

# An adaptive control scheme for switched nonlinear cyber-physical systems against state-dependent sensor attacks and input-dependent actuator attacks

Xiao Wang<sup>1</sup> and Ping Zhao<sup>1</sup>

<sup>1</sup>Shandong Normal University

December 28, 2022

## Abstract

This paper considers the problem of adaptive control against deception attacks for a class of switched nonlinear cyber-physical systems (CPSs), in which each subsystem has more general and unknown nonlinearities. Specifically, an adaptive controller is designed for CPSs with unknown switching mechanisms to mitigate the impact of state-dependent sensor attacks and input-dependent actuator attacks. Compared with the existing researches, the actuator attacks considered in our paper are input-dependent, which means the controller is substantially attacked, besides, the signs of unknown time-varying gains caused by state-dependent sensor attacks and input-dependent actuator attacks are all unknown. To deal with these scenarios, Nussbaum-type functions are introduced. In addition, by constructing a common Lyapunov function for all subsystems, the closed-loop system signals are proved to be globally bounded under arbitrary switchings. Finally, we give a simulation example of a continuously stirred tank reactor system with state-dependent sensor attacks and input-dependent actuator attacks to illustrate the effectiveness of our results.

# An adaptive control scheme for switched nonlinear cyber-physical systems against state-dependent sensor attacks and input-dependent actuator attacks

Xiao Wang<sup>a</sup>, Ping Zhao<sup>a</sup>

<sup>a</sup>*School of Information Science and Engineering, Shandong Normal University, Jinan, Shandong, 250014, PR China*

---

## Abstract

This paper considers the problem of adaptive control against deception attacks for a class of switched nonlinear cyber-physical systems (CPSs), in which each subsystem has more general and unknown nonlinearities. Specifically, an adaptive controller is designed for CPSs with unknown switching mechanisms to mitigate the impact of state-dependent sensor attacks and input-dependent actuator attacks. Compared with the existing researches, the actuator attacks considered in our paper are input-dependent, which means the controller is substantially attacked, besides, the signs of unknown time-varying gains caused by state-dependent sensor attacks and input-dependent actuator attacks are all unknown. To deal with these scenarios, Nussbaum-type functions are introduced. In addition, by constructing a common Lyapunov function for all subsystems, the closed-loop system signals are proved to be globally bounded under arbitrary switchings. Finally, we give a simulation example of a continuously stirred tank reactor system with state-dependent sensor attacks and input-dependent actuator attacks to illustrate the effectiveness of our results.

*Key words:* cyber-physical systems, sensor attack, actuator attack, adaptive control.

---

## 1. Introduction

Cyber-physical systems (CPSs) are large-scale engineering systems which realize real-time perception, dynamic control and information service, they are widely used in environment perception, embedded computing, network communication and network control, such systems include large-scale manufacturing systems, transportation systems, power systems and so on. The physical equipments in CPSs communicate through the heterogeneous networks and have great requirements for safety. Due to the structure and control mode of CPSs, they are vulnerable to be damaged.

Cyber attacks are the main threats to CPSs and have great impacts on data and security, and hence, it is crucial to develop strategies that can mitigate their effects on CPSs. Recently, many strategies for mitigating the impact of attacks have been developed for various types of cyber attacks, for example, strategies for denial of service (DOS) attacks[1]-[7], deception attacks[8]-[23] and replay attacks[24]-[27]. Deception attacks that inject false information into sensors or actuators, cause property loss and even endanger personal safety. Many adaptive mechanisms have been designed for deception attacks in recent years, which can automatically adjust the settings to adapt to the changes and disturbances of the dynamic systems when systems suffer deception attacks. Yucelen, Haddad and Feron provided an adaptive control architecture for linear system in [29] to mitigate sensor attacks, where the sign of state gain caused by sensor attack is assumed to be known. Further, Jin, Haddad and Yucelen designed an adaptive controller for linear system with both sensor attack and actuator attack in [30], where the actuator attack is state-dependent. An and Yang constructed an improved adaptive resilient control mechanism for linear system in [28] to

---

\* Corresponding author: Ping Zhao.

Funding information: Shandong Provincial Natural Science Foundation, China, Grant/Award Number: ZR2017JL028; National Natural Science Foundation of China, Grant/Award Number: 61873152

Email addresses: [sdnuwangxiao@163.com](mailto:sdnuwangxiao@163.com) (Xiao Wang), [zhaoping@amss.ac.cn](mailto:zhaoping@amss.ac.cn) (Ping Zhao).

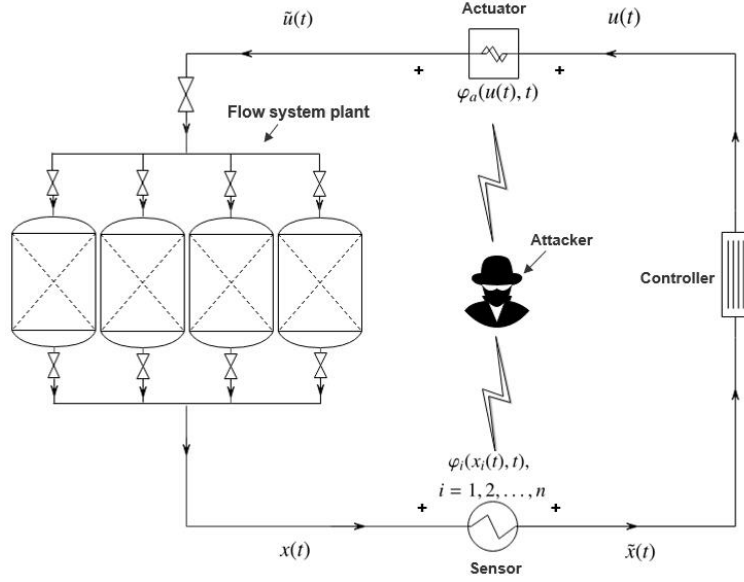


Fig 1. Flow system in the presence of state-dependent sensor attacks and input-dependent actuator attacks.

reduce the impact of sensor and actuator attacks, which used Nussbaum function to handle the unknown sign of state gain caused by sensor attack.

For nonlinear CPSs with deception attacks, in [9], an adaptive resilient control strategy was designed for mitigating the impact of sensor attack and actuator attack in the lower triangular form, which used dynamic surface design method to avoid complex explosion problems. Ren and Yang provided an adaptive control method in [15] for nonlinear CPSs with sensor attacks, which used new types of Nussbaum functions to deal with the time-varying gains and the Nussbaum-type ( $\tilde{N}$ ) functions are proved to be always effective in the scenarios of time-varying control coefficients and/or multivariable with unknown signs. In [31], an adaptive control mechanism was proposed for second-order nonlinear strict-feedback CPSs, which introduced nonlinear functions to deal with unknown time-varying gains. Further, for switched nonlinear CPSs with sensor attacks and actuator attacks, Li and Zhao [33] provided a resilient adaptive control scheme, where the switched nonlinear CPSs can describe the industry manufacturing process more accurately, the effectiveness of the proposed scheme is verified by a class of continuously stirred tank reactor (CSTR) system. However, under the above adaptive control strategies for mitigating deception attacks, many strategies only consider sensor attack or actuator attack. When two attacks co-exist, the actuator attack is depicted as state-dependent, has no substantial impact on the controller. Relatively speaking, it is more necessary to study input-dependent actuator attacks, which aim at the whole control input  $u$ , are more tally with the actual situation. For example, if the flow system suffers actuator attacks, will affect the whole control input, not the system state, which is shown in Fig. 1. In addition, in the previous results, the forms of nonlinear functions are usually linear-like or strict-feedback. Correspondingly, for the nonlinear functions in the system, there are few studies on non-triangular forms, which are more complex and can describe many practical industrial processes. As far as we know, developing an adaptive control strategy for switched nonlinear CPSs with state-dependent sensor attacks and input-dependent actuator attacks, remains a challenging problem.

Inspired by the researches above, an adaptive controller is proposed for switched nonlinear CPSs with state-dependent sensor attacks and input-dependent actuator attacks. Firstly, we assume that each subsystem of the switched nonlinear system encounters state-dependent sensor attacks and input-dependent actuator attacks, and the switching mechanism is unknown. Then, we design an adaptive control mechanism for the compromised system to recover the system performance as ideal as possible. For better understanding, the following summary of our contributions are given.

(1) An adaptive controller is designed for switched nonlinear systems in this paper, and the considered systems encounter both state-dependent sensor attacks and input-dependent actuator attacks simultaneously, in addition, external disturbances are taken into considered, too. By developing coordinate transformation in the backstepping design, the provided adaptive control mechanism can effectively ensure the closed-loop system operation steadily.

(2) The considered actuator attacks are modeled as input-dependent actuator attacks related to control input  $u$ , which means that the system encounters multiplicative actuator attacks, not state disturbances adding to the controller. Further, the proposed control scheme is more tolerant to the occurrence of attacks; it is not only effective when only sensor attacks or actuator attacks exists, but also when both sensor attacks and actuator attacks coexist.

(3) By introducing  $\tilde{N}$  functions, the problems of unknown time-varying gain signs are solved, which are caused by state-dependent sensor attacks and input-dependent actuator attacks. Different from the works of Ren et al. [15], An et al. [28], where only the sign of unknown time-varying state feedback coefficient caused by sensor attack is assumed to be unknown, the signs of unknown time-varying gains caused by state-dependent sensor attacks and input-dependent actuator attacks considered in our paper are all unknown, which make more drastic attacks be tolerated.

The content of this article is organized as follows. The main features of state-dependent sensor attacks and input-dependent actuator attacks are described in Section 2. Section 3 describes the design of controller using backstepping method. Section 4 gives the simulation results on CSTR system. Finally, in Section 5, some conclusions are drawn.

## 2. System description and preliminaries

Consider the following switched nonlinear CPSs whose subsystems are described as

$$\begin{aligned}\dot{x}_m(t) &= x_{m+1}(t) + \phi_{\sigma,m}(\tilde{x}_{m+1}(t)) + d_{\sigma,m}(t), \quad m = 1, 2, \dots, n-1, \\ \dot{x}_n(t) &= u_\sigma(t) + \phi_{\sigma,n}(x(t)) + d_{\sigma,n}(t),\end{aligned}\tag{1}$$

where  $x(t) = [x_1, x_2, \dots, x_n]^T \in \mathbb{R}^n$  denotes system state; for each  $m$ ,  $\tilde{x}_m = [x_1, x_2, \dots, x_m]^T \in \mathbb{R}^m$ ;  $u_\sigma(t)$  represents the control input.  $\phi_{\sigma,i}(\cdot)$ ,  $i = 1, 2, \dots, n$ , are unknown and continuous nonlinear functions with  $\phi_{\sigma,i}(0) = 0$ .  $d_{\sigma,i}(t)$  are disturbances with upper bounds  $\bar{d}_{\sigma,i}$ .  $\sigma := \sigma(t) : [0, \infty) \rightarrow \mathbb{S} = \{1, 2, \dots, Q\}$  is the switching signal, where  $Q$  is the number of subsystems. For clarity, the index  $\sigma(t) = s \in \mathbb{S}$  denotes the  $s$ -th subsystem is activated at time instant  $t$ . Without causing confusion, the argument  $t$  of variable  $x(t)$  is sometimes dropped for simplifying process.

