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Abstract

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Reviewing The Lorentz Factor In Terms of Heisenberg's Uncertainty Principle: The Uniqueness of The Velocity Vector and Time Perception

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1- Introduction

The distance change between two objects in Space can be determined by multiplying their relative speed and the time (duration) they experience together.

$$\Delta\text{distance} = v_{relative} \times \Delta\text{time}$$

$$\Delta x = v_{relative} \times \Delta t$$

If the distance equation above and Heisenberg's Uncertainty Principle are put together side by side:

$$\Delta x = v_{relative} \times \Delta t \qquad \qquad \Delta p \times \Delta x \geq \hbar / 2$$

According to Heisenberg's Uncertainty Principle, Δx can not be zero. Therefore in the distance equation neither $v_{relative}$ nor Δt can be zero. If $v_{relative}$ can not be zero, the Lorentz Factor (γ) between any two objects can never be equal to 1. This can be generalized as:

"" At any given moment, on any chosen reference frame, the velocity vector of an object is unique in whole Universe. The relative speed between any two objects can never be zero. ""

Otherwise, if the velocity vectors ($\vec{v}_{object}$) of any two objects were the same with respect to a chosen reference frame, the relative speed between these two objects would have to be zero. And this violates the Heisenberg's Uncertainty Principle. If an object is chosen to be the center of the reference frame, all other objects in whole Universe must have unique relative velocity vectors with respect to this chosen object. Otherwise, the Uncertainty Principle is violated again. Assume three objects moving in Space as shown in Figure-1.1:

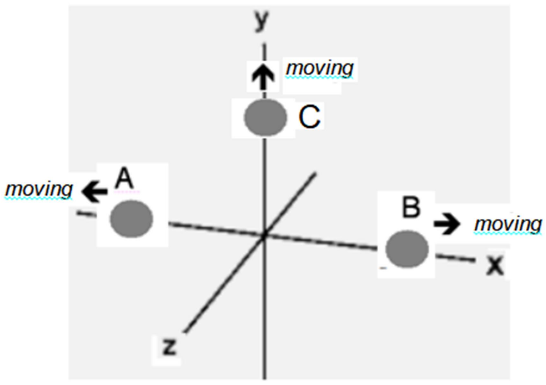

Figure-1.1: Three Objects Moving in Space

At any given moment:

i. None of the objects can have exactly the same velocity vector (in terms of magnitude and direction) with respect to this reference frame.
ii. None of the relative speeds between any two of A, B or C (v_{AB},v_{AC},v_{AC}) can be zero.
iii. In terms of speed; v_{AC} can be equal to v_{BC}. But in terms of velocity; $\vec{v}_{AC}$ can not be equal to $\vec{v}_{BC}$ (where v_{AC} is the relative speed of A with respect to C and $\vec{v}_{AC}$ İs the relative velocity of A with respect to C).

The Lorentz Factor (γ) is determined with respect to the relative speed between two objects:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Equation-1.1: The Lorentz Factor

Since the relative speed between any two objects can not be zero at any given moment, the Lorentz Factor (γ) can never be equal to 1.

The two Δx values in the distance and uncertainty equations above may be claimed to be different quantities. But Heisenberg's Uncertainty Principle is not just about the preciseness of a measurement but it reflects the physical reality. According to the principle, for a particular object Δx in Space can not be zero. Since Δx is a relative quantity defined with respect to a second object (reference frame) in Space which is assumed to be stationary, then the two Δx values in the equations above are the same quantities. So, the conclusions above are direct results of Heisenberg's Uncertainty Principle.

Moreover, since momentum is "p = mv", Δp becomes $m_x\Delta v$. Thus, according to Heisenberg's Uncertainty Principle not only the relative speed but the relative acceleration ($a_{relative}$) also can not be zero.

At any moment, for two different objects in Space where one of them is chosen to be the center of the reference frame:

1) $\vec{v}_{relative}$ is unique.
2) $0 < v_{relative} < c$
3) $1 < \gamma < \infty$
4) $a_{relative} \neq 0$
5) $\vec{a}_{relative}$ is unique.
6) The term "object" refers to a single fermion or a body that is made up of fermions.

