# Simulating Fatigue Crack Growth including Thermal Effects Using the Phase Field Method

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#### Abstract

Putting a mechanical structure under repeated cyclic loading can lead to fatigue crack initiation and propagation in materials. In engineering processes, a fatigue crack evolution behavior can be very complicated when the structure is in a complex environment. It has been shown that internal friction is one of the most important factors for the fatigue behavior of materials. However, there is still a lack of studies on how to predict the influences of internal friction on fatigue crack evolution. In this work, we show that the phase field model provides an elegant solution for structures under different loading frequencies and temperatures. Although the phase field fatigue model has been studied for years for mechanical loading situations, the question of how to include thermal fatigue effect remains open. In this work, we add thermal stresses to the phase field model as the second fatigue driving force. It is shown that the model is able to predict thermal fatigue behavior.

# ORIGINAL ARTICLE

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Putting a mechanical structure under repeated cyclic loading can lead to fatigue crack initiation and propagation in materials. In engineering processes, a fatigue crack evolution behavior can be very complicated when the structure is in a complex environment. It has been shown that internal friction is one of the most important factors for the fatigue behavior of materials. However, there is still a lack of studies on how to predict the influences of internal friction on fatigue crack evolution. In this work, we show that the phase field model provides an elegant solution for structures under different loading frequencies and temperatures. Although the phase field fatigue model has been studied for years for mechanical loading situations, the question of how to include thermal fatigue effect remains open. In this work, we add thermal stresses to the phase field model as the second fatigue driving force. It is shown that the model is able to predict thermal fatigue behavior.

#### KEYWORDS

Phase field; Fatigue fracture; Thermal effect

## 1 | INTRODUCTION

Finding ways to deal with fatigue failure is a crucial issue in mechanical design and manufacturing processes. Fatigue failure in structures is difficult to predict since unexpected cracks can occur at stress levels lower than the yield stress of the material. Moreover, unlike other fracture scenarios, fatigue failure does not happen immediately, but rather

after a huge number of repeated load cycles. Depending on the types of loading, geometry, and various complex environments for the technical components, the crack evolution behavior can be very complicated. Traditionally, fatigue life and crack patterns are determined by experiments, which are time-consuming and unfavorable in terms of economic and ecological aspects. Due to those natural properties of the fatigue fracture, an innovative numerical tool is desired that can cover the mentioned fatigue features in a simple way.

In the past decade, the phase field method has been developed to simulate complicated fracture processes because of its simple form and numerous application possibilities [1] [2] [3]. The phase field model can reproduce all crack evolutions, including initiation, propagation, branching, and kinking, in one single equation. Moreover, it can be simulated on a fixed mesh. Historically speaking, the phase field model is based on an extension of the Griffith fracture theory [4], where an energetic criterion is used to predict the onset of crack propagation. Following this idea, Francfort and Marigo [1] proposed a variational formulation to predict the crack evolution of brittle fractures. In order to numerically resolve the fracture behavior, a regularized formulation was devised by Bourdin et al. [2] [3] [5], where a crack field variable was introduced to indicate the crack status. This diffusive representation of cracks by a scalar field resembles the phase field method. Kuhn and Müller [6] proposed a phase field model for quasi-static fracture, where the variational problem yields an equation of the stress equilibrium and an equation of crack evolution. Amor et al. [7] distinguished the fracture cases in tension and compression by decomposing the elastic energy density to avoid unrealistic crack behavior. Later, an additional history field variable was introduced by Miehe et al. [8] in order to handle the irreversibility of the crack field, and this formulation enables a staggered scheme for phase field computation.

The phase field method has been applied in different fracture scenarios. For the fatigue phase field model, there are two main strategies: to reduce the fracture toughness or to accumulate the fatigue damage. For the first variation: Alessi et al. [9] introduced a fatigue degradation function related to an accumulated strain history variable, which is applied to the fracture energy accounting for the fatigue effect. The fracture energy, which is associated with the fatigue degradation function, will decrease when the strain history variable increases. In the other works like [10] [11], the fatigue degradation function and history variable are chosen differently, but they are still used to reduce the fracture energy term. Different from those phase field formulations for fatigue fracture, Schreiber et al. [12] [13], Yan et al. [14] follow the path from Kuhn and Müller [6] by extending the total energy with an additional fatigue damage. One major advantage of this formulation is that it directly couples the phase field model with the fatigue parameters of experiments, allowing us to handle complex environmental influences on crack growth.