In fact, the system (1) in CPSs is easily to suffer state-dependent sensor attacks and input-dependent actuator attacks. However, in the actual industrial process with state-dependent sensor attacks and input-dependent actuator attacks,  $x_i(t)$ ,  $i = 1, 2, \dots, n$ ,  $u_\sigma(t)$ ,  $\varphi_i(x_i(t), t)$  and  $\varphi_{a,\sigma}(u_\sigma(t), t)$  cannot be obtained accurately, where  $\varphi_i(x_i(t), t)$  are state-dependent sensor attacks and  $\varphi_{a,\sigma}(u_\sigma(t), t)$  are input-dependent actuator attacks. Correspondingly, the compromised system state  $\tilde{x}_i(t)$  and control input  $\tilde{u}_\sigma(t)$  can be obtained and used in feedback. Next, we give the compromised system state as

$$\tilde{x}_i(t) = x_i(t) + \varphi_i(x_i(t), t), \quad i = 1, 2, \dots, n.\tag{2}$$

In the actual attack scenario, the state-dependent sensor attacks  $\varphi_i(x_i(t), t)$ ,  $t \geq 0$  are time-varying, state-dependent and parameterized with  $\varphi_i(x_i(t), t) = \mu_\sigma(t)x_i(t)$ , where  $\mu_\sigma(t) \neq -1$ ,  $t \geq 0$ . The time-varying weights  $\mu_\sigma(t)$  are bounded with  $|\mu_\sigma(t)| \leq \bar{\mu}_\sigma$  and have bounded rate of change with  $|\dot{\mu}_\sigma(t)| \leq \bar{\dot{\mu}}_\sigma$ , where  $\bar{\mu}_\sigma$  and  $\bar{\dot{\mu}}_\sigma$  are unknown positive constants. Further, the compromised control input can be specifically described as

$$\tilde{u}_\sigma(t) = u_\sigma(t) + \varphi_{a,\sigma}(u_\sigma(t), t).\tag{3}$$

Correspondingly, the input-dependent actuator attacks  $\varphi_{a,\sigma}(u_\sigma(t), t)$ ,  $t \geq 0$  are time-varying, input-dependent and parameterized with  $\varphi_{a,\sigma}(u_\sigma(t), t) = \rho_{a,\sigma}(t)u_\sigma(t)$ , where  $\rho_{a,\sigma}(t) \neq -1$ ,  $t \geq 0$ . The time-varying weights  $\rho_{a,\sigma}(t)$  are bounded with  $|\rho_{a,\sigma}(t)| \leq \bar{\rho}_{a,\sigma}$  and have bounded rate of change with  $|\dot{\rho}_{a,\sigma}(t)| \leq \bar{\dot{\rho}}_{a,\sigma}$ , where  $\bar{\rho}_{a,\sigma}$  and  $\bar{\dot{\rho}}_{a,\sigma}$  are unknown positive constants.

**Remark 1:** For non-switched nonlinear CPSs with deception attacks, Ren and Yang provided an adaptive control method in [15] when sensor attack exist. In [9], an adaptive resilient control strategy was designed to mitigate the impact of sensor attacks and actuator attacks for the systems in lower triangular form. Comparing with the triangular form, the nonlinear functions in our system (1) are in non-triangular forms, which have more general nonlinearities and can describe many practical industrial processes. For instance, the CSTR systems in Section 4 can be described by our system (1).

This paper aims to design an adaptive controller for switched nonlinear CPS with state-dependent sensor attacks and input-dependent actuator attacks, which can ensure system operation steadily under arbitrary switchings.

In order to achieve the above objectives, the following analysis will be provided. For state-dependent sensor attacks, suppose  $\eta_\sigma(t) = (1 + \mu_\sigma(t))^{-1}$ ,  $x_i(t)$  and  $\eta_\sigma(t)$  have the following relationship

$$x_i(t) = \eta_\sigma(t)\tilde{x}_i(t).\tag{4}$$

Since  $\mu_\sigma(t)$  are bounded, there exist positive constants  $\eta_{\sigma,m}$ ,  $\eta_{\sigma,M}$  and  $\bar{\eta}_\sigma$  such that  $\eta_{\sigma,m} \leq |\eta_\sigma(t)| \leq \eta_{\sigma,M}$  and  $|\dot{\eta}_\sigma(t)| \leq \bar{\eta}_\sigma$ .

Further, for input-dependent actuator attacks, suppose  $\omega_\sigma(t) = 1 + \rho_{a,\sigma}(t)$ , then we can get

$$\tilde{u}_\sigma(t) = \omega_\sigma(t)u_\sigma(t).$$

Similarly, since  $\rho_{a,\sigma}(t)$  are bounded, there exist positive constants  $\omega_{\sigma,m}$ ,  $\omega_{\sigma,M}$  and  $\bar{\omega}_\sigma$  such that  $\omega_{\sigma,m} \leq |\omega_\sigma(t)| \leq \omega_{\sigma,M}$  and  $|\dot{\omega}_\sigma(t)| \leq \bar{\omega}_\sigma$ .

After being attacked, the signs of unknown time-varying gains  $(1 + \mu_\sigma(t))$  and  $(1 + \rho_{a,c}(t))$  caused by sensor attacks and actuator attacks may be changed. The signs of unknown time-varying gains caused by sensor attacks in [29] and [30] are assumed to be known. Further, Ren and Yang [15], An and Yang [28] used Nussbaum functions to deal with unknown time-varying gains caused by sensor attacks. Different from the cases that the unknown signs of time-varying gains considered in them, the signs of unknown time-varying gains  $(1 + \mu_\sigma(t))$  and  $(1 + \rho_{a,c}(t))$  caused by state-dependent sensor attacks and input-dependent actuator attacks in our work are all unknown.

In this paper,  $\tilde{\mathbb{N}}$  functions will be introduced for dealing with the unknown signs caused by the unknown time-varying gains, the definition of  $\tilde{\mathbb{N}}$  functions and some useful lemmas are as follows.

**Definition 1** [35]: If a continuous function  $N(\lambda) \in \mathbb{N}$  satisfies

$$\lim_{h \rightarrow \infty} \inf \frac{h - \int_0^h N^-(\lambda) d(\lambda)}{\int_0^h N^+(\lambda) d(\lambda)} = 0,$$

$$\lim_{h \rightarrow \infty} \inf \frac{h + \int_0^h N^+(\lambda) d(\lambda)}{-\int_0^h N^-(\lambda) d(\lambda)} = 0,$$

then  $N(\lambda)$  is named as  $\tilde{\mathbb{N}}$  function and expressed as  $N(\lambda) \in \tilde{\mathbb{N}} \subset \mathbb{N}$ , where  $\mathbb{N}$  is the set of Nussbaum functions,  $N^+(\lambda) \geq 0$  and  $N^-(\lambda) \leq 0$  are truncated functions with  $N(\lambda) = N^+(\lambda) + N^-(\lambda)$ .

**Lemma 1:** [36] Let  $V(\cdot)$  and  $\lambda(\cdot)$  be smooth functions defined on  $[0, \infty)$  with  $V(t) > 0$ ,  $\dot{\lambda}(t) \geq 0$ ,  $\forall t \geq 0$ , if there exist time-varying functions  $\varsigma_j(t) \in \mathbb{L} = [l_m^-, l_m^+]$  with  $0 \notin \mathbb{L}$ , positive constants  $A$  and  $B$  that make the following inequality holds:

$$\dot{V}(t) \leq -AV + B + \sum_{j=1}^n (\varsigma_j(t)N(\lambda(t)) + 1)\dot{\lambda}(t),$$

where  $l_m^-$  and  $l_m^+$  are constants,  $N(\cdot) \in \tilde{\mathbb{N}}$ , then  $V(t)$  and  $\lambda(t)$  are bounded for  $t \in [0, \infty)$ .

**Remark 2:** Nussbaum functions are used to solve the stabilization problem of systems with uncertain control coefficients, for example, they can be used for missile guidance system to deal with gain issues. However, for dealing with systems which have time-varying and/or multivariable control coefficients with unknown signs, not all Nussbaum functions are effective [34]. This paper introduce  $\tilde{\mathbb{N}} \subset \mathbb{N}$  functions to deal with the unknown signs caused by unknown time-varying gains, and  $\tilde{\mathbb{N}}$  are proved to be effective when dealing with systems which have time-varying and/or multivariable control coefficients with unknown signs. For example, functions  $e^{\lambda^2} \cos(\frac{\pi}{2}\lambda)$  and  $e^{\lambda^2} \sin(\frac{\pi}{2}\lambda)$  are all  $\tilde{\mathbb{N}}$  functions.

**Remark 3:** For clarity, we give a simple example to illustrate the principle of  $\tilde{\mathbb{N}}$ , Fig. 2 shows  $N(\lambda) = \cos(2\pi\lambda)\lambda^2$ . From the example, we know that with the continuous switching of symbols and the increase of amplitude, the system state is driven to swing constantly, so that the state can swing up and down again and again. When the system state is close to 0, the system state and the derivative of Nussbamm function become 0. In this case, it is no longer necessary to know whether the control direction of the system is positive or negative.

The following lemma will be presented to handle the nonlinear terms  $\phi_{\sigma,m}(\cdot)$  which related to the unavailable state  $\tilde{x}_{m+1}(t)$ , where  $\phi_{\sigma,m}(\cdot)$  in system (1) are non-triangular.

**Lemma 2:** [37] For any continuous function  $\phi(z, x)$ , there always exist smooth functions  $r(z) \geq 1$  and  $\bar{l}(x) \geq 1$  such that  $|\phi(z, x)| \leq r(z)\bar{l}(x)$ .

