Labeling On Pentagonal Pyramidal Graceful Graph

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Abstract

Numbers that can be expressed as (r ^{2} (r+1)) /2 for all r [?] 1 are called pentagonal pyramidal numbers. Assume G to be a graph with p vertices and q edges. Let Φ: V(G) -{0, 1, 2... B $_c$} where B $_c$ is the c number with a pentagonal pyramid, be an injective function. Define the function Φ* :E(G) -{1,6,18,.., B $_c$ } such that Φ * (ab) = |Φ(a)- Φ(b)| which is true for each and every edge ab E(G). If Φ*(E(G)) represents a sequential arrangement of non-identical successive pentagonal pyramidal numbers {B $_1$, B $_2$, . . . , B $_c$}, then Φ can be regarded as the pentagonal pyramidal graceful labeling. The graph permitting labeling of such kind can be referred to as a pentagonal pyramidal graceful graph. This study examines some unique pentagonal pyramidal elegant graph labeling outcomes.

1. Introduction

Consider a simple finite graph. Terms and terminologies have been referred from the book [1]. A graph G' with graceful labeling is given by a function g. It contains an edge with injective function g assigned from V(G') to {0,1,2,. . . ,r}. When all the edges 'ab' is reserved to be the edge label |g(a)- g(b)|, at that point the subsequent edge names are particular. The graceful labeling accepted by a graph is known as a graceful graph. Some classes of trees using graceful labeling were studied by A. Rosa 1966 [2]-paths and caterpillars, G. Sethuraman et al 2009 [3]-attaching caterpillars with other caterpillars, I. Cahit 2002 [4]-canonic labeling technique, R.E. Aldred et al 1998 [5] -lobsters, Jesintha et al 2011 [6]- constructed certain infinite families of graceful Acharya 1982 [7], A. Gallian 2019 [8]-surveyed on graceful tree conjecture [9-22]. A star graph can be defined as the complete bipartite graph $?_{1,r}$ containing r + 1 vertices and r edges. A simple graph can be defined as a graph that does not contain a self-loop or parallel edges. A path $?_?$ is acquired by signing up for $?_?$ to the consecutive vertices $?_{?+1}$ for 1 [?] ? [?] ? - 1.

In mathematical models, In a wide variety of mathematical models, Labeled graphs have played a remarkable usage. In the application of designing X-Ray crystallography, formulating an addressing system using a communication network, determining the layouts of a perfect circuit, problems based on the addition of numbers, etc. are used.

2. Star Graph

Theorem 2.1: For all values of s, the star graph $?_{1,s}$ is a pentagonal pyramidal graceful.

Proof: Consider V($?_{1,s}$) = {u_i: 1[?] i[?] s+1}.

Assume E($?_{1,s}$) = { $u_{n+1}u_i$: 1[?] i[?] s}.

Define an injection Φ : V($?_{1,s}$)-{0,1,2,. . . , B_c } by

Φ(u_i) = $B_?$ if 1[?] i[?] s and

$\Phi(u_{n+1}) = 0$.

Thus Φ prompts a bijection Φ_p : E(?$_{1,s}$) -{1,6,18,. . . B$_c$ }.

Hence the star graph ?$_{1,s}$ is a pentagonal pyramidal graceful for all values of s.

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Example 2.2: A pentagonal pyramidal graceful labeling of star graph ?$_{1,8}$ is shown in Fig. 1.

Fig 1. ?$_{1,8}$-Star Graph

Definition 2.3: Let $G_1 = (V_1, E_1)$ and$G_2 = (V_2, E_2)$ be two graphs. Then the disjoint union of two graphs is G_3 such that$G_3 = G_1 \cup G_2$ whose vertex set$V_3 = V_1 \cup V_2$ and the edge set$E_3 = E_1 \cup E_2$

Theorem 2.4: ?$_{1,m}$ [?] ?$_{1\ ,s}$ is a pentagonal pyramidal graceful labeling graph for all values of m, n satisfying m, s > 2.

Proof: Let G be a ?$_{1,\ a}$ [?] ?$_{1\ ,b}$graph for all a, b > 2.

Let V(G) = {? , ?$_p$, ?, ?$_q$: 1 [?] p [?] a , 1 [?] q [?] b} and

E(G) = {??$_p$, ??$_q$: 1 [?] p [?] a , 1 [?] q [?] b}

Here G contains a + b + 2 vertices with a + b edges.

Let c = a + b

Define f : V(G) - {0,1,. . . , B$_c$} as follows

f (u) = 0

f (?$_p$) = B$_p$, 1 [?] p [?] a

f (v) = f (?$_{?\text{-}1}$) − 1.

f (?$_q$) = B$_{?+q}$ - f (v) , 1 [?] q [?] b.

