



# Theoretical value for the strong coupling constant

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1 January 2.022

## Abstract

In this paper we will study the strong coupling constant. The recommended theoretical value for the strong coupling constant is  $\alpha_s = \text{Euler's number} / \text{Gelfond's constant}$ . It will turn out that this value is the key that solves many problems of Physics. We will find a lot of formulas for the strong coupling constant. All these equations prove that the value of the strong coupling constant depends on the energy scale. First we will find the beautiful unity formulas that connect the strong coupling constant and the fine-structure constant. These equations are the simple unification of the strong nuclear and the electromagnetic interactions. It will be presented the mathematical formulas that connect the dimensionless physical constants with the strong coupling constant. From these equations we reached the formula of the unification of the strong nuclear, the electromagnetic and the gravitational interactions. Also we will find the formula for the Gravitational constant. Finally we will be presented the formula for the cosmological constant. This unity formula is a simple analogy between atomic physics and cosmology. All these equations are simple, elegant and symmetrical in a great physical meaning.

## Keywords

Fine-structure constant , Proton to electron mass ratio , Dimensionless physical constants , Strong coupling constant , Gravitational constant , Strong interaction , Cosmological constant , Euler's number , Gelfond's constant

## 1. Introduction

In physics, the fundamental interactions, also known as fundamental forces, are the interactions that do not appear to be reducible to more basic interactions. There are four fundamental interactions known to exist: the gravitational and electromagnetic interactions, which produce significant long-range forces whose effects can be seen directly in everyday life, and the strong and weak interactions, which produce forces at minuscule, subatomic distances and govern nuclear interactions. Some scientists hypothesize that a fifth force might exist, but these hypotheses remain speculative. Each of the known fundamental interactions can be described mathematically as a field. The gravitational force is attributed to the curvature of spacetime, described by Einstein's general theory of relativity. The other three are discrete quantum fields, and their interactions are mediated by elementary particles described by the Standard Model of particle physics. The Standard Model of particle physics was developed throughout the latter half of the 20th century. In the Standard Model, the electromagnetic, strong, and weak interactions are associated with elementary particles, whose behaviors are modeled in quantum mechanics (QM). For predictive success with QM's probabilistic outcomes, particle physics conventionally models QM events across a field set to special relativity, altogether relativistic quantum field theory (QFT).

A coupling constant is a parameter in field theory, which determines the relative strength of interaction between particles or fields. In the quantum field theory the coupling constants are associated with the vertices of the corresponding Feynman diagrams. Dimensionless parameters are used as coupling constants, as well as the quantities associated with them that characterize the interaction and have dimensions. A coupling constant is a number that determines the strength of an interaction. Usually the Lagrangian or the Hamiltonian of a system can be separated into a kinetic part and an interaction part. The coupling constant determines the strength of the interaction part with respect to the kinetic part, or between two sectors of the interaction part. For example, the electric charge of a particle is a coupling constant. A coupling constant plays an important role in dynamics. For example, one often sets up hierarchies of approximation based on the importance of various coupling constants. In the motion of a large lump of

magnetized iron,the magnetic forces are more important than the gravitational force.

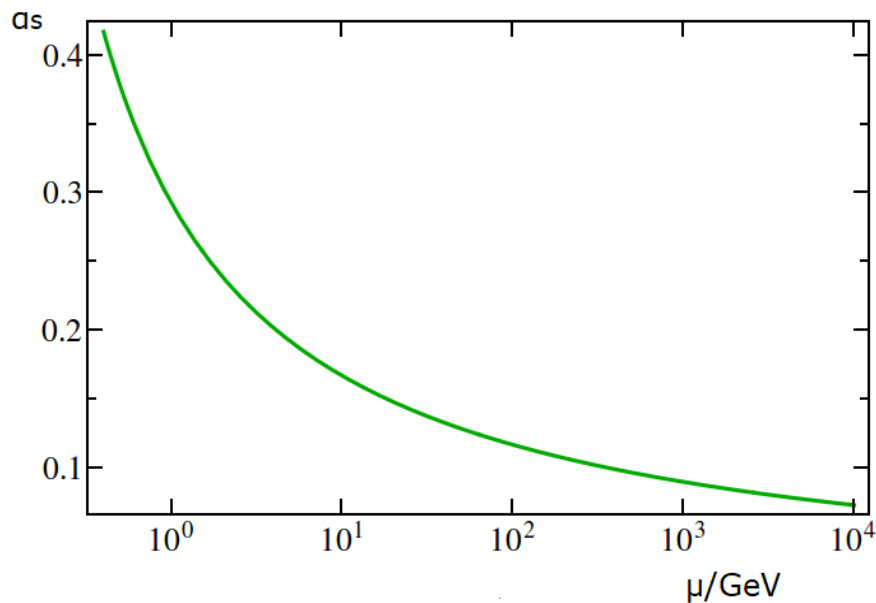
In nuclear physics and particle physics,the strong interaction is one of the four known fundamental interactions,with the others being electromagnetism,the weak interaction,and gravitation. Strong force involves the exchange of huge particles and therefore has a very small range. It is clear that strong force is much stronger simply than the fact that the nuclear magnitude (dominant strong force) is about  $10^{-15}$  m while the atom (dominant electromagnetic force) has a size of about  $10^{-10}$  m. At the range of  $10^{-15}$  m,the strong force is approximately 137 times as strong as electromagnetism, $10^6$  times as strong as the weak interaction,and  $10^{38}$  times as strong as gravitation. The strong coupling constant  $\alpha_s$  is one of the fundamental parameters of the typical model of particle physics.

Euler's number  $e$  is an important mathematical constant,which is the base of the natural logarithm. All five of these numbers play important and repetitive roles in mathematics and these five constants appear in a formulation of Euler's identity. Euler's number has many practical uses,especially in higher level mathematics such as calculus,differential equations,trigonometry,complex analysis,statistics,etc. Euler's number frequently appears in problems related to growth or decay,where the rate of change is determined by the present value of the number being measured. One example is in biology,where bacterial populations are expected to double at reliable intervals. Another case is radiometric dating,where the number of radioactive atoms is expected to decline over the fixed half-life of the element being measured. From Euler's identity the following relation of the mathematical constant  $e$  can emerge  $e=i^{-2i/\pi}$ .

Gelfond's constant,in mathematics,is the number  $e^\pi$ , $e$  raised to the power  $\pi$ . Like  $e$  and  $\pi$ ,this constant is a transcendental number. It was named after the Soviet mathematician Aleksandr Gelfond. Gelfond's constant were singled out in Hilbert's 7th problem as an example of numbers whose excess was an open problem. This was first established by Gelfond and may now be considered as an application of the Gelfond–Schneider theorem,noting that  $e^\pi=(e^{i\pi})^{-i}=(-1)^{-i}=i^{-2i}$ . In [8] we presented exact and approximate expressions between the Archimedes constant  $\pi$ ,the golden ratio  $\phi$ ,the Euler's number  $e$  and the imaginary number  $i$ .

## 2. Measurement of the strong coupling constant

The strong coupling constant  $\alpha_s$  is one of the fundamental parameters of the typical model of particle physics. The strong nuclear force confines quarks into hadron particles such as the proton and neutron. In addition,the strong force binds these neutrons and protons to create atomic nuclei,where it is called the nuclear force. Most of the mass of a common proton or neutron is the result of the strong force field energy;the individual quarks provide only about 1% of the mass of a proton. The electromagnetic force is infinite in range and obeys the inverse square law,while the strong force involves the exchange of massive particles and it therefore has a very short range. The value of the strong coupling constant,like other coupling constants,depends on the energy scale. As the energy increases,this constant decreases.