**Remark 4:** For the non-triangular nonlinear item  $\phi_{s,m}(\tilde{x}_{m+1}(t))$ , the state vector  $\tilde{x}_{m+1}(t)$  in which are the abbreviation of  $(x_1, x_2, \dots, x_m, x_{m+1})$ , i.e. the first  $m+1$  component of state  $x$ . Further, the system state suffered attacked and the  $(m+1)$ -th component of compromised system state are  $\tilde{x}_{m+1}(t)$ . From the relationships between  $x_{m+1}$  and  $\tilde{x}_{m+1}(t)$ , (4), then  $\phi_{s,m}(\tilde{x}_{m+1}(t))$  can be regarded as functions of  $\eta_s$  and  $\tilde{x}_{m+1}$ , where  $\tilde{x}_{m+1} = (\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_{m+1})$ . It is denoted by  $\phi_{s,m}(\eta_s, \tilde{x}_{m+1}) \triangleq \phi_{s,m}(\tilde{x}_{m+1}(t))$ . From Lemma 2 and the boundedness of  $\eta_s$ , there exist smooth functions  $r_s(\eta_s)$ ,  $\bar{l}_{s,m}(\tilde{x}_{m+1})$  and unknown constants  $\bar{r}_s$  such that  $\phi_{s,m}(\eta_s, \tilde{x}_{m+1}) \leq r_s(\eta_s)\bar{l}_{s,m}(\tilde{x}_{m+1}) \leq \bar{r}_s\bar{l}_{s,m}(\tilde{x}_{m+1})$ , where  $\bar{r}_s$  are the upper bound of  $r_s(\eta_s)$ .

**Remark 5:** The works in [28] and [30] have studied adaptive control strategies for linear systems with sensor attacks and actuator attacks. Further, the works in [9], [33] and [31] have studied adaptive control strategies for nonlinear system with sensor attacks and actuator attacks, which, formally, the actuator attacks in their studies can be regarded as state disturbances adding to the controller. The system we are considering is fundamentally

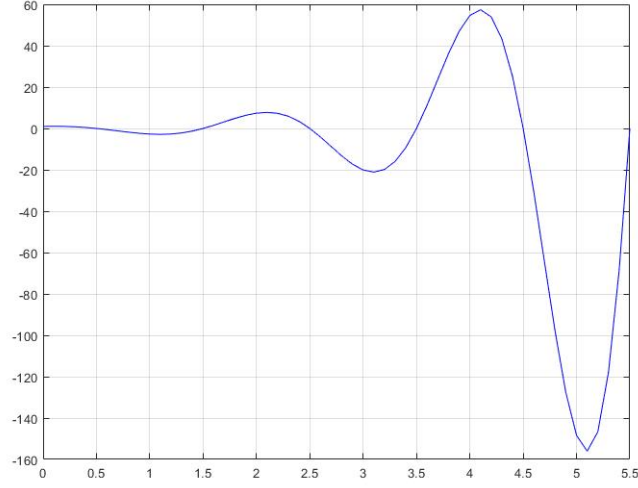


Fig 2. The imagine of Nussbaum function.

different from the system in the work of [9] and [33]. The actuator attacks in our system are input-dependent, which have direct multiplier effects on the controller itself. In fact, it is more realistic to represent actuator attacks as direct multiplier effects on the controller as a whole. Further, the unknown gains caused by the input-dependent actuator attacks need to be considered separately, which makes the design of step  $n$  more complicated. Therefore, the controller design we are considering is much more difficult.

### 3. Adaptive control design for input-dependent actuator attacks

Without loss of generality, we introduce the backstepping design process for the  $s$ -th subsystem. First, the following coordinate transformations are introduced to serve the design of the controller,

$$e_1 = x_1, \quad e_i = x_i - \eta_s \alpha_{i-1}, \quad i = 2, 3, \dots, n, \quad (5)$$

where, for  $\eta_s$ , the bound of it is denoted by  $0 < \eta_{s,m} \leq |\eta_s| \leq \eta_{s,M}$ , and the bounded rate of change is denoted by  $|\dot{\eta}_s| \leq \bar{\eta}_s$ ,  $\eta_{s,m}$ ,  $\eta_{s,M}$  and  $\bar{\eta}_s$  are unknown positive constants. Combining (4) and (5), the following equations are obtained,

$$\tilde{e}_1 = \tilde{x}_1, \quad \tilde{e}_i = \tilde{x}_i - \alpha_{i-1}, \quad (6)$$

where  $\alpha_{i-1}$  are virtual controllers to be designed. Further, according to (4), (5) and (6), the relation  $e_i = \eta_s \tilde{e}_i$  holds. In the following, we will give the steps of designing the adaptive controller for the  $s$ -th subsystem.

**Step 1:** From (1) and (5), we have

$$\dot{e}_1 = \dot{x}_1 = x_2 + \phi_{s,1}(\tilde{x}_2(t)) + d_{s,1}(t) = e_2 + \eta_s \alpha_1 + \phi_{s,1}(\tilde{x}_2(t)) + d_{s,1}(t).$$

Similar to Remark 4,  $\phi_{s,1}(\tilde{x}_2(t))$  can be regarded as functions of  $\eta_s$  and  $\tilde{x}_2$ , where  $\tilde{x}_2 = (\tilde{x}_1, \tilde{x}_2)$ . And it is denoted by  $\phi_{s,1}(\eta_s, \tilde{x}_2) \triangleq \phi_{s,1}(\tilde{x}_2(t))$ . Further, from Lemma 2 and the boundedness of  $\eta_s$ , there exist smooth functions  $r_s(\eta_s)$ ,  $\bar{l}_{s,1}(\tilde{x}_2)$  and unknown constants  $\bar{r}_s$  such that  $\phi_{s,1}(\eta_s, \tilde{x}_2) \leq r_s(\eta_s) \bar{l}_{s,1}(\tilde{x}_2) \leq \bar{r}_s \bar{l}_{s,1}(\tilde{x}_2)$ .

By choosing the following Lyapunov function candidate

$$V_{s,1} = \frac{1}{2} e_1^2 + \frac{1}{2} \tilde{\delta}_1^2,$$

we can get

$$\begin{aligned} \dot{V}_{s,1} &= e_1(e_2 + \eta_s \alpha_1 + \phi_{s,1}(\tilde{x}_2(t)) + d_{s,1}(t)) + \tilde{\delta}_1 \dot{\tilde{\delta}}_1 \\ &\leq e_1 e_2 + e_1 \eta_s \alpha_1 + e_1 \bar{r}_s \bar{l}_{s,1}(\tilde{x}_2) + e_1 d_{s,1}(t) + \tilde{\delta}_1 \dot{\tilde{\delta}}_1 \end{aligned}$$

$$\begin{aligned}
&\leq \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + e_1\eta_s\alpha_1 + \frac{1}{2}\epsilon_1 e_1^2 \bar{r}_s^2 \bar{l}_{s,1}^2(\check{x}_2) + \frac{1}{2}\epsilon_1 e_1^2 \bar{d}_{s,1}^2 + \tilde{\delta}_1 \dot{\delta}_1 + \frac{1}{\epsilon_1}, \\
&\leq \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + e_1\eta_s\alpha_1 + \frac{1}{2}\epsilon_1 \tilde{e}_1^2 \delta_1 \bar{l}_{s,1}^2(\check{x}_2) + \frac{1}{2}\epsilon_1 e_1^2 \bar{d}_{s,1}^2 + \tilde{\delta}_1 \dot{\delta}_1 + \frac{1}{\epsilon_1},
\end{aligned} \tag{7}$$

where  $\epsilon_1$  is a positive constant to be determined and  $\delta_1 = \eta_{s,M}^2 \bar{r}_s^2$  is an unknown constant.

Design the virtual controller as

$$\alpha_1 = -b_1 \tilde{e}_1 + N(\lambda_1)\beta_1 - \frac{1}{2}\epsilon_1 \tilde{e}_1 \bar{d}_{s,1}^2, \tag{8}$$

with

$$\beta_1 = \frac{1}{2}\epsilon_1 \tilde{e}_1 \hat{\delta}_1 \bar{l}_{s,1}^2(\check{x}_2),$$

where  $\hat{\delta}_1$  is the estimate of  $\delta_1$ ,  $\tilde{\delta}_1 = \hat{\delta}_1 - \delta_1$ ,  $b_1$  is a positive constant to be designed,  $\lambda_1$  is a smooth function with  $\dot{\lambda}_1 = \tilde{e}_1 \beta_1$ . Substitute (8) into (7), we have

$$\begin{aligned}
\dot{V}_{s,1} &\leq \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + e_1\eta_s(-b_1 \tilde{e}_1 + N(\lambda_1)\beta_1 - \frac{1}{2}\epsilon_1 \tilde{e}_1 \bar{d}_{s,1}^2) + \frac{1}{2}\epsilon_1 \tilde{e}_1^2 \delta_1 \bar{l}_{s,1}^2(\check{x}_2) \\
&\quad + \frac{1}{2}\epsilon_1 e_1^2 \bar{d}_{s,1}^2 + \tilde{\delta}_1 \dot{\delta}_1 + \frac{1}{\epsilon_1} \\
&= \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 - b_1 e_1^2 + \frac{1}{2}\epsilon_1 N(\lambda_1) \tilde{e}_1^2 \eta_s^2 \hat{\delta}_1 \bar{l}_{s,1}^2(\check{x}_2) - \frac{1}{2}\epsilon_1 e_1^2 \bar{d}_{s,1}^2 \\
&\quad + \frac{1}{2}\epsilon_1 \tilde{e}_1^2 \delta_1 \bar{l}_{s,1}^2(\check{x}_2) + \frac{1}{2}\epsilon_1 e_1^2 \bar{d}_{s,1}^2 + \tilde{\delta}_1 \dot{\delta}_1 + \frac{1}{\epsilon_1} \\
&= -(b_1 - \frac{1}{2})e_1^2 + \frac{1}{2}e_2^2 + (\eta_s^2 N(\lambda_1) + 1)\tilde{e}_1 \frac{1}{2}\epsilon_1 \tilde{e}_1 \hat{\delta}_1 \bar{l}_{s,1}^2(\check{x}_2) - \frac{1}{2}\epsilon_1 \tilde{e}_1^2 \tilde{\delta}_1 \bar{l}_{s,1}^2(\check{x}_2) + \tilde{\delta}_1 \dot{\delta}_1 + \frac{1}{\epsilon_1} \\
&= -\bar{b}_1 e_1^2 + \frac{1}{2}e_2^2 + (\varsigma_1(t)N(\lambda_1) + 1)\dot{\lambda}_1 + \tilde{\delta}_1(\dot{\delta}_1 - \frac{1}{2}\epsilon_1 \tilde{e}_1^2 \bar{l}_{s,1}^2(\check{x}_2) + k\hat{\delta}_1) - k\tilde{\delta}_1 \hat{\delta}_1 + \frac{1}{\epsilon_1} \\
&= -\bar{b}_1 e_1^2 + \frac{1}{2}e_2^2 + (\varsigma_1(t)N(\lambda_1) + 1)\dot{\lambda}_1 + \tilde{\delta}_1(\dot{\delta}_1 - \pi_{1,1}) - k\tilde{\delta}_1 \hat{\delta}_1 + \frac{1}{\epsilon_1},
\end{aligned} \tag{9}$$

where  $\bar{b}_1 = b_1 - \frac{1}{2}$ ,  $b_1 \geq \frac{1}{2}$  and  $\varsigma_1(t) = \eta_s^2$ ,  $\pi_{1,1} = \xi_{1,1} - k\hat{\delta}_1$ ,  $k$  is a constant. The intermediate variable  $\xi_{1,1} = \frac{1}{2}\epsilon_1 \tilde{e}_1^2 \bar{l}_{s,1}^2(\check{x}_2)$ .