Clearly, f is injective and f induces a bijective function f *: E(G) -{1,6,. . . B$_c$} as

f * (??$_p$) = B$_p$, 1 [?] p [?] a

f * (??$_q$) = B$_{?+q}$, 1 [?] q [?] b

Hence the edge labels are 1,6,18,. . . B$_c$.

Thus f is a pentagonal pyramidal graceful labeling of G.

Therefore, G = ?$_{1,a}$ [?] ?$_{1\ ,b}$ is a pentagonal pyramidal graceful labeling graph.

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Example 2.5: In Figure 2 shows the pentagonal pyramidal graceful labeling of ?$_{1,4}$ [?] ?$_{1,4}$

Fig 2. ?$_{1,4}$ [?] ?$_{1,4}$-Star Graph

Theorem 2.6: ?$_{1,r}$ [?] ?$_{n\ ,s}$ is a pentagonal pyramidal graceful labeling the entire graph r [?] 3 and n, s [?] 1

Proof: Assume G be a ?$_{1,r}$ [?] ?$_{n\ ,s}$ graph for all r [?] 3 and n, s [?] 1.

Consider V(G) = {? , $?_p$, ?, $?_q$, ?, $?_?$: 1 [?] p [?] r , 1 [?] q [?] n and 1 [?] k [?] s} and

E(G) = { $??_p$, $??_q$, ??, $??_?$: 1 [?] p [?] r, 1 [?] q [?] n and 1 [?] ? [?] s}

Here G contains r + n + s + 3 vertices with r+ n + s + 1 edges.

Let c = r + n + s + 1.

Define f : V(G) - {0,1,. . . , B_c} as follows

f (u) = 0

f ($?_p$) = B_p , 1 [?] p [?] r

f (v) = f ($?_{?\text{-}1}$) − 1.

f ($?_q$) = $B_{?+q+1}$- f (v) , 1 [?] q [?] n

f (?) = $B_{?+1}$- f (?),

f ($?_?$) = $B_{n+r+1+k}$ - f (?) , 1 [?] ? [?] s

It is evident that f is injective and

f prompts a bijective function f * : E(G) -{1,6,. . . , B_c} as

f * ($??_p$) = $B_?$, 1 [?] p [?] r

f * ($??_q$) = $B_{?+1+q}$, 1 [?] q [?] n

f * (??) = $B_{?+1}$

f * ($??_?$) = $B_{r+n+1+k}$, 1 [?] ? [?] s

As a result, the edge labels are 1,6,. . . , B_c .

Thus f is a pentagonal pyramidal graceful labeling of G.

Therefore, G = $?_{1,r}$ [?] $?_{n\ ,s}$ is a pentagonal pyramidal graceful labeling graph.

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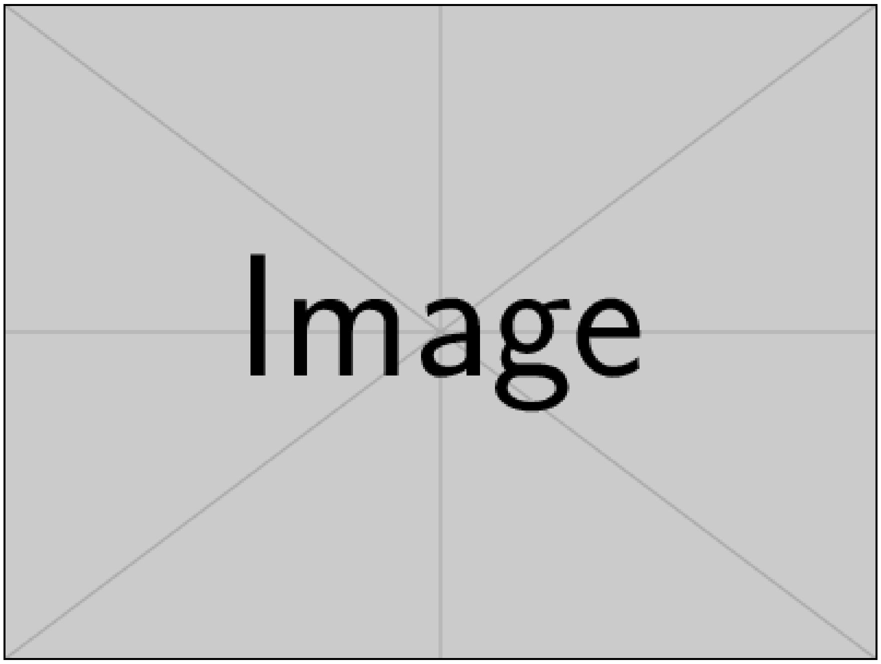

Example 2.7: The pentagonal pyramidal graceful labeling of $?_{1,4}$[?] $?_{4\ ,5}$ is shown in Fig. 3.