**Figure 1.** Strong coupling constant as a function of the energy.

The last measurement [16] in 23 November 2021 of European organization for nuclear research (CERN) is used in a comprehensive QCD analysis at next-to next-to-leading order,which results in significant improvement in the accuracy of the parton distributions in the proton. Simultaneously,the value of the strong coupling constant at the Z boson mass is extracted as:

$$\alpha_s(m_Z)=0,1170\pm0,0019$$

A measurement of the inclusive jet production in proton-proton collisions at the LHC at  $\sqrt{s}=13$  TeV is presented. The double-differential cross sections are measured as a function of the jet transverse momentum  $p_T$  and the absolute jet rapidity  $|y|$ . The anti-kT clustering algorithm is used with distance parameter of 0.4 (0.7) in a phase space region with jet  $p_T$  from 97 GeV up to 3.1 TeV and  $|y|<2.0$ . Data collected with the CMS detector are used, corresponding to an integrated luminosity of 36.3 fb<sup>-1</sup> (33.5 fb<sup>-1</sup>). The measurement is used in a comprehensive QCD analysis at next-to next-to-leading order, which results in significant improvement in the accuracy of the parton distributions in the proton. Simultaneously, the value of the strong coupling constant at the Z boson mass is extracted as  $\alpha_s(m_Z)=0,1170\pm0,0019$ . For the first time, these data are used in a standard model effective field theory analysis at next to-leading order, where parton distributions and the QCD parameters are extracted simultaneously with imposed constraints on the Wilson coefficient  $c_1$  of 4-quark contact interactions.

### 3. Theoretical value for the strong coupling constant

Interaction phenomena in field theory are often defined using perturbation theory, in which the functions in the equations are extended to forces of constant interaction. Usually, for all interactions except the strong one, the coupling constant is much smaller than the unit. This makes the application of perturbation theory effective, as the contribution from the main terms of the extensions decreases rapidly and their calculation becomes redundant. In the case of strong interactions, perturbation theory becomes useless and other calculation methods are required. One of the predictions of quantum field theory is the so-called "floating constants" phenomenon, according to which interaction constants change slowly with the increase of energy transferred during the interaction of particles. Thus, the constant of the electromagnetic interaction increases, and the constant of the strong interaction decreases with increasing energy. For quarks in quantum chromodynamics, a strong interaction constant is introduced:

$$\alpha_s = \frac{q_{qg}^2}{4\pi\hbar c} = \frac{q_{qg}^2 \varepsilon_0 \alpha}{q_e^2} = \frac{\varepsilon_0 q_{qg}^2}{q_{pl}^2}$$

where  $q_{qg}$  is the active color charge of a quark that emits virtual gluons to interact with another quark. By reducing the distance between the quarks, which is achieved in high-energy particle collisions, a logarithmic reduction of  $\alpha_s$  and a weakening of the strong interaction (the effect of the asymptotic freedom of the quarks) is expected. The recommended value for the strong coupling constant is:

$$\alpha_s = \frac{\text{Eulers' number}}{\text{Gerford's constant}}$$

$$\alpha_s = \frac{e}{e^\pi}$$

$$\alpha_s = e^{1-\pi} \quad (1)$$

with numerical value:

$$\alpha_s=0,11746...$$

This value is the current world average value for the coupling evaluated at the Z-boson mass scale. It fits perfectly in the measurement of the strong coupling constant of the European organization for nuclear research (CERN). Also for the value of the strong coupling constant we have the equivalent expressions:

$$\alpha_s = e \cdot e^{-\pi} = e \cdot i^{2i} = i^{-2i/n} \cdot i^{2i} = i^{2i-(2i/n)} = i^{2i(n-1)/n}$$

The series representations for the strong coupling constant is:

$$e^{1-\pi} = \left( \sum_{k=0}^{\infty} \frac{1}{k!} \right)^{1-\pi}$$

$$e^{1-\pi} = \left( \frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^k}{k!}} \right)^{1-4 \sum_{k=0}^{\infty} (-1)^k / (1+2k)}$$

[0; 8, 1, 1, 18, 1, 3, 21, 2, 39, 3, 2, 1, 2, 1, 14, 1, 8, 1, 1, 101, 1, 1, 1, 7, 3, 2, 2, 1, 3, 1, 1, 4, 2, 1, 1, 1, 5, 16, 2, 2, 3, 69, 1, 2, 5, 1, 4, 9, 6, 1, 1, 1, 1, 1, 9, 10, 3, 4, 5, 1, 3, 1, 1, 4, 5, 2, 1, 2, 23, 1, 1, 2, 2, 2, 2, 142, 3, 1, 1, 1, 1, 4, 1, 1, 37, 10, 2, 2, 3, 4, 1, 1, 1, 8, 2, 1, 115, 1, 4, 1, 4, 11, 1, 3, 1, 5, 1, 1, 5, 1, 1, 4, 2, 3, 1, 1, 28, 2, 7, 1, 5, 7, 2, 2, 1, 3, 1, 14, 1, 1, 4, 1, 22, 4, 6, 3, 5, 2, 2, 1, 5, 2, 8, 2, 2, 14, 9, 9, 14, 18, 10, 2, 1, 7, 3, 1, 1, 3, 3, 1, 17, 1, 1, 15, 1, 1, 1, 1, 1, 1, 7, 3, 1, 7, 2, 4750, 5, 1, 4, 1, 1, 1, 1, 12, 1, 1, 4, 4, 4, 1, 3, 9, 2, 8, 1, 2, 2, 1, 3, 1, 3, 3, 4, 1, 2, 1, 12, 1, 11, 2, 11, 4, 4, 1, 5, 8, 2, 2, 1, 1, 3, 1, 1, 2, 19, 1, 2, 3, 1, 1, 8, 2, 80, 1, 2, 1, 1, 1, 4, 3, 1, 2, 1, 4, 5, 1, 2, 1, 2, 3, 3, 1, 4, 1, 1, 3, 38, 1, 1, 3, 1, 1, 1, 1, 12, 1, 1, 10, 7, 4, 1, 3, 16, 4, 9, 1, 1, 8, 1, 1, 7, 85, 1, 2, 1, 7, 1, 3, 11, 1, 13, 4, 5, 31, 1, 2, 1, 1, 1, 1, 690, 4, 3, 2, 2, 1, 2, 5, 1, 2, 4, 2, 4, 5, 1, 1, 1, 2, 1, 1, 2, 4, 19, 2, 1, 12, 1, 1, 6, 1, 2, 5, 2, 1, 2, 1, 145, 1, 1, 1, 2, 1, 73, 2, 3, 2, 1, 1, 5, 3, 1, 2, 12, 9, 1, 1, 33, 1194, 2, 5, 1, 24, 1, 1, 26, 3, 6, 3, 11, 2, 2, 10, 118, 2, 1, 56, 2, 5, 1, 13, 18, 1, 1, 3, 1, 2, 1, 2, 2, 4, 1, 7, 1, 3, 3, 2, 8, 4, 5, 1, 7, 7, 2, 4, 1, 1, 8, 1, 1, 1, 4, 1, 1, 2, 1, 3, 1, 13, 18, 8, 20, 1, 1, 3, 2, 2, 1, 5, 1, 12, 2, 1, 16, 1, 1, 1, 6, 1, 2, 6, ...]