**Step 2:** Based on (1) and (5), we have

$$\begin{aligned}
\dot{e}_2 &= \dot{x}_2 - \dot{\eta}_s \alpha_1 - \eta_s \dot{\alpha}_1 \\
&= x_3 + \phi_{s,2}(\check{x}_3(t)) + d_{s,2}(t) - \dot{\eta}_s \alpha_1 - \eta_s \dot{\alpha}_1 \\
&= e_3 + \eta_s \alpha_2 + \phi_{s,2}(\check{x}_3(t)) + d_{s,2}(t) - \dot{\eta}_s \alpha_1 - \eta_s \left( \frac{\partial \alpha_1}{\partial \tilde{x}_1} \frac{\eta_s \dot{x}_1 - \dot{\eta}_s x_1}{\eta_s^2} + \frac{\partial \alpha_1}{\partial \hat{\delta}_1} \dot{\delta}_1 + \frac{\partial \alpha_1}{\partial \lambda_1} \dot{\lambda}_1 \right).
\end{aligned}$$

Similar to Remark 4 and the proof described above,  $\phi_{s,2}(\check{x}_3(t))$  can be regarded as functions of  $\eta_s$  and  $\check{x}_3$ , where  $\check{x}_3 = (\tilde{x}_1, \tilde{x}_2, \tilde{x}_3)$ . And it is denoted by  $\phi_{s,2}(\eta_s, \check{x}_3) \triangleq \phi_{s,2}(\check{x}_3(t))$ . Further, from Lemma 2 and the boundedness of  $\eta_s$ , there exist smooth functions  $r_s(\eta_s)$ ,  $\bar{l}_{s,2}(\check{x}_3)$  and unknown constants  $\bar{r}_s$  such that  $\phi_{s,2}(\eta_s, \check{x}_3) \leq r_s(\eta_s) \bar{l}_{s,2}(\check{x}_3) \leq \bar{r}_s \bar{l}_{s,2}(\check{x}_3)$ .

The Lyapunov function candidate is chosen as

$$V_{s,2} = V_{s,1} + \frac{1}{2}e_2^2 + \frac{1}{2}\tilde{\delta}_2^2 + \frac{1}{2}\tilde{\delta}_0^2,$$

then

$$\dot{V}_{s,2} = \dot{V}_{s,1} + e_2(e_3 + \eta_s \alpha_2 + \phi_{s,2}(\check{x}_3(t)) + d_{s,2}(t) - \dot{\eta}_s \alpha_1$$

$$- \eta_s \left( \frac{\partial \alpha_1}{\partial \tilde{x}_1} \frac{\eta_s \dot{x}_1 - \dot{\eta}_s x_1}{\eta_s^2} + \frac{\partial \alpha_1}{\partial \hat{\delta}_1} \dot{\delta}_1 + \frac{\partial \alpha_1}{\partial \lambda_1} \dot{\lambda}_1 \right) + \tilde{\delta}_2 \dot{\delta}_2 + \tilde{\delta}_0 \dot{\delta}_0. \quad (10)$$

Using Young's inequality, the following inequality holds,

$$e_2 \phi_{s,2}(\tilde{x}_3(t)) \leq \frac{1}{2} \epsilon_1 \eta_s^2 \tilde{e}_2^2 \bar{r}_s^2 \bar{l}_{s,2}^2(\tilde{x}_3) + \frac{1}{2\epsilon_1}. \quad (11)$$

Based on (1) and Lemma 2, we have

$$\begin{aligned} & e_2 \eta_s \frac{\partial \alpha_1}{\partial \tilde{x}_1} \frac{\eta_s \dot{x}_1 - \dot{\eta}_s x_1}{\eta_s^2} \\ &= e_2 \frac{\partial \alpha_1}{\partial \tilde{x}_1} \dot{x}_1 - e_2 \frac{\partial \alpha_1}{\partial \tilde{x}_1} \frac{\dot{\eta}_s x_1}{\eta_s} \\ &= e_2 \frac{\partial \alpha_1}{\partial \tilde{x}_1} (x_2 + \phi_{s,1}(\tilde{x}_2(t))) - e_2 \frac{\partial \alpha_1}{\partial \tilde{x}_1} \frac{\dot{\eta}_s x_1}{\eta_s} \\ &\leq e_2 \eta_s \frac{\partial \alpha_1}{\partial \tilde{x}_1} \tilde{x}_2 + \frac{1}{2} \epsilon_1 \eta_s^2 \tilde{e}_2^2 \left( \frac{\partial \alpha_1}{\partial \tilde{x}_1} \right)^2 \bar{r}_{s,1}^2 \bar{l}_{s,1}^2(\tilde{x}_2) + \frac{1}{2\epsilon_1} - \dot{\eta}_s e_2 \frac{\partial \alpha_1}{\partial \tilde{x}_1} \tilde{x}_1. \end{aligned} \quad (12)$$

Substituting (11) and (12) into (10),

$$\begin{aligned} \dot{V}_{s,2} &\leq \dot{V}_{s,1} + \frac{1}{2} e_2^2 + \frac{1}{2} e_3^2 + e_2 \eta_s \alpha_2 + \frac{1}{2} \epsilon_1 \tilde{e}_2^2 \delta_2 \bar{l}_{s,2}^2(\tilde{x}_3) + \frac{1}{2} \epsilon_1 e_2^2 \bar{d}_{s,2}^2 \\ &\quad - e_2 \eta_s \left( \frac{\partial \alpha_1}{\partial \tilde{x}_1} \tilde{x}_2 + \frac{\partial \alpha_1}{\partial \hat{\delta}_1} \dot{\delta}_1 + \frac{\partial \alpha_1}{\partial \lambda_1} \dot{\lambda}_1 \right) + \frac{1}{2} \epsilon_1 \tilde{e}_2^2 \left( \frac{\partial \alpha_1}{\partial \tilde{x}_1} \right)^2 \delta_1 \bar{l}_{s,1}^2(\tilde{x}_2) \\ &\quad + \frac{1}{2} \epsilon_1 \tilde{e}_2^2 \left( \frac{\partial \alpha_1}{\partial \tilde{x}_1} \tilde{x}_1 - \alpha_1 \right)^2 \delta_0 + \tilde{\delta}_2 \dot{\delta}_2 + \tilde{\delta}_0 \dot{\delta}_0 + \frac{2}{\epsilon_1}, \end{aligned} \quad (13)$$

where  $\delta_2 = \eta_{s,M}^2 \bar{r}_s^2$  and  $\delta_0 = \bar{\eta}_s^2 \eta_{s,M}^2$  are unknown constants.

Design the virtual controller as

$$\alpha_2 = -b_2 \tilde{e}_2 + \frac{\partial \alpha_1}{\partial \tilde{x}_1} \tilde{x}_2 + \frac{\partial \alpha_1}{\partial \hat{\delta}_1} \pi_{1,2} + \frac{\partial \alpha_1}{\partial \lambda_1} \dot{\lambda}_1 + N(\lambda_2) \beta_2 - \frac{1}{2} \epsilon_1 \tilde{e}_2 \bar{d}_{s,2}^2, \quad (14)$$

with

$$\beta_2 = \frac{1}{2\tilde{e}_2} \epsilon_1 (\hat{\delta}_1 \xi_{1,2} + \hat{\delta}_2 \xi_{2,1} + \hat{\delta}_0 \xi_{0,1}),$$

where  $\hat{\delta}_0$  and  $\hat{\delta}_2$  are the estimates of  $\delta_0$  and  $\delta_2$ , respectively,  $b_2$  is a positive constant to be designed,  $\xi_{1,2} = \tilde{e}_2^2 \left( \frac{\partial \alpha_1}{\partial \tilde{x}_1} \right)^2 \bar{l}_{s,1}^2(\tilde{x}_2)$ ,  $\xi_{2,1} = \tilde{e}_2^2 \bar{l}_{s,2}^2(\tilde{x}_3)$ ,  $\xi_{0,1} = \tilde{e}_2^2 \left( \frac{\partial \alpha_1}{\partial \tilde{x}_1} \tilde{x}_1 - \alpha_1 \right)^2$ ,  $\pi_{1,2} = \xi_{1,2} + \pi_{1,1}$  and  $\lambda_2$  is a smooth function with  $\dot{\lambda}_2 = \tilde{e}_2 \beta_2$ . Further, denote  $\pi_{2,1} = \xi_{2,1} - k \hat{\delta}_2$  and  $\pi_{0,1} = \xi_{0,1} - k \hat{\delta}_0$ , where  $k$  is a constant. Substitute (14) into (13), we have

$$\begin{aligned} \dot{V}_{s,2} &\leq -\bar{b}_1 e_1^2 - \bar{b}_2 e_2^2 + \frac{1}{2} e_3^2 + \sum_{j=1}^2 (\varsigma_j(t) N(\lambda_j) + 1) \dot{\lambda}_j + \sum_{j=1}^2 \tilde{\delta}_j (\dot{\delta}_j - \pi_{j,2-j+1}) \\ &\quad + k \tilde{\delta}_0 (\dot{\delta}_0 - \pi_{0,1}) - e_2 \eta_s \frac{\partial \alpha_1}{\partial \hat{\delta}_1} (\dot{\delta}_1 - \pi_{1,2}) - k \sum_{j=0}^2 \tilde{\delta}_j \dot{\delta}_j + 6 \times \frac{1}{2\epsilon_1}, \end{aligned}$$

with  $\bar{b}_2 = b_2 - 1$ ,  $b_2 \geq 1$  and  $\varsigma_2(t) = \eta_s^2$ .