And the two objects observe the relative speed between each other the same. Time dilation or length contraction phenomenons do not change this fact. But if this is the case, they can not observe their relative acceleration the same because of time dilation. They can not agree on the relative speed and the relative acceleration at the same time if time dilation is a reality.

According to the Special Relativity Theory of Einstein, γ is both the time dilation and the length contraction rate between two bodies:

$$\gamma = \frac{t_{stationary}}{t_{moving}} \quad \text{(time dilation)}$$

$$\gamma = \frac{L_{stationary}}{L_{moving}} \quad \text{(length contraction)}$$

Therefore, another conclusion can be derived: The time perceptions of any two objects must always be different with respect to each other because γ can not be equal to 1. Even two quarks in a proton can never experience the time similarly. The time perceptions of different objects get closer to each other as their relative speed decreases but can never be exactly the same because their relative speed can never be 0.

These conclusions have a significant effect on our understanding of physical reality, especally on our comprehension about the Special and General Relativity Theories of Albert Einstein. Big masses (like planets, stars, black holes) do not create a time flow rate field (time flow rate gradients) around themselves. But this does not mean that they do not effect the time perception of other objects via their gravity. They effect the time perceptions of other objects by changing the speed and acceleration of objects but not by creating time flow rate gradients in Space around.

Assume, two stationary electrons (or bowling balls) equally distant away from the center of mass of Earth: The current common understanding of relativity even among many Physicists states that these two electrons should experience the time similarly because their masses are equal, they are stationary with respect to each other and their position in Earth's gravitational field are the same. But according to Heisenberg's Uncertainty Principle as shown aoove, this understanding does not reflect the whole reality: Firstly, these two electrons can not be stationary wirth respect to each other or with respect to Earth; there must always be a non-zero relative velocity (or speed) in between. Secondly, the time experienced by these three objects (2 electrons and the Earth) has to be different because the time dilation ratio $\gamma \neq 1$ for any two of them. Therefore, the time perception of an object is not just relative but unique with respect to all other objects in Universe.

It is well known and experimentally observed that time flows faster for satellites around the Earth and time corrections are taken into account in GPS calculations. This is true but not the whole truth. The time experienced between two given moments by even any two fermions of the clock in the Satellite can not be exactly the same.

To express more precisely; according to Heisenberg's Uncertainty principle not only the relative speed but also the relative acceleration between two objects must be nonzero; therefore the Lorentz Factor (γ) also has to change with time. But for simplicity if it is assumed to be constant between two moments, then the two objects can not experience exactly the same amount of time between these moments.

Since the relative acceleration must also be nonzero; if düe to the acceleration or deceleration of these two objects, the Lorentz Factor may change momentarily in such a way that the accumulated amount of time experienced by the first object may become equal to the accumulated amount of time experienced by the second object between two moments. But this does not change the fact that their time perception rates can not be equal at any moment because γ can never be equal to 1.

Exactly the same conclusions can be re-drawn in terms of the distance (length between two points in Space) perception of an object since according to relativity the length conrtaction is defined as $L_{\text{moving}} = L_{\text{stationary}} / \gamma$.

2- Results In Terms of Energy and Mass

Similar results can also be obtained in terms of energy and mass concepts: Thanks to Albert Einstein, the total energy content (mass + kinetic energy) of objects can be determined by:

$$E = \gamma mc^2 \qquad \text{(Equation-2.1: Relativistic Energy)}$$

"$E = \gamma mc^2$" is a relative energy definition because it includes γ (Lorentz Factor) which means it determines the energy of an object with respect to a second reference frame (second object) in Space.

Assume, there are two objects A and B with masses m_A and m_B in Space with a relative velocity of $\vec{v}_{\text{relative}}$ (or relative speed v_{relative}) in between. Both can claim to be stationary and the other to be moving:

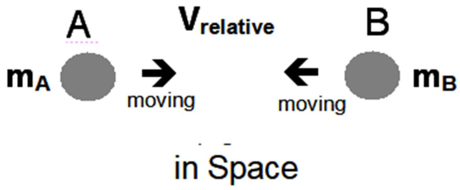

Figure-2:1: Energy and Mass

With respect to A, energy of B is equal to $E_B = \gamma m_B c^2$ and the total energy of this 2-object system is:
$E_{\_of\_the\_system\_withrespectto\_A} = E_{SA} = \gamma m_B c^2 + m_A c^2 = c^2 (\gamma m_B + m_A)$

