boldsymbol{X}}'^T \bar{\boldsymbol{\omega}}\|_\infty \quad (11)$$

Regardless of the error at the transmitter or receiver, we integrates its modeling as a virtual aperture steering vector mismatch. Considering the target angle of arrival(AOA)

$$\theta_0 = \theta_a + \theta_e \quad (12)$$

where θ_0 is the actual AOA of the target, θ_a is the assumed AOA of the target, and θ_e is the AOA error of the target. Thus, we model the virtual aperture steering vector mismatch as follows

$$\boldsymbol{v}_0 = \boldsymbol{v}_a + \boldsymbol{v}_e \quad (13)$$

where $\|\boldsymbol{v}_e\|_1 \leq \boldsymbol{\varepsilon}$, and $\boldsymbol{\varepsilon}$ is the size of uncertainty region $\boldsymbol{U}$. So the constraints can be written as

$$|(\boldsymbol{v}_a^H + \boldsymbol{v}_e^H)\boldsymbol{\omega}|_\infty \geq 1, \text{ for all } \boldsymbol{v}_e \in \boldsymbol{U} \quad (14)$$

Due to $|\boldsymbol{v}_a|_\infty = \lim_{p\to\infty} |\boldsymbol{v}_a|_p = \max(|\mathrm{Re}[\boldsymbol{v}_a]|, |\mathrm{Im}[\boldsymbol{v}_a]|)$, and according to the Minkowski inequality [13], we have

$$|\boldsymbol{v}_a^H \boldsymbol{\omega} + \boldsymbol{v}_e^H \boldsymbol{\omega}|_\infty \geq |\boldsymbol{v}_a^H \boldsymbol{\omega}|_\infty - |\boldsymbol{v}_e^H \boldsymbol{\omega}|_\infty \quad (15)$$

Similarly, we can construct the expanded real-valued matrix of $\boldsymbol{v}_e$, that is,

$$\bar{\boldsymbol{V}}_e = \begin{bmatrix} \mathrm{Re}[\boldsymbol{v}_e] & -\mathrm{Im}[\boldsymbol{v}_e] \\ \mathrm{Im}[\boldsymbol{v}_e] & \mathrm{Re}[\boldsymbol{v}_e] \end{bmatrix} \tag{16}$$

Then, by Holder inequality [14], it follows

$$\begin{aligned} |\boldsymbol{v}_e^H\omega| &= \|\bar{\boldsymbol{V}}_e^T\bar{\omega}\|_\infty \le \|\bar{\boldsymbol{V}}_e^T\|_\infty\|\bar{\omega}\|_\infty = \|\bar{\boldsymbol{V}}_e\|_1\|\bar{\omega}\|_\infty \\ &= \max\left(\left\|\begin{bmatrix} \mathrm{Re}[\boldsymbol{v}_e] \\ \mathrm{Im}[\boldsymbol{v}_e] \end{bmatrix}\right\|_1, \left\|\begin{bmatrix} -\mathrm{Im}[\boldsymbol{v}_e] \\ \mathrm{Re}[\boldsymbol{v}_e] \end{bmatrix}\right\|_1\right)\|\bar{\omega}\|_\infty \\ &= \left\|\begin{bmatrix} \mathrm{Re}[\boldsymbol{v}_e] \\ \mathrm{Im}[\boldsymbol{v}_e] \end{bmatrix}\right\|_1 \|\bar{\omega}\|_\infty = \|\boldsymbol{v}_e\|_1\|\bar{\omega}\|_\infty \le \varepsilon\|\bar{\omega}\|_\infty \end{aligned} \tag{17}$$

and

$$\begin{aligned} |\boldsymbol{v}_a\omega|_\infty &= \max(|\mathrm{Re}(\boldsymbol{v}_a^H\omega)|, |\mathrm{Im}(\boldsymbol{v}_a^H\omega)|) \\ &\ge \mathrm{Re}(\boldsymbol{v}_a^H\omega) = \bar{\boldsymbol{v}}_a^T\omega \end{aligned} \tag{18}$$

From (11), (17) and (18), the rubost MIMO radar beamformer is

$$\begin{aligned} &\min_{\bar{\omega}\in\mathbb{R}^{2N_tN_r}} \|\bar{\boldsymbol{X}}'^T\bar{\omega}\|_\infty \\ &s.t.\ \bar{\boldsymbol{v}}_a^T\bar{\omega} - \varepsilon\|\bar{\omega}\|_\infty \ge 1 \end{aligned} \tag{19}$$

Selection of the ε: ε will affect the performance of the robust MIMO radar beamformer. When the ε is larger, the robustness of the robust MIMO radar beamformer is better, that is, more bigger mismatch angle of AOA can be tolerated. In the proposed beamformer, we employ the constant ε which determined by MIMO radar arrays.