**Step  $i$  ( $3 \leq i \leq n-1$ ):** Based on (1) and (5), we have

$$\dot{e}_i = \dot{x}_i - \dot{\eta}_s \alpha_{i-1} - \eta_s \dot{\alpha}_{i-1}$$



$$\begin{aligned}
&= x_{i+1} + \phi_{s,i}(\tilde{x}_{i+1}(t)) + d_{s,i}(t) - \dot{\eta}_s \alpha_{i-1} - \eta_s \dot{\alpha}_{i-1} \\
&= e_{i+1} + \eta_s \alpha_i + \phi_{s,i}(\tilde{x}_{i+1}(t)) + d_{s,i}(t) - \dot{\eta}_s \alpha_{i-1} - \eta_s \left( \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \tilde{x}_j} \frac{\eta_s \dot{x}_j - \dot{\eta}_s x_j}{\eta_s^2} + \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\delta}_j} \dot{\delta}_j + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \lambda_j} \dot{\lambda}_j \right).
\end{aligned}$$

From Remark 4,  $\phi_{s,i}(\tilde{x}_{i+1}(t))$  can be regarded as functions of  $\eta_s$  and  $\tilde{x}_{i+1}$ , where  $\tilde{x}_{i+1} = (\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_{i+1})$ . And it is denoted by  $\phi_{s,i}(\eta_s, \tilde{x}_{i+1}) \triangleq \phi_{s,i}(\tilde{x}_{i+1}(t))$ . By Lemma 2 and the boundedness of  $\eta_s$ , there exist smooth functions  $r_s(\eta_s)$ ,  $\bar{l}_{s,i}(\tilde{x}_{i+1})$  and unknown constants  $\bar{r}_s$  such that  $\phi_{s,i}(\eta_s, \tilde{x}_{i+1}) \leq r_s(\eta) \bar{l}_s(\tilde{x}_{i+1}) \leq \bar{r}_s \bar{l}_{s,i}(\tilde{x}_{i+1})$ , where  $\bar{r}_s$  are the upper bound of  $r_s(\eta_s)$ .

Construct the following Lyapunov function

$$V_{s,i} = V_{s,i-1} + \frac{1}{2} e_i^2 + \frac{1}{2} \tilde{\delta}_i^2,$$

then,

$$\begin{aligned}
\dot{V}_{s,i} &= \dot{V}_{s,i-1} + e_i(e_{i+1} + \eta_s \alpha_i + \phi_{s,i}(\tilde{x}_{i+1}(t)) + d_{s,i}(t) - \dot{\eta}_s \alpha_{i-1} \\
&\quad - \eta_s \left( \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \tilde{x}_j} \frac{\eta_s \dot{x}_j - \dot{\eta}_s x_j}{\eta_s^2} + \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\delta}_j} \dot{\delta}_j + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \lambda_j} \dot{\lambda}_j \right)) + \tilde{\delta}_i \dot{\delta}_i.
\end{aligned} \tag{15}$$

Using Young's inequality, the following inequality holds,

$$e_i \phi_{s,i}(\tilde{x}_{i+1}(t)) \leq \frac{1}{2} \epsilon_1 \eta_s^2 \tilde{e}_i^2 \bar{r}_s^2 \bar{l}_{s,i}^2(\tilde{x}_{i+1}) + \frac{1}{2\epsilon_1}. \tag{16}$$

Based on (1) and Lemma 2, one has

$$\begin{aligned}
&e_i \eta_s \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \tilde{x}_j} \frac{\eta_s \dot{x}_j - \dot{\eta}_s x_j}{\eta_s^2} \\
&= e_i \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \tilde{x}_j} \dot{x}_j - e_i \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \tilde{x}_j} \frac{\dot{\eta}_s x_j}{\eta_s} \\
&= e_i \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \tilde{x}_j} (x_{j+1} + \phi_{s,j}(\tilde{x}_{j+1}(t))) - e_i \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \tilde{x}_j} \frac{\dot{\eta}_s x_j}{\eta_s} \\
&\leq e_i \eta_s \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \tilde{x}_j} \tilde{x}_{j+1} + \frac{1}{2} \epsilon_1 \eta_s^2 \tilde{e}_i^2 \sum_{j=1}^{i-1} \left( \frac{\partial \alpha_{i-1}}{\partial \tilde{x}_j} \right)^2 \bar{r}_s^2 \bar{l}_{s,j}^2(\tilde{x}_{j+1}) + \frac{1}{2\epsilon_1} - \dot{\eta}_s e_i \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \tilde{x}_j} \tilde{x}_j \\
&\leq \dot{V}_{i-1} + \frac{1}{2} e_i^2 + \frac{1}{2} e_{i+1}^2 + e_i \eta_s \alpha_i + \frac{1}{2} \epsilon_1 \tilde{e}_i^2 \bar{d}_{s,i}^2 \\
&\quad - e_i \eta_s \left( \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \tilde{x}_j} \tilde{x}_{j+1} + \sum_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\delta}_j} \dot{\delta}_j + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \lambda_j} \dot{\lambda}_j \right) \\
&\quad + \frac{1}{2} \epsilon_1 \tilde{e}_i^2 \delta_i \bar{l}_{s,i}^2(\tilde{x}_{i+1}) + \frac{1}{2} \epsilon_1 \tilde{e}_i^2 \delta_0 \left( \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \tilde{x}_j} \tilde{x}_j - \alpha_{i-1} \right)^2 \\
&\quad + \frac{1}{2} \epsilon_1 \tilde{e}_i^2 \sum_{j=1}^{i-1} \left( \frac{\partial \alpha_{i-1}}{\partial \tilde{x}_j} \right)^2 \delta_j \bar{l}_{s,j}^2(\tilde{x}_{j+1}) + \tilde{\delta}_i \dot{\delta}_i + (i+2) \times \frac{1}{2\epsilon_1},
\end{aligned} \tag{17}$$

where  $\delta_i = \eta_{s,M}^2 \bar{r}_s^2$ .

The virtual controller is designed as

$$\begin{aligned}\alpha_i = & -b_i \tilde{e}_i - \frac{1}{2} \epsilon_1 \tilde{e}_i \bar{d}_{s,i}^2 + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \tilde{x}_j} \tilde{x}_{j+1} + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\delta}_j} \pi_{j,i-j+1} + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \lambda_j} \dot{\lambda}_j + \frac{\partial \alpha_{i-1}}{\partial \hat{\delta}_0} \pi_{0,i-1} \\ & + N(\lambda_i) \beta_i + \frac{1}{\tilde{e}_i} \sum_{q=1}^{i-2} \sum_{j=1+q}^{i-1} \tilde{e}_j \frac{\partial \alpha_{j-1}}{\partial \hat{\delta}_q} \xi_{q,i-q+1} + \frac{1}{\tilde{e}_i} \sum_{j=3}^{i-1} \tilde{e}_j \frac{\partial \alpha_{j-1}}{\partial \hat{\delta}_0} \xi_{0,i-1},\end{aligned}\quad (18)$$

with

$$\beta_i = \frac{1}{2\tilde{e}_i} \epsilon_1 \left( \sum_{j=1}^i \hat{\delta}_j \xi_{j,i-j+1} + \hat{\delta}_0 \xi_{0,i-1} \right),$$

where  $\hat{\delta}_i$  is the estimate of  $\delta_i$ ,  $\tilde{\delta}_i = \hat{\delta}_i - \delta_i$ ,  $\lambda_i$  is a smooth function with  $\dot{\lambda}_i = \tilde{e}_i \beta_i$ ,  $b_i$  is a positive constant to be designed, denote  $\xi_{j,i-j+1} = \tilde{e}_i^2 \left( \frac{\partial \alpha_{i-1}}{\partial \tilde{x}_j} \right)^2 \bar{l}_{s,j}^2(\tilde{x}_{j+1})$ ,  $\xi_{0,i-1} = \tilde{e}_i^2 \left( \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \tilde{x}_j} \tilde{x}_j - \alpha_{i-1} \right)^2$ ,  $\xi_{i,1} = \tilde{e}_i^2 \bar{l}_{s,i}^2(\tilde{x}_{i+1})$ ,  $\pi_{j,i-j+1} = \xi_{j,i-j+1} + \pi_{j,i-j}$ ,  $\pi_{0,i-1} = \xi_{0,i-1} + \pi_{0,i-2}$  and  $\pi_{i,1} = \xi_{i,1} - k\hat{\delta}_i$ . Substitute (16)-(18) into (15), we have

$$\begin{aligned}\dot{V}_{s,i} \leq & -\sum_{j=1}^i \bar{b}_j e_j^2 + \frac{1}{2} e_{i+1}^2 + \sum_{j=1}^i \tilde{\delta}_j (\dot{\tilde{\delta}}_j - \pi_{j,i-j+1}) + \sum_{j=1}^i (\varsigma_j(t) N(\lambda_j) + 1) \dot{\lambda}_j \\ & + \tilde{\delta}_0 (\dot{\tilde{\delta}}_0 - \pi_{0,i-1}) - \eta_s \sum_{q=1}^{i-1} \sum_{j=q+1}^i e_j \frac{\partial \alpha_{i-1}}{\partial \hat{\delta}_q} (\dot{\tilde{\delta}}_q - \pi_{q,i-q+1}) - \eta_s \sum_{j=3}^i e_j \frac{\partial \alpha_{i-1}}{\partial \hat{\delta}_0} (\dot{\tilde{\delta}}_0 - \pi_{0,i-1}) \\ & - k \sum_{j=0}^i \tilde{\delta}_j \hat{\delta}_j + \left( \frac{(i+1)(i+4)}{2} - 3 \right) \times \frac{1}{2\epsilon_1},\end{aligned}$$

where  $\bar{b}_j = b_j - 1$ ,  $b_j \geq 1$  and  $\varsigma_j(t) = \eta_s^2$  for  $j = 3, 4, \dots, n-1$ .