Diagram illustrating a sequence of horizontal lines with labels on the left and right, representing a sequence of values or indices. The labels on the left are 8+, 1+, 1+, 18+, 1+, 3+, 21+, 2+, 39+, 3+, 2+, 1+, 2+, 1+, 14+, 1+, 8+, 1+, 1+. The labels on the right are 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1. The lines are connected by a series of horizontal and vertical segments, forming a staircase-like pattern.

#### 4. Fine-structure constant and the proton to electron mass ratio

The fine-structure constant is one of the most fundamental constants of physics. It describes the strength of the force of electromagnetism between elementary particles in what is known as the standard model of particle physics. In particular, the fine-structure constant sets the strength of electromagnetic interaction between light (photons) and charged elementary particles such as electrons and muons. The quantity  $\alpha$  was introduced into physics by A. Sommerfeld in 1916 and in the past has often been referred to as the Sommerfeld fine-structure constant. In order to explain the observed splitting or fine structure of the energy levels of the hydrogen atom, Sommerfeld extended the Bohr theory to include elliptical orbits and the relativistic dependence of mass on velocity.

One of the most important numbers in physics is the fine-structure constant  $\alpha$  which defines the strength of the electro-magnetic field. It is a dimensionless number independent of how we define our units of mass, length, time or electric charge. A change in these units of measurement leaves the dimensionless constant unchanged. The number can be seen as the chance that an electron emits or absorbs a photon. It's a pure number that shapes the universe to an astonishing degree. Paul Dirac considered the origin of the number «the most fundamental unsolved problem of physics». The constant is everywhere because it characterizes the strength of the electromagnetic force affecting charged particles such as electrons and protons. Many eminent physicists and philosophers of science have pondered why  $\alpha$  itself has the value that it does, because the value shows up in so many important scenarios and aspects of physics. Nobody has come up with any ideas that are even remotely convincing. A similar situation occurs with the proton-electron mass ratio  $\mu$ , not because of its ubiquity, but rather how chemistry can be based on two key electrically charged particles of opposite electric charge that are opposite but of seemingly identical magnitude while their masses have a ratio that is quite large yet finite. These two questions have a huge bearing on why physics and chemistry behave the way they do. The product of the two quantities appears, at least at first glance, not to be so important. The fine-structure constant  $\alpha$  is defined as:

$$\alpha = \frac{q_e^2}{4\pi\epsilon_0\hbar c}$$

The 2018 CODATA recommended value of the fine-structure constant is  $\alpha=0,0072973525693(11)$  with standard uncertainty  $0,0000000011\times 10^{-3}$  and relative standard uncertainty  $1,5\times 10^{-10}$ . Also the fine-structure constant is universal scaling factor:

$$\alpha = \frac{2\pi r_e}{\lambda_e} = \frac{\lambda_e}{2\pi\alpha_0} = \frac{r_e}{l_{pl}} \frac{m_e}{m_{pl}} = \sqrt{\frac{r_e}{\alpha_0}}$$

There is a dream, which, albeit more often not confessed, occupies the most secret aspirations of theoreticians and is that of reducing the various constants of Physics to simple formula involving integers and transcendent numbers. The fine-structure constant plays an important role in modern physics. Yet it continues to be a mystery as to exactly what it represents and why it has the mystical value it has. The purpose of many sciences is to find the most accurate mathematical formula that can be found in the experimental value of fine-structure constant. Attempts to find a mathematical basis for this dimensionless constant have continued up to the present time. However, no numerological explanation has ever been accepted by the physics community. We propose in [9] the exact formula for the fine-structure constant  $\alpha$  with the golden angle, the relativity factor and the fifth power of the golden mean:

$$\alpha^{-1}=360\cdot\varphi^{-2}-2\cdot\varphi^{-3}+(3\cdot\varphi)^{-5} \quad (2)$$

with numerical value:

$$\alpha^{-1}=137,035999164...$$

Another beautiful forms of the equations are:

$$\begin{aligned} \frac{1}{\alpha} &= \frac{360}{\varphi^2} - \frac{2}{\varphi^3} + \frac{1}{3^5\varphi^5} \\ \frac{1}{\alpha} &= \frac{360}{\varphi^2} - \frac{2}{\varphi^3} + \frac{3^{-5}}{\varphi^5} \end{aligned} \quad (3)$$

Other equivalent expressions for the fine-structure constant are:

$$\alpha^{-1} = (362 - 3^{-4}) \cdot \varphi^{-2} - (1 - 3^{-5}) \cdot \varphi^{-1}$$

$$\alpha^{-1} = (362 - 3^{-4}) + (3^{-4} + 2 \cdot 3^{-5} - 364) \cdot \varphi^{-1}$$

$$\alpha^{-1} = 1 - 2 \cdot \varphi^{-1} + 360 \cdot \varphi^{-2} - \varphi^{-3} + (3 \cdot \varphi)^{-5}$$

Also we propose in [10] a simple and accurate expression for the fine-structure constant  $\alpha$  in terms of the Archimedes constant  $\pi$ :

$$\alpha^{-1} = \frac{2.706}{43} \pi \ln 2 \quad (4)$$

with numerical value:

$$\alpha^{-1} = 137,035999078...$$

Other equivalent expression for the fine-structure constant is:

$$\alpha^{-1} = 2 \cdot 3 \cdot 11 \cdot 41 \cdot 43^{-1} \cdot \pi \cdot \ln 2 \quad (5)$$

The equivalent expressions for the fine-structure constant with the madelung constant  $b_2(2)$  are:

$$\alpha^{-1} = \frac{2.706}{43} b_2(2) \quad (6)$$

$$\alpha^{-1} = 2 \cdot 3 \cdot 11 \cdot 41 \cdot 43^{-1} \cdot b_2(2) \quad (7)$$

In Physics, the ratio of the mass of a proton to an electron is simply the remainder of the mass of the proton divided by that of the electron, from the system of units. Two of the great mysteries of physics are the origin of mass and the mysterious ratio of mass between a proton and an electron. The values of  $m_e$  and  $m_p$ , and the equilibrium between them, govern nuclear reactions such as the decay of protons and the nuclear synthesis of stars, leading to the formation of basic biochemical elements, including carbon. The space where stars and planets form and support life and molecular structures can appear. The mass ratio of protons to electrons, two constant particles that make up about 95% of the visible Universe, may be related to the total computational value of the Universe. Thus, as pure numbers they are supposed to be associated with prime numbers, entropy, binary and complexity. The proton to electron mass ratio  $\mu$  is a ratio of like-dimensioned physical quantities, it is a dimensionless quantity, a function of the dimensionless physical constants, and has numerical value independent of the system of units. Two of the great mysteries of physics are the origin of mass and the mysterious mass ratio between the proton and electron. The numerical challenge of the mass ratio of proton to electron in the field of elementary particle physics began with the discovery of the electron by JJ Thomson in 1.897, and with the identification of the point nature of the proton by E. Rutherford in 1.911. These two particles have electric charges that are identical in size but opposite charges. The 2.018 CODATA recommended value of the proton to electron mass ratio is  $\mu = 1.836,15267343$  with standard uncertainty 0,00000011 and relative standard uncertainty  $6,0 \times 10^{-11}$ . The value of  $\mu$  is a solution of the equation:

$$3 \cdot \mu^4 - 5.508 \cdot \mu^3 - 841 \cdot \mu^2 + 10 \cdot \mu - 2.111 = 0$$