With respect to B, energy of A is equal to $E_A = \gamma m_A c^2$ and the total energy of the same 2-object system:
$E_{\_of\_the\_system\_withrespectto\_B} = E_{SB} = \gamma m_A c^2 + m_B c^2 = c^2 (\gamma m_A + m_B)$

If $m_A \neq m_B$ or $\gamma \neq 1$ (the Lorentz Factor can not be 1), they will not be able to agree on the total energy of the system. By leaving γ alone in both equations:

$$\gamma = \frac{\frac{E_{SA}}{c^2} - m_A}{m_B} \quad \text{and} \quad \gamma = \frac{\frac{E_{SB}}{c^2} - m_B}{m_A}$$

Objects can not agree on the total energy of the system but they would agree on the relative speed in between, hence on γ. If even $m_A = m_B$ (like two electrons, two quarks etc), they can not observe each others energy to be exactly the same because $\gamma \neq 1$. In other words, E_A and E_B can not be equal with respect to each other. Even two electrons in an atom or any two electrons in whole Universe can not have equal amount of total energy with respect to each other.

This result takes us to Pauli's Exclusion Principle. Two electrons in an atom can not have the same energy state. Since Pauli's Exclusion Principle is not just a proposal but a firm principle of Physics, the conclusions expressed above are also solid.

Actually, not only two electrons in an atom but any two electrons in whole Universe or not only two quarks in a proton but any two quarks in whole Universe can not have exactly same energy state with respect to each other. This is also valid for any fermions or bigger objects. Therefore the results obtained in section-1 and 2 can be expressed together as a general exlusion principle:

The General Exclusion Principle:

At any moment, for two different objects in Space where one of them is chosen to be the center of the reference frame:

1) $\vec{v}_{relative}$ is unique.
2) $0 < v_{relative} < c$
3) $1 < \gamma < \infty$
4) $a_{relative} \neq 0$
5) $\vec{a}_{relative}$ is unique.
6) Any two objects in Universe can not have the same total energy with respect to each other at any given moment.
7) $\Delta E_{object} \neq 0$ since the relative acceleration can not be zero.
8) Both the time perception and the distance perception of an object are unique with respect to all other objects in Universe at a given moment since $v_{relative} \neq 0$ and $\gamma \neq 1$.
9) The term "object" refers to a single fermion or a body that is made up of fermions.

The results can also be summarized as: (assuming $t_A > t_B$)

$$\gamma = \frac{\frac{E_{SA}}{c^2} - m_A}{m_B} = \frac{\frac{E_{SB}}{c^2} - m_B}{m_A} = \frac{t_A}{t_B}$$

Equation-2.1: Time Dilation vs γ vs Energy Relation

3- Results In Terms of The Attributes of An Object

In modern Physics, an object has 3 fundamental attributes: total energy, charge and spin. All of them are scalar quantities. The charge and spin attributes are absolute; they are not relative to a second object or reference frame. And these two attributes are not unique to the object. The total energy attribute of an object includes its mass ($E = mc^2$) and kinetic energy. It is neither unique nor absolute. It is a relative quantity which is determined with respect to a reference frame in Space by using Einstein's $E = \gamma mc^2$ equation. As a result, these three attributes together are not enough to define an object uniquely in whole Universe.

As shown in the preceding sections, at any moment, on any chosen reference frame, the velocity vector ($\vec{v}_{object}$) of an object is unique. The total energy attribute includes its relative speed in the kinetic energy component. Therefore, if we define the total energy attribute (E_{object}) of an object as a vector ($\vec{E}_{object}$) in the same direction with its velocity vector, it turns into a unique attribute with respect to the chosen reference frame. $\vec{E}_{object}$ is unique for any chosen reference frame (because $a_{relative} \neq 0$ for any two objects) but its value is still relative to the reference.

As a discussion on the attributes of objects; since the total energy vector ($\vec{E}_{object}$) is not only unique for a particular reference frame but for any chosen reference frame in whole Universe, there may be a more fundamental vectoreal attribute of the object which makes its velocity vector ($\vec{v}_{object}$, hence $\vec{E}_{object}$) unique on any reference frame. This proposed attribute is supposed to be a vector, defines the object uniquely in Universe and its magnitude must be related to the total energy (mass + kinetic energy) of the object at a given moment. It must be absolute for any reference frame. Such a more fundamental attribute may also resolve the symmetry problem in Twin Paradox of Special Relativity which is still not clarified satisfactorily even by Albert Einstein.

