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No phase transition for SU(2) black holes

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In this article, we fabricate a black hole metric that conforms to the SU(2) group structure and compute the scalar curvature of the thermodynamic geometry of this black hole. We conclude that there is no phase transition for SU(2) black holes.
Keywords : phase transition, SU(2) black hole, Chern-Simons theory

1. INTRODUCTION

The SU(2) linear non-autonomous quantum system refers to the linear functional whose Hamiltonian is the SU(2) generator and its superposition coefficient is related to time. It is a time-dependent quantum system with important practical value. One of the most important achievements of nonlinear science at present lies in understanding chaotic phenomena. Therefore, discussing the chaos of SU(2) linear non-autonomous quantum systems has certain theoretical significance and practical value.

The chaotic problem in SU(2) linear non-autonomous quantum systems is discussed using the SU(2) algebraic dynamics equation, and a very important and interesting result is found: Complementary chaos exists in $mathrmSU(2)$ linear non-autonomous quantum system, and the box dimension of the fractal graph is calculated.

Example: SU(2) group: $\mathrm{SU}(2)=\{J_0, J_+, J_-\}, \mathrm{n}=3, \mathrm{l}=1$. The group chain is $su(2) \supset u(1)$, CSCO II $=\hat{J}^2, \hat{J}_0$, their common eigenfunction y_{Jm} is the basis vector of the entire Hilbert space.

The Chern-Simons theory related to the group $SU(2)$ is a gauge theory for the three-dimensional manifold M governed by action[1]

$$S_k[A]=\frac{k}{4\pi}\int_M \left\langle A\wedge dA+\frac{2}{3}A\wedge A\wedge A\right\rangle \tag{1}$$

where k is called the level of the action, A is the local $SU(2)$-connection field, and $\langle\cdot,\cdot\rangle$ is the notation for $\mathfrak{su}(2)$ kill form. The Chern-Simons theory became very important when it was first shown to be closely related to gravity in three dimensions, especially when Witten demonstrated [16] its surprising relationship to manifold and knot invariants. The Chen-Simons path integral is manifold invariant, and the mean of quantum observables naturally leads to Jones polynomials. For all these reasons, Chen-Simons theory has been the center of much interest, and its quantification is now very famous when the gauge group is compact, especially when the gauge group is SU(2).

In this article, we fabricate a black hole metric that conforms to the SU(2) group structure and compute the scalar curvature of the thermodynamic geometry of this black hole. We conclude that there is no phase transition for SU(2) black holes.

2. SU(2) BLACK HOLE IN GENERAL FORM

SU(2) black hole metric in general form[1, 2]

$$\mathrm{d}s^2=-e^{2z_1}\mathrm{d}t^2+e^{-2z_1}\ \mathrm{d}r^2+r^2e^{2z_2}d\theta^2+r^2e^{-2z_2}\sin^2\theta d\varphi^2, \tag{2}$$

where z1 and z2 are complex numbers modulo 1 or less, they are both functions of entropy.

Convert the metric to a simple form by taking a specific value

$$\mathrm{d}s^2=-e^{2z_1}\mathrm{d}t^2+e^{-2z_1}\ \mathrm{d}r^2+e^{2z_2}d\theta^2+e^{-2z_2}d\varphi^2, \tag{3}$$

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The non-zero Christoffel symbols are

$$
\begin{aligned}
\Gamma^1_{00} &= {z_1}' e^{4z_1} \\
\Gamma^0_{10} &= {z_1}' \\
\Gamma^1_{11} &= -{z_1}' \\
\Gamma^2_{21} &= {z_2}' \\
\Gamma^1_{22} &= -{z_2}' e^{2(z_2+z_1)} \\
\Gamma^3_{31} &= -{z_2}' \\
\Gamma^1_{33} &= {z_2}' e^{2(z_1-z_2)}
\end{aligned}
\tag{4}
$$

The Ricci tensors are

$$
\begin{aligned}
R_{00} &= \left({z_1}'' + {z_1}'^2 - {z_1}'(-z_1)' + {z_1}'{z_2}' + {z_1}'(-z_2)'\right) e^{4(z_1)} \\
R_{11} &= {z_1}'(-z_1)' + {z_2}'(-z_1)' + -{z_2}'(-z_1)' - {z_2}'' - (-z_2)'' - {z_1}'^2 - {z_2}'^2 - (-z_2)'^2 - {z_1}'' \\
R_{22} &= \left(-{z_2}'' + {z_2}'(-z_1)' - {z_1}'(z_2)' - (-z_2)'{z_2}' - {z_2}'^2\right) e^{2(z_1+z_2)} \\
R_{33} &= \left(-(-z_2)'' + -{z_2}'(-z_1)' - {z_1}'(-z_2)' - {z_2}'(-z_2)' - (-z_2)'^2\right) e^{2(z_1-z_2)}
\end{aligned}
\tag{5}
$$

The Ricci scalar is

$$
\begin{aligned}
R = (&-2{z_1}'' - 2{z_1}'^2 + 2{z_1}'(-z_1)' - 2{z_1}'{z_2}' - 2{z_1}'(-z_2)' + 2{z_2}'(-z_1)' + 2 - {z_2}'(-z_1)' \\
&-2{z_2}'' - 2(-z_2)'' - 2{z_2}'^2 - 2(-z_2)'^2 - 2(-z_2)'{z_2}')e^{2z_1}
\end{aligned}
\tag{6}
$$

3. SUMMARY AND DISCUSSION

For z1 and z2 are functions of entropy, we perform a generalized thermodynamic geometric analysis on them and see that there is no divergence term for this curvature scalar.

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