We propose in [11] the exact mathematical expression for the proton to electron mass ratio using Fibonacci and Lucas numbers:

$$\mu^{32} = \varphi^{-42} \cdot F_5^{160} \cdot L_5^{47} \cdot L_{19}^{40/19} \quad (8)$$

with numerical value:

$$\mu = 1.836,15267343...$$

Also we propose in [11] the exact mathematical expression for the proton to electron mass ratio:

$$\mu = 165 \sqrt[3]{\frac{\ln^{11} 10}{7}} \quad (9)$$

with numerical value:

$$\mu = 1836,15267392...$$

Other equivalent expressions for the proton to electron mass ratio are:

$$\begin{aligned} \mu^3 &= 7^{-1} \cdot 165^3 \cdot \ln^{11} 10 \\ 7 \cdot \mu^3 &= (3 \cdot 5 \cdot 11)^3 \cdot \ln^{11} (2 \cdot 5) \end{aligned} \quad (10)$$

Other exact mathematical expression in [11] for the proton to electron mass ratio is:

$$\mu = 6 \cdot \pi^5 + \pi^{-3} + 2 \cdot \pi^{-6} + 2 \cdot \pi^{-8} + 2 \cdot \pi^{-10} + 2 \cdot \pi^{-13} + \pi^{-15} \quad (11)$$

with numerical value:

$$\mu = 1.836,15267343...$$

Also in [11] was presented the exact mathematical expressions that connects the proton to electron mass ratio  $\mu$  and the fine-structure constant  $\alpha$ :

$$9 \cdot \mu - 119 \cdot \alpha^{-1} = 5 \cdot (\varphi + 42) \quad (12)$$

$$\mu - 6 \cdot \alpha^{-1} = 360 \cdot \varphi - 165 \cdot \pi + 345 \cdot e + 12 \quad (13)$$

$$\mu - 182 \cdot \alpha = 141 \cdot \varphi + 495 \cdot \pi - 66 \cdot e + 231 \quad (14)$$

$$\mu - 807 \cdot \alpha = 1.205 \cdot \pi - 518 \cdot \varphi - 411 \cdot e \quad (15)$$

In [13] was presented the unity formula that connects the fine-structure constant and the proton to electron mass ratio. It was explained that  $\mu \cdot \alpha^{-1}$  is one of the roots of the following trigonometric equation:

$$2 \cdot 10^2 \cdot \cos(\mu \cdot \alpha^{-1}) + 13^2 = 0 \quad (16)$$

The exponential form of this equation is:

$$10^2 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha}) + 13^2 = 0 \quad (17)$$

This exponential form can also be written with the beautiful form:

$$10^2 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha}) = 13^2 \cdot e^{i\pi} \quad (18)$$

Also this unity formula can also be written in the form:

$$10 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha})^{1/2} = 13 \cdot i \quad (19)$$

So other beautiful formula that connects the fine-structure constant, the proton to electron mass ratio and the fifth power of the golden mean is:

$$5^2 \cdot (5 \cdot \varphi^{-2} + \varphi^5)^2 \cdot (e^{i\mu/\alpha} + e^{-i\mu/\alpha}) + (5 \cdot \varphi^2 - \varphi^5)^2 = 0 \quad (20)$$

The formula that connects the fine-structure constant, the proton to electron mass ratio and the mathematical constants  $\pi, \varphi, e$  and  $i$  is:

$$10^2 \cdot (e^{i\mu/a} + e^{-i\mu/a}) = (5 \cdot \varphi^2 - \varphi^{-5})^2 \cdot e^{in} \quad (21)$$

## 5. Unity formulas that connect the strong coupling constant and the fine-structure constant

Based on Einstein's light quantum hypothesis, the duality of the photon was confirmed through quantum-mechanical experiments and examination. The photon is now regarded as a particle in fields related to the interaction of material with light that is absorbed and emitted; and regarded as a wave in regions relating to light propagation. It is known that among the four forces constituting the universe, the photon serves to convey electromagnetic force. The other three forces are gravitational force, strong force, and weak force. The photon plays an important role in the structure of the world where we live and is deeply involved with sources of matter and life. Through the work of Max Planck, Albert Einstein, Louis de Broglie, Arthur Compton, Niels Bohr, Erwin Schrödinger and many others, current scientific theory holds that all particles exhibit a wave nature and vice versa. This phenomenon has been verified not only for elementary particles, but also for compound particles like atoms and even molecules. For macroscopic particles, because of their extremely short wavelengths, wave properties usually cannot be detected.

Bohr regarded the "duality paradox" as a fundamental or metaphysical fact of nature. He regarded renunciation of the cause-effect relation, or complementarity, of the space-time picture, as essential to the quantum mechanical account. Werner Heisenberg considered the question further. He saw the duality as present for all quantum entities, but not quite in the usual quantum mechanical account considered by Bohr. He saw it in what is called second quantization, which generates an entirely new concept of fields that exist in ordinary space-time, causality still being visualizable. Jesús Sánchez in [14] explained that the fine-structure constant is one of the roots of the following trigonometric equation:

$$\cos \alpha^{-1} = e^{-1} \quad (22)$$

Another elegant expression is the following exponential form equations:

$$\begin{aligned} e^{i/a} - e^{-1} &= -e^{-i/a} + e^{-1} \\ e^{i/a} + e^{-i/a} &= 2 \cdot e^{-1} \end{aligned} \quad (23)$$

These expressions show the wave nature of light. The modern theory of quantum mechanics came to picture light as both a particle and a wave and, as a phenomenon which is neither a particle or a wave. Instead, modern physics sees light as something that can be described sometimes with mathematics appropriate to one type of macroscopic metaphor (particles) and sometimes another macroscopic metaphor (water waves), but is actually something that cannot be fully imagined. Also from [17] the fine-structure constant is one of the roots of the following trigonometric equation:

$$\begin{aligned} \cos(10^3 \cdot \alpha^{-1}) &= \varphi^2 \cdot e^{-1} \\ e \cdot \cos(10^3 \cdot \alpha^{-1}) &= \varphi^2 \end{aligned} \quad (24)$$

Another elegant expression is the following exponential form equation:

$$e^{1000i/a} + e^{-1000i/a} = 2 \cdot \varphi^2 \cdot e^{-1} \quad (25)$$

From these expressions resulting the following equations:

$$\begin{aligned} \cos^{-1} \alpha^{-1} \cdot \cos(10^3 \cdot \alpha^{-1}) &= \varphi^2 \\ \cos(10^3 \cdot \alpha^{-1}) &= \varphi^2 \cdot \cos \alpha^{-1} \end{aligned} \quad (26)$$

We will use the expressions (1) and (22) to resulting the unity formulas that connects the strong coupling constant  $\alpha_s$  and the fine-structure constant  $\alpha$ :

$$\begin{aligned} \cos \alpha^{-1} &= e^{-1} \\ \alpha_s &= e^{1-n} \end{aligned}$$

$$\cos \alpha^{-1} = (e^n \cdot \alpha_s)^{-1}$$

$$\cos \alpha^{-1} = e^{-n} \cdot \alpha_s^{-1}$$

$$e^n \cdot \alpha_s \cdot \cos \alpha^{-1} = 1 \quad (27)$$

Other forms of the equations are:

$$\cos \alpha^{-1} = (i^{-2i} \cdot \alpha_s)^{-1}$$

$$i^{-2i} \cdot \alpha_s \cdot \cos \alpha^{-1} = 1$$

$$\cos \alpha^{-1} = i^{2i} \cdot \alpha_s^{-1}$$

$$\alpha_s \cdot \cos \alpha^{-1} = i^{2i} \quad (28)$$

So the beautiful formulas for the strong coupling constant  $\alpha_s$  are:

$$\alpha_s = e^{-n} \cdot \cos^{-1} \alpha^{-1} \quad (29)$$

$$\alpha_s = i^{2i} \cdot \cos^{-1} \alpha^{-1} \quad (30)$$

Now we need to study the following equivalent equations:

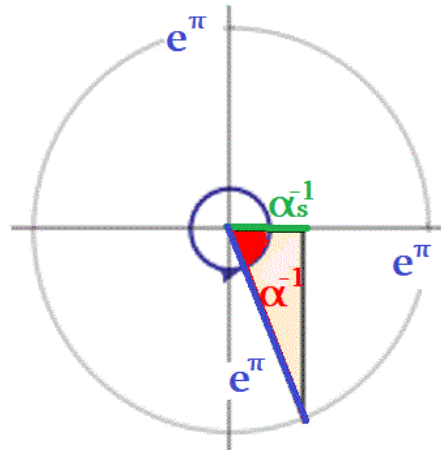
$$\cos \alpha^{-1} = \frac{e^{-\pi}}{\alpha_s}$$

$$\cos \alpha^{-1} = \frac{i^{2i}}{\alpha_s}$$

$$\cos \alpha^{-1} = \frac{\alpha_s^{-1}}{e^{\pi}}$$

$$\cos \alpha^{-1} = \frac{\alpha_s^{-1}}{i^{-2i}}$$

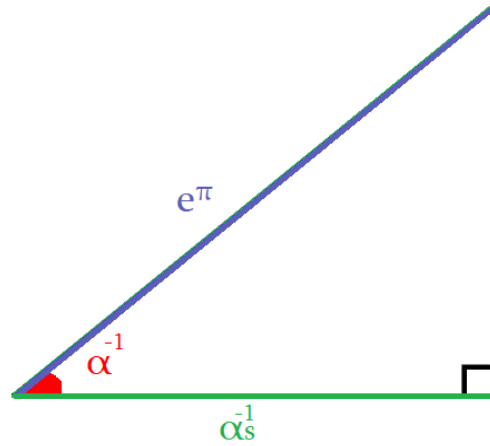
The figure below shows the angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $e^n$ . The strong coupling constant  $\alpha_s$  and the fine-structure constant  $\alpha$  are in a right triangle with the variable acute angle  $\alpha^{-1}$  radians. The adjacent side is the variable side  $\alpha_s^{-1}$  while the hypotenuse is constant  $e^n$ .



**Figure 2.** The angle in  $\alpha^{-1}$  radians. The rotation vector moves in a circle of radius  $e^n$

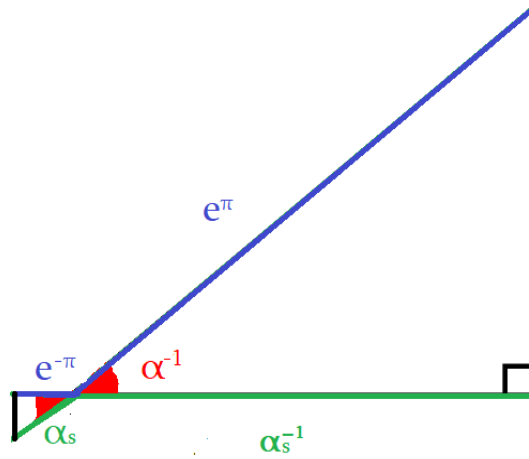
The fine structure constant is the ratio of the speed of the electron compared to the speed of light, in the first level of an atom. It is also related to the ratio of the Bohr radius of an atom to the Compton wavelength of an electron. We

could try to relate it to the electromagnetic interactions in the atom. The nucleus emits a photon that has an intrinsic property associated with the electromagnetic interaction represented by a vector. This vector can rotate as the photon moves along. The electron has an additional property related to the electromagnetic interaction that is also represented by a vector.



**Figure 3.** The strong coupling constant  $\alpha_s$  and the fine-structure constant  $\alpha$  are in a right triangle with the variable acute angle  $\alpha^{-1}$  radians. The adjacent side is the variable side  $\alpha_s^{-1}$  while the hypotenuse is constant  $e^\pi$ .

Thus, when the photon reaches the electron, the electromagnetic interaction between them is related to the relative position of these vectors at the time of interaction. When we talk about relative placement between vectors, we can talk about a projection of one of them onto the other. This means that  $\cos \alpha^{-1}$  will be related to the interaction of these two properties of the photon and the electron. It would be related to their relative vector position at the time of interaction.



**Figure 4.** Geometric representation of the simple unification of the strong nuclear and the electromagnetic interactions.

The angle in  $\alpha^{-1}$  radians is not only the final interaction angle, but also includes, for example, the number of rotations the photon or electron vector has made before the interactions. From expressions (1) and (23) resulting the formulas that connects the strong coupling constant  $\alpha_s$  and the fine-structure constant  $\alpha$ :

$$e^{i/\alpha} + e^{-i/\alpha} = 2 \cdot e^{-1}$$

$$e^{i/\alpha} + e^{-i/\alpha} = 2 \cdot (e^\pi \cdot \alpha_s)^{-1}$$

$$e^{i/\alpha} - (e^\pi \cdot \alpha_s)^{-1} = -e^{-i/\alpha} + (e^\pi \cdot \alpha_s)^{-1}$$

$$e^{i/\alpha} + e^{-i/\alpha} = 2 \cdot e^{-\pi} \cdot \alpha_s^{-1}$$

$$e^n \cdot a_s \cdot (e^{i/a} + e^{-i/a}) = 2 \quad (31)$$

Other forms of the equations are:

$$e^{i/a} + e^{-i/a} = 2 \cdot (i^{-2i} \cdot a_s)^{-1}$$

$$e^{i/a} + e^{-i/a} = 2 \cdot i^{2i} \cdot a_s^{-1}$$

$$e^{i/a} - i^{2i} \cdot a_s^{-1} = -e^{-i/a} + i^{2i} \cdot a_s^{-1}$$

$$a_s \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot i^{2i} \quad (32)$$

These equations are applicable for all energy scales. So the beautiful formulas for the strong coupling constant  $a_s$  are:

$$a_s = 2 \cdot e^{-n} \cdot (e^{i/a} + e^{-i/a})^{-1} \quad (33)$$

$$a_s = 2 \cdot i^{2i} \cdot (e^{i/a} + e^{-i/a})^{-1} \quad (34)$$

We reached the conclusion of the simple unification of the strong nuclear and the electromagnetic forces:

$$a_s \cdot (e^{i/a} + e^{-i/a}) = 2 \cdot i^{2i}$$

(Simple unification of the strong nuclear and the electromagnetic interactions)

## 6. Mathematical formulas that connects dimensionless physical constants

In physics, a dimensionless physical constant is a physical constant that is dimensionless, a pure number having no units attached and having a numerical value that is independent of whatever system of units may be used. The term fundamental physical constant is used to refer to some universal dimensionless constants. A long-sought goal of theoretical physics is to find first principles from which all of the fundamental dimensionless constants can be calculated and compared to the measured values.

In the 1.920s and 1.930s, Arthur Eddington embarked upon extensive mathematical investigation into the relations between the fundamental quantities in basic physical theories, later used as part of his effort to construct an overarching theory unifying quantum mechanics and cosmological physics. The mathematician Simon Plouffe has made an extensive search of computer databases of mathematical formulas, seeking formulas for the mass ratios of the fundamental particles. An empirical relation between the masses of the electron, muon and tau has been discovered by physicist Yoshio Koide, but this formula remains unexplained.