Assume, there are two free electrons scattering away from each other due to electrical repulsion as shown in figüre-3.1 below:

Figure-3.1: Two Electrons

The conditions are totally symmetric for these two electrons. The relative speed and distance between them will increase by the time. Therefore, the Lorentz Factor (γ) can never be 1. Then, which one of these two electrons will experience less time than the other between two moments according to Special or General Theory of Relativity? The question is valid not just for two fermions but for any two bodies. If they experience the time similarly, should the increasing relative speed (hence increasing γ) between them be ignored?

In the scattering electrons scenario, these questions can be resolved by taking the continuous boson exchange between the two electrons into account. The total energy of these two electrons can never be equal at any moment due to general exclusion principle. The boson (photon) exchange will sustatin the inequality. The continuous boson exchange may change the total energy of these two electrons momentarily, hence the their acceleration, the relative speed and the Lorentz Factor.

As a prediction for the particular example, the continuous boson (photon in this case) exchange may determine the total energy attributes of these two electrons (hence γ) momentarily in such a way that the accumulated time experienced by these two electrons will become the same during the scattering movement although γ≠1 at any moment. In some segments of the movement electron-1 will experience less time than electron-2 while in some other segments electron-2 will experience less time than electron-1; the accumulated amount of time for electron-1 and the accumulated time for electron-2 will balance each other and be equal.

4- Results In Terms of Volume

In Physics, mass without a volume is impossible and it is called the “singularity” as a concept. It is known that a significant percentage of the volume of a proton is empty space; the significant percentage of the volume of an atom or a massive object is also free space. But a proton can not pass through another proton or one can not pass his/her hand through a wall because of exclusion.

As a thought experiment, assume there are two objects A and B again with masses m_A and m_B in Space with a relative speed of $v_{relative}$ in between. But this time, these objects both have zero volume. They are singularities; volumeless but massive points in Space:

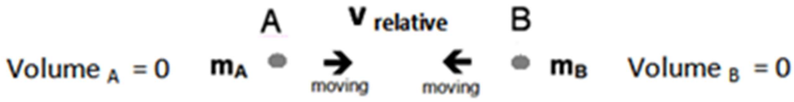

Figure-4:1: Objects without Volume

They are point masses without volume, so the center of mass of this 2-object system can not be within any of these objects. It must be somewhere in Space between A and B.

Since the relative speed between these two singular objects can not be zero and therefore γ can not be 1 according to general exclusion principle defined in section-2, there is no any kind of force or interaction in Universe (graivty, EM, strong force, weak force) that can force them to touch or stick to each other. There must always be a non-zero distance and non-zero $v_{relative}$ between A and B. Nothing can turn these two singularities into a single singularity. And, A and B can not have the same time perception since γ ≠ 1.

Discussing from this point of view, elementary fermions like quarks, electrons, neutrinos do not really have to occupy any volume in Space to show the characteristics they present in scientific observations. They may really be singularities. But it may be the general exclusion principle which creates the perception of volume for fermions and as a result for all massive objects. Singularity, mass without a volume is not impossible in terms of general exclusion principle. And considering the fermions as volumeless but massive singularities makes it easier to comprehend the particle-wave duality. Otherwise, it is harder to comprehend a mass with a volüme to be a volumeless wave at the same time. This may also explain why an electron can not stick to a proton in an atom although there are both electrical and gravitational attraction in between.

5- Discussion In Terms of The Nature of Light

For an object, existing without a time perception is impossible. It has to experience time between moments. But this is not the case for bosons like photons. Light, comprised of photons, does not experience time since the Lorentz Factor (γ) goes to infinity (∞) for the speed of c. In other words, time does not flow for light, so its time perception between any two events (moments) is always zero.

If light has no time perception, how long would it take for it to travel from one edge of the Universe to the other?

Having a zero time perception means that as soon as the light is generated, it is immediately in every point of the whole Universe, resonating with its frequency f. Since light is unaware of time, it does not take any time for it to spread out to whole Universe. Here, the first event (moment) is generation of light and the second event (moment) is the light beam reaching to the furthest point in Universe. Light does not experience any time between these two events.

