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A novel f(R)-form solution for coupled gravity

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When Λ is the cosmological constant concerning $(g^{\mu\nu})^2$[3, 10], we use the constructed new action to get the new cosmological constant and deduce a new form of f(R) using the coupled effects of gravity and quantum fields.

Keywords:f(R), coupled gravity ,cosmological constant

1. INTRODUCTION

In the case of minimum coupling, general relativity has a unique theoretical form, that is, its motion equation and the minimum coupling form of the action are unified, and the corresponding motion equation after the minimum coupling generalization of the equivalent action in flat spacetime is also the same. Interestingly, in the case of considering more degrees of freedom, the form of the theory of gravity becomes rich, and this richness mainly comes from two aspects: 1. The equation of motion and the action cannot take the minimum coupling form at the same time, 2. The equations of motion corresponding to the minimum coupling generalization of the equivalent action in flat space-time are inconsistent. From the perspective of gauge principle, this paper examines the uniqueness of the minimal coupling form of gravitational theory and discusses the limitation of Poincaré symmetry on gravitational theory. We find that the form of the gravitational theory is not unique under the constraints of gauge symmetry, and the method of taking the covariant derivative in the theory is still very arbitrary, that is, affine connection plus a suitable third-order tensor field is still an allowable affine contact.

When the gravitational field action of Poincaré gauge theory takes its simplest form, it degenerates to Einstein-Cartan theory. In the Einstein-Cartan theory, the introduction of more degrees of freedom results in the equation of motion, and the action cannot take the minimum coupling form at the same time. This paper discusses the modification scheme of Einstein-Cartan theory from three perspectives: modifying the integral measure, modifying the form of the covariant derivative, and modifying the space-time background, to coordinate the equation of motion and the form of action. On the other hand, unlike gauge transformations in field theory, Poincaré gauge transformations are essentially coordinate transformations. To keep the action specification unchanged, finding an integral measure whose coordinate transformation is invariant is necessary. This leads to the fact that the minimum coupling form of the material field is not unique, and there are two choices of the first-order derivative form and the second-order derivative form. In this paper, different gravitational theories are discussed based on the uniqueness of the least coupled form, and it is found that the least coupled form of Poincaré gauge theory and Einstein-Cartan theory is not unique; while general relativity, parallel gauge theory The equations of motion and actions of the (Parallel gauge theory) and the modified Einstein-Cartan theory can simultaneously take the unique minimal coupling form.

When Λ is the cosmological constant about $(g^{\mu\nu})^2$[3, 10].We use the constructed new action to obtain the structure of the new cosmological constant, and use the coupling effect of gravity and a quantum field to deduce the new form of f(R).

2. NEW GRAVITATIONAL COUPLING EQUATION

We can pre-set the boundary conditions $\mu = y\omega$[8, 9].

Spherical quantum solution in vacuum state.

In this theory, the general relativity theory's field equation is written completely.

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{8\pi G}{c^4}T_{\mu\nu} \tag{1}$$

The Ricci tensor is by $T_{\mu v} = 0$ in vacuum state.

$$R_{\mu v} = 0 \tag{2}$$

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The proper time of spherical coordinates is

$$d\tau^2 = A(t,r)dt^2 - \frac{1}{c^2}\left[B(t,r)dr^2 + r^2 d\theta^2 + r^2 \ \sin\theta d\phi^2\right] \tag{3}$$

If we use Eq, we obtain the Ricci-tensor equations.

$$R_{tt} = -\frac{A''}{2B} + \frac{A'B'}{4B^2} - \frac{A'}{Br} + \frac{A'^2}{4AB} + \frac{\ddot{B}}{2B} - \frac{\dot{B}^2}{4B^2} - \frac{\dot{A}\dot{B}}{4AB} = 0 \tag{4}$$

$$R_{rr} = \frac{A''}{2A} - \frac{A'^2}{4A^2} - \frac{A'B'}{4AB} - \frac{B'}{Br} - \frac{\ddot{B}}{2A} + \frac{\dot{A}\dot{B}}{4A^2} + \frac{\dot{B}^2}{4AB} = 0, \tag{5}$$

$$R_{\theta\theta} = -1 + \frac{1}{B} - \frac{rB'}{2B^2} + \frac{rA'}{2AB} = 0, R_{\phi\phi} = R_{\theta\theta}\sin^2\theta = 0, R_{tr} = -\frac{\dot{B}}{Br} = 0, R_{t\theta} = R_{t\phi} = R_{r\theta} = R_{r\phi} = R_{\theta\phi} = 0 \tag{6}$$

In this time, $\ ' = \frac{\partial}{\partial r} \quad , \cdot = \frac{1}{c}\frac{\partial}{\partial t},$

$$\dot{B} = 0 \tag{7}$$

We see that,

$$\frac{R_{tt}}{A} + \frac{R_{rr}}{B} = -\frac{1}{Br}\left(\frac{A'}{A} + \frac{B'}{B}\right) = -\frac{(AB)'}{rAB^2} = 0 \tag{8}$$

Hence, we obtain this result.