Simulation: Assume that a MIMO radar equipped with transmitting antennas $N_t = 4$ and receiving antennas $N_r = 4$. Both the transmitting array spacing and the receiving array spaceing are halfwavelength. The signal carrier frequency is $f_0 = 50$MHz, the interference carrier frequencies are $f_1 = 90$MHz and $f_2 = 0.7$GHz, and the sampling frequency is $f_s = 2.1$GHz. Monte Carlo method is adopted for simulation, and the number of Monte Carlo is 100. The size of uncertainty region is $\varepsilon = 3$, the number of snapshots is $N = 200$, the AOA of the target signal is $\theta_0 = 43°$, the AOA of the interferences are $\theta_1 = 30°$ and $\theta_2 = 75°$, and the intereference-to-noise ratio(INR) are $\mathrm{INR}_1 = 30$dB and $\mathrm{INR}_2 = 50$dB. For convenience, we refer to the proposed MIMO beamformer against the virtual array steering vector mismatch as MIMO-AVM.

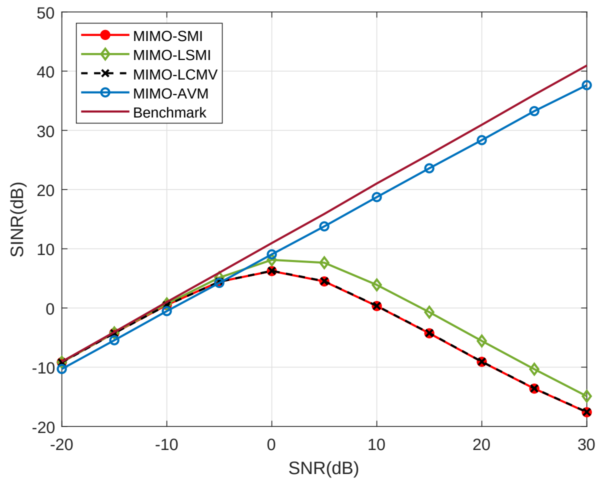

Fig 1 *Output SINR versus SNR with 2° AOA mismatch.*

Figure 1 shows the curves of the output SINR versus SNR, when SNR > 0, the output SINR, for conventional MIMO radar beamformers, decreases rapidly with the increase of SNR. This indicates that the conventional one are sensitive to the virtual array steering vector mismatch. However, it is seen that our proposed MIMO radar beamformer is more robust among all SNR values. Figure 2 shows that the relationship between the number of snapshots and SINR for SNR = 25dB. The

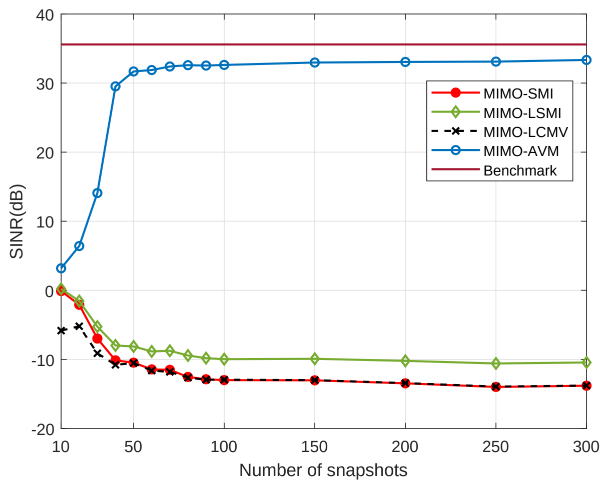

Fig 2 *Output SINR versus number of snapshots with 2° AOA mismatch.*

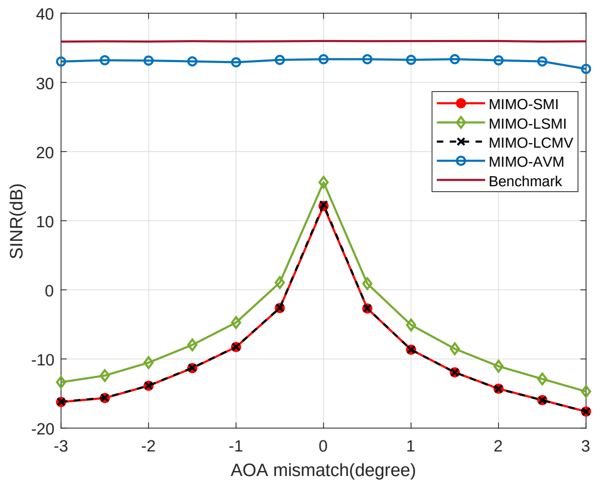

Fig 3 *Output SINR versus AOA mismatch.*

SINR for the proposed beamformer increase as the number of snapshots increase. When the number of snapshots is 50, the change trend of the SINR with the number of snapshots begins to flatten. However, the SINR for the counterparts are still low even for large snapshots. This is because they are affected by the virtual array steering vector mismatch. When $N = 200$, $\theta_0 = 43°$, and SNR = 25dB, the relationship between the AOA mismatch and SINR is shown in Figure 3. From Figure 3, we can see that the SINR curve fluctuates little with the change of the AOA mismatch, and it further explains the robustness of our proposed MIMO radar beamformer.