At first glance, this may seem controversial and in total contradiction with current understanding. Then, why do we wait for millions of years for a light beam from a star to reach Earth? Why do not we see it immediately? Before discussing these questions, please recall equation-2.1 which associates γ (Lorentz Factor) with time dilation rate and determine the relative speed of light with respect to an arbitrary object-A in Space:

t_A → time experienced by A: $t_A > 0$
t_{light} → time experienced by the light: $t_{light} = 0$

$$\gamma = \text{ time dliation } = \frac{t_A}{t_{light}} = \frac{1}{\sqrt{1 - \frac{v_{relative}^2}{c^2}}}$$

$$\gamma = \frac{t_A}{0} = \frac{1}{\sqrt{1 - \frac{v_{relative}^2}{c^2}}} \quad \text{and then,} \quad \gamma = \infty = \frac{1}{\sqrt{1 - \frac{v_{relative}^2}{c^2}}}$$

$$v_{relative} = c$$

This unsurprising result means, any object in Space has to observe the relative speed of light as c in any direction which was stated by Maxwell and Einstein. We and all other objects in Space observe the relative speed of light as c but it is not the light but objects themselves who are travelling through time and space and experiencing duration between moments. In contrast, light does not travel in time. It is immediately in every point of Universe as soon as it is generated. But, we have to travel in time (experience time) to be able to see the light beam coming from a star. From light's perspective, there is no time between the two events (moments) where event-1 is leaving the star and event-2 is reaching the Earth. But for objects, there may be millions of years of time between event-1 and event-2. That is why we can not see the light immediately, although it spreads out to every point of whole Universe within no time following its generation.

As an analogy, think of a diver in the depths of a light ocean. She perceives light coming to her and going away from her with a relative speed of c in any direction. But actually, it is not the light but the diver herself who is moving through the Ocean with a speed of c with respect to the Ocean itself.

As shown in the preceding sections, time is not a flowing river behind the scene of the Universe or a 4th dimension of Spacetime. It is a scalar quantity, experienced by every object uniquely. So it is intuitive to consider that time experienced between two events (moments) may be different with respect to different objects or entities.

From this point of view, the absolute speed of light with respect to Space itself is infinite ($v_{light\_absolute} = \infty$) and the absolute speed of a fermion or an object with respect to Space itself is c in any direction ($v_{object\_absolute} = c$, 3×10^8 m/sec approximately). But objects are unaware of their absolute speeds; they can only observe the relative velocities between the objects. This point is discussed further in the next section.

Can this way of thinking provide an improvement in understanding the nature of light or its behaviour in particular circumstances?

Firstly, the "absolute speed of a fermion or object with respect to Space itself is c in any direction" statement is compatible with Einstein's "$E = mc^2$" equation which determines the rest mass energy of an object. If the absolute speed of an object is c with respect to Space itself and it is unaware of this speed but perceives itself to be at rest, than its rest energy should be equal to $E = mc^2$.

Secondly, if the absolute speed of light with respect to Space itself is infinite as stated, the behaviour of light in the "Delayed Choice Experiment" can be comprehended:

The Behaviour of Light in The Delayed Choice Experiment:

In Wheeler's delayed choice experiment; light spreads out to whole Universe as soon as it is generated within no time and is immediately aware of both the screen and the detector placed far away from the screen. But only the observer and the detector (as objects made up of fermions) experience some amount of time between the moment (event-1) of generation of light and the moments light hits the detector (event-2) or screen (event-3).

From the perspective of light, there is no time betweeen the moment the first beam hitting the screen (event-2) and the moment the second beam hitting the detector (event-3). The two beams hit their targets as soon as they are first generated in zero time. Light does not experince any time either between event-1 and event-2 or event-1 and event-3 or event-2 and event-3. But the observer and the detector do. The observer measures different amounts of time between these events. There is not a causality problem in the delayed choice experiment. Events in the future can not effect events in the past. And the observed results can be predicted by the general exclusion principle as explained above.

Moreover, since the Lorentz Factor (γ) for light is infinite (∞), because of length contraction the distance perception of light must also be zero. Therefore, from the perspective of light, the Universe may be a timeless singularity. But that is not the case for fermions or objects that are made up of fermions.

