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Einstein's Mass-Energy Equivalence and Relativistic Mass derived from Newton's Second Law of Motion

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Abstract: The Newton's second law of motion states that the force acting on a body is equal to the rate of change of its momentum. This paper presents the derivation of Einstein's mass-energy equivalence and relativistic mass from the Newton's second law of motion.

Keywords: differential equation**,** momentum, kinetic energy

1. Introduction

The equation of Einstein's mass-energy equivalence [1-3] is $E = mc^2$, where E, m, and c denote electromagnetic/light energy, mass of light, and speed of light respectively. The derivation of the equations of relativistic mass and mass-energy equivalence is obtained from the Newton's second law of motion by differentiation and integration [4].

2. Relativistic Mass

In the Einstein's theory of special relativity, relativistic mass is the mass, which is assigned to a body in motion. Let us derive the relativistic mass as follows:

The Newton's second law of motion states that the force (F) acting on a body is equal to the rate of change of its momentum (p).

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} + v\frac{dv}{dt}, \text{where } v \text{ is velocity.}$$

The differential equation for work and kinetic energy is derived as follows:

$$dK = dW = Fds$$

$$dK = Fds = \left(m\frac{dv}{dt} + v\frac{dv}{dt}\right)ds$$

$$dK = Fds = m\frac{ds}{dt}dv + v\frac{ds}{dt}dv, \text{where } \frac{ds}{dt} = v$$

$$dK = mvdv + v^2dm$$

Note that the term v^2dm allows the hypothesis of a variable mass as it actually occurs at high speed.

If, instead of the added kinetic energy dK the equivalent term of mass c^2dm is assigned, then the resulting differential equation is:

$$c^2dm = mvdv + v^2dm$$

$$\frac{dm}{m} = \frac{v}{c^2 - v^2} dv$$

$$\int_{m_0}^{m} \frac{dm}{m} = \int_0^v \frac{v}{c^2 - v^2} dv$$

$$[\ln(m)]_{m_0}^{m} = -\frac{1}{2} [\ln(c^2 - v^2)]_0^v$$

$$\ln m - \ln m_0 = -\frac{1}{2} \ln(c^2 - v^2) + \frac{1}{2} \ln c^2$$

$$\ln \frac{m}{m_0} = \frac{1}{2} \ln \frac{c^2}{c^2 - v^2}$$

$$\frac{m}{m_0} = \sqrt{\frac{c^2}{c^2 - v^2}}$$

$$\text{Relativistic mass } (m) = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Where,

- the rest mass of the body is m_0
- the velocity of the body in motion is v
- the speed of the light is c

Hence, the equations of relativistic mass are derived from Newton's Second Law of Motion.

3. Mass-Energy Equivalence

$$\text{Relativistic mass } (m) = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$m^2 = \frac{{m_0}^2}{1 - \frac{v^2}{c^2}} \Longrightarrow m^2(c^2 - v^2) = {m_0}^2 c^2, \text{where rest mass energy} = {m_0}^2 c^2$$

$$m^2 c^2 - m^2 v^2 = {m_0}^2 c^2$$

By differentiating the equation, we get

$$2mc^2 dm - 2mv^2 dm - m^2 2v dv = 0,$$

where the rest mass (m_0) and the speed of light (c) are constant.

We know that the kinetic energy (dK) is equivalent to the mass $c^2 dm$.

$$dK = c^2 dm$$

$$\int_0^K dK = \int_{m_0}^{m} c^2 dm$$

$$K = c^2(m - m_0), \text{where } K \text{ is kinetic energy.}$$

Total Energy (E) = Kinetic Energy (K) + Rest Mass-Energy (${m_0}^2c^2$)

$$E = c^2(m - m_0) + {m_0}^2c^2$$

$$E = c^2m - c^2m_0 + {m_0}^2c^2$$

$$E = c^2m$$

Hence, the equation of Einstein's mass-energy equivalence is derived from Newton's Second Law of Motion.

4. Conclusion

In the Einstein's theory of special relativity, mass-energy equivalence and relativistic mass play an important role. In this article, the equations of Einstein's mass-energy equivalence and relativistic mass have been derived from Newton's Second Law of Motion.

References

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