Recent studies [15] [16] showed that there is a relation between internal friction and material fatigue: different kinds of particle migration can be seen as a source of fatigue processes. However, there is still no physical model that can simulate all the effects of internal friction on fatigue behavior in complex circumstances. For a loaded body, the internal friction of the solid is determined by the loading frequency and the loading temperature. In this work, we show that the phase field fatigue model can take internal friction into consideration. It is shown that results from phase field simulation agree with experiments.

Besides mechanical loading, the fatigue of material can also be caused by rapid heating and cooling, known as thermal fatigue. The thermal fatigue is a special type of fatigue failure where the macroscopic cracks result from cyclic thermal stresses due to the repetitive fluctuations of material temperature. The other key novelty of this work is that we include the phase field fracture model with thermal fatigue effects. The thermal stresses related to temperature gradients are considered as the second fatigue driving force in the phase field model. It is shown that the model can predict structures under thermal fatigue. In the Sec. 2 of the paper, the phase field model for fatigue fracture is stated. In Sec. 3, simulation results are compared with experiment data. In Sec. 4, a thermal fatigue phase field model is

proposed and in Sec. 5, the conclusion is drawn and directions for future work are given.

#### 2 | A PHASE FIELD MODEL FOR CYCLIC FATIGUE

The phase field fracture model introduces an additional field variable to represent cracks. The crack field *s* is 1 if the material is intact and is 0 where cracks occur [6]. The crack field *s* varies continuously from 0 to 1, modeling the transition zone similar to the Landau-Ginzburg phase transition [17]. Following the variational principle, it is to be postulated that the displacement field **u** and crack field *s* locally minimize the total energy of a loaded body  $\Omega$ . This assumption yields two coupled equations—the equilibrium of the stress field and the evolution of the crack field—to describe fatigue fracturing.

Let t be the external traction on the body, the total energy  ${\mathcal E}$  for a case of fatigue fracture is given as

$$\mathcal{E} = \int_{\Omega} \psi(\varepsilon, s, \nabla s, D) dV = \int_{\Omega} \left[ (g(s) + \eta) \psi^{\varepsilon}(\varepsilon) + \psi^{s}(s, \nabla s) + h(s) \psi^{ad}(D) \right] dV - \int_{\partial \Omega} \mathbf{t} \cdot \mathbf{u} dA,$$
(1)

where  $\psi$  denotes the total energy density. The stain energy density

$$\psi^{e}(\varepsilon) = \frac{1}{2}\varepsilon : (\mathbb{C}\varepsilon)$$
<sup>(2)</sup>

models the energy stored inside of a body, where  $\mathbb{C}$  is the 4<sup>th</sup> order stiffness tensor and  $\varepsilon = \frac{1}{2}(\nabla \mathbf{u} + \nabla^{\mathsf{T}}\mathbf{u})$  is the infinitesimal strain tensor. The function g(s) is the degradation function, which models the loss of stiffness in the broken material. In this work, the degradation function is taken from [6] as  $g(s) = s^2$ . In Eq. (2), the parameter  $\eta$  describes a residual stiffness to avoid numerical difficulties.

The fracture surface energy density  $\psi^s$  denotes the energy required to separate the material and to generate cracks, which is assumed to be proportional to the crack surface. The formulation of the crack surface area is adapted from the work of Mumford and Shah in image processing [18]. Mumford and Shah proposed minimizing a functional to segment the image into nearly homogeneous regions separated by smooth boundaries. This functional captures the length of the boundary, the gradient of the image, and the image itself. As an analogy to this proposal, the surface density functional is given in relation to the gradient of the crack  $\nabla s$  and the crack *s* itself. Therefore, the fracture surface energy density is given as

$$\psi^{s}(s,\nabla s) = \mathcal{G}_{c}\left(\frac{(1-s)^{2}}{4\epsilon} + \epsilon |\nabla s|^{2}\right),\tag{3}$$

where the length parameter e controls the width of the smooth transition zone between the broken (s = 0) and undamaged material (s = 1). When the length parameter e goes to zero, the crack surface density functional approximates the crack surface area. The parameter  $G_c$  is the crack energy density, related to the fracture toughness. It is the ability of a material to resist fracturing.