**Step n:** From (1) and (5), one has

$$\begin{aligned}\dot{e}_n = & u_s(t) + \varphi_{a,s}(u_s(t), t) + \phi_{s,n}(x(t)) + d_{s,n}(t) - \dot{\eta}_s \alpha_{n-1} - \eta_s \dot{\alpha}_{n-1} \\ = & (1 + \rho_{a,s}(t)) u_s(t) + \phi_{s,n}(x(t)) + d_{s,n}(t) - \dot{\eta}_s \alpha_{n-1} - \eta_s \dot{\alpha}_{n-1} \\ = & \omega_s(t) u_s(t) + \phi_{s,n}(x(t)) + d_{s,n}(t) - \dot{\eta}_s \alpha_{n-1} - \eta_s \left( \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \tilde{x}_j} \frac{\eta_s \dot{x}_j - \dot{\eta}_s x_j}{\eta_s^2} + \sum_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\delta}_j} \dot{\tilde{\delta}}_j + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \lambda_j} \dot{\lambda}_j \right).\end{aligned}$$

Similar to the proof described above, we can also get that  $\phi_{s,n}(x(t))$  can be regarded as functions of  $\eta_s$  and  $\tilde{x}_n$ , where  $\tilde{x}_n = (\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n)$ . And it is denoted by  $\phi_{s,n}(\eta_s, \tilde{x}_n) \triangleq \phi_{s,n}(x_n(t))$ . Further, from Lemma 2 and the boundedness of  $\eta_s$ , there exist smooth functions  $r_s(\eta_s)$ ,  $\bar{l}_{s,n}(\tilde{x}(t))$  and unknown constants  $\bar{r}_s$  such that  $\phi_{s,n}(\eta_s, \tilde{x}(t)) \leq r_s(\eta_s) \bar{l}_{s,n}(\tilde{x}(t)) \leq \bar{r}_s \bar{l}_{s,n}(\tilde{x}(t))$ .

Construct the following Lyapunov function

$$V_{s,n} = V_{s,n-1} + \frac{1}{2} e_n^2 + \frac{1}{2} \tilde{\delta}_n^2 + \frac{1}{2} \tilde{\delta}_{n+1}^2,$$

then,

$$\begin{aligned}\dot{V}_{s,n} \leq & \dot{V}_{s,n-1} + e_n (\omega_s u_s - \eta_s \alpha_n + \eta_s \alpha_n) - e_n \eta_s \left( \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \tilde{x}_j} \tilde{x}_{j+1} + \sum_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\delta}_j} \dot{\tilde{\delta}}_j + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \lambda_j} \dot{\lambda}_j \right) \\ & + \frac{1}{2} \epsilon_1 e_n^2 \bar{d}_{s,n}^2 + \frac{1}{2} \epsilon_1 \tilde{e}_n^2 \delta_n \bar{l}_{s,n}^2(\tilde{x}(t)) + \frac{1}{2} \epsilon_1 \tilde{e}_i^2 \delta_0 \left( \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \tilde{x}_j} \tilde{x}_j - \alpha_{n-1} \right)^2\end{aligned}\quad (19)$$

$$+ \frac{1}{2}\epsilon_1 \tilde{e}_n^2 \sum_{j=1}^{n-1} \left( \frac{\partial \alpha_{n-1}}{\partial \tilde{x}_j} \right)^2 \delta_j \bar{l}_{s,j}^2 (\tilde{x}_{j+1}) + \tilde{\delta}_n \dot{\tilde{\delta}}_n + \tilde{\delta}_{n+1} \dot{\tilde{\delta}}_{n+1} + \frac{(n+2)}{2\epsilon_1},$$

where  $\delta_n = \eta_{s,M}^2 \bar{r}_s^2$  and  $\delta_{n+1} = \eta_{s,M}^4$ .

The virtual controller is designed as

$$\begin{aligned} \alpha_n = & -b_n \tilde{e}_n - \frac{1}{2}\epsilon_1 \tilde{e}_n \bar{d}_{s,n}^2 + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \tilde{x}_j} \tilde{x}_{j+1} + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\delta}_j} \pi_{j,n-j+1} + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \lambda_j} \dot{\lambda}_j + \frac{\partial \alpha_{n-1}}{\partial \hat{\delta}_0} \pi_{0,n-1} \\ & + N(\lambda_n) \beta_n + \frac{1}{\tilde{e}_n} \sum_{q=1}^{n-2} \sum_{j=1+q}^{n-1} \tilde{e}_j \frac{\partial \alpha_{j-1}}{\partial \hat{\delta}_q} \xi_{q,n-q+1} + \frac{1}{\tilde{e}_n} \sum_{j=3}^{n-1} \tilde{e}_j \frac{\partial \alpha_{j-1}}{\partial \hat{\delta}_0} \xi_{0,n-1}, \end{aligned} \quad (20)$$

with

$$\beta_n = \frac{1}{2\tilde{e}_n} \epsilon_1 \left( \sum_{j=1}^n \hat{\delta}_j \xi_{j,n-j+1} + \hat{\delta}_0 \xi_{0,n-1} \right),$$

where  $\hat{\delta}_n$  is the estimate of  $\delta_n$ ,  $\tilde{\delta}_n = \hat{\delta}_n - \delta_n$ ,  $b_n$  is a positive constant to be designed,  $\lambda_n$  is a smooth function with  $\dot{\lambda}_n = \tilde{e}_n \beta_n$ ,  $\xi_{j,n-j+1} = \tilde{e}_n^2 \left( \frac{\partial \alpha_{n-1}}{\partial \tilde{x}_j} \right)^2 \bar{l}_{s,j}^2 (\tilde{x}_j)$ ,  $j = 1, 2, \dots, n-1$ ,  $\xi_{0,n-1} = \tilde{e}_n^2 \left( \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \tilde{x}_j} \tilde{x}_j - \alpha_{n-1} \right)^2$ ,  $\xi_{n,1} = \tilde{e}_n^2 \bar{l}_{s,n}^2 (\tilde{x}(t))$ ,  $\pi_{j,n-j+1} = \xi_{j,n-j+1} + \pi_{j,n-j}$  and  $\pi_{0,n-1} = \xi_{0,n-1} + \pi_{0,n-2}$ .

The adaptive laws of the parameters are designed as

$$\dot{\hat{\delta}}_0 = \pi_{0,n-1}, \quad \dot{\hat{\delta}}_j = \pi_{j,n-j+1}, \quad j = 1, 2, \dots, n, \quad (21)$$

where  $\pi_{n,1} = \xi_{n,1} - k\hat{\delta}_n$ .

Then,

$$\dot{V}_{s,n} \leq \dot{\tilde{V}}_{s,n} + e_n(\omega_s u_s - \eta_s \alpha_n) + \tilde{\delta}_{n+1} \dot{\tilde{\delta}}_{n+1},$$

where

$$\dot{\tilde{V}}_{s,n} \leq - \sum_{j=1}^n \bar{b}_j e_j^2 - k \sum_{j=0}^n \tilde{\delta}_j \hat{\delta}_j + \sum_{j=1}^n (\varsigma_j(t) N(\lambda_j) + 1) \dot{\lambda}_j + \frac{n^2 + 5n - 2}{4\epsilon_1},$$

with  $\bar{b}_n = b_n - 1$ ,  $b_n \geq 1$  and  $\varsigma_n(t) = \eta_s^2$ .

Then we have the actual control law

$$\begin{aligned} u = & N(\lambda_{n+1}) \beta_{n+1}, \\ \beta_{n+1} = & \tilde{e}_n \hat{\delta}_{n+1} \alpha_n^2, \\ \dot{\lambda}_{n+1} = & \tilde{e}_n \beta_{n+1}, \end{aligned} \quad (22)$$

and the adaptive law of  $\hat{\delta}_{n+1}$

$$\dot{\hat{\delta}}_{n+1} = \tilde{e}_n^2 \alpha_n^2 - k\hat{\delta}_{n+1}, \quad (23)$$

where  $\tilde{\delta}_{n+1} = \hat{\delta}_{n+1} - \delta_{n+1}$  and  $\hat{\delta}_{n+1}$  is the estimate of  $\delta_{n+1}$ .

Then,

$$\dot{V}_{s,n} \leq - \sum_{j=1}^n \bar{b}_j e_j^2 - k \sum_{j=0}^{n+1} \tilde{\delta}_j \hat{\delta}_j + \sum_{j=1}^{n+1} (\varsigma_j(t) N(\lambda_j) + 1) \dot{\lambda}_j + \frac{n^2 + 5n - 2}{4\epsilon_1},$$

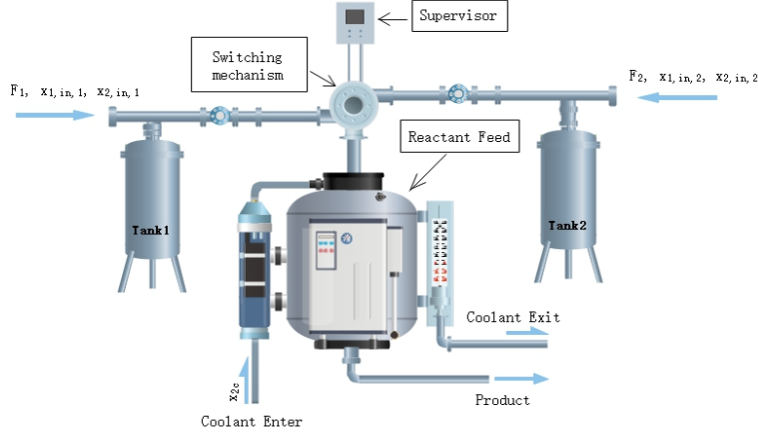


Fig 3. The schematic diagram of CSTR.

where  $\varsigma_{n+1}(t) = \omega_s \eta_s$ .  
Define

$$A = \min\{2\bar{b}_1, 2\bar{b}_2, \dots, 2\bar{b}_n, k\} \text{ and } B = \sum_{j=0}^{n+1} \frac{1}{2} k \delta_j^2 + \frac{n^2 + 5n - 2}{4\epsilon_1}.$$

Using the Young's inequality, the following relations hold

$$-k\tilde{\delta}_j\hat{\delta}_j \leq -\frac{1}{2}k\tilde{\delta}_j^2 + \frac{1}{2}k\hat{\delta}_j^2, \quad j = 0, 1, 2, \dots, n+1.$$

Then, we can get

$$\dot{V}_{s,n}(t) \leq -AV_{s,n} + B + \sum_{j=1}^{n+1} (\varsigma_j(t)N(\lambda_j))\dot{\lambda}_j(t).$$

From Lemma 1, the following theorem can be obtained.

**Theorem 1:** For system (1) with state-independent sensor attacks  $\varphi_i(x_i(t), t)$  and input-independent actuator attacks  $\varphi_{a,s}(u_s(t), t)$ , the proposed controller (22) with adaptive laws (21), (23) and constants  $b_1 \geq \frac{1}{2}, b_j \geq 1, j = 2, 3, \dots, n$  ensure that all signals of the closed-loop system are bounded under the arbitrary switchings.