Dimensionless physical constants cannot be derived and have to be measured. Developments in physics may lead to either a reduction or an extension of their number: discovery of new particles, or new relationships between physical phenomena, would introduce new constants, while the development of a more fundamental theory might allow the derivation of several constants from a more fundamental constant. A long-sought goal of theoretical physics is to find first principles from which all of the fundamental dimensionless constants can be calculated and compared to the measured values.

The laws of physics have a set of fundamental constants, and it is generally admitted that only dimensionless combinations of constants have physical significance. These combinations include the electromagnetic and gravitational fine structure, along with the ratios of elementary particles masses. Cosmological measurements clearly depend on the values of these constants in the past and can therefore give information on their time dependence if the effects of time-varying constants can be separated from the effects of cosmological parameters.

It was presented in [12] the mathematical formulas that connects the proton to electron mass ratio  $\mu$ , the fine-structure constant  $\alpha$ , the ratio  $N_1$  of electric force to gravitational force between electron and proton, the Avogadro's number  $N_A$ , the gravitational coupling constant  $a_G$  of the electron and the gravitational coupling constant of the proton  $a_G(p)$ :

$$a_G(p) = \mu^2 \cdot a_G \quad (35)$$

$$\alpha = \mu \cdot N_1 \cdot a_G \quad (36)$$

$$\alpha \cdot \mu = N_1 \cdot a_G(p) \quad (37)$$

$$\alpha^2 = N_1^2 \cdot a_G \cdot a_G(p) \quad (38)$$

$$4 \cdot e^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1 \quad (39)$$

$$\mu^2 = 4 \cdot e^2 \cdot a^2 \cdot a_{G(p)} \cdot N_A^2 \quad (40)$$

$$\mu \cdot N_1 = 4 \cdot e^2 \cdot a^3 \cdot N_A^2 \quad (41)$$

$$4 \cdot e^2 \cdot a \cdot \mu \cdot a_G^2 \cdot N_A^2 \cdot N_1 = 1 \quad (42)$$

$$\mu^3 = 4 \cdot e^2 \cdot a \cdot a_{G(p)}^2 \cdot N_A^2 \cdot N_1 \quad (43)$$

$$\mu^2 = 4 \cdot e^2 \cdot a_G \cdot a_{G(p)}^2 \cdot N_A^2 \cdot N_1^2 \quad (44)$$

$$\mu = 4 \cdot e^2 \cdot a \cdot a_G \cdot a_{G(p)} \cdot N_A^2 \cdot N_1 \quad (45)$$

Also from these expressions and (1) resulting the mathematical formulas that connects the strong coupling constant  $a_s$ , the proton to electron mass ratio  $\mu$ , the fine-structure constant  $a$ , the ratio  $N_1$  of electric force to gravitational force between electron and proton, the Avogadro number  $N_A$ , the gravitational coupling constant  $a_G$  of the electron and the gravitational coupling constant of the proton  $a_{G(p)}$ :

$$4 \cdot e^{2n} \cdot a_s^2 \cdot a^2 \cdot a_G \cdot N_A^2 = 1 \quad (46)$$

$$\mu^2 = 4 \cdot e^{2n} \cdot a_s^2 \cdot a^2 \cdot a_{G(p)} \cdot N_A^2 \quad (47)$$

$$\mu \cdot N_1 = 4 \cdot e^{2n} \cdot a_s^2 \cdot a^3 \cdot N_A^2 \quad (48)$$

$$4 \cdot e^{2n} \cdot a_s^2 \cdot a \cdot \mu \cdot a_G^2 \cdot N_A^2 \cdot N_1 = 1 \quad (49)$$

$$\mu^3 = 4 \cdot e^{2n} \cdot a_s^2 \cdot a \cdot a_{G(p)}^2 \cdot N_A^2 \cdot N_1 \quad (50)$$

$$\mu^2 = 4 \cdot e^{2n} \cdot a_s^2 \cdot a_G \cdot a_{G(p)}^2 \cdot N_A^2 \cdot N_1^2 \quad (51)$$

$$\mu = 4 \cdot e^{2n} \cdot a_s^2 \cdot a \cdot a_G \cdot a_{G(p)} \cdot N_A^2 \cdot N_1 \quad (52)$$

Other equivalent forms of the equations are:

$$4 \cdot a_s^2 \cdot a^2 \cdot a_G \cdot N_A^2 = i^{4i} \quad (53)$$

$$i^{4i} \cdot \mu = a_s^2 \cdot a^2 \cdot a_{G(p)} \cdot N_A^2 \quad (54)$$

$$i^{4i} \cdot \mu \cdot N_1 = 4 \cdot a_s^2 \cdot a^3 \cdot N_A^2 \quad (55)$$

$$4 \cdot a_s^2 \cdot a \cdot \mu \cdot a_G^2 \cdot N_A^2 \cdot N_1 = i^{4i} \quad (56)$$

$$i^{4i} \cdot \mu^3 = 4 \cdot a_s^2 \cdot a \cdot a_{G(p)}^2 \cdot N_A^2 \cdot N_1 \quad (57)$$

$$i^{4i} \cdot \mu^2 = 4 \cdot e^{2n} \cdot a_s^2 \cdot a_G \cdot a_{G(p)}^2 \cdot N_A^2 \cdot N_1^2 \quad (58)$$

$$i^{4i} \cdot \mu = 4 \cdot a_s^2 \cdot a \cdot a_G \cdot a_{G(p)} \cdot N_A^2 \cdot N_1 \quad (59)$$

In his experiments of 1.849–50, Michael Faraday was the first to search for a unification of gravity with electricity and magnetism. However, he found no connection. In 1.900, David Hilbert published a famous list of mathematical problems. In Hilbert's sixth problem, he challenged researchers to find an axiomatic basis for all of physics.

We reached the conclusion of the simple unification of the strong nuclear, the electromagnetic and the gravitational forces:

$$4 \cdot a_s^2 \cdot a^2 \cdot a_G \cdot N_A^2 = i^{4i}$$

(Simple unification of the strong nuclear, the electromagnetic and the gravitational interactions)

## 7. Gravitational Constant

The gravitational constant is an empirical physical constant that participates in the calculation of gravitational force between two bodies and is denoted by the letter  $G$ . It usually appears in Isaac Newton's law of universal gravitation and Albert Einstein's general theory of relativity. The physicist Sir Isaac Newton in 1687 published his book "Philosophiae Naturalis Principia Mathematica" where he presented the law of universal gravity to describe and calculate the mutual attraction of particles and huge objects in the universe. In this paper, Isaac Newton concluded that the attraction between two bodies is proportional to the product of their masses and inversely proportional to the square of the distance separating them. However, these must be adjusted by introducing the gravity constant  $G$ . The gravitational constant  $G$  occupies an anomalous position among the other constants of physics. The mass  $M$  of any celestial object cannot be determined independently of the gravitational attraction that it exerts. Thus, the combination  $G \cdot M$ , not the separate value of  $M$ , is the only meaningful property of a star, planet, or galaxy. According to general relativity and the principle of equivalence,  $G$  does not depend on material properties but is in a sense a geometric factor. The gravitational constant is defined as:

$$G = \alpha_G \frac{\hbar c}{m_e^2}$$

The 2018 CODATA recommended value of gravitational constant  $G = 6,67430 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$  with standard uncertainty  $0,00015 \times 10^{-11} \text{ m}^3/\text{kg} \cdot \text{s}^2$  and relative standard uncertainty  $2,2 \times 10^{-5}$ . Now we will find the formulas for the gravitational constant  $G$  using the unity formulas for the coupling constants that we calculated. From expression (39) the gravitational coupling constant  $\alpha_G$  can be written in the form:

$$\begin{aligned} 4 \cdot e^2 \cdot N_A^2 \cdot a^2 \cdot \alpha_G &= 1 \\ \alpha_G &= (2 \cdot e \cdot a \cdot N_A)^{-2} \end{aligned} \quad (60)$$