Fig 3. $?_{1,4}$ [?] $?_{4\ ,5}$-Star Graph with Bi-star Graph

3. Power of three- acyclic graph

Definition 3.1: A power of three- acyclic graph H_r consists of acyclic graph H_{r-1} for every r [?] 0 which is associated together with the one's root being the leftmost child of the other's root, the power of three acyclic graphs is denoted by H_r. H_0 comprises a solitary vertex. The power of three-acyclic graph H_r is an arranged non-cyclic diagram characterized recursively. H_r consists of the power of three- acyclic graph $H_{r\ -1}$ which is associated with each other, ie., the leftmost child of the other root. Note that the vertices in H_r are 3^r.

Theorem 3.2: Every power of three trees is a pentagonal pyramidal graceful labeling graph.

Proof: Assume G to be a tree containing s vertices.

Assume V (G) = {v_p : 1[?] p [?] s} as the set of vertex G and

E(G) = {$?_p?_{p+1}$:1 [?] p [?] s-1} as the set of edge of G.

Hence G has s vertices and s-1 edges.

Let c = s-1.

Consider a function Φ: V(G) -{0,1,2,. . . , B_c} defined as stated below.

Φ ($?_1$) = 0

Φ($?_2$) = B_c

Φ (v_p) = Φ ($?_{p-1}$) – $B_{c-(p-2)}$ if p is odd and 3 [?] p [?] s.

= Φ ($?_{p-1}$) + $B_{c-(p-2)}$ if p is even and 3 [?] p [?] s

Let Φ * be the induced edge labeling of f

Then Φ ($?_1?_2$) = B

$\Phi^*(v_p v_{p+1}) = B_{c\text{-}(p\text{-}1)}$; 2 [?] p [?] s-1.

The induced edge labels B_1, B_2,..., B_c are separate, sequential pentagonal pyramidal numbers.

Hence G, the graph is proved to be a pentagonal pyramidal graceful.

Example 3.3: Figure 2 shows the Pentagonal pyramidal graceful labeling of power of three trees.

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Fig 4.H_r- (r = 3) -Power of three tree Graph

4. Comb Graph

Definition 4.1: Consider graph G. From the graph each edge is broken into precisely two segments. This is done by the insertion of intermediate vertices in between two ends. The new graph obtained is known as a graph of subdivisions, which is denoted as S(G).

Definition 4.2: ?? is a graph possessing n copies of the graph G, which means, ?? = $[?]_{i\text{-}1}{}^{n}G_i$ where each $?_? = ?$.

Definition 4.3: Comb graphs is the graph that is formed by connecting a single pendant edge to every vertex of a path which is represented by $?_? ??_1$

Theorem 4.4: The comb graph $?_? ??_1$ is said to be a pentagonal pyramidal graceful graph for all 2[?]n.

Proof: Assume ? to be a comb graph $?_? ??_1$.Then V(?) = { u_i, w_i : ???? 1 [?] ? [?] n}

E(G) = {$?_? ?_{?+1}$: ???? 1 [?] ? [?] r - 1} [?] {$?_? ?_?$: ???? 1 [?] ? [?] r}

Hence ? contains 2? vertices and 2? - 1 edge.

Let ? = 2? - 1.

Define Φ : (?) - {0,1,2, ... , $B_?$ } as follows.

$\Phi(?_1)$= 0

$\Phi(?_?) = \Phi(?_{?\text{-}1})$ - $B_{?\text{-}(?\text{-}2)}$?? ? ?? ??? 2 [?] ? [?] r

$\Phi(?_{?\text{-}1})$ + $B_{?\text{-}(?\text{-}2)}$?? ? ?? ???? 2 [?] ? [?] r

$\Phi(?_1) = B_{2?+1}$

$\Phi(?_?) = \Phi(?_?) + B_{?+(?\text{-}1)}$, 2 [?] ? [?] r.

Φ is injective.

Φ^* the induced edge function defined from V(?) - {B_1, B_2, ..., $B_{2?\text{-}1}$} is as given below.

Φ^* ($?_? ?_{?+1}$) = $B_{?\text{-}?}$, 1 [?] ? [?] r – 1

$\Phi(?_1 ?_1$) = $B_{2?+1}$

$\Phi(?_? ?_?$) = $B_{?+(?\text{-}1)}$, 2 [?] ? [?] r.

Clearly Φ^* is a bijection and Φ^*((?)) = {B_1, B_2, ... , $B_{2?\text{-}1}$}.

Hence ? permits pentagonal pyramidal graceful labeling.

Hence the comb $?_? ??_1$ is a pentagonal pyramidal graceful graph for all ? [?] 2.

Example 4.5: Figure 3 is a representation of pentagonal pyramidal graceful labeling of $?_5 ??_1$

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Fig 5. $?_5??_1$-Comb graph

5. Conclusion:

We have proved that some graphs like star graph, comb graph, disjoint union of the finite number of star graph say G_1, G_2, ..., G_n is the pentagonal pyramidal graceful graph and also the power of three acyclic graphs H_r is a pentagonal pyramidal graceful graph.

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