The fatigue energy density

$$\psi^{\mathrm{ad}}(D) = q < D - D_c >^b \tag{4}$$

is introduced to account for the accumulated fatigue driving forces, which is associated with a fatigue damage param-

eter *D*. This parameter *D* is introduced to model the damage related to fatigue. Inspired by Miner's rule [19], the damage parameter will be accumulated during the entire simulation

$$D = D_0 + \mathrm{d}D. \tag{5}$$

The parameter  $D_0$  is the previous damage from the simulation step n and

$$dD = \frac{dN}{n_D} \left(\frac{\hat{\sigma}}{A_D}\right)^k \tag{6}$$

is the damage increment. The damage increment is associated with the cycle increment dN, which provides a potential to reduce the computational cost and simulation time by applying the adaptive cycle number adjustment algorithm [14]. The parameters  $n_D$ ,  $A_D$  and k are read from the Wöhler curve of experiments. Those parameters allow the phase model to incorporate the fatigue experiments directly, such that the information for complicated fatigue crack evolution can be integrated into a simple form. In the phase field model, the fatigue driving force is the first principal stress of the undegraded stress field  $\hat{\sigma} = [\mathbb{C}\epsilon]_1$ . Moreover, the mean stress corrector can be applied to include the mean stress effect in the fatigue crack propagation [13]. The parameter  $D_c$  is a damage threshold, which models the crack nucleation process since the fatigue cracks can occur only after a large number of cycles. The Macauley brackets

$$\langle x \rangle^n = \begin{cases} 0 & \text{if } x \le 0 \\ x^n & \text{if } x > 0, \end{cases}$$
(7)

enforces the fatigue energy to be zero when the damage parameter is smaller than this threshold  $D_c$ . After the damage parameter overcomes this threshold  $D_c$ , the numerical parameters q and b control how fast the fatigue energy grows. The function h(s) is, similarly, a degradation function modeling the loss of stiffness of the material due to cyclic fatigue. Fig. 1 reports different choices of the degradation functions on the fatigue crack growth rate. It is to be noted that the numerical stability is sensitive to the choice of the degradation function of the fatigue energy term. In this work, the degradation function h(s) is taken in agreement with g(s) because of its simple form and better numerical robustness.

Applying the variational principal on the total energy yields four coupled Euler-Lagrange equations

$$\operatorname{div}\frac{\partial\psi}{\partial\nabla\mathbf{u}} = 0 \tag{8}$$

$$\frac{\partial \psi}{\partial s} - \operatorname{div} \frac{\partial \psi}{\partial \nabla s} = 0 \tag{9}$$

$$\frac{\partial \psi}{\partial \nabla s} \cdot \mathbf{n} = 0 \quad \text{on } \partial \Omega_{\nabla s} \tag{10}$$

$$\left(\frac{\partial \psi}{\partial \nabla \mathbf{u}}\right)^T \cdot \mathbf{n} = \mathbf{t} \quad \text{on } \partial \Omega_{\mathbf{t}}.$$
(11)

Eq. (8) represents the equilibrium condition of the stress field

div
$$\sigma = 0$$
 with  $\frac{\partial \psi}{\partial \nabla u} = \sigma$ . (12)

Eq. (9) can be extended to a regularized form consistent with a mechanical view of the second law of thermodynam-



**FIGURE 1** Different choices of the degradation function on the crack growth rate.

ics [20], providing the evolution equation of the crack field

$$\frac{\mathrm{d}s}{\mathrm{d}N} = -M\frac{\delta\psi}{\delta s} = -M\left(\frac{\partial\psi}{\partial s} - \mathrm{div}\frac{\partial\psi}{\partial\nabla s}\right),\tag{13}$$

where M > 0 is a mobility parameter, which models the "viscosity" (rate dependency) of the phase field fracture model. The last two equations (Eq. (10) and (11)) are the Neumann boundary conditions for the crack field and the stress field.

#### 3 | PHASE FIELD FATIGUE SIMULATION OF ENGINEERING PROBLEMS

The fatigue test is a mechanical experiment to determine fatigue life. In the fatigue test, one side of the specimen is fixed, and the other side is applied with a periodic loading [21]. In order to compare materials in an adequate way, different cyclic loads are converted to respective stresses acting on the cross-section of the specimen. The respective stresses alternate sinusoidally around the mean stress  $\sigma_m$  with a stress amplitude  $\sigma_a$ .

