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Abstract

Sea waves represents a very promising energy source. Pioneered by Wiercigroch a pendulum system would be feasible for such an energy harvesting purpose. These devices consist of a pendulum with a vertical motion induced by the sea waves. As it is well known, pendulum's stable rotations generate enough energy able to be extracted by an electrical generator attached to its axis. In this paper, using a brushless dc motor as an input torque u(t) to maintain stable pendulum rotations, a singular optimal control formalism provides a very simple control law using a mechanical model.

Besides the main objective in this paper: stable controlled rotations, in order to control every motion possibility: rotation, stability or even chaos, a singular optimal control policy is defined

A salient property of the the resulting control law lies on its very simple hardware implementation bang-bang control: only sign is needed using Arduino and operational amplifiers (see Figure 1). While Matlab/Simulink simulations will be presented, the possibility of being tuned for stable/unstable rotations or even asymptotic stability is also an interesting analysis. Conclusions and future work are also depicted.

ENERGY HARVESTING FROM A ROTATING PARAMETRIC PENDULUM: SINGULAR OPTIMAL CONTROL

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ABSTRACT

Sea waves represents a very promising energy source. Pioneered by Wiercigroch a pendulum system would be feasible for such an energy harvesting purpose (see for instance[1] and [2]).

These devices consist of a pendulum with a vertical motion induced by the sea waves. As it is well known, pendulum's stable rotations generate enough energy able to be extracted by an electrical generator attached to its axis [2].

In this paper, using a brushless dc motor as an input torque $u(t)$ to maintain stable pendulum rotations, following the ideas implemented in [3], a singular optimal control formalism provides a very simple control law using the mechanical model given by equation (1):

$$\ddot{\theta}(t) + \beta \cdot \dot{\theta}(t) + \left(R \cdot cos(\omega \cdot t) + \lambda \cdot R \cdot \frac{\Lambda_3}{\Lambda_1^3} + \lambda \cdot \frac{\Lambda_2}{\Lambda_1} + 1 \right) + u(t) = 0 \tag{1}$$

Besides the main objective in this paper: stable controlled rotations, in order to control every motion possibility: rotation, stability or even chaos, a singular optimal control policy is defined given in equation (2) rewriting in state-space form equation (1):

$$min_{u \in U} \quad \frac{1}{2}\left(\dot{\theta}(t) - \phi(t)\right)^2 \tag{2}$$

such that:

$$\dot{X}(t) = \begin{bmatrix} x_2(t) \\ h(x_1, x_2, t) \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} \cdot u(t)$$

where $x_1 = \theta(t), x_2(t) = \dot{\theta}(t)$ and $\phi(t)$ acts as a set-point (desired pendulum's trajectory). Notice that tuning the function $\phi(t)$, different controlled behaviours for the pendulum's orbits can be achieved.

Then, Pontryagin's principle solve this singular optimal control problem, providing a very simple controller in equation (3):

$$u(t) = -K \cdot sign\left(\dot{\theta}(t) - \phi(t)\right) \tag{3}$$

with $K \in \Re^+$ and arbitrary constant and $sign(.)$ the classic sign function. A salient property of the the resulting control law lies on its very simple hardware implementation bang-bang control: only sign is needed using Arduino and operational amplifiers (see Figure 1).

While Matlab/Simulink simulations will be presented, the possibility of being tuned for stable/unstable rotations or even asymptotic stability is also an interesting analysis. Conclusions and future work are also depicted.

Keywords: Parametric pendulum, Singular optimal control, Energy harvesting

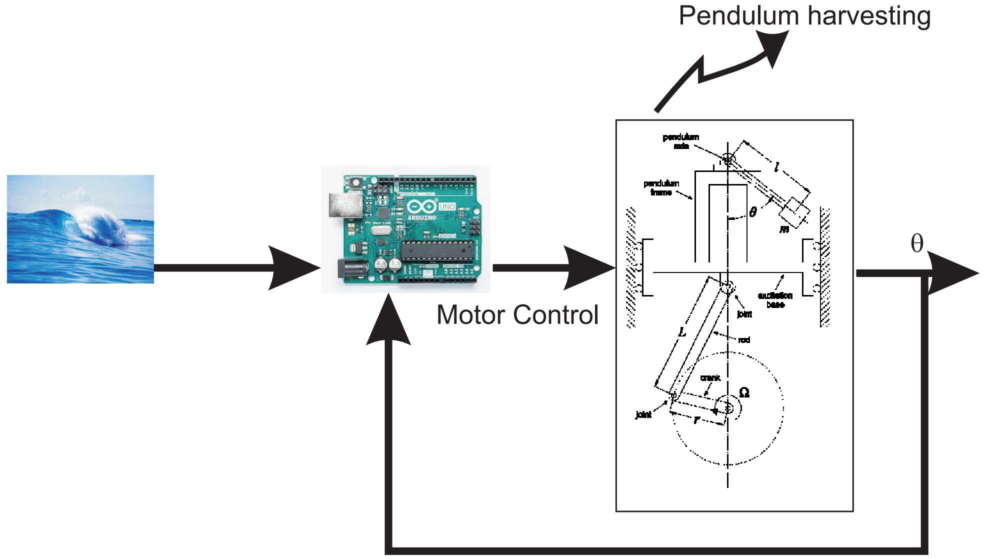

Figure 1: Matlab-Simulink model with singular optimal control

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