To sum up the discussion:

(a) $t_{light} = 0$ *(time perception of light)*
(b) $L_{light} = 0$ *(distance perception of light)*
(c) $t_{object} \neq 0$ and $L_{object} \neq 0$
(d) $v_{light\_absolute} = \infty$ and $v_{object\_absolute} = c$ *(with respect to Space itself)*

6- Discussion On Fermions As Singularities In Terms of Particle-Wave Duality

In section-4, it is stated that considering fermions as singularities (volumeless point masses) makes it easier to comprehend the particle-wave duality and does not effect their characteristics. In section-5, it is stated that the absolute speed of a fermion with respect to Space itself is c in any direction and the absolute speed of light with respect to Space itself is infinite.

These two statements can be combined to understand the particle-wave duality and movement of a fermion in Space. Assume there are two neutrinos (A and B) moving with a relative speed of $v_{relative}$ with respect to each other. Assume, they are singularities (point particles) and waves at the same time as shown in figüre-6.1 below:

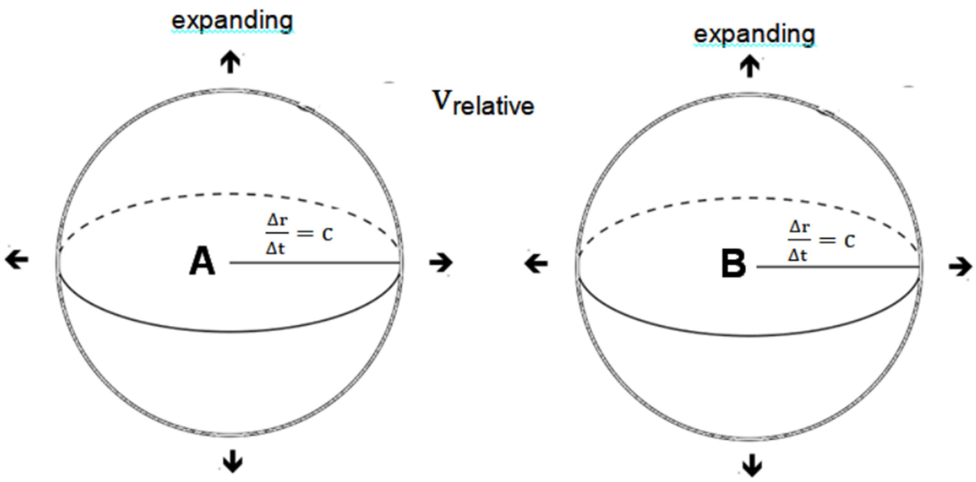

Figure-6.1: Two Neutrinos, both singularities and waves simultaneously

These neutrinos are particles as singularity points A and B respectively. And at the same time, they are waves propagating to the Universe as a 3-dimensional sphere with the speed of c. The expansion rate of their wave spheres is:

$$\frac{\Delta r}{\Delta t} = c \qquad \textit{(c = 3x10^8 m/sec approximately)}$$

Therefore, since light is immediately in every point of the Universe resonating with its frequency f, as the wave sphere of the neutrino expands with the speed of c, it experiences light coming from any direction into the sphere with the speed of c. And that is why fermions and other objects observe the speed of light as c in any direction although the absolute speed of light with respect to Space itself is actually infinite. On the other hand, as singularity points (particles), these two neutrinos move with a relative speed of $v_{relative}$ with respect the each other. The relative speed between point-A (neutrino-A) and point-B (neutrino-B) is $v_{relative}$. That is how an objects can both move with an absolute speed of c in any direction with respect to Space itself while moving with a relative speed of $v_{relative}$ with respect to another object at the same time and perceives the speed of light as c in any direction.

7- Conclusion

Heisenberg's Uncertainty Principle, the Lorentz Factor, Einstein's time dilation, length contraction, relativistic energy equations and the general exclusion principle introduced in this paper together may be the links between the Standard Model and Theory of Relativity. At a given moment the velocity vector of an object is unique with respect to any chosen reference frame. And the time perception of any object in Universe is also unique with respect to others. The comprehension about relativity, singularity, particle-wave duality and the nature of light may be refined and the general exclusion principle can provide the sort of advance in general understanding.

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