Figure. 2 reports the fatigue life of different materials (a: AISI316L; b: Ti6Al4V; c: Al6061T6) using the phase field model. It should be noted that the tested materials possess to different fatigue strengths, which yield different fatigue behaviors until the material is broken. The specimen with Ti6Al4V appears to have a maximal fatigue strength among all samples, where it fails first after around 100,000 cycles under a stress amplitude of 505MPa. It is shown that the number of cycles to failure using the presented methods is slightly lower compared to the experimental data. For further investigations, Fig. 3 reports the fatigue life of AISI316L under different maximum stresses. The experiment and simulation are set up with a stress ratio of R = 0.1 and a loading frequency of 5Hz. It is noted that the results from the phase field model have a similar degradation tendency of fatigue life compared to the experiment. Moreover, it is shown that the results from the phase field simulation lie between the experiment data and the Basquin model. Generally speaking, the results obtained from phase field model yield a smaller number of cycles to failure compared



**FIGURE 2** Fatigue test using phase field model (**a**: AISI316L (236MPa); **b**: Ti6Al4V (505MPa); **c**: Al6061T6 (107MPa)).

to experiment data; however this provides a safe estimate of the lifetime of engineering and manufacturing processes.

#### 3.1 | Internal friction of the fatigue life

Internal friction happens inside of a deformed body, and it is the internal force resisting the movement between the particles making up a solid body. Internal friction can be influenced by the frequency of the cyclic loading and the thermal state of the loaded material [23] [15] [16]. Recently, Gräfe [15] proposed that there exists a relation between internal friction and fatigue of solids. On the one hand, the internal friction can be caused by the motion of defects by the stress field [16]. The dislocations of atoms in the solid destroy chemical bonds, which is shown macroscopically as the fatigue of material [16]. On the other hand, the temperature gradient, generated by the internal friction, can be also seen as a driving force for the movement of the unspecified defects in the solid. This kind of particle migration can also lead to the fatigue process of a solid. It is noted that the internal friction grows monotonously with the increase of the temperature or with the decrease of the frequency.

Differing from Gräfe's model, which microscopically explains the internal friction and the fatigue of solids, the phase field model provides a macroscopic procedure to incorporate the influences of internal friction into the fatigue process. Inspired by the Miner rule [19], where the fatigue failure is expected when the accumulated damages sum to 1. The phase field model introduces the damage threshold  $D_c$  to describe the fatigue damage initiation. The fatigue energy term takes contributions into the total energy only when the fatigue damage parameter D reaches the threshold  $D_c$ . The fatigue propagation behavior is mainly determined by the fatigue parameters k,  $n_D$ , and  $A_D$ 

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FIGURE 3 Fatigue life of AISI316L from simulation compared to experiment datas and Basquin-model [22].

obtained from the Wöhler curve of the additional fatigue energy term. The Wöhler curve describes the relationship between stress amplitude and the number of cycles to failure; thus, the effects of internal friction (temperature and frequency) are integrated within those parameters. In the following section, we will discuss how the loading frequency and temperature influence the fatigue behavior of the solids using the phase field model.

### 3.2 | Fatigue test under different loading frequency

In general, it is still unclear how the loading frequency affects the fatigue performance of all materials. Recent research [24] discusses the loading frequency and its influence on the dislocation movement, damage formation, and crack propagation for some specific metals. For those metals, the fatigue damage is triggered by the activation energy for the dislocation movement of the atoms in a solid. The fatigue crack evolution process involves cyclic plastic deformation of the material around the crack tip, and a critical shear stress is required for the plastic deformation. Furthermore, the shear stress can be sensitive to the strain rate, where high loading frequency may be attributed to the high strain rate of the material. For more details, see [25] [26].

We provide here a different way to model the loading frequency effect by the phase field method. The material of the phase field simulation is considered with low carbon steel S15C. The material fatigue properties are read from the Wöhler curve in the literature [27] [28] <sup>1</sup>

Fig. 4 shows the fatigue life and the crack propagation rate of low carbon steel with different loading frequencies.

<sup>1</sup>The fatigue limit  $A_D$  and the knee point cycle number  $n_D$  of 0.2Hz frequency loading are not provided by [28].