#### 4. Simulation results

To verify the theoretical results of the provided adaptive control mechanism for switched nonlinear CPSs in the presence of state-dependent sensor attacks and input-dependent actuator attacks, simulation example of a class of CSTR system with state-dependent sensor attacks and input-dependent actuator attacks is introduced in this section. Fig. 3 shows the schematic diagram of CSTR system. The kinetic equation of CSTR system in the work of Li and Zhao [33] is employed in this section, which is described as

$$\begin{aligned} \dot{x}_1 &= \frac{F_s(x_{1,in,s} - x_1)}{V} + K_s\psi_s(x_1, x_2), \\ \dot{x}_2 &= \frac{F_s(x_{2,in,s} - x_2)}{V} + \chi(x_{2c} - x_2) - \Delta H_s(x_1, x_2)\psi_s(x_1, x_2), \end{aligned} \quad (24)$$

where  $s$  takes value from the set  $S = \{1, 2\}$ ,  $x_{1,in,s}, x_1 \in \mathbb{R}$  are the chemical species concentration in the input flow and in the reactor respectively,  $x_{2,in,s}, x_2 \in \mathbb{R}$  are the reactor and the input flow temperatures respectively. The physical meaning of the rest part of the system (24) can be found in [32]. To convert the dynamic system (24) into the form of System (1), the following variable transformations are introduced as  $\Gamma_1 = x_1 - x_1^*$ ,  $\Gamma_2 = x_2 - x_2^*$  and  $u = x_{2c} - x_{2c}^*$ , where  $x_1^*, x_2^*$  and  $x_{2c}^*$  are the steady-state values. Further, suppose that the system (24) encounters state-dependent sensor attacks and input-dependent actuator attacks, then (24) is rewritten as a switched nonlinear

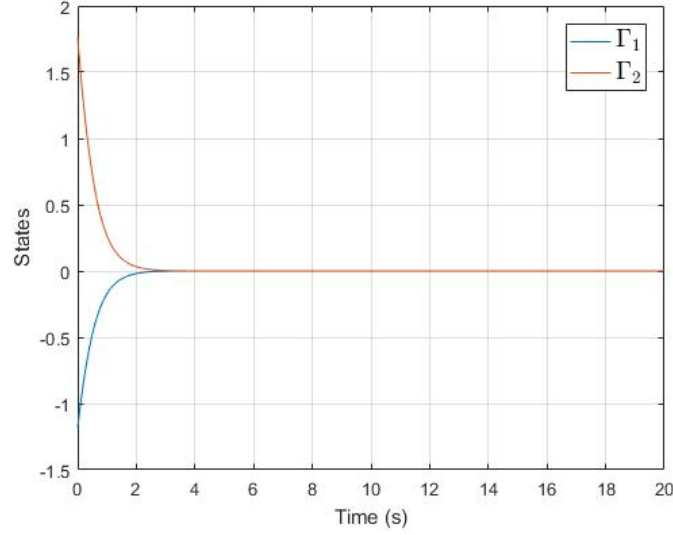


Fig 4. The state of the switched system.

system in the following form

$$\begin{aligned}\dot{\Gamma}_1 &= \Gamma_2 + \phi_{s,1}(\Gamma_1, \Gamma_2), \\ \dot{\Gamma}_2 &= \chi u_s + \varphi_{a,s}(u_s(t), t) + \phi_{s,2}(\Gamma_1, \Gamma_2), \\ \tilde{\Gamma}_i &= \Gamma_i + \varphi_i(\Gamma_i, t),\end{aligned}\tag{25}$$

which is exactly the switched nonlinear CPSs form described in our system (1), where  $x(t) = [\Gamma_1, \Gamma_2]^T$ . Substitute the above variable transformations into (24), we can get  $\phi_{s,1} = \frac{F_s(x_{1,in,s} - x_1^* - \Gamma_1)}{V} + K_s \psi_s(\Gamma_1 + x_1^*, \Gamma_2 + x_2^*) - \Gamma_2$  and  $\phi_{s,2} = \frac{F_s(x_{2,in,s} - x_2^* - \Gamma_2)}{V} + \chi(x_{2c}^* - x_2^* - \Gamma_2) - \Delta H_s(\Gamma_1 + x_1^*, \Gamma_2 + x_2^*) \psi_s(\Gamma_1 + x_1^*, \Gamma_2 + x_2^*) - \Gamma_2$  with  $\phi_{s,1}(0, 0) = 0$ ,  $\phi_{s,2}(0, 0) = 0$ . Inspired by [32] and [38], the following nonlinear terms are taken into considered, for the first subsystem,

$$\begin{aligned}\phi_{1,1} &= \Gamma_2 e^{-1-\Gamma_1^2} + (1 - e^{-\Gamma_1}) / (1 + e^{-\Gamma_1}), \\ \phi_{1,2} &= \Gamma_1^2 - \Gamma_2,\end{aligned}$$

and for the second subsystem,

$$\begin{aligned}\phi_{2,1} &= -0.2\Gamma_2 + 0.4 \tanh(\Gamma_1 + \Gamma_2)\Gamma_2 - 2\Gamma_1, \\ \phi_{2,2} &= \Gamma_1 \cos(\Gamma_1).\end{aligned}$$

Further, we assume (25) encounters state-dependent sensor attacks (2) and input-dependent actuator attacks (3) as  $\varphi_1 = -3 - 0.5 \cos(t)\Gamma_1$ ,  $\varphi_2 = 0.5 + 0.75 \sin(t)\Gamma_2$  and  $\varphi_{a,1} = e^{\sin(t)}u_1$ ,  $\varphi_{a,2} = \cos(t)u_2$ .

Next, let the proposed adaptive control mechanism in Section 3 be applied to the system (25), then, the provided controller and adaptive laws are

$$\begin{aligned}\alpha_1 &= -b_1 \tilde{e}_1 + N(\lambda_1)\beta_1 - \frac{1}{2}\epsilon_1 \tilde{e}_1 \tilde{d}_{s,1}^2, \\ \alpha_2 &= -b_2 \tilde{e}_2 + \frac{\partial \alpha_1}{\partial \tilde{\Gamma}_1} \tilde{\Gamma}_2 + \frac{\partial \alpha_1}{\partial \tilde{\delta}_1} (\tilde{e}_2^2 (\frac{\partial \alpha_1}{\partial \tilde{x}_1})^2 \tilde{l}_{s,1}^2(\tilde{x}_2) + \frac{1}{2}\epsilon_1 \tilde{e}_1^2 \tilde{l}_{s,1}^2(\tilde{x}_2) - k\hat{\delta}_1) \\ &\quad + \frac{\partial \alpha_1}{\partial \lambda_1} \dot{\lambda}_1 + N(\lambda_2)\beta_2 - \frac{1}{2}\epsilon_1 \tilde{e}_2 \tilde{d}_{s,2}^2, \\ u &= N(\lambda_3)\beta_3,\end{aligned}$$

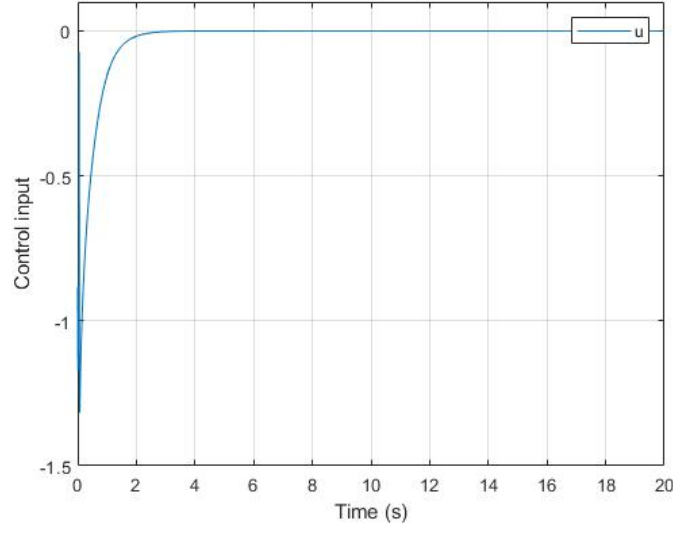


Fig 5. The control input.

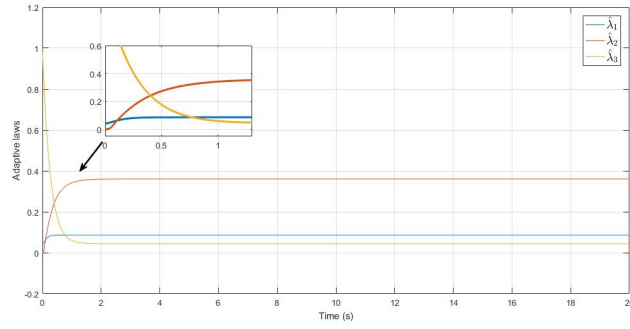


Fig 6. The adaptive laws.

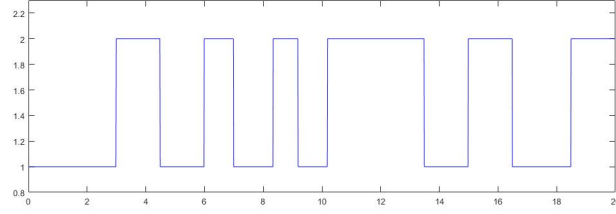


Fig 7. The switching signal.

with

$$\begin{aligned}
\beta_1 &= \frac{1}{2} \epsilon_1 \tilde{e}_1 \hat{\delta}_1 \bar{l}_{s,1}^2(\tilde{x}_2), \\
\beta_2 &= \frac{1}{2 \tilde{e}_2} \epsilon_1 (\hat{\delta}_1 \tilde{e}_2^2 \left( \frac{\partial \alpha_1}{\partial \tilde{x}_1} \right)^2 \bar{l}_{s,1}^2(\tilde{x}_2) + \hat{\delta}_2 \tilde{e}_2^2 \bar{l}_{s,2}^2(\tilde{x}_3) + \hat{\delta}_0 \tilde{e}_2^2 \left( \frac{\partial \alpha_1}{\partial \tilde{x}_1} \tilde{x}_1 - \alpha_1 \right)^2), \\
\beta_3 &= \tilde{e}_2 \hat{\delta}_3 \alpha_2^2, \\
\dot{\lambda}_1 &= \tilde{e}_1 \beta_1, \quad \dot{\lambda}_2 = \tilde{e}_2 \beta_2, \quad \dot{\lambda}_3 = \tilde{e}_2 \beta_3, \\
\dot{\delta}_0 &= \tilde{e}_2^2 \left( \frac{\partial \alpha_1}{\partial \tilde{x}_1} \tilde{x}_1 - \alpha_1 \right)^2, \quad \dot{\delta}_1 = \tilde{e}_2^2 \left( \frac{\partial \alpha_1}{\partial \tilde{x}_1} \right)^2 \bar{l}_{s,1}^2(\tilde{x}_1) + \frac{1}{2} \epsilon_1 \tilde{e}_1 \bar{l}_{s,1}^2(\tilde{x}_2) - k \hat{\delta}_1, \\
\dot{\delta}_2 &= \tilde{e}_2^2 \bar{l}_{s,2}^2 - k \hat{\delta}_2, \quad \dot{\delta}_3 = \tilde{e}_2^2 \alpha_2^2 - k \hat{\delta}_3.
\end{aligned}$$

The parameters are taken as  $b_1 = 1.5$ ,  $b_2 = 3$ ,  $k = 0.001$ ,  $d_{s,1} = d_{s,2} = 0.1 \sin(t)$ ,  $s \in \{1, 2\}$ , and the  $\tilde{N}$  functions are chosen as  $N(\lambda_i) = e^{\lambda_i^2} \cos(\frac{\pi}{2} \lambda_i)$ ,  $i = 1, 2, 3$ . Based on the above design, through Theorem 1, we get that all signals of the closed-loop system are bounded under the switching signal  $s$ .