Then, we study the influence of three different MIMO radar arrays on the proposed MIMO radar beamformer, which are represented by MIMO-AVM-1, MIMO-AVM-2, and MIMO-AVM-3. Assume that MIMO-AVM-1 has four transmitting antennas and four receiving antennas, MIMO-AVM-2 has one transmitting antenna and four receiving antennas, and MIMO-AVM-3 has four transmitting antennas and one receiving antenna. In addition, the uncertainty region sizes of three linear programming MIMO radar beamformers are $\varepsilon_1 = 3$, $\varepsilon_2 = 0.5$, and $\varepsilon_3 = 0.5$, and other parameters remain the same as the previous simulation.

From Figure 4–6, we can see that the performance of the proposed MIMO radar beamformer is related to the MIMO radar arrays. The larger the MIMO radar array is, the greater the total SNR of the signal is, and the better the output of the MIMO radar beamformer is. It is not difficult to understand that the larger the MIMO radar arrays, the more signals will be processed. But the MIMO radar arrays do not affect the overall trend of the curve.

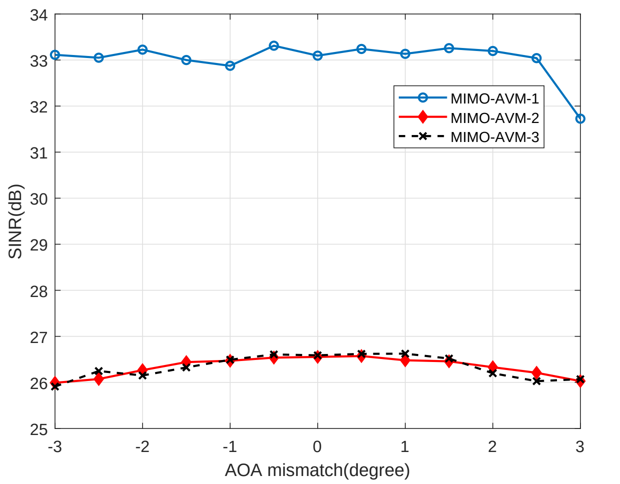

Fig 6 *Output SINR versus AOA mismatch under three different MIMO radar arrays.*

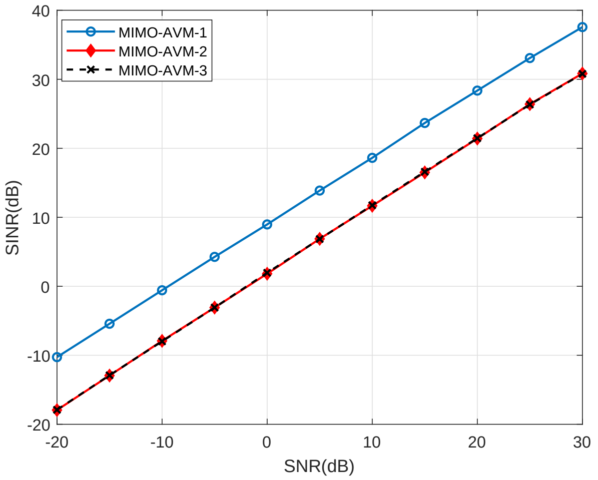

Fig 4 *Output SINR versus SNR with 2° AOA mismatch under three different MIMO radar arrays.*

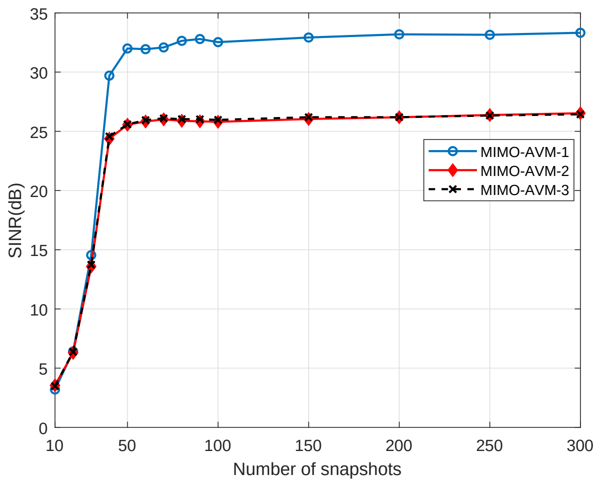

Fig 5 *Output SINR versus number of snapshots with 2° AOA mismatch under three different MIMO radar arrays.*

Conclusion: In order to solve the problem of low output SINR caused by the virtual array steering vector mismatch of the MIMO radar, we propose a robust beamformer for MIMO radar. Regardless of the error at the transmitter or receiver, we integrate its modeling as a virtual aperture steering vector mismatch. The object function of this robust MIMO radar beamformer is constructed from an infinite norm of the output data, which is solved by linear programming. Furthermore, we also studied the influence of the MIMO radar arrays on the performance of the proposed MIMO radar beamformer. We find that MIMO arrays did not greatly affect the trend of beamforming. Numerical results shows the proposed beamformer outperforms conventional counterparts.

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