Material property	Value					
Modulus of Elasticity E	207GPa					
Poissons ratio v	0.285					
Fracture toughness $K_I$	75 MPa/m <sup>2</sup>					
Critical energy release rate $\mathcal{G}_c$	29576 N/m <sup>2</sup>					
Test frequency	0.2Hz	2Hz	20Hz	140Hz	20kHz	
Fatigue limit A <sub>D</sub>	(185MPa)	185 MPa	185 MPa	200 MPa	248 MPa	
Knee point cycle number $n_D$	(5 · 10 <sup>5</sup> )	$9\cdot 10^5$ cycle	$1.1 \cdot 10^{6}$ cycle	$2 \cdot 10^6$ cycle	$6 \cdot 10^7$ cycle	
Slope Factor k	4.09	9.55	11.01	18.57	37.91	

It is shown that increasing the loading frequency, the material has a longer fatigue life. In contrast, simulations on CT-Specimen [29] show that the fatigue crack growth rate of low carbon steel decreases with higher loading frequency. Those results can be verified by the experiment findings in [27] [28].



FIGURE 4 Loading frequency influence on the a : fatigue life and b : crack growth rate

## 3.3 | Fatigue test under different loading temperatures

The difficulty to determine the temperature influence on fatigue is due to the complexity of the fatigue mechanism. Some semi-empirical approaches have been proposed to evaluate the effect of temperature on fatigue under some specific circumstances. Generally speaking, the effect of loading temperature on fatigue for polycrystalline materials is the result of the influence of temperature on the microcracks' origination and propagation. For more details, see [30]. Without exploring the details, the phase field model assumes fatigue parameters (k,  $n_D$ , and  $A_D$ ) cover all the environmental effects of the fatigue evolution. In the presented example, an Al6061T6 specimen is considered. The material fatigue properties are read from [31].

Material property	Value				
Test temperature	27° <i>C</i>	70° <i>C</i>	150° <i>C</i>	250° <i>C</i>	
Fatigue limit A <sub>D</sub>	100MPa	100 MPa	100 MPa	100 MPa	
Knee point cycle number $n_D$	$9\cdot 10^5$	1.1 · 10 <sup>6</sup> cycle	$2 \cdot 10^6$ cycle	$2.5 \cdot 10^{6}$ cycle	
Slope Factor k	9.57	7.99	7.64	7.44	



FIGURE 5 Loading temperature influence on the a : fatigue life and b : crack growth rate.

As shown in Fig. 5, the loading temperature has similar effects on both the fatigue life and the crack growth rate. Higher temperature allows for a higher fatigue life of the aluminum alloy; whereas for a given loading of 125MPa, the aluminum specimen breaks first after 23,935 number of cycles at  $27^{\circ}C$  but after 1,477,226 number of cycles at  $270^{\circ}C$ . It is also interesting to notice that loading temperature has a positive correlation with the crack growth rate with simulations on CT-specimen: a high temperature can accelerate the speed of the crack propagation. Similar fatigue behavior of aluminum alloy can be found in the literature [31][32].

# 4 | A PHASE FIELD MODEL FOR THERMAL FATIGUE FRACTURE

When a material is under rapidly alternating heating and cooling, the temperature of the material's surface and interior will be different, leading to expansion or contraction of the material. This localized deformation of the material generates thermal stresses. In general, compressive stresses are produced when the loading process is at high temperatures; alternatively, tensile stresses are occuring when the material becomes cool [33]. This cyclic expansion and contraction of material causes material fatigue. In order to include the thermal aspect of in the phase field model, instead of using Eq. (2), the strain energy density is given by an modified term for thermal stress

$$\psi^{e} = \frac{1}{2}(\varepsilon - \varepsilon_{\mathsf{T}}) : \mathbb{C}(\varepsilon - \varepsilon_{\mathsf{T}}), \tag{14}$$

where  $\varepsilon_{T}$  is the thermal strain, which can be calculated by [34]

$$\varepsilon_{\rm T} = \kappa 1 \Delta T$$
. (15)

The constant  $\kappa$  is the thermal expansion coefficient, which describes the size of material expansion in relation to a temperature change. The tensor 1 denotes the second order unit tensor and  $\Delta T$  is the temperature difference to a reference temperature.

With the help of the variational principle, minimizing the total energy  $\psi$  yields Eq. (8) to Eq. (11) to describe the thermal fatigue problem. Now the stress field with the thermal effect taken into account is given by

$$\sigma = \frac{\delta \psi}{\delta \nabla \mathbf{u}} = (g(s) + \eta) \mathbb{C}(\varepsilon - \varepsilon_{\mathsf{T}}), \tag{16}$$

and the fatigue driving force is taken as the first principal stress of the sum of mechanical stresses and thermal stresses, i.e.

$$\hat{\sigma} = \sigma_1 = [\mathbb{C}(\varepsilon - \varepsilon_{\mathsf{T}})]_1. \tag{17}$$