The results show that under the switching signal shown in Fig. 7, the state of system (25) is shown in Fig. 4, the control input is shown in Fig. 5, and the adaptive law of parameter  $\hat{\lambda}_k, k = 1, 2, 3$  is shown in Fig. 6. The results demonstrate the designed mechanism is effective when the system encounters state-dependent sensor attacks and input-dependent actuator attacks.

**Remark 6:** The work of Li and Zhao [33] proposed an adaptive controller for state-dependent sensor attacks and state-dependent actuator attacks, where the signs of time-varying gains are assumed to be known. In fact, the signs of unknown time-varying gains caused by sensor attack and actuator attack may be changed after being attacked. Different from the work of Li and Zhao [33], the input-dependent actuator attacks in our experiment are more complex. In addition, the signs of unknown time-varying gains caused by state-dependent sensor attacks and input-dependent actuator attacks are all unknown in our study, which is different from the work in [33].

## 5. Conclusions

This article develops an adaptive controller for switched nonlinear system with state-dependent sensor attacks and input-dependent actuator attacks, especially when the controller is also attacked. The proposed controller can mitigate the impact of attacks effectively. Specifically, a new coordinate transformation is used in backstepping design process and new types of Nussbaum functions are introduced to deal with unknown time-varying gains caused by state-dependent sensor attacks and input-dependent actuator attacks. A common Lyapunov function is constructed for all subsystems, which can ensure that the signals are globally bounded under arbitrary switchings. In the future, we can further consider sampling control design, event-driven control design and other methods to study the systems' security control strategy.

## References

- [1] V. S. Dolk, P. Tesi, C. De Persis, et al., Event-triggered control systems under denial-of-service attacks, *IEEE Transactions on Control of Network Systems*. 4 (1) (2017) 93-105.
- [2] Y. Zhu, W. X. Zheng, Observer-based control for cyber-physical systems with periodic dos attacks via a cyclic switching strategy, *IEEE Transactions on Automatic Control*. 65 (8) (2020) 3714-3721.
- [3] X. Wang, D. Ding, X. Ge, Q. L. Han, Neural-network-based control for discrete-time nonlinear systems with denial-of-service attack: The adaptive event-triggered case, *International Journal of Robust and Nonlinear Control*. 32(5) (2022) 2760-2779.
- [4] Z. Feng, G. Hu, Secure cooperative event-triggered control of linear multiagent systems under dos attacks, *IEEE Transactions on Control Systems Technology*. 28 (3) (2020) 741-752.
- [5] T. Li, B. Chen, L. Yu, et al., Active security control approach against dos attacks in cyber-physical systems, *IEEE Transactions on Automatic Control*. 66 (9) (2021) 4303-4310.
- [6] J. Q. Wang, J. F. Gao, P. Wu, Attack-resilient event-triggered formation control of multi-agent systems under periodic dos attacks using complex Laplacian, *ISA Transactions*. 128 (2022) 10-16.
- [7] S. L. Hu, D. Yue, Z. H. Cheng, et al., Co-design of dynamic event-triggered communication scheme and resilient observer-based control under aperiodic DoS attacks, *IEEE Transactions on Cybernetics*. 51 (9) (2020) 4591-4601.
- [8] C. Wang, J. Huang, D. Wang, et al., A secure strategy for a cyber physical system with multi-sensor under linear deception attack, *Journal of the Franklin Institute*. 358 (2021) 6666-6683.
- [9] S. J. Yoo, Neural-network-based adaptive resilient dynamic surface control against unknown deception attacks of uncertain nonlinear time-delay cyber physical systems, *IEEE Transactions on Neural Networks and Learning Systems*. 31 (10) (2020) 4341-4353.
- [10] S. Song, B. Y. Zhang, J. H. Park, X. N. Song, Adaptive resilient control design for nonlinear time-delay systems against unknown state-dependent deception attacks, *International Journal of Robust and Nonlinear Control*. 32(4) (2022) 2159-2182.
- [11] H. Fawzi, P. Tabuada, S. Diggavi, Security for control systems under sensor and actuator attacks, *IEEE Conference on Decision and Control, Maui, Hawaii, USA*. 2012, pp. 3412-3417.
- [12] X. Huang, J. X. Dong, An adaptive secure control scheme for T-S fuzzy systems against simultaneous stealthy sensor and actuator attacks, *IEEE Transactions on Fuzzy Systems*. 2020, pp. 1-14.
- [13] M. Showkatbakhsh, Y. Shoukry, S. N. Diggavi, et al., Securing state reconstruction under sensor and actuator attacks: Theory and design, *Automatica*. 116 (2020) 1-11.
- [14] Y. Zhou, K. G. Vamvoudakis, W. M. Haddad, et al., A secure control learning framework for cyber-physical systems under sensor and actuator attacks, *IEEE Transactions on Cybernetics*. 51 (9) (2021) 4648-4660.
- [15] X. X. Ren, G. H. Yang, Adaptive control for nonlinear cyber-physical systems under false data injection attacks through sensor networks, *International Journal of Robust and Nonlinear Control*. 30 (2020) 65-79.

- [16] Y. M. Dong, N. Gupta, N. Chopra, False data injection attacks in bilateral teleoperation systems, *IEEE Transactions on Control Systems Technology*. 28 (3) (2020) 1168-1176.
- [17] J. C. Wu, C. Peng, J. Zhang, et al., Event-triggered finite-time  $H_\infty$  filtering for networked systems under deception attacks, *Journal of the Franklin Institute*. 357 (6) (2020) 3792-3808.
- [18] H. Fawzi, P. Tabuada, S. Diggavi, Secure estimation and control for cyber-physical systems under adversarial attacks, *IEEE Transactions on Automatic Control*. 59 (6) (2014) 1454-1467.
- [19] X. Jin, W. M. Haddad, T. Hayakawa, An adaptive control architecture for cyber-physical system security in the face of sensor and actuator attacks and exogenous stochastic disturbances, *Cyber-Physical Systems*. 4 (1) (2018) 39-56.
- [20] H. B. Sun, L. L. Hou, Adaptive attitude control for spacecraft systems with sensor and actuator attacks, *International Journal of Adaptive Control and Signal Processing*. 36 (3) (2021) 448-468.
- [21] M. S. Chong, M. Wakaiki, J. P. Hespanha, Observability of linear systems under adversarial attacks, *American Control Conference, Palmer House Hilton, Chicago, USA*. 2015, pp. 2439-2444.
- [22] L. W. An, G. H. Yang, Secure state estimation against sparse sensor attacks with adaptive switching mechanism, *IEEE Transactions on Automatic Control*. 63 (8) (2018) 2596-2603.
- [23] Y. H. Chang, Q. Hu, C. J. Tomlin, Secure estimation based Kalman filter for cyber-physical systems against sensor attacks, *Automatica*. 95 (2018) 399-412.
- [24] D. Ye, T. Y. Zhang, G. Guo, Stochastic coding detection scheme in cyber-physical systems against replay attack, *Information Sciences*. 481 (2019) 432-444.
- [25] B. Chen, D. Ho, G. Q. Hu, et al., Secure fusion estimation for bandwidth constrained cyber-physical systems under replay attacks, *IEEE Transactions on Cybernetics*. 48 (6) (2018) 1862-1876.
- [26] L. Su, D. Ye, X. G. Zhao, Static output feedback secure control for cyber-physical systems based on multisensor scheme against replay attacks, *International Journal of Robust and Nonlinear Control*. 30 (2020) 8313-8326.
- [27] M. H. Zhu, S. Martinez, On the performance analysis of resilient networked control systems under replay attacks, *IEEE Transactions on Automatic Control*. 59 (3) (2014) 804-808.
- [28] L. W. An, G. H. Yang, Improved adaptive resilient control against sensor and actuator attacks, *Information Sciences*. 423 (2018) 145-156.
- [29] T. Yucelen, W. M. Haddad, E. M. Feron, Adaptive control architectures for mitigating sensor attacks in cyber-physical systems, *Cyber-Physical Systems*. 2 (1-4) (2016) 1-23.
- [30] X. Jin, W. M. Haddad, T. Yucelen, An adaptive control architecture for mitigating sensor and actuator attacks in cyber-physical systems, *IEEE Transactions on Automatic Control*. 62 (11) (2017) 6058-6064.
- [31] Y. Yang, J. S. Huang, X. J. Su, Adaptive control of second-order nonlinear systems with injection and deception attacks, *IEEE Transactions on Systems Man and Cybernetics Systems*. 52 (1) (2020) 574-581.
- [32] L. J. Long, J. Zhao, Global stabilization of switched nonlinear systems in non-triangular form and its application, *Journal of the Franklin Institute*. 351(2) (2014) 1161-1178.
- [33] Z. J. Li, J. Zhao, Resilient adaptive control of switched nonlinear cyber-physical systems under uncertain deception attacks, *Information Sciences*. 543 (2021) 398-409.
- [34] Z. Y. Chen, Nussbaum functions in adaptive control with time-varying unknown control coefficients, *Automatica*. 102 (2019) 72-79.
- [35] Z. Y. Chen, J. Huang, Stabilization and regulation of nonlinear systems a robust and adaptive approach, Cham, Switzerland: SpringerInternational Publishing; 2015.
- [36] S. S. Ge, J. Wang, Robust adaptive tracking for time-varying uncertain nonlinear systems with unknown control coefficients, *IEEE Transactions on Automatic Control*. 48 (8) (2003) 1463-1469.
- [37] W. Lin, C. J. Qian, Adaptive control of nonlinearly parameterized systems: the smooth feedback case, *IEEE Transactions on Automatic Control*. 47 (8) (2002) 1249-1266.
- [38] A. R. Jose, F. Ricardo, Robust PI stabilization of a class of chemical reactors, *Systems and Control Letters*. 38 (4-5) (1999) 219-225.