Therefore from this expression the formula for the gravitational constant is:

$$G = (2e\alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (61)$$

From equivalent expressions (46) and (53) the gravitational coupling constant  $\alpha_G$  can be written in the forms:

$$\begin{aligned} 4 \cdot e^{2n} \cdot a_s^2 \cdot a^2 \cdot \alpha_G \cdot N_A^2 &= 1 \\ \alpha_G &= (2 \cdot e^n \cdot a_s \cdot a \cdot N_A)^{-2} \end{aligned} \quad (62)$$

$$\begin{aligned} 4 \cdot a_s^2 \cdot a^2 \cdot \alpha_G \cdot N_A^2 &= i^{4i} \\ \alpha_G &= i^{4i} \cdot (2 \cdot a_s \cdot a \cdot N_A)^{-2} \end{aligned} \quad (63)$$

Therefore from these expressions the equivalent formulas for the gravitational constant are:

$$G = (2e^\pi \alpha_s \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (64)$$

$$G = i^{4i} (2\alpha_s \alpha N_A)^{-2} \frac{\hbar c}{m_e^2} \quad (65)$$

From this expression resulting the formulas for the strong coupling constant  $\alpha_s$ :

$$\alpha_s = \frac{1}{2e^\pi \alpha N_A} \sqrt{\frac{\hbar c}{G m_e^2}} \quad (66)$$

$$\alpha_s = \frac{i^{2i}}{2\alpha N_A} \sqrt{\frac{\hbar c}{G m_e^2}} \quad (67)$$

## 8. Cosmological constant

The relevant constant in atomic physics is the fine-structure constant  $\alpha$ , which plays a fundamental role in atomic physics and quantum electrodynamics. The analogous constant in cosmology is the gravitational fine-structure constant  $\alpha_g$ . It plays a fundamental role in cosmology. The mysterious value of the gravitational fine-structure constant  $\alpha_g$  is an equivalent way to express the biggest issue in theoretical physics. In [12] we found the new formula for the Planck length  $l_{pl}$ :

$$l_{pl} = a\sqrt{\alpha_G}\alpha_0$$

The fine-structure constant equals:

$$\alpha^2 = \frac{r_e}{a_0}$$

From these expressions we have:

$$l_{pl} = \frac{\alpha\sqrt{\alpha_G}r_e}{\alpha^2}$$

$$l_{pl} = \frac{\sqrt{\alpha_G}}{\alpha}r_e$$

$$\frac{l_{pl}^3}{r_e^3} = \frac{\sqrt{\alpha_G^3}}{\alpha^3}$$

The gravitational fine structure constant  $\alpha_g$  is defined as:

$$\alpha_g = \frac{l_{pl}^3}{r_e^3}$$

$$\alpha_g = \frac{\sqrt{\alpha_G^3}}{\alpha^3}$$

$$\alpha_g = \sqrt{\frac{\alpha_G^3}{\alpha^6}} \quad (68)$$

with numerical value:

$$\alpha_g = 1,886837 \times 10^{-61}$$

Also equals:

$$\alpha_g^2 \cdot \alpha^6 = \alpha_G^3$$

$$\alpha_g^2 = \alpha_G^3 \cdot \alpha^{-6}$$

$$\alpha_g^2 = \left( \frac{\alpha_G}{\alpha^2} \right)^3$$

From the expressions (39) and (68) resulting the unity formula for the gravitational fine-structure constant  $a_g$ :

$$a_g = (2 \cdot e \cdot a^2 \cdot N_A)^{-3} \quad (69)$$

Also apply the expressions:

$$(2 \cdot e \cdot a^2 \cdot N_A)^3 \cdot a_g = 1 \quad (70)$$

$$8 \cdot e^3 \cdot a^6 \cdot a_g \cdot N_A^3 = 1 \quad (71)$$

From the expressions (46),(53) and (68) resulting the unity formula for the gravitational fine-structure constant  $a_g$ :

$$a_g = (2 \cdot e^n \cdot a_s \cdot a^2 \cdot N_A)^{-3} \quad (72)$$

$$a_g = i^{6i} \cdot (2 \cdot a_s \cdot a^2 \cdot N_A)^{-3} \quad (73)$$

From expression (72) apply the expressions:

$$(2 \cdot e^n \cdot a_s \cdot a^2 \cdot N_A)^3 \cdot a_g = 1 \quad (74)$$

$$8 \cdot e^{3n} a_s^3 \cdot a^6 \cdot a_g \cdot N_A^3 = 1 \quad (75)$$

Also from expression (73) apply the expressions:

$$(2 \cdot a_s \cdot a^2 \cdot N_A)^3 \cdot a_g = i^{6i} \quad (76)$$

$$8 \cdot a_s^3 \cdot a^6 \cdot a_g \cdot N_A^3 = i^{6i} \quad (77)$$

In the context of cosmology the cosmological constant is a homogeneous energy density that causes the expansion of the universe to accelerate. Originally proposed early in the development of general relativity in order to allow a static universe solution it was subsequently abandoned when the universe was found to be expanding. Now the cosmological constant is invoked to explain the observed acceleration of the expansion of the universe. The cosmological constant is the simplest realization of dark energy, which is the more generic name given to the unknown cause of the acceleration of the universe. Its existence is also predicted by quantum physics, where it enters as a form of vacuum energy, although the magnitude predicted by quantum theory does not match that observed in cosmology.

The cosmological constant  $\Lambda$  is presumably an enigmatic form of matter or energy that acts in opposition to gravity and is considered by many physicists to be equivalent to dark energy. Nobody really knows what the cosmological constant is exactly, but it is required in cosmological equations in order to reconcile theory with our observations of the universe. One potential explanation for the cosmological constant lies in the realm of modern particle physics. Experiments have verified that empty space is permeated by countless virtual particles constantly popping in and out of existence. It is commonly believed that the cosmological constant problem can only be solved ultimately in a unified theory of quantum gravity and the standard model of electroweak and strong interactions, which is still absent so far. But connecting vacuum energy to the cosmological constant is not straightforward. Based on their observations of supernovas, astronomers estimate that dark energy should have a small and sedate value, just enough to push everything in the universe apart over billions of years. Yet when scientists try to calculate the amount of energy that should arise from virtual particle motion, they come up with a result that's 120 orders of magnitude greater than what the supernova data suggest. The cosmological constant has the same effect as an intrinsic energy density of the vacuum,  $\rho_{vac}$  and an associated pressure. In this context, it is commonly moved onto the right-hand side of the equation, and defined with a proportionality factor of  $\Lambda = 8 \cdot \pi \cdot \rho_{vac}$  where unit conventions of general relativity are used (otherwise factors of  $G$  and  $c$  would also appear, i.e.:

$$\Lambda = 8\pi\rho_{vac} \frac{G}{c^4} = \kappa\rho_{vac}$$

where  $\kappa$  is Einstein's rescaled version of the gravitational constant  $G$ . The cosmological constant has been introduced in gravitational field equations by Einstein in 1917 in order to satisfy Mach's principle of the relativity of inertia. Then it was demonstrated by Cartan in 1922 that the Einstein field tensor including a cosmological constant  $\Lambda$ :

$$E_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu}$$

,is the most general tensor in Riemannian geometry having null divergence like the energy momentum tensor  $T_{\mu\nu}$ . This theorem has set the general form of Einstein's gravitational field equations as  $E_{\mu\nu}=\kappa\cdot T_{\mu\nu}$  and established from first principles the existence of  $\Lambda$  as an unvarying true constant. The cosmological constant problem dates back to the realization that it is equivalent to a vacuum energy density. One of the main consequences in cosmology of a positive cosmological constant is an acceleration of the expansion of the universe. Such an acceleration has been first detected in 1.981 in the Hubble diagram of infrared elliptical galaxies,yielding a positive value close to the presently measured one,but with still large uncertainties. Accurate measurements of the acceleration of the expansion since 20 years have reinforced the problem. The cosmological constant  $\Lambda$ ,as it appears in Einstein's equations,is a curvature. As such,besides being an energy density,it is also the inverse of the square of an invariant cosmic length  $L$ .