It is to be noted that a positive temperature difference  $(T_{high} - T_{low})$  for a single heating process generates compressive stresses in the body, which will not lead to a fracture for many material [8]. However, in a cyclic thermal loading scenario, the material is subjected to repeated heating and cooling. As the temperature increases, the material becomes more ductile [35]; when it cools down, the thermal tensile stresses are developed and lead to thermal fatigue. For the phase field model, each single fluctuating temperature loading is not explicitly modeled; instead, the temperature change is taken as the difference between the enveloping high- and low temperatures, and several temperature loads are bundled into one simulation step.

$$\Delta T = T_{\rm low} - T_{\rm high}.$$
 (18)

However, it is not claimed that this model holds for any thermal boundary conditions. Our model is suitable for a thermal fatigue problem with an inhomogeneous temperature distribution over the body. Figure 6 displays how the crack propagates under a point thermal source. In this numerical example, a regular quadrilateral geometry is given, where the upper surface is loaded with uniform stresses of frequency 200Hz and the bottom is fixed. In addition, a point of fluctuating thermal source with a frequency of 0.2Hz is applied in the middle of the left side of the square geometry, providing continuous heating and cooling in the specimen. The results show that the first crack initialized after 4,610 cycles of thermal loading, and then the crack propagated in a straight line horizontally to the right. For further investigation of the crack evolution behavior, the magnitude of the fatigue driving force  $\hat{\sigma}$ , the mechanical stress  $|C\sigma|$  and the thermal stress  $|C\varepsilon_{T}|$  on the line of the crack propagation direction are provided. At the cycle  $N_1$ , where the cracks are still in nucleation and not macroscopically visible, the mechanical stress is at almost zero along the evaluated line. However, the thermal stress peaks at the left side of the line (x = 0), which provides the

fatigue driving force for the crack nucleation. At later cycles  $N_2$  and  $N_3$ , the mechanical stresses can be found to increase with the material breaking; at the same time, a small mechanical stress impulse is observed at the crack fronts, which is the driving force for the crack extending. On the other hand, the thermal stress remains unaltered during the simulation, because the heat transfer is not considered in the presented framework. Thermal point sources in different temperature gradients are used with the same numerical setup for further investigation of the effect of thermal stress. Fig. 7a shows that the thermal source with lower temperature gradient results in a longer fatigue life, where as a temperature difference  $\Delta T = 500$ K requires nearly 35% more number of cycles then  $\Delta T = 800$ K to break the specimen. Although the temperature gradients are different, it is shown in Fig. 7b that the crack growth rate in those thermal conditions remains almost the same. It can be concluded that the thermal stress is mainly responsible for the fatigue crack nucleation due to the high local stresses and the mechanical load is the driving force for the crack extension due to the global action of this load.

Thermal expansion coefficient $\kappa$	1e-6 K <sup>-1</sup>
Temperature difference $ \Delta T $	500 K

#### 5 | CONCLUSION

In this work, challenging fatigue problems are simulated by the phase field method. The phase field model introduces a scalar variable to represent the crack state. Two coupled equations for the displace field and the crack field are derived to describe the crack evolution. In the phase field fatigue model, an additional energy term is introduced to represent the accumulated fatigue driving force related to fatigue damage. In addition, a damage parameter, which will be accumulated during the simulation, combines the entire external influences on the fatigue crack evolution. Studies show that fatigue failure can be influenced by the internal friction of the material, where the fatigue processes can be microscopically interpreted as the migration of interstitial particles [15] [16]. The loading frequency and loading temperature influence the internal friction of the solid. It is shown that internal friction could be generally considered in the phase field simulation without further modification, where a similar tendency of fatigue life and crack growth rate compared to the experiment is observed. The fatigue cracks can also be a result of fluctuating temperatures, known as the thermal fatigue phenomenon. Another important contribution of this work is the consideration of the thermal stresses can be seen as a second fatigue driving force for the fatigue crack evolution, where higher temperature differences lead to a shorter fatigue life of the material. In future work, heat transfer will be considered in the thermal fatigue phase field method for modeling e.g. problems in manufacturing processes more realistically.

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**FIGURE 6** A example of thermal fatigue with the stresses development (**a**: driving force; **b**: mechanical stress; **c**: thermal stress) during the crack propagation.

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**FIGURE 7** Different temperature gradients differences results in different fatigue life (in **a**), but the crack growth rate remains almost the same (in **b**).

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