

Multiple nodal solutions for the Schrödinger-Poisson system with an asymptotically cubic term

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Abstract

This paper deals with the following Schrödinger-Poisson system
$$\begin{cases} -\Delta u + \lambda \phi u = f(u) & \text{in } \mathbb{R}^3, \\ -\Delta \phi = u^2 & \text{in } \mathbb{R}^3, \end{cases}$$
 where $\lambda > 0$ and $f(u)$ is a nonlinear term asymptotically cubic at the infinity. Taking advantage of the Miranda theorem and deformation lemma, we combine some new analytic techniques to prove that for each positive integer k , the system admits a radial nodal solution $u_k(\lambda)$, which has exactly $k+1$ nodal domains and the corresponding energy is strictly increasing in k . Moreover, for any sequence $\{\lambda_n\}$ to 0_+ as $n \rightarrow \infty$, up to a subsequence, $u_k(\lambda_n)$ converges to some $u_k^0 \in H^1(\mathbb{R}^3)$, which is a radial nodal solution with exactly $k+1$ nodal domains of $\begin{cases} -\Delta u = f(u) & \text{in } \mathbb{R}^3, \\ -\Delta \phi = 0 & \text{in } \mathbb{R}^3. \end{cases}$ for $\lambda = 0$. These results give an affirmative answer to the open problem proposed in [Kim S, Seok J. Commun. Contemp. Math., 2012] for the Schrödinger-Poisson system with an asymptotically cubic term.

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