Existence theory and stability analysis of switched coupled system of nonlinear implicit impulsive Langevin equations with mixed derivatives

Rizwan Rizwan[1], Akbar Zada[1], Syed Omar Shah[2], and Manzoor Ahmad[1]

[1]University of Peshawar
[2]Qurtuba University of Sciences and Information Technology - Peshawar Campus

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Abstract

In this paper, we consider switched coupled system of nonlinear implicit impulsive Langevin equations with mixed derivatives. Some sufficient conditions are constructed to observe the existence, uniqueness and generalized Ulam–Hyers–Rassias stability of our proposed model, with the help of Generalized Diaz–Margolis's fixed point approach, over generalized complete metric space. We give an example which supports our main result.

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Existence theory and stability analysis of switched coupled system of nonlinear implicit impulsive Langevin equations with mixed derivatives

Rizwan Rizwan‡, Akbar Zada†, Manzoor Ahmad† and Syed Omar Shah†

‡,†Department of Mathematics, University of Buner, Buner, Pakistan
†Department of Mathematics, University of Peshawar, Peshawar 25000, Pakistan
†Department of Mathematics, University of Peshawar, Peshawar 25000, Pakistan
†Department of Physical and Numerical Sciences Qurtaba University of Science and Technology Peshawar, DI Khan, Pakistan

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1 INTRODUCTION

At Wisconsin university, Ulam raised a question about the stability of functional equations in the year 1940. The question of Ulam was: under what conditions does there exist an additive mapping near an approximately additive mapping? [31]. In 1941, Hyers was the first mathematician who gave partial answer to Ulam's question [13], over Banach space. Afterwards, stability of such form is known as Ulam–Hyers stability. In 1978, Rassias [22], provided a remarkable generalization of the Ulam–Hyers stability of mappings by considering variables. For more information about the topic, we refer the reader to [14,26,28,32,38,39,41].

An equation of the form $m\frac{d^2x}{dt^2} = \lambda\frac{dx}{dt} + \eta(t)$ is called Langevin equation, introduced by Paul Langevin in 1908. Langevin equations are broadly used to described stochastic problems in image processing, physics, astronomy, chemistry, defence system, electrical and mechanical engineering. Brownian motion is well describe by the Langevin equations when the random oscillation force is supposed to be Gaussian noise. For the removal of noise, mathematicians used fractional order differential equations, also it performs well in reducing the staircase effects as compare to ordinery differential equations. Thus it is very important to study fractional Langevin equations, for more details see [2,11,19,20,23].

Fractional order differential equations are the generalizations of the classical integer order differential equations. Fractional calculus has become a speedily developing area and its applications can be found in diverse fields ranging from physical sciences, porous media, electrochemistry, economics, electromagnetics, medicine and engineering to biological sciences. Progressively, fractional differential equations play a very important role in fields such as thermodynamics, statistical physics, nonlinear oscillation of earthquakes, viscoelasticity, defence, optics, control, signal processing, electrical circuits, astronomy etc. There are some outstanding articles which provide the main theoretical tools for the qual-

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[0]*****Corresponding author: Rizwan Rizwan (Email: rixwan4630@gmail.com)**
[0]E-mail: ‡rixwan4630@gmail.com, †zadababo@yahoo.com, †manzoor230ahmad@gmail.com, †omarshah89@yahoo.com