$$A = \frac{1}{B} \tag{9}$$

If,

$$R_{\theta\theta} = -1 + \frac{1}{B} - \frac{rB'}{2B^2} + \frac{rA'}{2AB} = -1 + \left(\frac{r}{B}\right)' = 0 \tag{10}$$

If we solve the Eq,

$$\frac{r}{B} = r + C \rightarrow \frac{1}{B} = 1 + \frac{C}{r} \tag{11}$$

When r tends to infinity, and we set C=ye^{-y}, Therefore,

$$A = \frac{1}{B} = 1 - \frac{y}{r}\Sigma, \Sigma = e^{-y} \tag{12}$$

$$d\tau^2 = \left(1 - \frac{y}{r}\sum\right)dt^2 \tag{13}$$

In this time, if particles' mass are m_i, the fusion energy is e,

$$E = Mc^2 = m_1c^2 + m_2c^2 + \ldots + m_nc^2 + T. \tag{14}$$

3. NEW CLASS OF ACTION AND FIELD EQUATIONS

The purpose of this theory is that we find that the modified Einstein equation of gravity has a coupled solution in a vacuum. First, we can consider the following equation (modified Einstein's equation of gravity).R1 is the curvature scalar of formula (3), and similarly defines $R1^{\mu\nu}$ and $R1_{\mu\nu}$. The proper time of spherical coordinates is[1–6](The metric in which is in exponential form)

$$ds^2 = -e^{G(t,r)}dt^2 + e^{-G(t,r)}dr^2 + \left[r^2 d\theta^2 + r^2\sin^2\theta d\varphi^2\right] \tag{15}$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda((g^{\mu\nu})^2)g_{\mu\nu} = -\frac{8\pi G}{C^4}T_{\mu v} \tag{16}$$

In this work, the action(we set $8G = c = 1$) is given by the following relation which in the special case,[10]

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}(-2\Lambda((g^{\mu\nu}) + R) \tag{17}$$

where Λ is a function of the Ricci scalar R, Φ is the representation of the inflation field, also analogous to f(R) (we will now consider the non-pathological functional form of $f(R)$) can be expanded in the form of Taylor series $f(R) = a_0 + R + a_2R^2 + a_3R^3 + \ldots a_nR^n + \ldots$ where we have normalized all coefficients according to the coefficients of the linear term). The following field equations are given for the action changes of the metric $g_{\mu\nu}$, the gauge A_μ and the expansion field Φ:

This leads to:[10, 11] In these equations we have:

$$\begin{aligned}
&\nabla_\mu\nabla^\mu\Lambda_R = \frac{1}{\sqrt{-g}}\partial_r\left(\sqrt{-g}\partial^r\right)\Lambda_R = \left(e^{G(t,r})'\Lambda_R' + e^{G(t,r)}\Lambda_R'' + \frac{e^{G(t,r)}}{r}\Lambda_R'\right)\\
&\nabla^t\nabla_t\Lambda_R = g^{tt}\left[(\Lambda_R)_{,t,t} - \Gamma_{tt}^m(\Lambda_R)_{,m}\right] = \frac{1}{2}(e^{G(t,r)})'\Lambda_R'\\
&\nabla^r\nabla_r\Lambda_R = g^{rr}\left[(\Lambda_R)_{,r,r} - \Gamma_{rr}^m(\Lambda_R)_{,m}\right] = \left(e^{G(t,r)}\Lambda_R'' + \frac{(e^{G(t,r)})'}{2}\Lambda_R'\right)\\
&\nabla^\theta\nabla_\theta\Lambda_R = g^{\theta\theta}\left[(\Lambda_R)_{,\theta,\theta} - \Gamma_{\theta\theta}^m(\Lambda_R)_{,m}\right] = \frac{e^{G(t,r)}}{r}\Lambda_R',
\end{aligned} \tag{18}$$

and

$$\left(R1^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R1\right)_{;\rho} = 0, \tag{19}$$

$$\begin{aligned}
&\left(R1_{\mu v} - \frac{1}{2}g_{\mu\nu}R1\right)\left(R1^{\mu v} - \frac{1}{2}g^{\mu v}R1\right)\\
&= R1_{\mu v}R1^{\mu v} - \frac{1}{2}g^{\mu v}R1_{\mu\nu}R1 - \frac{1}{2}g_{\mu\nu}R1^{\mu v}R1 + \frac{1}{4}g_{\mu\nu}g^{\mu v}R1^2, g^{\mu v}R1_{\mu v} = R1, g_{\mu v}R1^{\mu v} = R1, g_{\mu v}g^{\mu v} = 4\\
&= R1_{\mu v}R1^{\mu v} - \frac{1}{2}R1^2 - \frac{1}{2}R1^2 + R1^2 = R1_{\mu\nu}R1^{\mu\nu}
\end{aligned} \tag{20}$$

This leads to:

$$\Lambda = R1_{\mu\nu}R1^{\mu\nu}. \tag{21}$$

4. SUMMARY

When Λ is the cosmological constant about $(g^{\mu\nu})^2$[3, 10].We use the constructed new action to obtain the structure of the new cosmological constant, and use the coupling effect of gravity and a quantum field to deduce the new form of f(R).

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