In the early-mid 20th century Dirac and Zel'dovich were among the first scientists to suggest an intimate connection between cosmology and atomic physics. Though a revolutionary proposal for its time,Dirac's Large Number Hypothesis (1.937) adopted a standard assumption of the non-existence of the cosmological constant term  $\Lambda=0$ . Zel'dovich insight (1.968) was to realize that a small but nonzero cosmological term  $\Lambda>0$  allowed the present day radius of the Universe to be identified with the de Sitter radius which removed the need for time dependence in the fundamental couplings. Thus,he obtained the formula:

$$\Lambda = \frac{m_p^6 G^2}{\hbar^6}$$

where  $m$  is a mass scale characterizing the relative strengths of the gravitational and electromagnetic interactions,which he identified with the proton mass  $m_p$ .

Laurent Nottale in [15] which,instead,suggests the identification  $m=m_e/\alpha$ . He assumed that the cosmological constant  $\Lambda$  is the sum of a general-relativistic term and of the quantum,scale-varying,gravitational self-energy of virtual pairs. A renormalization group approach is used to describe its scale-dependence. We argue that the large scale value of  $\Lambda$  is reached at the classical electron scale. This reasoning provides with a large-number relation:

$$\alpha \frac{m_{pl}}{m_e} = \left( \frac{L}{l_{pl}} \right)^{\frac{1}{3}}$$

The cosmological constant  $\Lambda$  has the dimension of an inverse length squared. The cosmological constant is the inverse of the square of a length  $L$ :

$$L = \sqrt{\Lambda^{-1}}$$

For the de Sitter radius equals:

$$R_d = \sqrt{3}L$$

So the de Sitter radius and the cosmological constant are related through a simple equation:

$$R_d = \sqrt{\frac{3}{\Lambda}}$$

From this equation resulting the expressions for the gravitational fine structure constant  $\alpha_g$ :

$$\alpha \frac{m_{pl}}{m_e} = \left( l_{pl} \sqrt{\Lambda} \right)^{-\frac{1}{3}}$$

$$\alpha_g = l_{pl} \sqrt{\Lambda}$$

$$\alpha_g = \sqrt{\frac{G\hbar\Lambda}{c^3}}$$

So the cosmological constant  $\Lambda$  equals:

$$\Lambda = \alpha_g^2 l_{pl}^{-2}$$

$$\Lambda = \frac{l_{pl}^4}{r_e^6}$$

$$\Lambda = \alpha_g^2 \frac{c^3}{G\hbar}$$

$$\Lambda = \frac{G}{\hbar^4} \left( \frac{m_e}{a} \right)^6$$

From the expression (69) resulting the simple unification of atomic physics and cosmology:

$$\alpha_g = (2 \cdot e \cdot a^2 \cdot N_A)^{-3}$$

$$l_{pl}^2 \cdot \Lambda = (2 \cdot e \cdot a^2 \cdot N_A)^{-6} \quad (78)$$

$$(2 \cdot e \cdot a^2 \cdot N_A)^6 \cdot l_{pl}^2 \cdot \Lambda = 1 \quad (79)$$

Now we will use the unity formulas of the simple unification of atomic physics and cosmology to find the equations of the cosmological constant. For the cosmological constant equals:

$$\Lambda = \left( 2e a^2 N_A \right)^{-6} \frac{c^3}{G\hbar} \quad (80)$$

From the expression (72) resulting the simple unification of atomic physics and cosmology:

$$\alpha_g = (2 \cdot e^n \cdot a_s \cdot a^2 \cdot N_A)^{-3}$$

$$l_{pl}^2 \cdot \Lambda = (2 \cdot e^n \cdot a_s \cdot a^2 \cdot N_A)^{-6} \quad (81)$$

$$(2 \cdot e^n \cdot a_s \cdot a^2 \cdot N_A)^6 \cdot l_{pl}^2 \cdot \Lambda = 1 \quad (82)$$

From the equivalent expression (73) resulting the simple unification of atomic physics and cosmology:

$$\alpha_g = i^{6i} \cdot (2 \cdot a_s \cdot a^2 \cdot N_A)^{-3}$$

$$l_{pl}^2 \cdot \Lambda = i^{12i} \cdot (2 \cdot a_s \cdot a^2 \cdot N_A)^{-6} \quad (83)$$

$$(2 \cdot a_s \cdot a^2 \cdot N_A)^6 \cdot l_{pl}^2 \cdot \Lambda = i^{12i} \quad (84)$$

Also we will use the unity formulas of the simple unification of atomic physics and cosmology to find the equations of the cosmological constant. So for the cosmological constant equals:

$$\Lambda = \left( 2e^\pi \alpha_s a^2 N_A \right)^{-6} \frac{c^3}{G\hbar} \quad (85)$$

$$\Lambda = i^{12i} \left( 2\alpha_s a^2 N_A \right)^{-6} \frac{c^3}{G\hbar} \quad (86)$$

## 9. Conclusions

We presented the recommended theoretical value for the strong coupling constant:

$$\alpha_s = \text{Euler's number} / \text{Gelfond's constant} = e / e^\pi = e^{1-\pi}$$

From this definition of the strong coupling constant we arrived at significant results. We found the beautiful unity formula that connect the strong coupling constant  $\alpha_s$  and the fine-structure constant  $\alpha$ :

$$\alpha_s \cdot (e^{i/\alpha} + e^{-i/\alpha}) = 2 \cdot i^{2i}$$

From this equation we reached the unification of the strong nuclear and the electromagnetic interactions. It was presented the formula of the simple unification of the strong nuclear, the electromagnetic and the gravitational interactions:

$$4 \cdot \alpha_s^2 \cdot \alpha^2 \cdot \alpha_G \cdot N_A^2 = i^{4i}$$

Also the formula for the gravitational constant is:

$$G = i^{4i} (2\alpha_s \alpha N_A)^{-2} \frac{\hbar c}{m_e^2}$$

The simple unification of atomic physics and cosmology:

$$\ell_{\text{pt}}^2 \cdot \Lambda = i^{12i} \cdot (2 \cdot \alpha_s \cdot \alpha^2 \cdot N_A)^{-6}$$

Finally the formula for the cosmological constant is:

$$\Lambda = i^{12i} (2\alpha_s \alpha^2 N_A)^{-6} \frac{c^3}{G \hbar}$$

All these equations are simple, elegant and symmetrical in a great physical meaning. Also the equations prove that the value of the strong coupling constant depends on the energy scale.

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