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Exotic localized vector waves in nonlinear systems with higher-order effects

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September 25, 2023

We mainly give a briefly summarizes including the Background, the main results as follows.

Background:

As is well-known, the study of freak waves attracts a great deal of attention and interest from scientific community, especially in nonlinear sciences. In fact, the research of freak waves has become a research focus in soliton theory. Freak waves are unpredictable, rare, localizable, impact with enormous force, and may appear suddenly or without warning, which have been intensively discussed in various fields such as oceanography, optics and etc, Nowadays collisions between elementary solitons on a condensate background are regarded consistently as the prototype that causes the formation of freak waves. The solitons after collisions transitory form a wave that is far larger than a single soliton in time (t) and space (s). The simplest soliton for the case of solitons on a condensate background was proposed by Kuznetsov as a theoretical model for a kind of rogue-wave phenomena seen on water waves, and was experimentally seen in optical fibre. Another important nonlinear wave (Akhmediev breather (AB)) which is periodic in s but localized in t was presented by Akhmediev, Eleonskii and Kulagin. It starts from a condensate and returns to the condensate with a different phase and the same amplitude. The Peregrine soliton (PS) was predicted to exist in 1983. It is localized in both t and s, and defines the limit of Kuznetsov-Ma (KM) breather and AB.

The standard nonlinear Schrödinger equation (NLSE) can provide a central description of the formation of rogue wave phenomenon in nonlinear media because it reveals MI. Therefore, many approaches been proposed over the past few years for studying the standard NLSE, such as the Darboux transformation and the inverse scattering method etc, which allow us to construct higher-order solutions from simple ones. In particular, we can also construct N-soliton solutions by starting with the trivial solution of the standard NLSE. However, because a number of papers and books have been published on this field in the past few decades, the application of these approaches has been well rounded and is not a novelty any more.

The standard NLSE is a crucial model in mathematical physics, which plays a key role in various fields of nonlinear science such as plasma physics, nonlinear optics, Bose-Einstein condensates, deep ocean, even finance. It is undoubted that the standard NLSE has extensive physical applications. However, To well describe other important types of nonlinear physical phenomena in a relevant way, it is necessary to add more terms into the NLSE in order to display the contributions of higher-order nonlinear effects in realistic physical systems. On the other hand, it is very important to keep the integrability in the above modifications of the NLSE, which requires a subtle balance between existing terms and the newly added terms on the basis of the physical considerations. Along this idea, more general forms of integrable equations with higher-order terms were proposed and studied. In this paper, we pay close attention to solutions of the following higher-order NLSE (HNLSE) with nonvanishing boundary conditions The HNLSE can be widely used to describe the dynamics of the ultrashort pulses in optical fibers. Although these freak waves resulting from the MI have been discussed for decades. To the best of our knowledge, there are few records about the demonstration describing these freak waves resulting from the MI in the frame of the higher-order NLSEs on a condensate background, which motivates our study on this subject.

It is well-known that the dressing method is an efficient and powerful technique, because it avoids strict requirements for initial conditions in solving the nonlinear problem. The main aim of this paper is to analyze the superregular solitonic solutions exist in the HNLSE with nonvanishing boundary conditions by utilizing the dressing method. Besides, the key features of those solutions are graphically discussed by seeking appropriate parameters.

Main results (Main innovation):

In this work, we study the superregular solitonic solutions of the higher-order nonlinear Schrödinger equation (HNLSE) with nonvanishing boundary conditions. Solving the HNLSE with the dressing method, we obtain the explicit form of one-and two-solitonic solutions and discuss them in detail. These solutions can be used to describe the nonlinear stage of the modulation instability (MI) of the condensate. Moreover, we also find some novel features of the nonlinear stage of the MI arising from higher-order effects. This family of novel solutions include Peregrine soliton, Akhmediev breather, Kuznetsov-Ma breather, a symmetrical quasi-Akhmediev breather, coexistence of two symmetrical quasi-Akhmediev breather, coexistence of two nonsymmetrical quasi-Akhmediev breather, coexistence of a quasi-Akhmediev breather and a soliton, a bipolar-freak-wave etc. Finally, the main characteristics of these rational solutions are discussed with some graphics. These solutions would be of much importance in understanding and explaining rogue wave phenomena arising in nonlinear and complex systems.

This method is also suitable for other nonlinear differential equations in the field of mathematical physics.

We would be deeply appreciated for all the work you have done for our manuscript.

We are looking forward to hearing from you.

Best wishes,

Yours Sincerely,
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